QB365

Important Questions - Application of Derivatives

12th Standard CBSE

Maths Reg.No. :

Time : 01:00:00 Hrs

Total Marks : 50

Section-A

1) The total cost C(x) associated with provision of free mid-day meals to x students of a school in primary classes is given by $C(x)=0.005x^3-0.02x^2+30x+50$ If the marginal cost is given by rate of change $\frac{dC}{dX}$ of total cost,write the marginal cost of food for 300students.What value is shown here? **1**

2) The total cost C(x)of producing x units of an article is given by $C(x)=0.005x^2-0.02x^2+30x+5000$.Find the marginal cost when 3 units are produced. **1**

3) Prove that the tangents on the curve $y=x^3+6$ at the points (-1,5) and (1,7) are parallel. **1**

4) Find the points on the curve $\frac{x^2}{4}+\frac{y^2}{25}=1$at which the tangents are (i) parallel to the x-axis.(ii)parallel to the y-axis. **1**

5) It is given that x=1,the function $f(x)=x^2-62x^2+ax+9$ attains its maximum value in the interval [0,2].Find the value of a. **1**

Section-B

6) If x changes from 4 to 4.01, then find the approximate change in log, x. **2**

7) The volume of a cube increasing at the rate of 9 cm^3/sec. How fast is the surface area increasing when the length of an edge is 10 cm. **2**

8) Find the slope and tangent and normal to the curve $x^2+2y+y^2=0\ at\ (-1,2)$. **2**

9) Show that the tangents to the curve $y=x^2-7x+18$ at (3, 0) and (4, 0) are at right angles. **2**

10) Use differentials find the approximate value of $(127)^{1/3}$. **2**

11) Find an angle θ which increases twice as fast as its sine. **2**

Section-C

12) A balloon,which always remains spherical, is being inflated by pumping in 900 cubic centimetres of gas per second.Find the rate at which the radius of the balloon is increasing when the radius is 15 cm. **3**

13) The total cost C(x) associated with the production of 'x' units of an item is given by: **3**

$$C(x)=0.007x^3-0.003x^2+15x+4000$$

Find the marginal cost when 17 units are produced.

14) Find the slope of the tangent to the curve $y=3x^4-4x\ at\ x=4$ **3**

Section-D

15) A stone is dropped into a quiet lake and waves move in circles at a speed of 5cm per second.At the instant when the radius of the circular wave is 8cm,how fast is the enclosed area increasing? **4**

16) A man 1.6m tall walks at the rate of 0.5m/sec away from a lamp post ,8 meters high.Find the rate at which his shadow is increasing and the rate with which the tip of shadow is moving away from the pole **4**

17) f(x)=$x^3+\frac{1}{x^3}, x\neq 0$ **4**

Section-E

18) Prove that the function f defined by $f(x) = x^2 - x + 1$ is neither increasing nor decreasing in (- 1, 1). Hence find the intervals in which f(x) is: **6**

(a) strictly increasing

b) strictly decreasing.

19) Find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ **6**

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Section-A

1) 1368 Concern for children health and nutrient food for every child. **1**

2) 30.015 **1**

3) 3.As slope at these points are equal,hence tangents are parallel. **1**

4) (i)$y = \pm 5. \therefore$ points are:$(0, \pm 5)$ (ii)$x = \pm 2 \therefore$ points are:$(\pm 2, 0)$ **1**

5) a=120 **1**

Section-B

6) $y - \log\ x,\ x = 4,\ \delta x = \cdot 01$ **2**

$\frac{\delta y}{\delta x} = (\frac{dy}{dx})_{x=4}$

$\delta y = (\frac{dy}{dx})_{x=4} \times \delta x$

$= \frac{1}{400} = \cdot 0025$

7) Let x denote the edge of cube, v denote the volume and s denotes the surface area of cube at instant t. **2**

$\frac{dv}{dt} = 9\ cm^2/sec. \quad x = 10cm$

$v = x^3$

$\frac{dv}{dt} = 3x^2 \frac{dx}{dt}$

$\frac{9}{3x^2} = \frac{dx}{dt}$

Now, s=6x^2

$\frac{ds}{dt} = 12x \times \frac{dx}{dt}$

$\frac{ds}{dt} = 12 \times 10 \times \frac{9}{3 \times 10 \times 10}$

$\frac{12 \times 9}{3 \times 10} = \frac{36}{10}$

$= 3.6\ cm^2/sec$

8) Given, $x^2 + 2y + y^2 = 0$ **2**

$2x + 2\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(2 + 2y) = -2x$

$\frac{dy}{dx} = \frac{-2x}{2(1+y)} = -\frac{x}{1+y}$

Slope of tangent at (-1,2)

$\frac{-1(-1)}{1+2} = \frac{1}{3}$

Slope of normal at (-1,2)

$= -\frac{3}{1} = -3$

9) Given, $y = x^2 - 7x + 18$ **2**

$\frac{dy}{dx} = 2x - 7$

$\frac{dy}{dx}\ at\ (3, 0) = 2(3) - 7$

$= 6 - 7 = -1$

$\frac{dy}{dx}\ at\ (4, 0) = 2(4) - 7$

$= 8 - 7 = 1$

$m_1 = -1, m_2 = 1$

$m_1 \times m_2 = -1 \times 1 = -1$

Hence tangent are at right angles to each other,

10) Let $y = x^{1/3}$ **2**

Let $x = 125\ and\ x + \Delta x = 127$

$\Rightarrow \quad \Delta x = 2$

$y = (125)^{1/3} \Rightarrow 5$

$y = x^{1/3}$

$\frac{dy}{dx} = \frac{1}{3}(x)^{-2/3}$

$\Rightarrow \quad \Delta y = \frac{x^{-2/3}}{3} \times \Delta x$

$= \frac{2}{3(5)^2} = \frac{2}{75} = 0.0266$

$\therefore \quad y + \Delta y = 5 + 0.0266 \Rightarrow 5.0266$

$\Rightarrow \quad 3\sqrt{127} = 5.0266$

11) Let θ denote the angle at instant t **2**

$\frac{d\theta}{dt} = 2\frac{d}{dt}(\sin\theta)$

$\frac{d\theta}{dt} = 2\cos\theta.(\frac{d\theta}{dt})$

$1 = 2\cos\ \theta$

$2\cos\theta = 1$

$\cos\theta = \frac{1}{2}$

$\Rightarrow \quad \theta = \cos^{-1}(\frac{1}{2})$

Hence required angles is $\frac{\pi}{3}$

Section-C

12) V, the volume of the balloon= $\frac{4}{3}\pi r^3$,where 'r' is the radius **3**

Given: $\frac{dV}{dt} = 900(cubic \quad cm/s.)$

$\therefore \frac{dV}{dt} = \frac{4}{3}\pi(3r^2)\frac{dr}{dt}$

$\Rightarrow 900 = 4\pi r^2\frac{dr}{dt}$

$\Rightarrow \frac{dr}{dt} = \frac{225}{\pi r^2}$

Hence, $\left[\frac{dr}{dt}\right]_{r=15} = \frac{225}{\pi X15X15} = \frac{1}{\pi}cm/s.$

13) Marginal cost is the rate of change of total cost with respect to output. **3**

Now $C(x) = 0.007x^3 - 0.003x^2 + 15x + 4000$

$\therefore$Marginal Cost (MC)=$\frac{dC}{dx}$ = $(0.007)(3x^2) - 0.003(2x) + 15$

When x=17.MC=(0.007)(317)^2)-0.003(2(17))+15

=6.069-0.102+15=20.967.

Hence the reqd marginal cost is Rs21 (nearly).

14) The given curve is $y = 3x^4 - 4x$ **3**

$\therefore \frac{dy}{dx} = 12x^3 - 4$

$\therefore$ Slope of the tangent at x=4 is:

$\left[\frac{dy}{dx}\right]_{x=4} = 12(4)^3 - 4 = 768 - 4 = 764$

Section-D

15) 80πcm^2/sec **4**

16) 0.14cm/sec **4**

17) Increasing:$(-\infty, 1) \cup (1, \infty)$;Decreasing:(-1,1)-{0} **4**

Section-E

18) $f'(x) = 2x - 1$ **6**

$f'(x) > 0, \forall x \in (\frac{1}{2}, 1)$

$f'(x) > 0, \forall x \in (-1, \frac{1}{2})$

∴ f(x) is neither increasing nor decreasing in (-1,1)

f(x) is strictly increasing on $(\frac{1}{2}, 1)$ and f(x) is strictly decreasing on $(-1, \frac{1}{2})$

19) Let ABCD be a rectangle having area A inscribed in an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ **6**

Let the co-ordinate of A be (α,β)

$\therefore$ Coordinate of $B\equiv(\alpha,-\beta)$

$C\equiv(-\alpha,-\beta)$

$D\equiv(-\alpha,\beta)$

Now A=Length x Breadth

$=2\alpha\times2\beta$

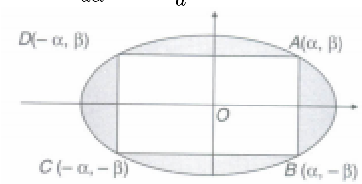
$\Rightarrow\quad A=4\alpha\beta$

$\Rightarrow\quad A=4\alpha\sqrt{b^2(1-\frac{\alpha^2}{a^2})}$

$\Rightarrow\quad A^2=16a^2\{b^2(1-\frac{\alpha^2}{a^2})\}$

$\Rightarrow\quad A^2=\frac{16b^2}{a^2}(a^2\alpha^2-\alpha^4)$

$\Rightarrow\quad \frac{d(A^2)}{d\alpha}=\frac{16b^2}{a^2}(2a^2\alpha^2-4\alpha^3)$



For maximum or minimum value

$\frac{d(A^2)}{d\alpha}=0$

$\Rightarrow\quad 2a^2\alpha-4\alpha^3=0$

$\Rightarrow\quad 2\alpha(a^2-2\alpha^3)=0$

$\Rightarrow\quad \alpha=0,\ \ a=\frac{a}{\sqrt{2}}$

Again $\Rightarrow\quad \frac{d(A^2)}{d\alpha}=\frac{16b^2}{a^2}(2a^2-12\alpha^2)$

$\Rightarrow$ For $\alpha=\frac{a}{\sqrt{2}}$ A is maximum.

i.e., for greatest area A $\alpha=\frac{a}{\sqrt{2}}$ and $\beta=\frac{b}{\sqrt{2}}$

$\therefore$ Greatest area=$4\alpha\cdot\beta=4\frac{a}{\sqrt{2}}\times\frac{b}{\sqrt{2}}=2ab$