QB365

Important Questions - Application of Integrals

12th Standard CBSE

Maths Reg.No. :

Time : 01:00:00 Hrs

Total Marks : 50

Section-A

1) Find the area of the region by the curve $y=\frac{1}{x}$, X-axis and between X = 1, X = 4. 1

2) On sketching the graph of $y=|x-2|$ and evaluating $\int_{-1}^{3}|x-2|dx$, what does $\int_{-1}^{3}|x-2|dx$ represent on the graph ? 1

Section-B

3) Find the area of the region enclosed between the two circles: $X^2 + Y^2 = 1$, $(X - 1)^2 + Y^2 = 1$. 4

4) Using integration find the area of the circle $X^2 + Y^2 = 16$ which is exterior to the parabola $Y^2 = 6X$. 4

5) Find the area of the region included between the parabola $y^2 = x$ and the line x + y =2. 4

6) Using integration find the area of the following region: $\left\{(x,y):|x-1|\le y\le\sqrt{5-x^2}\right\}$ 4

7) Using integration, find the area of the region enclosed between the two circles $x^2 + y^2 = 4$ and $(x-2)^2 + y^2 = 4$. 4

8) Find the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the ordinates x=ae and x=0, where $b^2=a^2(1-e^2)$ and e<1. 4

Section-C

9) Using integration, find the area of the triangle formed by positive x-axis and tangent and normal to the circle x+y = 4 at $(1,\sqrt{3})$. 6

10) Find the area of the region in the first quadrant enclosed by the y-axis, the line y = x and the circle $x^2+y^2 = 32$, using integration. 6

11) Using integration, find the area bounded by the tangent to the curve $4y = x^2$ at the at the point (2, 1) and the lines whose equations are x = 2y and x = 3y-3. 6

12) Using integration, find the area of the region bounded by the curves: y = |x+1| + 1, x = -3, x = 3, y = 0. 6

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Section-A

1) log 4 sq units. 1

2) 1

On the graph it represents the area bounded by the curve $y=|x-2|$, x-axis and between the ordinates at x = -1 and x = 3.

Section-B

3) $\left(\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right)$ sq units 4

4) $\frac{4}{3}\left(8\pi-\sqrt{3}\right)$ sq units 4

5) $\frac{9}{2}$ sq units **4**

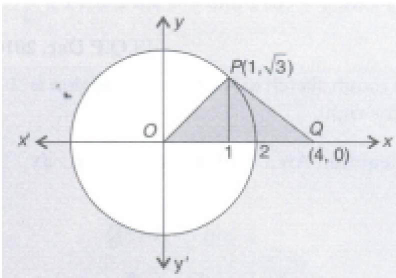
6) $\frac{1}{4}(5\pi - 2)$ sq units **4**

7) $\left(\frac{8\pi}{3} - 2\sqrt{3}\right)$ sq units **4**

8) ($b^2e + ab\sin^{-1}e$) sq units. **4**

Section-C

9) **6**



Equation of normal (OP) $\Rightarrow$ y = $\sqrt{3}$ x

Equation of tangent (PQ) is

y- $\sqrt{3}$ = $\frac{1}{\sqrt{3}}$ (x-1)

$\Rightarrow$ y = $\frac{1}{\sqrt{3}}$(4-x)

Co-ordinates of point Q is (4, 0).

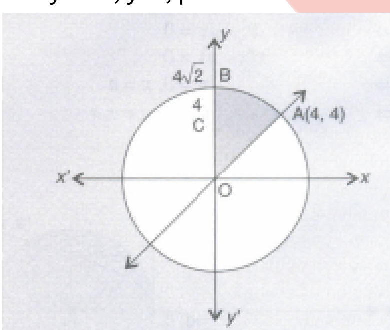
$\therefore$ Required Area = $\int_0^1 \sqrt{3}x\, dx + \int_1^4 4\, \frac{1}{\sqrt{3}}(4-x)\, dx$

$= \sqrt{3}\left[\frac{x^2}{2}\right]_0^1 + \frac{1}{\sqrt{3}}\left[4x - \frac{x^2}{2}\right]_1^4$

$= \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}}\left[16 - 8 - 4 + \frac{1}{2}\right]$

$= 2\sqrt{3}\ sq.units$

10) $x^2+y^2=32$; y=x, point of intersection is y=4. **6**



Required Area = $\int_0^4 y \quad dy + \int_4^{4\sqrt{2}} \sqrt{32 - y^2 dy}$

$= \left[\frac{y^2}{2}\right]_0^4 + \left[\frac{y}{2}\sqrt{32 - y^2} + 16sin^{-1}\frac{y}{4\sqrt{2}}\right]_{-4}^{4\sqrt{2}}$

$\Rightarrow \qquad = 8 + \left(0 + 16.\frac{\pi}{2}\right) - \left(8 + 16.\frac{\pi}{4}\right) = \ 4\pi$

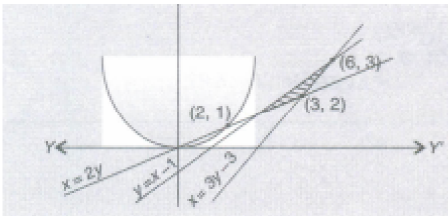
11) Given $4y = x^2$ **6**

$$\Rightarrow \quad 4\frac{dy}{dx} = 2x$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x}{2}$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{x=2} = 1$$

The equation of tangent is y=x-1.



The required Area = Shaded Area of graph

$$\Rightarrow -\left[\int_2^3 \left\{(x-1) - \frac{x}{2}\right\} dx + \int_3^6 \left[\frac{(x+3)}{3} - \frac{x}{2}\right] dx\right]$$

$$\Rightarrow -\left[\int_2^3 (x-1)\, dx + \frac{1}{3}\int_3^6 (x+3)\, dx - \frac{1}{2}\int_2^6 x dx\right]$$

$$\Rightarrow -\left[\left[\frac{x^2}{2} - x\right]_2^3 + \frac{1}{3}\left[\frac{x^2}{2} + 3x\right]_3^6 - \frac{1}{4}\left[x^2\right]_2^6\right]$$

$$\Rightarrow \left[\frac{9}{2} - 3 - 2 + 2\right] - \frac{1}{3}\left[18 + 18 - \frac{9}{2} - 9\right] + \frac{1}{4}[36-4]$$

= 1 sq.unit

12) Area (EABCDE) = Area (ABFE) + Area (CBFD) **6**

$$= \int_{-3}^{-1} (|x+1| + 1)\, dx + \int_{-1}^{3} (|x+1| + 1)\, dx$$

$$= \int_{-3}^{-1} (-x)\, dx + \int_{-1}^{3} (x+2)\, dx$$

$$= \left[-\frac{x^2}{2}\right]_{-3}^{-1} + \left[\frac{(x+2)^2}{2}\right]_{-1}^{3}$$

$$= -\frac{1}{2}(1-9) + \frac{1}{2}(25-1) = 16\ sq.unit.$$