QB365

Important Questions - Determinants

12th Standard CBSE

Maths

Reg.No. :

Time : 01:00:00 Hrs

Total Marks : 50

Section-A

1) Find the minor of the element of second row and third column (a_{23})in the following determinant: $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & 7 \end{vmatrix}$ **1**

2) If A is a square matrix of order 3 and |3A|=k|A|, then write the value of k. **1**

3) Find x, if $\begin{vmatrix} -1 & 2 \\ 4 & 8 \end{vmatrix} = \begin{vmatrix} 2 & x \\ x & -4 \end{vmatrix}$ **1**

4) Given $\begin{vmatrix} 4 & -1 & 0 \\ 2 & 1 & 4 \\ 1 & 0 & 3 \end{vmatrix}$find (i)minor of an element a_{23} (ii)cofactor of an element a_{32} **1**

5) Area of a triangle with vertices (k,0),(1,1) and (0,3) is 5 sq units. Find the value(s) of k. **1**

Section-B

6) If $A = \begin{vmatrix} 2 & 3 & -1 \\ 4 & 1 & 0 \\ 3 & 3 & 2 \end{vmatrix}$, find $M_{12} \times M_{21} + C_{21} \times C_{12}$ **2**

when M_{ij} called minor and C_{ij} called co-factors of A.

7) If $A = \begin{vmatrix} 6 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix}$, then show that |2A|=8|A|: **2**

Section-C

8) Evaluate:$\begin{vmatrix} x & x+1 \\ x-1 & x \end{vmatrix}$ **3**

9) Evaluate: $\Delta = \begin{vmatrix} 0 & sin\alpha & -cos\alpha \\ -sin\alpha & 0 & sin\beta \\ cos\alpha & -sin\beta & 0 \end{vmatrix}$ **3**

10) Find the co - factors of the elements of the determinant: $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$ and verify that $a_{11}A_{31}+a_{12}A_{32}+a_{13}A_{33}=0$. **3**

11) Show that the matrix $A = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix}$satisfies the equation A^2-4A+I=0, where I is 2x2 identity matrix and O is 2x2 zero matix.Using this equation, find A^{-1}. **3**

12) solve the system of equations: **3**

2x+5y=1

3x+2y=7

Section-D

13) Using properties of determinants solve for x $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$ **4**

14) Using properties of determinants solve the following for X: $\begin{vmatrix} x-a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0, a \neq 0$ **4**

15) Find the value of x,y and z if $A = \begin{vmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{vmatrix}$ satisfies $A' = A^{-1}$. **4**

Section-E

16) If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1} Hence solve the system of equations: **6**

2x-3y+5z=11, 3x+2y-4z=-5, x+y-2z=-3.

17) Two schools P and Q want to award their selected students on the values of Tolerance, Kindness and Leadership. The school P wants to award Rs.x each, Rs.y each and Rs.z each for the three respective values to its 3, 2 and 1 students, respectively with a total award money of Rs.2,200. School Q wants to spend Rs.3,100 to award its 4, 1 and 3 students on the respective values (by giving the same award money for the three values as school P). If the total amount of award for one prize on each value is Rs.1,200. Using matrices, find the award money for each value. Apart from the above these three values, suggest one more value which should be considered for award. **6**

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Section-A

1) 13 **1**

2) 27 **1**

3) $x = \pm 2\sqrt{2}$ **1**

4) (i)1 (ii)-16 **1**

5) -7/2 or 13/2 **1**

Section-B

6) **2**

We have,

$$A = \begin{vmatrix} 2 & 3 & -1 \\ 4 & 1 & 0 \\ 3 & 3 & 2 \end{vmatrix}$$

$$M_{12}\begin{vmatrix} 4 & 0 \\ 3 & 2 \end{vmatrix} = 8 - 0 = 8M_{21}\begin{vmatrix} 3 & -1 \\ 3 & 2 \end{vmatrix} = 6 + 3 = 9C_{21} = -\begin{vmatrix} 3 & -1 \\ 3 & 2 \end{vmatrix} = -(6+3) = -9C_{12} = -\begin{vmatrix} 4 & 0 \\ 3 & 2 \end{vmatrix} = -(8-0) = -8M_{12} \times M_{21} + C_{21} \times C_{12} 8 \times (9) + (-9)(-8) = 72 + 72 = 1$$

7) We have, **2**

$$A = \begin{vmatrix} 6 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix}$$

|A|=6(4-0)-0(0-0)+1(0-0)

|2A|= $\begin{vmatrix} 12 & 0 & 2 \\ 0 & 2 & 4 \\ 0 & 0 & 8 \end{vmatrix}$

=12(16-0)-0(0-0)+2(0-0)

=192

|2A|=8x24=8|A|

Section-C

8) $\begin{vmatrix} x & x+1 \\ x-1 & x \end{vmatrix} = (x)(x) - (x-1)(x-1)$ **3**

9) Expanding R_1, we get: **3**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Delta = 0.\begin{vmatrix} 0 & sin\beta \\ -sin\beta & 0 \end{vmatrix} - sin\alpha \begin{vmatrix} -sin\alpha & sin\beta \\ cos\alpha & 0 \end{vmatrix} - cos\alpha \begin{vmatrix} -sin\alpha & 0 \\ cos\alpha & -sin\beta \end{vmatrix}$$

=0-sinα(-0-sinβ cosα)-cosα(sinαsinβ-0)

=sinαsinβ cosα-cosα sinαsinβ=0

10) 3

$M_{11}=\begin{vmatrix}0 & 4\\5 & -7\end{vmatrix}=-0-20=-20$

$A_{11}=(-1)^{1+1}M_{11}=(-1)^2(-20)=-20$

$M_{12}=\begin{vmatrix}6 & 4\\1 & -7\end{vmatrix}=-42-4=-46$

$A_{12}=(-1)^{1+2}M_{12}=(-1)^3(-46)=(-1)(-46)=46$

$M_{13}=\begin{vmatrix}6 & 0\\1 & 5\end{vmatrix}=30-0=30$

$A_{13}=(-1)^{1+3}M_{13}=(-1)^4(30)=30$

$M_{21}=\begin{vmatrix}-3 & 5\\5 & -7\end{vmatrix}=21-25=-4$

$A_{21}=(-1)^{2+1}M_{21}=(-1)^3(-4=(-1)(-4)=4)$

$M_{22}=\begin{vmatrix}2 & 5\\1 & -7\end{vmatrix}=-14-15=-19$

$A_{22}=(-1)^{2+2}M_{22}=(1)^4(-19)=-19$

$M_{23}=\begin{vmatrix}2 & -3\\1 & 5\end{vmatrix}=10+3=13$

$A_{23}=(-1)^{2+3}M_{23}=(-1)^5 13=-13$

$M_{31}=\begin{vmatrix}-3 & 5\\0 & 4\end{vmatrix}=-12-0=-12$

$A_{31}=(-1)^{3+1}M_{31}=(-1)^4(-12)=-12$

$M_{32}=\begin{vmatrix}2 & 5\\6 & 4\end{vmatrix}=8-30=-22$

$A_{32}=(-1)^{3+2}M_{32}=(-1)^5(-22)=(-1)(-22)=22$

$M_{33}=\begin{vmatrix}2 & -3\\6 & 0\end{vmatrix}=0+18=18$

$A_{33}=(-1)^{3+3}M_{33}=(-1)^6(18)=18$

(ii)$a_{11}A_{31}+a_{12}A_{32}+a_{13}A_{32}$

$=(2)(-12)+(-3)(22)+(5)(18)=-24-66+90=0$

11) 3

We have: $A=\begin{vmatrix}2 & 3\\1 & 2\end{vmatrix}$ ....(i)

$A^2=AA=\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}$

$=\begin{bmatrix}4+3 & 6+6\\2+2 & 3+4\end{bmatrix}=\begin{bmatrix}7 & 12\\4 & 7\end{bmatrix}$. ....(ii)

$=A^2-4A+I=\begin{bmatrix}7 & 12\\4 & 7\end{bmatrix}-4\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}+\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

$=\begin{bmatrix}7 & 12\\4 & 7\end{bmatrix}+\begin{bmatrix}-8 & -12\\-4 & -8\end{bmatrix}+\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

$=\begin{bmatrix}7-8+1 & 12-12+0\\4-4+0 & 7-8+1\end{bmatrix}=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}=0$

(II)Now $A^2-4A+I=0$

$\Rightarrow AA-4A=-I$

$\Rightarrow AA(A^{-1})-4AA^{-1}=-IA^{-1}$

$\Rightarrow A(AA^{-1})-4I=-A^{-1}$

$\Rightarrow AI-4I=A^{-1}$

$\Rightarrow A^{-1}=4I-A$

$=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}-\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}$

$=\begin{bmatrix}4 & 0\\0 & 4\end{bmatrix}+\begin{bmatrix}-2 & -3\\-1 & -2\end{bmatrix}$

$\Rightarrow\begin{bmatrix}4-2 & 0-3\\0-1 & 4-2\end{bmatrix}=\begin{bmatrix}2 & -3\\-1 & 2\end{bmatrix}$

12) The given system of equations can be expressed in the matrix form as AX=B **3**

Where
$$A = \begin{bmatrix} 2 & 5 \\ 3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} and\ B = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$$

Now
$$|A| = \begin{vmatrix} 2 & 5 \\ 3 & 2 \end{vmatrix} = 4 - 15 = -11 \neq 0$$

A is non-singular matrix $\Rightarrow$ A^{-1}exists

From (1), $A^{-1}(AX)=A^{-1}B \Rightarrow (A^{-1}A)X=A^{-1}B$

$\Rightarrow IX=A^{-1}B \qquad \Rightarrow X=A^{-1}B$

Now $A_{11}=2$; $A_{12}=-3$;$A_{21}=-5$ and $A_{22}=2$

$$adj\ A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{adj\ A}{|A|} = \frac{1}{-11}\begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2/11 & 5/11 \\ 3/11 & -2/11 \end{bmatrix}$$

$$\therefore From(2), \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2/11 & 5/11 \\ 3/11 & -2/11 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-2}{11} + \frac{35}{11} \\ \frac{3}{11} - \frac{14}{11} \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

Hence, x=3, y=-1

Section-D

13) x=0 or x=3a **4**

14) x= -a/3 **4**

15) The relation $A' = A^{-1}$ $gives$ $A'A = A^{-1}A = I$ **4**

Thus,
$$\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -z & z \end{bmatrix}\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 + x^2 + x^2 & 0 + xy - xy & z - xz + xz \\ 0 + xy - xy & 4y^2 + y^2 + y^2 & 2yz - yz - yz \\ 0 - zx + zx & 2yz - yz - yz & z^2 + z^2 + z^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 2y^2 & 0 \\ 0 & 0 & 2z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad 2x^2 = 1; 6y^2 = 1; \quad and \quad 3z^2 = 1$$

$$\Rightarrow \quad x = \pm\frac{1}{\sqrt{2}}; \quad y = \pm\frac{1}{\sqrt{6}}; \quad z = \pm\frac{1}{\sqrt{3}};$$

Section-E

16) 6

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

|A|=2 x 0-(-3) x -2+5 x 1

=-6+5=-1

a_{11}=(-4+4)=0

a_{12}=-(-6+4)=2

a_{13}=(3-2)=1

a_{21}=-(6-5)=-1

a_{22}=(-4-5)=-9

a_{23}=-(2+3)=-5

a_{31}=(12-10)=2

a_{32}=-(-8-15)=23

a_{33}=(4+9)=13

$$A^{-1} = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

Given set of equations may be written as

$AX = B,$

where

$$B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$\Rightarrow \quad X = A^{-1}B$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 0-5+6 \\ -22-45+69 \\ 11-25+39 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence x=1, y=2 and x=3.

17) Here, 3x + 2y + Z = 2,200 6

4x + y + 3z = 3,100

x + y + z = 1,200

Matrix equation is

$$\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2,200 \\ 3,100 \\ 1,200 \end{bmatrix} \quad or \quad AX = B$$

|A|=3(-2)-2(1)+1(3)

$=-5 \neq 0$

$\therefore$ $X=A^{-1}B$

cofactors are:

$$\begin{bmatrix} A_{11} = -2 & A_{12} = -1 & A_{13} = 3 \\ A_{21} = -1 & A_{22} = 2 & A_{23} = -1 \\ A_{31} = 5 & A_{32} = -5 & A_{33} = -5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}\begin{bmatrix} 2,200 \\ 3,100 \\ 1,200 \end{bmatrix}$$

x=300,y=400,z=500

One more value punctuality, honesty etc.