QB365

Important Questions - Differential Equations

12th Standard CBSE

Maths Reg.No. :

Time : 01:00:00 Hrs

Total Marks : 50

Section-A

1) Write the order and degree of the differential equation $(\frac{d^2y}{dx^2})^3$ -5$\frac{dy}{dx}$+6=0. **1**

2) Write the other and degree of the differential equation $\frac{dy}{dx}+sin(\frac{dy}{dx})=0.$ **1**

3) Write the degree of the differential equation $\left(\frac{d^2s}{dt^2}\right)^2+\left(\frac{ds}{dt}\right)^3+4=0$ **1**

4) Write the degree of the differential equation $x^3\left(\frac{d^2y}{dx^2}\right)^2+x\left(\frac{dy}{dx}\right)^4=0$ **1**

5) Write the degree of the differential equation : $x\left(\frac{d^2y}{dx^2}\right)^3+y\left(\frac{dy}{dx}\right)^4+x^3=0$ **1**

Section-B

6) Obtain the differential equation of the family of circles passing through the points (a,0) and (- a, 0). **2**

7) Solve the differential equation $\frac{dy}{dx}=e^{x-y}+xe^{-y}$. **2**

8) Solve the differential equation $\frac{dy}{dx}=\sqrt{\frac{1-\cos x}{1+\cos x}}$ **2**

9) Find the general solution of differential equation log $\left(\frac{dy}{dx}\right)=x+1$ **2**

10) Find the general solution of differential equation **2**
$\frac{dy}{dx}+y=e^{-x}$

11) Find the general solution of differential equation **2**
$x\frac{dy}{dx}+y=x^2$.

Section-C

12) Find the particular solution of the differential equation: $log\left(\frac{dy}{dx}\right)=3x+4y$, given that $y=0$ when $x=0.$ **3**

13) Form the differential equation of the family of circles having centre on y-axis and radius 3 units. **3**

14) A Homogeneous differential equation of the form $\frac{dx}{dy}=h\left(\frac{x}{y}\right)$ can be solved by making the substitution: **(A)** $y=vx$ **(B)** $y=yx$ **(C)** $x=vy$ **(D)** $x=y.$ **3**

Section-D

15) Solve: cos x$\frac{dy}{dx}$+y = sinx, given that y =2 when x = 0. **4**

16) Solve the differential equation : $2ye^{x/y}$ dx + (y - $2xe^{x/y}$) dy = 0 **4**

17) Solve the differential equation : (x - y^3) dy + y dx = 0. **4**

Section-E

18) Find the particular solution of the differential equation : x^2dy = y(x + y)dx = 0, when x = 1, y = 1. **6**

19) Show that the differential equation **6**
$x\frac{dy}{dx}\sin\left(\frac{y}{x}\right)+x-y\sin\left(\frac{y}{x}\right)=0$is homogeneous. Find particular solution of this differential equation, given that x = 1 when y = $\frac{\pi}{2}$.

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Section-A

1) Order =2; degree = 3. **1**

2) **1**

Highest order derivative is $\frac{dy}{dx}$. Hence, order of differential equation is 1. Equation cannot be written as a polymial in derivatives. Hence, degree is not defined.

3) **1**

$\left(\frac{d^2s}{dt^2}\right)^2+\left(\frac{ds}{dt}\right)^3+4=0$ In the given differential equation, the power of highest order derivative i.e., $\frac{d^2s}{dt^2}=2$ $\therefore$ Its degree = 2

4) **1**

Given $x^3\left(\frac{d^2y}{dx^2}\right)^2+x\left(\frac{dy}{dx}\right)^4=0$ The highest order derivative is $\left(\frac{d^2y}{dx^2}\right)$ and its power is 2. So, the degree of differential equation is 2.

5) The highest order derivative is $\frac{d^2y}{dx^2}$ and its power is 3. So, the degree of differential equation is 3. **1**

Section-B

6) $x^2+(y-b)^2=a^2+b^2$ **2**

or $x^2+y^2-2by=a^2$ ... (i)

$\Rightarrow \quad 2x+2y\frac{dy}{dx}-2b\frac{dy}{dx}=2b$

$\Rightarrow \quad 2b=\frac{2x+2y\frac{dy}{dx}}{\frac{dy}{dx}}$

Substituting the value in eqn. (i),

$\left(x^2-y^2-a^2\right)\frac{dy}{dx}-2xy=0$

7) $\frac{dy}{dx}=e^{x-y}+xe^{-y}$ **2**

$=e^{-y}\left[e^x+x\right]$

$\Rightarrow \quad \frac{dy}{dx}=\left(e^x+x\right)e^{-y}$

$\Rightarrow \quad \int e^y dy=\int\left(e^x+x\right)dx$

$\Rightarrow \quad e^y=e^x+\frac{x^2}{2}+C$

8) $\frac{dy}{dx}=\sqrt{\frac{1-\cos x}{1+\cos x}}$ **2**

Integrating both the sides,

$\int dy=\int\sqrt{\frac{1-\cos x}{1+\cos x}}dx$

$\Rightarrow \quad \int dy=\int\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}dx$

$\{\because \quad \cos x=2\cos^2\frac{x}{2}-1 \quad and \quad \cos x=1-2\sin^2\frac{x}{2}\}$

$\Rightarrow \quad \int dy=\int\tan\frac{x}{2}dx$

$\Rightarrow \quad y=-2\log\cos\frac{x}{2}+c$

$\Rightarrow \quad \log\cos\frac{x}{2}+c$

9) $\log\left(\frac{dy}{dx}\right) = x + 1$ **2**

$\Rightarrow \quad \frac{dy}{dx} = e^{x+1}$

$\Rightarrow \quad dy = e^{x+1}dx$

Integrating both the sides,

$\int dy = \int e^{x+1}dx$

$\Rightarrow \quad y = e^{x+1} + C$

10) $\frac{dy}{dx} + y = e^{-x}$ **2**

Let $\frac{dy}{dx} + Py = Q$

Where P = 1, Q = e^{-x}

$y(I.F.) = \int I.F. \times Qdx$

$\Rightarrow \quad I.F. = e^{\int Pdx} = e^{\int Pdx} = e^{x}$

$\Rightarrow \quad ye^{x} = \int e^{x}.e^{-x}dx = \int dx$

$\Rightarrow \quad ye^{x} = x + C$

11) $x\frac{dy}{dx} + y = e^{2}$ **2**

Divide by x both the sides,

$\frac{dy}{dx} + \frac{y}{x} = x$

$\frac{dy}{dx} + Py = Q,$

Where $P = \frac{1}{x}, Q = x$

$I.F. = e^{\int Pdx} = e^{\int \frac{dx}{x}} = e^{logx} = x$

$y(I.F.) = \int Q.I.F.dx$

$xy = \int x.xdx = \int x^{2}dx$

$\Rightarrow \quad yx = \frac{x^{3}}{3} + C$

$\Rightarrow \quad y = \frac{1}{3}x^{2} + \frac{C}{x}$

Section-C

12) **3**

The given differential equation can be written as: $\frac{dy}{dx} = e^{3x+4y} \Rightarrow$

$\frac{dy}{dx} = e^{3x}.e^{4y} \quad \Rightarrow \quad e^{-4y}.dy = e^{3x}dx$ **| Variables Separable**

Integretating, $\int e^{-4y}dy = \int e^{3x}dx + c \Rightarrow \quad \frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + c \Rightarrow$

$4e^{3x} + 3e^{-4y} + 12c = 0$ ....(1) When x=0,y=0. $\therefore$

$4(1) + 3(1) + 12c = 0 \quad \Rightarrow \quad c = \frac{-7}{12}$ Putting in (1),$4e^{3x} + 3e^{-4y} - 7 = 0,$ Which is the reqd. particular solution.

13) **3**

Let the equation of the family of circles be: $x^{2} + (y-\alpha)^{2} = 9$ ...(1) Diff. w.r.t x, $2x + 2(y-\alpha)\frac{dy}{dx} = 0 \Rightarrow \quad y - \alpha = -\frac{x}{\frac{dy}{dx}}$. Putting in (1),$x^{2} + \frac{x^{2}}{\left(\frac{dy}{dx}\right)^{2}} = 9 \Rightarrow$

$x^{2} = (9 - x^{2})y'^{2} \Rightarrow \quad (9 - x^{2})y'^{2} - x^{2} = 0 \Rightarrow \quad (x^{2} - 9)y'^{2} + x^{2} = 0,$ Which is required differential equation.

14) Part (C) is the correct answer. **3**

Section-D

15) Substituting in (i) we get (sec x + tan x)y = sec x + tan x - x + 1 is the required nsolution. **4**

16) $e^v = -\frac{1}{2}log|y| + c \Rightarrow e^{x/y} = \frac{1}{2}log|y| + c$ is the required solution. **4**

17) $\Rightarrow$ yx = $\frac{y^4}{4} + c \Rightarrow$ x = $\frac{y^3}{4} + \frac{c}{y}$ is the required solution. **4**

Section-E

18) The given D.E. is $x^2dy + y(x + y)dx = 0$ ... (i) **6**

when x = 1, y = 1

From (i), $\frac{dy}{dx} = -\frac{xy+y^2}{x^2}$ ... (ii)

Put, y = vx

$\Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

From (ii), $v + x\frac{dv}{dx} = \frac{xvx+v^2x^2}{x^2}$

$\Rightarrow \quad x\frac{dv}{dx} = \frac{-vx^2-v^2x^2-vx^2}{x^2}$

$\Rightarrow \quad x\frac{dv}{dx} = -2v - v^2$

$\Rightarrow \quad \frac{dv}{v(v+2)} = -\frac{dx}{x}$

$\Rightarrow \quad \frac{1}{2}\left[\frac{1}{v} - \frac{1}{v+2}\right]dv + \frac{dx}{x} = 0$

Integrating it we get

$\frac{1}{2}[\log v - \log|v+2|] + \log x = C_1$

$\Rightarrow \quad \log\frac{v}{|v+2|} + \log x^2 = 2C_1$

$\Rightarrow \quad \log\left[\frac{y}{y+2x}x^2\right] = C_1$

$\Rightarrow \quad \log\left[\frac{yx^2}{y+2x}\right] = C$

$\Rightarrow \quad \log\left[\frac{1}{1+2}\right] = C \Rightarrow \quad C = \log\frac{1}{3}$

$\therefore \quad \log\left[\frac{yx^2}{y+2x}\right] = \log\frac{1}{3}$

$\Rightarrow 3yx^2 = y + 2x$

$3yx^2 - y = 2x$

$y(3x^2 - 1) = 2x$

$y = \frac{2x}{3x^2-1}$

19) The given differential equation is :

$x\frac{dy}{dx}\sin\left(\frac{y}{x}\right)+x-y\sin\left(\frac{y}{x}\right)=0$

$\Rightarrow \quad y\sin\left(\frac{y}{x}\right)-x=\frac{xdy}{dx}\sin\left(\frac{y}{x}\right)$

$\Rightarrow \quad \frac{dy}{dx}=\frac{y\sin\left(\frac{y}{x}\right)-x}{x\sin\left(\frac{y}{x}\right)}$ ... (i)

$f(x,y)=\frac{y}{x}-cosec\left(\frac{y}{x}\right)$

Now, put $x=\lambda x, y=\lambda y$, then

$f(\lambda x,\lambda y)=\frac{\lambda y}{\lambda x}-cosec\left(\frac{\lambda y}{\lambda x}\right)$

$=\frac{y}{x}-cosec\left(\frac{y}{x}\right)$

= f (x, y)

So, it is homogeneous

Now, put y = vx in equation (i).

On differentiating both sides w.r.t. x, we get

$\frac{dy}{dx}=v+x\frac{dv}{dx}$

Now from (i), $v+x\frac{dv}{dx}=\frac{vx}{x}-\frac{1}{\sin\left(\frac{vx}{x}\right)}$

$\Rightarrow \quad x\frac{dv}{dx}=-\frac{1}{\sin v}$

$\Rightarrow \quad -\int\sin v dv=\int\frac{dx}{x}$

$\Rightarrow \quad \cos v=\log|x|+C$

$\Rightarrow \quad \cos\left(\frac{y}{x}\right)=\log|x|+C$

It is given that x = 1 and y = $y=\pi/2$

So, $\cos\frac{\left(\frac{\pi}{2}\right)}{1}=C+\log|1|$

$\Rightarrow \quad C=0$

$\Rightarrow \quad \cos\left(\frac{y}{x}\right)=\log|x|$

Hence, required solution is

$\cos\left(\frac{y}{x}\right)=\log|x|$