QB365

Important Questions - Integrals

12th Standard CBSE

Maths

Reg.No. :

Time : 01:00:00 Hrs

Total Marks : 50

Section-A

1) $\int \frac{1+tan\,x}{1-tan\,x}dx.$ **1**

2) $\int cos^2x\,cosec^2x\,dx.$ **1**

3) $\int \frac{x^2-1}{x^2+1}dx.$ **1**

4) $\int \frac{1}{(2-x)^2+1}dx.$ **1**

5) $\int tan^{-1}(cot\,x)dx.$ **1**

Section-B

6) $\int sin^2x \quad cos^2xdx$ **2**

7) $\int logxdx$ **2**

8) $\int tan^{-1}xdx$ **2**

9) $\int \frac{(x+1)e^x}{cos^2(xe^x)}dx$ **2**

10) Evaluate : $\int_0^{\frac{\pi}{4}} secxdx$ **2**

11) Evaluate : $\int_e^{e^2} \frac{dx}{xlogx}$ **2**

Section-C

12) If $x = \int_0^x tan$ sin t dt, then write the value of $f'(x)$ **3**

13) Write an antiderivative for each of the followings functions, using method of inspection : (i) $\cos 2x$ (ii)$3x^2+4x^3$ (iii) $\frac{1}{x}, x \neq 0$ **3**

14) Find: $\int \frac{x^2+1}{x^2-5x+6}dx$ **3**

Section-D

15) Find: $\int \frac{\sqrt{x}}{\sqrt{a^3-x^3}}dx$ **4**

16) Evaluate : $\int x^2\tan^{-1}xdx$ **4**

17) Integrate: $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x}+e^{-\cos x}}dx.$ **4**

Section-E

18) Evaluate the following: $\int_0^{\frac{\pi}{2}} \frac{x\sin x\cos x}{\sin^4 x+\cos^4 x}dx$ **6**

19) Evaluate : $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\cot x}}$ **6**

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Section-A

1) Consider, = -log |t| + c = log |cos x - sin x|+ c **1**

2) $= -\frac{1}{3}cot^3 x + c$ **1**

3) = x-2 $\tan^{-1}$x + c **1**

4) = -$\tan^{-1}$ (2-x)+c **1**

5) $= \frac{\pi}{2} \times - \frac{x^2}{2} + c$ **1**

Section-B

6) **2**

$$\int \left[\frac{2sinxcosx}{2}\right]^2 dx$$

$$= \frac{1}{4}\int(sin2x)^2dx$$

$$= \frac{1}{4}\int sin^2 2xdx$$

$$\frac{1}{4}\int \frac{1-cos4x}{2}dx = \frac{1}{8}\int 1dx - \frac{1}{8}\int cos4xdx$$

$$= \frac{x}{8} - \frac{1}{8}\frac{sin4x}{4}$$

$$= \frac{1}{8}\left(x - \frac{1}{4}sin4x\right) + c$$

7) $\int logxdx$ **2**

$= log\int 1dx - \int\left[\frac{d}{dx}logx\int 1dx\right]dx$

$= logx(x) - \int\frac{1}{x}\times xdx$

$= logx(x) - \int\frac{1}{x}\times xdx$

$= xlogx - \int 1dx$

$= xlogx - x + C$

8) $\int tan^{-1}x\times 1dx$ **2**

$= tan^{-1}x\int 1dx - \int\left[\frac{d}{dx}tan^{-1}x\int 1dx\right]dx$

$= tan^{-1}x(x) - \int\frac{1}{1+x^2}\times xdx$

$= xtan^{-1}x - \int\frac{x}{1+x^2}dx$

Put $1 + x^2 = t$

$\Rightarrow\quad 2x\quad dx = dt$

$\Rightarrow\quad xdx = \frac{1}{2}dt$

$= xtan^{-1} - \frac{1}{2}\int\frac{dt}{t}$

$= xtan^{-1}x - \frac{1}{2}log|t|$

$= xtan^{-1}x - \frac{1}{2}log\left|1 + x^2\right| + C$

9) $\int\frac{(x+1)e^x}{cos^2(xe^x)}dx$ **2**

Put, $xe^x = t$

$\Rightarrow\quad (xe^x + e^x)dx = dt$

$\Rightarrow\quad e^x(x+1)dx = dt$

$\int\frac{dt}{cos^2t} = \int sec^2tdt$

$= tant = tan(xe^x) + C$

10) $I = \int_0^{\frac{\pi}{4}}secxdx$ **2**

$\Rightarrow\quad I = [log(secx + tanx]_0^{\frac{\pi}{4}}$

$\Rightarrow\quad I = \left[log\left(sec\frac{\pi}{4} + tan\frac{\pi}{4}\right) - log(sec0 + tan0)\right]$

$\Rightarrow\quad I = log\left|\sqrt{2} + 1\right| - log(1 + 0)$

$\Rightarrow\quad I = log\left|\sqrt{2} + 1\right| - log1$

$\Rightarrow\quad I = log\left|\sqrt{2} + 1\right| - 0$

$\Rightarrow\quad I = log\left|\sqrt{2} + 1\right|$

11) $I = \int_e^{e^2}\frac{dx}{xlogx}$ **2**

Put log x=t

$\Rightarrow\quad \frac{dx}{t} = dt$

On Integration

log x=t

$loge = t \Rightarrow 1 = t$

$\Rightarrow$ 1=t lower limit

log e^2=t

$\Rightarrow$ 2=t upper limit

$I = \int_1^2\frac{dx}{t}$

$\Rightarrow\quad I = [logt]_1^2$

$\Rightarrow\quad I = [log2 - log1]$

$\Rightarrow\quad I = [log2 - 0]$

$\Rightarrow\quad I = log2$

Section-C

12) We have : $f(x) = \int_0^x tsint\quad dt\quad\therefore\quad f'(x) = x\sin\frac{d}{dx}(x) - 0 = x(1) = x\sin x.$ **3**

13) **3**

(i) We know that $\frac{d}{dx}(\sin 2x) = 2\cos 2x\quad\Rightarrow\quad \cos 2x = \frac{1}{2}\frac{d}{dx}(\sin 2x) = \frac{d}{dx}(\frac{1}{2}\sin 2x)$ Hence, antiderivative of $\cos 2x = \frac{1}{2}\sin 2x$ (ii) We know that $\frac{d}{dx}(x^3 + x^4) = 3x^2 + 4x^3.$ Hence, antiderivative of $3x^2 + 4x^3 + x^3 + x^4$ (iii) We know that $\frac{d}{dx}(\log x) = \frac{1}{x}, x > 0$ and $\frac{d}{dx}[\log(-x)] = \frac{1}{-x}(-1) = \frac{1}{x}, x > 0.$ combining, $\frac{d}{dx}(\log 1\quad x1) = \frac{1}{x}, x \neq 0.$

$\Rightarrow \int\frac{1}{x}dx = \log 1\quad x1.$ which is one of antiderivative of $\frac{1}{x}.$

14) 3

Here $\frac{x^2+1}{x^2-5x+6}$ is not a proper rational function, so dividing $(x^2+1) by (x^2-5x+6)$, we get:

$\frac{x^2+1}{x^2-5x+6} \equiv 1 + \frac{5x-5}{x^2-5x+6}$ $\equiv 1 + \frac{5x-5}{(x-2)(x-3)}$ $\equiv 1 + \frac{A}{x-1} + \frac{B}{x-1}$ $.......(1) Multiplying \quad (x-2)(x-3), \quad we \quad get:$

Section-D

15) 4

Let $I = \frac{\sqrt{x}}{\sqrt{a^3-x^3}}dx \quad Put \quad x^{3/2} = t$ so that $\frac{3}{2}x^{1/2}dx = dt \quad i.e. \quad \sqrt{x}dx = \frac{2}{3}dt.$

$\therefore \quad I = \int \frac{\frac{2}{3}dt}{\sqrt{a^3-t^2}} = \frac{2}{3}\int \frac{dt}{\sqrt{(a^{3/2})^2-t^2}} \quad | \quad From: \quad \int \frac{dx}{\sqrt{a^3-x^2}} = \frac{2}{3}\sin^{-1}\frac{t}{a^{3/2}} + c = \frac{2}{3}\left[\sin^{-1}\left(\frac{x^{3/2}}{a^{3/2}}\right)\right] + c = \frac{2}{3}\sin^{-1}\left(\frac{x}{a}\right)^{3/2} + c.$

16) 4

$= \int x^2\tan^{-1}x dx = \int \tan^{-1}x.\, x^2 \quad dx = \tan^{-1}x.\, \frac{x^3}{3} - \int \frac{1}{1+x^2}.\, \frac{x^3}{3}dx$ [Integrating by Parts]

$= \frac{x^3}{3}\tan^{-1}x - \frac{1}{3}\int \frac{x^3}{1+x^2}dx = \frac{x^3}{3}\tan^{-1}x - \frac{1}{3}\int \left(x - \frac{x}{1+x^2}\right)dx = \frac{x^3}{3}\tan^{-1}x - \frac{1}{3}\int x.\, dx + \frac{1}{6}\int \frac{2x}{1+x^2}dx = \frac{x^3}{3}\tan^{-1}x - \frac{1}{3}.\, \frac{x^2}{2} + \frac{1}{6}\log\left|1+x^2\right| + c\left[\because \frac{d}{dx}(1+x^2) = 2x\right] = \frac{1}{3}x^3\tan^{-1}x - \frac{1}{6}x^2 + \frac{1}{6}\log\left|1\right.$

17) 4

Let $I = \int_0^{\pi}\frac{e^{\cos x}}{e^{\cos x}+e^{-\cos x}}dx \quad .....(1) \therefore \quad I = \int_0^{\pi}\frac{e^{\cos(0+\pi-x)}}{e^{\cos(0+\pi-x)}+e^{-\cos(0+\pi-x)}}dx\left[\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right] = \int_0^{\pi}\frac{e^{-\cos x}}{e^{-\cos x}+e^{\cos x}}dx \quad .....(2)$ Adding(1) and (2), $2I = \int_0^{\pi}\frac{e^{\cos x}+e^{-\cos x}}{e^{\cos x}+e^{-\cos x}}dx = \int_0^{\pi}1.dx = [x]_0^{\pi} = \pi - 0 = \pi.$ Hence, $I = \frac{\pi}{2}.$

Section-E

18) 6

$I = \int_0^{\frac{\pi}{2}}\frac{x\sin x\cos x}{\sin^4 x+\cos^4 x}dx$

$= \int_0^{\frac{\pi}{2}}\frac{\left(\frac{\pi}{2}-x\right)\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)}{\sin^4\left(\frac{\pi}{2}-x\right)+\cos^4\left(\frac{\pi}{2}-x\right)}dx$

$= \int_0^{\frac{\pi}{2}}\frac{\left(\frac{\pi}{2}-x\right)\cos x\sin x}{\cos^4 x+\sin^4 x}dx$

$= \int_0^{\frac{\pi}{2}}\frac{\left(\frac{\pi}{2}\right)\cos x\sin x}{\cos^4 x+\sin^4 x}dx$

$= \left(\frac{\pi}{2}\right)\left[\int_0^{\frac{\pi}{4}}\frac{\cos x\sin x}{\cos^4 x+\sin^4 x}dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{\cos x\sin x}{\cos^4 x+\sin^4 x}dx\right]$

$= \left(\frac{\pi}{2}\right)\left[\int_0^{\frac{\pi}{4}}\frac{\tan x\sec^2 x}{1+\tan^4 x}dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{cosec^2 x\cot x}{\cot^4 x+1}dx\right]$

$= 2I = \left(\frac{\pi}{2}\right)\left[\int_0^1\frac{1}{1+t^2}dx + \int_0^1\frac{1}{1+p^2}dp\right]$

$\Rightarrow \quad 2I = \left(\frac{\pi}{4}\right)\left[\tan^{-1}t\right]_0^1 + \left(\frac{\pi}{4}\right)\left[\tan^{-1}p\right]_0^1$

$= \frac{\pi^2}{8}$

$\Rightarrow \quad I = \frac{\pi^2}{16}$

19) 6

$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{dx}{1+\sqrt{\cot x}}$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$

Apply property $\int_a^b f(x) = \int_a^b f(a+b-x)$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sqrt{\sin \frac{\pi}{3}+\frac{\pi}{6}-x}}{\sqrt{\sin \frac{\pi}{3}+\frac{\pi}{6}-x}+\sqrt{\cos \frac{\pi}{3}+\frac{\pi}{6}-x}}dx$

$\therefore \quad I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$

Adding, we get $2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}}dx = [x]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$