QB365

Important Questions - Inverse Trigonometric Functions

12th Standard CBSE

Maths Reg.No. : | | | | | | |

Time : 01:00:00 Hrs

Total Marks : 50

Section-A

1) Evaluate $sin\left[\frac{\pi}{3} - sin^{-1}\left(-\frac{1}{2}\right)\right]$. **1**

2) Show that $sin^{-1}\left(2X\sqrt{1-X^2}\right) = 2sin^{-1}X$ **1**

3) Write the principal value of $cos^{-1}\left(\frac{1}{2}\right) - 2sin^{-1}\left(-\frac{1}{2}\right)$ **1**

4) Write the value of $tan^{-1}\left[2sin\left(2cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$. **1**

5) Write the value of $sin\left(2sin^{-1}\frac{3}{5}\right)$ **1**

Section-B

6) Write in the simplest form : $tan^{-1}\left[\frac{cos\quad x}{1+sin\quad x}\right], x\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ **2**

7) Show that $sin^{-1}\frac{5}{13} + cos^{-1}\frac{3}{5} = tan^{-1}\frac{63}{16}$ **2**

8) Evaluate : $4tan^{-1}\frac{1}{5}$ **2**

9) show that : $tan^{-1}\frac{1}{4} + tan^{-1}\frac{2}{9} = \frac{1}{2}tan^{-1}\frac{4}{3}$ **2**

10) Prove that : $sin^{-1}x + cos^{-1}x = \frac{\pi}{2}; if \in x[-1, \quad 1]$ **2**

11) Prove that $sin^{-1}x + sin^{-1}y = sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2}) if x^2 - y^2 \leq 1$ **2**

12) Prove that $cos^{-1}x = 2sin^{-1}\left(\sqrt{\frac{1-x}{2}}\right)$ **2**

Section-C

13) Find the principal value of $sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ **3**

14) Show that : **3**

$sin^{-1}\frac{3}{5} - sin^{-1}\frac{8}{17} = cos^{-1}\frac{84}{85}$

15) Find te principal values of the following: **3**

$cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

16) Find te principal values of the following: $sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ **3**

17) Find te principal values of the following: $cot^{-1}\left(\sqrt{3}\right)$ **3**

Section-D

18) Solve for X, $\tan^{-1}(X+1) + \tan^{-1}(X-1) = tan^{-1}\frac{8}{31}$ **4**

19) Solve for X, $tan^{-1}\left(\frac{X-1}{X-2}\right) + tan^{-1}\left(\frac{X+1}{X+2}\right) = \frac{\pi}{4}$ **4**

20) Solve for X, $tan^{-1}3X + tan^{-1}2X = \frac{\pi}{4}$ **4**

21) Solve the following for X: $cos^{-1}\left(\frac{X^2-1}{X^2+1}\right) + tan^{-1}\left(\frac{2X}{X^2-1}\right) = \frac{2\pi}{3}$ **4**

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Section-A

1) 1.[As $\sin^{-1}(-X) = -\sin^{-1}X$] **1**

2) $sin^{-1}(sin2\theta)$ **1**

3) $\frac{2\pi}{3}$ **1**

4) $\frac{\pi}{3}$ **1**

5) $\frac{24}{25}$ **1**

Section-B

6)

$$\tan^{-1}\left[\frac{\cos\ \ x}{1+\sin\ \ s}\right] \quad \because \begin{cases} \cos\ \ x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2} \\ \text{and}\ \ 1+\sin\ \ x = \left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2 \end{cases}$$

$$= \tan^{-1}\left[\frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2}\right]$$

$$= \tan^{-1}\left[\frac{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2}\right]$$

$$= \tan^{-1}\left[\frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}\right] \text{Divide by } \cos\frac{x}{2}, \text{ we get}$$

$$= \tan^{-1}\left[\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right] = \frac{\pi}{4} - \frac{x}{2}$$

7) $L.H.S. = sin^{-1}\frac{5}{13} + cos^{-1}\frac{3}{5}$ **2**

$$\because\ sin^{-1}x = tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

$$sin^{-1}\frac{5}{13} = tan^{-1}\left(\frac{5/13}{\sqrt{1-25/169}}\right)$$

$$= tan^{-1}\frac{(5/13)}{\sqrt{\frac{144}{169}}} = tan^{-1}\frac{5}{12}$$

$$and\quad cos^{-1}x = tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$$

$$\therefore\quad cos^{-1}\frac{3}{5} = tan^{-1}\left(\frac{\sqrt{1-\frac{9}{25}}}{\frac{3}{5}}\right)$$

$$= tan^{-1}\left(\frac{\sqrt{\frac{16}{25}}}{\frac{3}{5}}\right) = tan^{-1}\left(\frac{4}{3}\right)$$

$$tan^{-1}\frac{5}{12} + tan^{-1}\frac{4}{3} = tan^{-1}\left[\frac{\frac{5}{12}+\frac{4}{2}}{1-\frac{5\times 4}{12\times 3}}\right]$$

$$\left[tan^{-1}x + tan^{-1}y = tan^{-1}\frac{x+y}{1-xy}\right]$$

$$= tan^{-1}\left|\frac{\frac{15+48}{36}}{\frac{36-20}{36}}\right|$$

$$= tan^{-1}\left(\frac{63}{16}\right) = R.H.S.$$

8)

$$4\quad tan^{-1}\frac{1}{5} = 2\left[2tan^{-1}\frac{1}{5}\right]$$

$$= 2\left[tan^{-1}\left(\frac{2\times\frac{1}{5}}{1-\frac{1}{25}}\right)\right]$$

$$\left[\because 2tan^{-1}x = tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$$

$$= 2tan^{-1}\left(\frac{\frac{2}{5}}{\frac{25-1}{25}}\right)$$

$$= 2tan^{-1}\left(\frac{2\times 25}{24\times 5}\right) = 2tan^{-1}\left(\frac{5}{12}\right)$$

$$= tan^{-1}\left(\frac{\frac{2\times 5}{12}}{1-\frac{25}{144}}\right)$$

$$= tan^{-1}\left(\frac{\frac{10}{12}}{\frac{144-25}{144}}\right)$$

$$= tan^{-1}\left(\frac{10\times 144}{119\times 12}\right)$$

$$= tan^{-1}\left(\frac{120}{119}\right)$$

9) $LHS = tan^{-1}\frac{1}{4} + tan^{-1}\frac{2}{9}$ **2**

$$= tan^{-1}\left[\frac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1\times 2}{4\times 9}}\right]$$

$$\left[tan^{-1}x + tan^{-1}y = tan^{-1}\frac{x+y}{1-xy}\right]$$

$$= tan^{-1}\left[\frac{\frac{9+8}{36}}{\frac{36-2}{36}}\right]$$

$$= tan^{-1}\left(\frac{17}{34}\right) = tan^{-1}\left(\frac{1}{2}\right)$$

$$= \frac{1}{2}\left(2tan^{-1}\frac{1}{2}\right) = \frac{1}{2}tan^{-1}\left(\frac{2\times\frac{1}{2}}{1-\frac{1}{4}}\right)$$

$$\because \left[2tan^{-1} = tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$$

$$= \frac{1}{2}tan^{-1}\left(\frac{1}{3/4}\right) = \frac{1}{2}tan^{-1}\left(\frac{4}{3}\right)$$

10) We have $x \in [-1, \quad 1]$ **2**

Let x = sin $\theta \quad \therefore \quad \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\Rightarrow \quad \theta = sin^{-1}x$

$\Rightarrow \quad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$\Rightarrow \quad \frac{\pi}{2} \ge -\theta \ge -\frac{\pi}{2}$

$\Rightarrow \quad \frac{\pi}{2}+\frac{\pi}{2} \ge \frac{\pi}{2}-\theta \ge -\frac{\pi}{2}+\frac{\pi}{2}$

$\Rightarrow \quad \pi \ge \frac{\pi}{2}-\theta \ge 0$

$\Rightarrow \quad cos\left(\frac{\pi}{2}-\theta\right) = sin \quad \theta = x$

$\Rightarrow \quad \frac{\pi}{2}-\theta = cos^{-1}x$

$\Rightarrow \quad cos^{-1}x = \frac{\pi}{2} - sin^{-1}x$

$\Rightarrow \quad sin^{-1}x + cos^{-1}x = \frac{\pi}{2} if \quad x \in [-1, \quad 1]$

11) $sin^{-1}x = A \quad and \quad sin^{-1}y = B$ **2**

$\Rightarrow$ x=sin A and y = sin B

$\therefore \quad cos \quad A = \sqrt{1-x^2}, cos \quad B = \sqrt{1-y^2}$

we have sin (A+B) = sin A cos B + cos A sin B

$\Rightarrow \quad sin(A+B) = x\sqrt{1-y^2} + \sqrt{1-x^2}y$

$\Rightarrow \quad sin(A+B) = x\sqrt{1-y^2} + y\sqrt{1-x^2}$

$\Rightarrow \quad A + B = sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$

$\therefore \quad sin^{-1}x + sin^{-1}y = sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$

12) **2**

$R.H.S. = 2sin^{-1}\sqrt{\frac{1-x}{2}} \qquad \begin{cases} Let \quad x = cos\theta \\ \Rightarrow \theta = cos^{-1}x \end{cases}$

$= 2sin^{-1}\left(\sqrt{\frac{1-cos\theta}{2}}\right)$

$\begin{cases} as \quad cos\theta = 1 = 2sin^2\frac{\theta}{2} \\ \Rightarrow 1 - cos\theta = 2sin^2\frac{\theta}{2} \end{cases}$

$= 2sin^{-1}\left(sin\frac{\theta}{2}\right)$

$= 2\left(\frac{\theta}{2}\right) = \theta = cos^{-1}x$

L.H.S=R.H.S

Section-C

13) Let $sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \theta$, where $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ **3**

$\therefore \quad \sin\theta = \frac{1}{\sqrt{2}} \Rightarrow \sin\theta\sin\frac{\pi}{4} \Rightarrow \quad \theta = \frac{\pi}{4}$

Hence, $sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

14) $sin^{-1}\frac{3}{5} - sin^{-1}\frac{8}{17} = cos^{-1}\frac{84}{85}$ **3**

15) Let $cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = y$, where $y \in [0, \pi]$ **3**

$\Rightarrow cosy = \frac{\sqrt{3}}{2} \Rightarrow cosy = cos\frac{\pi}{6} \Rightarrow \quad y = \frac{\pi}{6}$

Hence, the reqd. Principal value = $\frac{\pi}{6}$

16) Let $sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = y$, where $y \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$ **3**

$\Rightarrow secy = \frac{2}{\sqrt{3}} = sec\frac{\pi}{6} \Rightarrow y = \frac{\pi}{6}$

Hence, the reqd. principal value $\frac{\pi}{6}$

17) Let \({ cot }^{ -1 }\left(\sqrt { 3 } \right) =y,0 **3**

$\Rightarrow coty = \sqrt{3} \Rightarrow y = \frac{\pi}{6}$

Hence, the reqd. principal value $\frac{\pi}{6}$

Section-D

18) $X = \frac{1}{4}$, X= -8 (rejected). So, $X = \frac{1}{4}$ **4**

19) $X = \pm\frac{1}{\sqrt{2}}$ **4**

20) $X = \frac{1}{6}$ [-1 rejected] **4**

21) $X = tan\frac{\pi}{12}$ **4**