QB365

Important Questions - Linear Programming

12th Standard CBSE

Maths

Reg.No. : | | | | | |

Time : 01:00:00 Hrs

Total Marks : 50

Section-A

1) There are two factories located one at place P and the other at place Q. From these locations, a certain commodity is to be delivered to each of the three depots situated at A,B and C. The weekly requirements of the depots are respectively 5,5 and 4 units of the commodity while the production capacity of the commodity while the production capacity of the factories at P and Q are respectively 8 and 6 units. The cost of transportation per unit is given below: **3**

From/To	Cost (in RS)		
	A	B	B
P	160	100	150
Q	100	120	100

How many units should be transported from each factory to each depot in order that the transportable cost is minimum. When will be the minimum transportation cost?

2) Minimise Z=-3x+4y **3**

Subject to $x+2y\leq 8,\quad 3x+2y\leq 12,\quad x\geq 0, y\geq 0.$

3) Maximise Z=5x+3y **3**

Subject to $3x+5y\leq 15, 5x+2y\leq 10, x\geq 0,\quad y\geq 0.$

Section-B

4) Verify that the following problem has no feasible solution: **3**

Maximize Z=4x_1+4x_2, subject to the constraints:

$2x_1 3x_2\leq 18, x_1+x_2\geq 12, x_1, x_2\geq 0.$

5) If a man rides his motor cycle at 25 km/hr., he has to spend RS.2 per km on petrol, if he rides at a faster speed of 40 km/hr., the petrol cost increases to RS.5 per km. He has RS.100 to spend on petrol and wishes to find maximum distance he can travel within one hour. Express this as a linear programming problem and then solve it graphically. **3**

6) (Diet Problem) Every gram of wheat produces 0.1 g of protein and 0.25 g of carbohydrates. The corresponding values for rice are 0.05 g and 0.5 g respectively. Wheat costs Rs.4 per kg and rice Rs.6 per kg. The minimum daily requirements of protein and carbohydrates for an average child are 50 g and 200 g respectively. In What quantities should wheat and rice be mixed in the daily diet so as to provide the maximum daily requirements of protein and carbohydrates at minimum cost? Frame an L.P.P. and solve it graphically. **3**

Section-C

7) (Manufacturing Problem) A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is atmost 24. It takes 1 hour to make a ring and 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is Rs.300 and that on a chain is Rs.190, find the number of rings and chains that should be manufactured per day, so as to earn the maximum profit. Make it as an LPP and solve it graphically. **4**

8) A cooperative society of farmers has 50 hectares of land to grow two crops A and B. The profits from crops A and B per hectare are estimated as Rs.10,500 and Rs.9,000 respectively. The control weeds, a liquid hectare, respectively. Further not more than 800 litres of herbicide should be used in order to protect fish and wild life using a pond which collects drainage from this land. Keeping in mind that the protection of fish and other wild life is more important than earning profit, how much land should be allocated to each crop so as to maximize the total profit? Form an LPP from the above and solve it graphically. Do you agree with the message that the protection of wild life is utmost necessary to preserve the balance in environment? **4**

9) A producer has 30 and 17 units of labour and capital respectively, which he can use to produce two types of goods X and Y. To produce one unit of X, 2 units of labour and 3 units of capital are required. Similarly 3 units of labour and 1 unit of capital is required to produce one unit of Y. If X and Y are priced at Rs.100 and Rs.120 per unit respectively, how should the producer use his resources to maximise the total revenue. Solve the problem graphically. **4**

10) A company uses 3 mechines to manufacture and sell two types of shirts--half sleeves and full sleeves. Machines M_1,M_2 and M_3 take 1 hour, 2 hours and $1\frac{3}{5}$ hours to make a full sleeve shirt. The profit on each half sleeve shirt is Rs.1.00 and on a full sleeve shirt is Rs.1.50. No machine can work for more than 40 hours per week. How many shirts of each type should be made to maximise the company's profit? Solve the problem graphically. **4**

11) Solve the following Linear Programming Problem graphically: **4**

Maximize Z = 3x + 4y

Subject to $x + y \leq 4, x \geq 0 andy \geq 0.$

Section-D

12) A manufacturer produces nuts and bolts. It takes 2 hours work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 2 hours on machine B to produce a package of bolts. He earns a profit of Rs 24 per package on nuts and Rs 18 per package on bolts. How many packages of each should be produced each day so as to maximize his profit, if he operates his machines both for at the most 10 hours a days. Make an LPP from above and solve it graphically? **6**

13) A manufacturing company makes two model A and B of a product. Each piece of mode} A requires 9 hours of labour for fabricating and 1 hour for finishing. Each piece of model B requires 12 hours of labour for fabricating and 3 hours for finishing. The maximum number of labour hours, available for fabricating and for finishing, are 180 and 30 respectively. The company makes a profit of Rs 8,000 and Rs 12,000 on each piece of model A and model B respectively. How many pieces of each model should be manufactured to get maximum profit? Also, find the maximum profit. **6**

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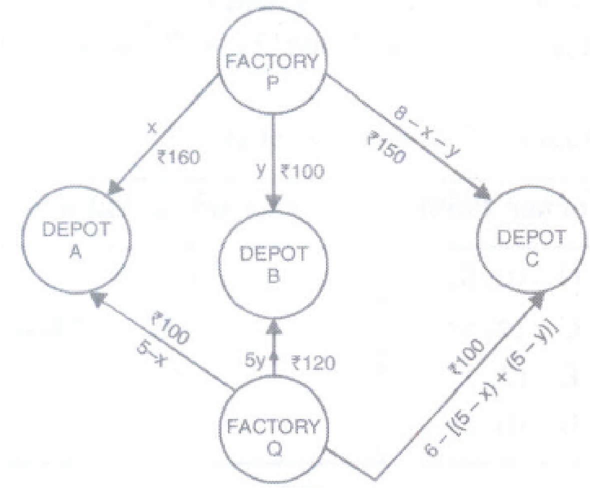
Section-A

1)

Let 'x' and 'y' units of the commodity bee transported from factory at P to the depots at A and B respectively. Then (8-x-y) units are to be transported to depot C.



Thus we have: $x \geq 0, y \geq 0$ and $8-x-y \geq 0$

i.e. $x \geq 0, y \geq 0$ and $x+y \leq 8$.since weekly requirement of depot A is 5 units of the commodity,

∴ (5-x) units are to be transported from the factory Q. obviously.$5-x \geq 0 \Rightarrow x \leq 5$

Again since (5-y) and 6-(5-x+5-y)=x+y-4 units are to be transported from factory at Q to depots at B and C respectively.

Now total cost Z is given by:

Z=160x+100y+100(5-x)+120(5-y)+100(x+y-4)+150(8-x-y)

=10(x-7y+190).

Thus the LPP problem is:

Minimise Z=10(x-7y+190) subject to the constraints:

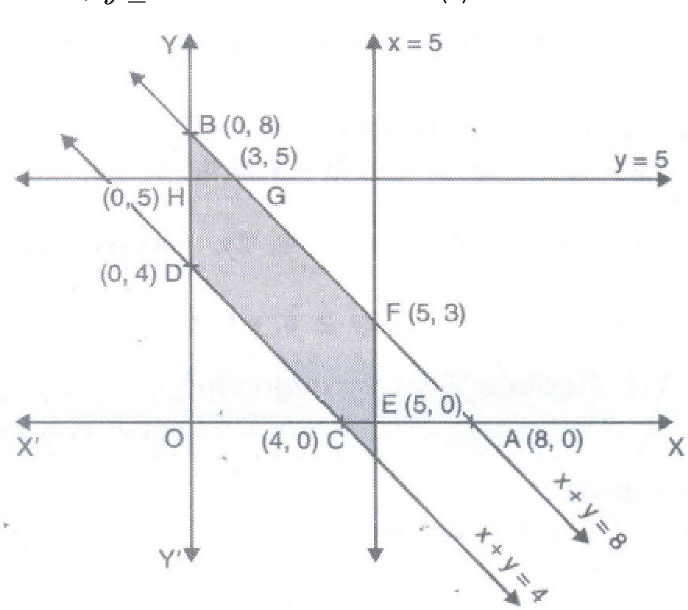
$x \geq 0, y \geq 0$ .....(1)

$x+y \leq 8$ .....(2)

$x \leq 5$ ......(3)

$y \leq 5$ ....(4)

and $x+y \geq 4$ ....(5)



The shaded portion is the feasible region, which is bounded.

Applying Corner Point Method, we have:

Corner Point	Z=10(x-7y+190)
C : (4,0)	1940
E : (5,0)	1950
F : (5,3)	1740
G : (3,5)	1580
H : (0,5)	1550 (Minimum)
D : (0,4)	1620

Hence, minimum transportation cost =Rs. 1550, when 0,5 and 3 units are delivered from factories at P and 5,0 and units are delivered from factory at Q.

2) 3

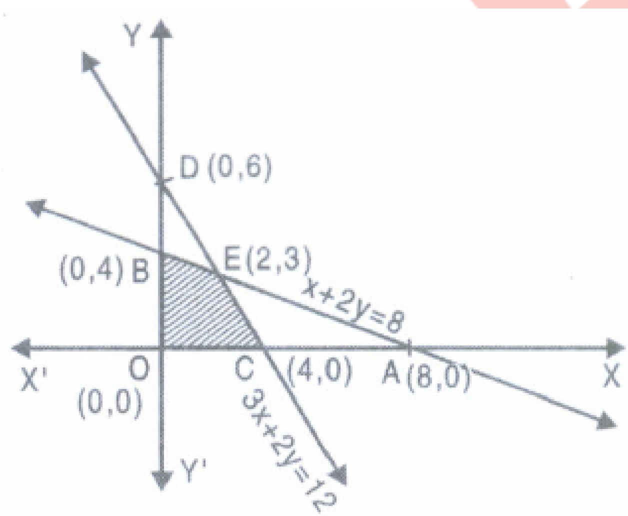
The system of constraints is:

$x+2y \leq 8$ ....(1)

$3x+2y \leq 12$ ....(2)

and $x \geq 0, y \geq 0$ ....(3)

The shaded region in the following figure is the feasible region determined by the system constraints(1)-(3).



It is observed that feasible region OCEB is bounded. Thus we use Corner Point Method to determine the minimum value of Z, where

Z=-3x+4y ......(4)

The co-ordinates of O,C,E and B are (0,0), (4,0), (2,3) (on solving x+2y=8 and 3x+2y=12) and (0,4) respectively.

We evaluate Z at each corner point.

Corner Point	Corresponding Value of Z
O : (0,0)	0
C : (4,0)	-12 (Minimum)
E : (2,3)	6
B : (0,4)	16

Hence, $Z_{min}=$ -12 at the point (4,0).

3)

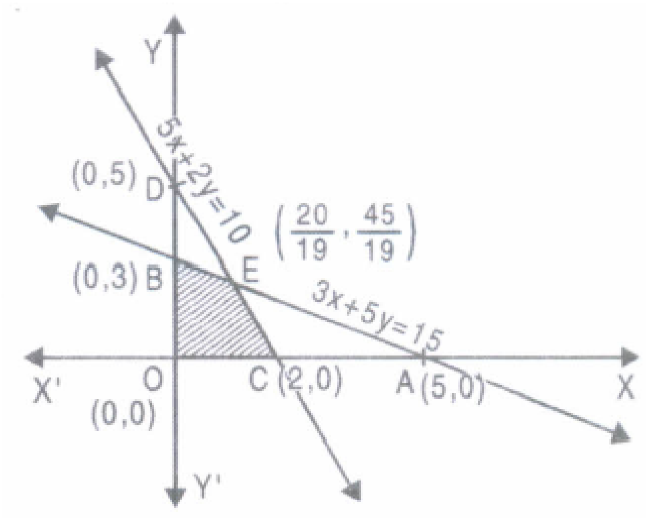
The system of constraints is:

$3x + 5y \leq 15$ ....(1)

$5x + 2y \leq 10$ ....(2)

and $x \geq 0, y \leq 0$ ....(3)

The shaded region in the following figure is the feasible region determined by the system of constraints (1)-(3)



It is observed that the feasible region OCEB is bounded.

Thus we use Corner Point Method to determine the maximum value of Z, where

Z=5x+3y ......(4)

The coordinates of O, C, E and B are (0,0), (2,0), $\left(\frac{20}{19}, \frac{45}{19}\right)$ (on solving 3x+5y=15,5x+2y=10) and (0,3) respectively.

We evaluate Z at each corner point.

Corner Point	Corresponding Value of Z
C : (2,0)	10
E : $\left(\frac{20}{19}, \frac{45}{19}\right)$	$\frac{235}{19}$ (Maximum)
B : (0,3)	9
O : (0,0)	0

Hence, $Z_{max} = \frac{235}{19}$ at $\left(\frac{20}{19}, \frac{45}{19}\right)$.

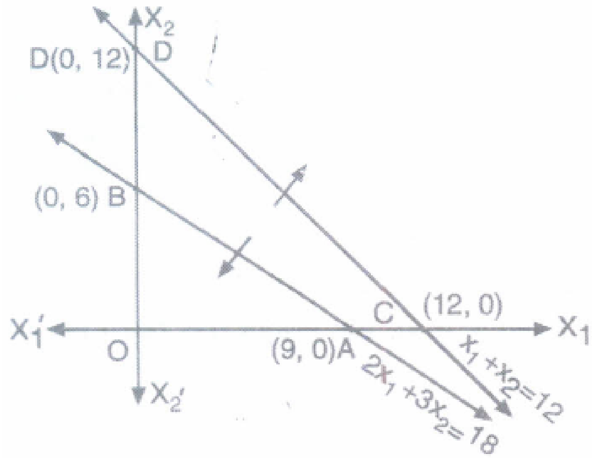
Section-B

4) The given system of constraints is:

$2x_1 3x_2 \leq 18$ ......(1)

$x_1+x_2 \geq 12$ ......(2)

$x_1, x_2 \geq 0$ ......(3)



Since there is no shaded portion, which is feasible region of the given constraints,

∴ the problem has no feasible solution.

5) Let the man travel 'x' travel 'x' km at 25 km/hr. and 'y' km at 40 km/hr.

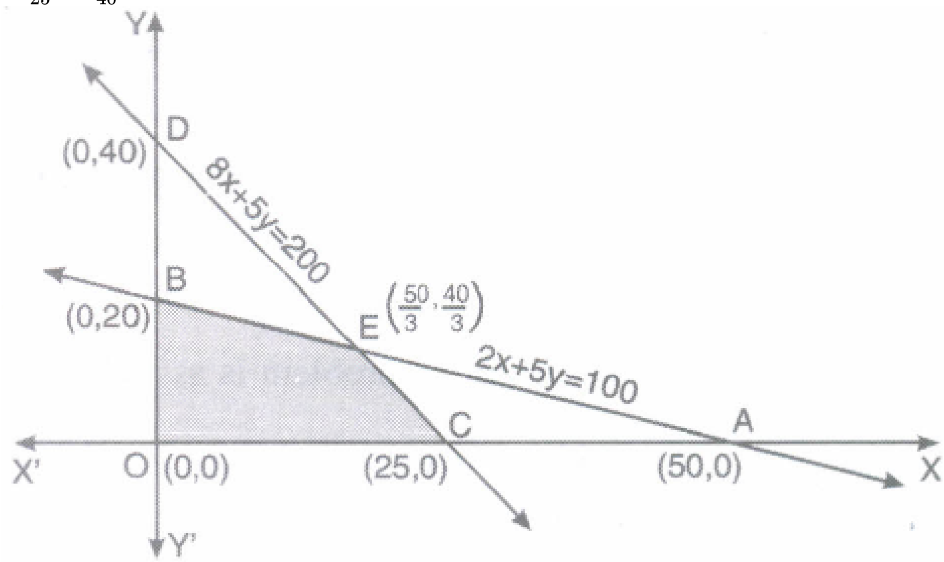
Clearly $x \geq 0$ ...(1) and $y \geq 0$ ...(2) ....(2)

Since the total money is RS.100,

$\therefore 2x + 5y \leq 100$ ....(3)

Since the total time is 1 hour,

$\therefore \frac{x}{25} + \frac{y}{40} \leq$ i.e. $8x + 5y \leq 200$ .....(4)



The objective function or the distance D, in one hour is:

D=X+Y .....(5)

Draw the lines:

x=0,y=0,2x+5y=100

and 8x+5y=200.

The feasible region (shown shaded) OCEB is bounded,

where O is (0,0), C is (25,0), B is (0,20) and E is $\left(\frac{50}{3}, \frac{40}{3}\right)$.

$$\left[\because Solving 2x + 5y = 100 and 8x + 5y = 200; x = \frac{50}{3}, y = \frac{40}{3}\right]$$

Applying Corner Point Method, we have:

Corner Point	D=x+y
O : (0,0)	0
C : (25,0)	25
E : $\left(\frac{50}{3}, \frac{40}{3}\right)$	30 (Maximum)
B : (0,20)	20

Hence, maximum distance travelled=30 km.

6)

Let quantity of wheat=x grams.

Quantity of rice=y grams.

Cost of wheat RS 4 per/kg and rice RS.6 per/kg.

Therefore, $Z=\frac{4x}{1000}+\frac{6y}{1000}$

The minimum daily requirement of protein =50 gm and wheat provides 0.1 gm and rice 0.05 gm protein.

Therefore $0.1x+0.05y\geq 50$.

The minimum daily requirements of carbohydrates=200 gm.

Carbohydrates in wheat =0.25 gm and in rice=0.5 gm.

Therefore, $0.25x+0.5y\geq 200$Thus the linear programming problem is as follows:

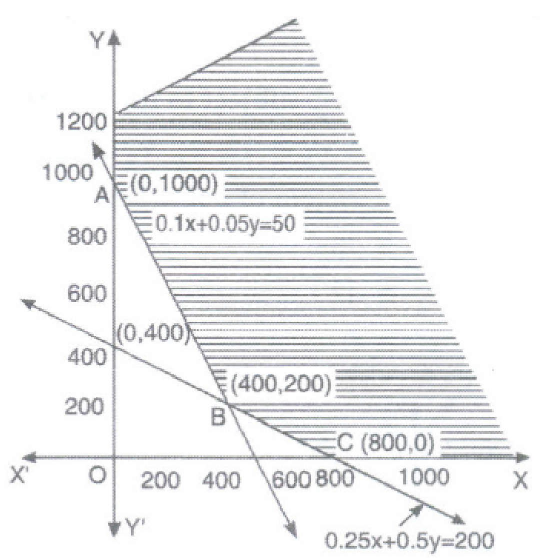
Minimise: $Z=\frac{4x}{1000}+\frac{6y}{1000}$

Subject to the constraints:

$0.1x+0.05y\geq 50$

$0.25x+0.5y\geq 200$

$x,y\geq 0.$



The graph of the constraints can be easily obtained. As $x\geq 0$ and $y\geq 0$, the graph has been drawn in the first quadrant only.

The coordinates of A are (0,1000), B are (400,200) and C are (800,0).

The feasible region is not bounded.

Applying Corner Point Method, we have:

Corner Point	$Z=\frac{4x}{1000}+\frac{6y}{1000}$
A : (0,1000)	$0+\frac{6}{1000}\times 1000=Rs.6$
B : (400,200)	$\frac{4\times 400}{1000}+\frac{6\times 200}{1000}$ $=RS.2.80\quad (Minimum)$
C : (800,0)	$\frac{4\times 800}{1000}+\frac{6\times 0}{1000}=RS.3.20$

Hence, minimum cost is RS.2.80 when x=400 g and y=200 g.

Section-C

7)

Let 'x' and 'y' be the number of gold rings and chains respectively.

We have:

$x \geq 0$ ...(1) $y \geq 0$ ...(2)

$x + y \leq 24$ ...(3) $x + \frac{y}{2} \leq 16$ ...(4)

The objective function, or the profit, Z is:

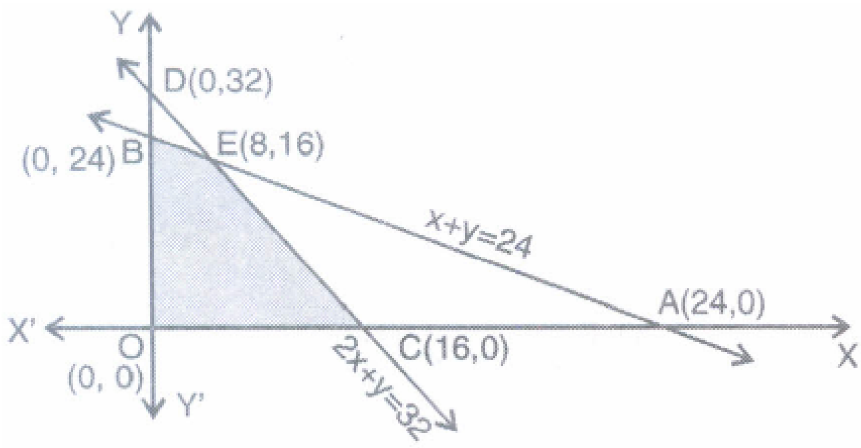
Z=300x+190y ....(5)

We have to maximise Z subject to (1)-(4).

For solution set, we draw the lines:

x=0, y=0, x+y=24, 2x+y=32.

The lines x+y=24 and 2x+y=32 meet at E (8,16).



The shaded portion represents the feasible region, which is bounded.

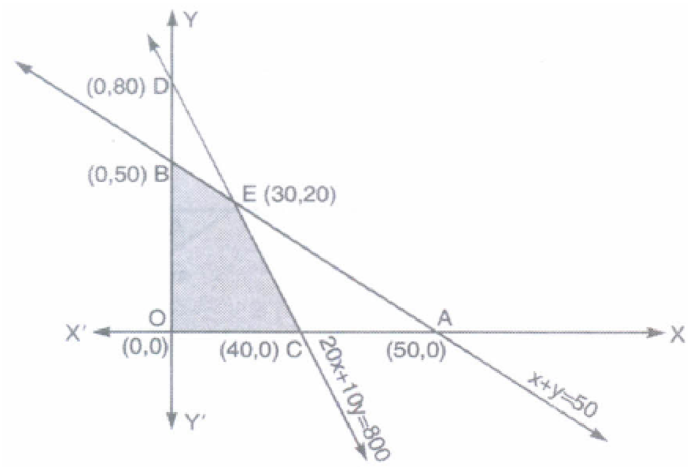
Applying Corner Point Method, we have:

Corner Point	Z=300x+190y
O : (0,0)	0
C : (16,0)	4800
E : (8,16)	5440 (Maximum
B : (0,24)	4560

Hence, the maximum profit is RS,5,440 and it is obtained when 8 gold rings and 16 chains are manufactured.

8) **4**

Let 'x' and 'y' hectares of land be allocated to crops A and B respectively. Then the problem is:
Maximize: Z=10,500x+9,000y subject to: $x \geq 0, y \geq 0, x+y \leq 50, 20x+10y \leq 800$.



For solution set, we draw the lines:

x=0,y=0,x+y=50 and 20x+10y=800.

The lines x+y=50 and 20x+10y=800 interest at E(30,20)

The shaded portion represents the feasible region, which is bounded.

Applying Corner Point Method, we have:

Corner Point	Z=10,500x+9,000y
O : (0,0)	0
C : (40,0)	4,20,000
E : (30,20)	4,95,000 (Maximum)
B : (0,50)	4,50,000

Hence, to earn maximum profit 30 and 20 hectares of land should be allocated to crops A and B respectively. We agree with the message that the protection of wildlife is utmost necessary in order to preserve the balance in environment.

9) 3 units of X, 8 units of Y and maximum revenue Rs.1260. **4**

10) 10 half sleeves, 15 full sleeves, Maximum profit Rs.32.50. **4**

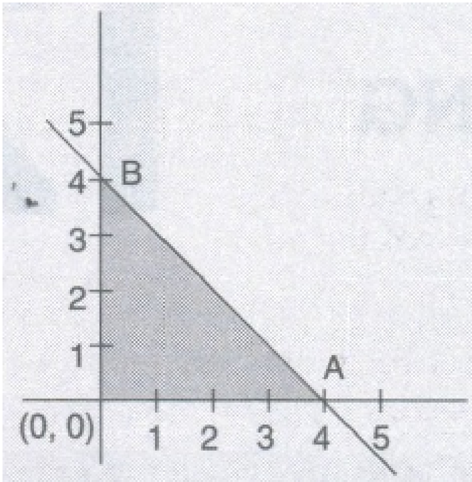
11) The feasible region is a triangle with vertices O(0,0).A(4,0) and B(0,4) **4**

$Z_a = 3 \times 0 + 4 \times 0 = 0$

$Z_A = 3 \times 4 + 4 \times 0 = 12$

$Z_B = 3 \times 0 + 4 \times 4 = 16$

Thus, maximum of Z is at B(0,4) and the maximum value is 16



Section-D

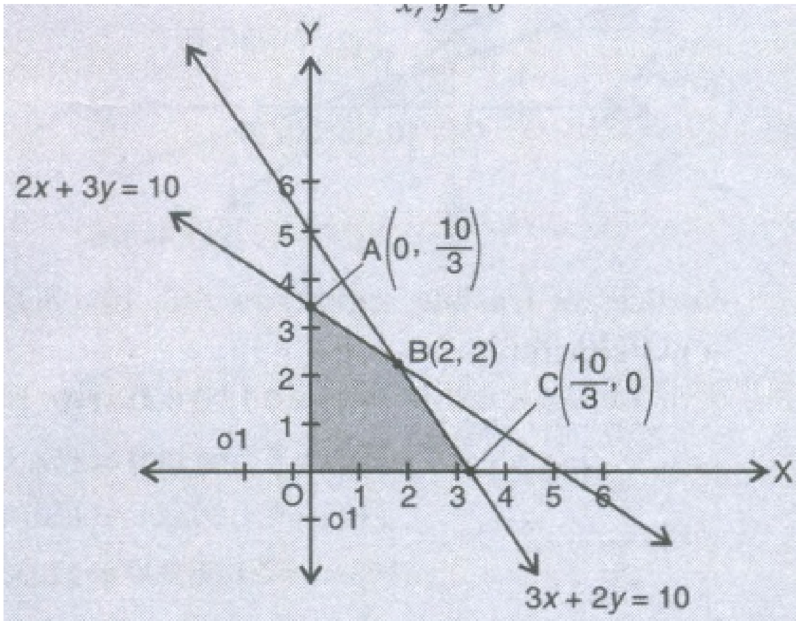
12) Let x and y be nut packages and bolt packages produced each day, respectively. **6**

$\therefore$ LPP is Maximize Z = 24x + 18y

Subject to $2x + 3y \leq 10$

$3x + 2y \leq 10$

$x, y \geq 0$



Vertices are

$A\left(0, \frac{10}{3}\right), B(2,2) \quad and \quad C\left(\frac{10}{3}, 0\right)$

Points	Z=24x + 18y
$0, \frac{10}{3}$	z = 0+ 60 = Rs 60
(2, 2)	z = 48 + 36 = Rs 84 (Max)
$\left(\frac{10}{3}, 0\right)$	z= 80 + 0 = rs 80

Hence, 2 nuts & 2 bolts to be produced to get max. profit of Rs 84.

13)

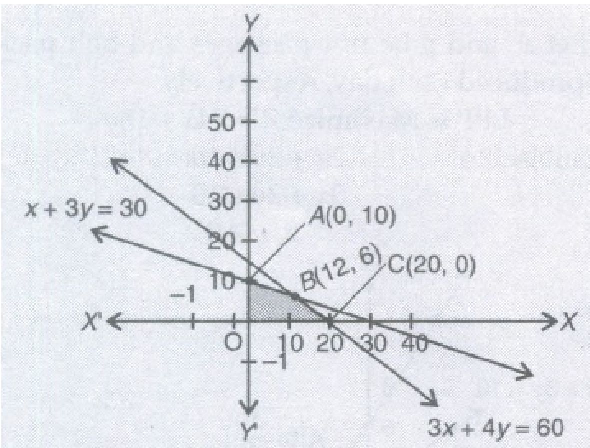
Let the number of pieces of model A to be manufactured be x and the number of pieces of model B to be manufactured be y.Then LPP is maximize P = 8,000 x + 12,000 y

S.T. $9x + 12y \leq 180$

$3x + 4y \leq 60$

$x + 3y \leq 30$

$x, y \geq 0$



Vertices of feasible region are A(0, 10), B(12, 6) and C(20, 0).

P(A) = Rs 1,20,000 at (0, 10)

P(B) = Rs 1,68,000 at (12, 6)

P(C) = Rs 1,68,000 at (20, 0)

Max. = Rs 1,68,000 at (12, 6)

Hence, the number of pieces of model A = 12, the number of pieces of model B = 6 and the maximum profit = Rs 1,68,000