QB365

Important Questions - Matrices

12th Standard CBSE

Maths

Reg.No. :

Time : 01:00:00 Hrs

Total Marks : 50

Section-A

1) If $\begin{bmatrix}3 & 4\\2 & x\end{bmatrix}\begin{bmatrix}x\\1\end{bmatrix}=\begin{bmatrix}19\\15\end{bmatrix}$, find the value of x **1**

2) |If matrix has 5 elements, write all possible orders it can have. **1**

3) Write the value of x-y+z from the following equation : $\begin{bmatrix}x+y+z\\x+z\\y+z\end{bmatrix}=\begin{bmatrix}9\\5\\7\end{bmatrix}$ **1**

4) If $\begin{bmatrix}2 & 3\\5 & 7\end{bmatrix}\begin{bmatrix}1 & -3\\-2 & 4\end{bmatrix}=\begin{bmatrix}-4 & 6\\-9 & x\end{bmatrix}$, write the value of x. **1**

5) Simplify $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\-\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\\cos\theta & \sin\theta\end{bmatrix}$. **1**

Section-B

6) Solve the matrix equation $\begin{bmatrix}x^2\\y^2\end{bmatrix}-3\begin{bmatrix}x\\2y\end{bmatrix}=\begin{bmatrix}-2\\-9\end{bmatrix}$ **2**

7) Find the value of X and Y if **2**

$X+Y=\begin{bmatrix}2 & 3\\5 & 1\end{bmatrix}, X-Y=\begin{bmatrix}6 & 5\\7 & 3\end{bmatrix}$

8) If $\begin{bmatrix}cos\theta & -sin\theta\\sin\theta & cos\theta\end{bmatrix}$, find the value of θ satisfying the equation $A + A^T = I_2$, where $0 \le 0 \le \frac{\pi}{2}$. **2**

9) Prove that the diagonal elements of a skew symmetric matrix are all zero. **2**

10) If $A=\begin{bmatrix}1 & 1 & 1\\1 & 1 & 1\\1 & 1 & 1\end{bmatrix}$ then prove that $A^n=\begin{bmatrix}3^{n-1} & 3^{n-1} & 3^{n-1}\\3^{n-1} & 3^{n-1} & 3^{n-1}\\3^{n-1} & 3^{n-1} & 3^{n-1}\end{bmatrix}$, then $n\epsilon N$. **2**

11) Prove the following by the principle of mathematical induction : **2**

if $A=\begin{bmatrix}3 & -4\\1 & -1\end{bmatrix}$, then $A^n=\begin{bmatrix}1+2n & -4n\\n & 1-2n\end{bmatrix}$ for every positive integer n.

Section-C

12) If $A = diag.\begin{bmatrix}3, & -5, & 7\end{bmatrix}$ then $B=\begin{bmatrix}-1, & 2, & 4\end{bmatrix}$, then find $(2A+3B)$. **3**

13) If $A=\begin{bmatrix}3 & 1 & -1\\0 & 1 & 2\end{bmatrix}$, then show that AA' is a symmetric matrix. **3**

14) Show that: $\left[\begin{pmatrix}1 & \omega & \omega^2\\\omega & \omega^2 & 1\\\omega^2 & 1 & \omega\end{pmatrix}+\begin{pmatrix}\omega & \omega^2 & 1\\\omega^2 & 1 & \omega\\\omega & \omega^2 & 1\end{pmatrix}\right]\begin{bmatrix}1\\\omega\\\omega^2\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$, where ω is a cube root of unity. **3**

Section-D

15) Express the following matrix as the sum of a symmetric and a skew symmetric matrix, and verify your result : $\begin{bmatrix}3 & -2 & -4\\3 & -2 & -5\\-1 & 1 & 2\end{bmatrix}$ **4**

16) Find x, if $[x \quad 1]\begin{bmatrix}1 & 0\\-2 & -3\end{bmatrix}\begin{bmatrix}x\\3\end{bmatrix}=O$ **4**

17) Construct a 3×3 matrix $[a_{ij}]$, whose elements are given by $a_{ij}=2i-3j$. **4**

Section-E

18) If $A=\begin{pmatrix}1 & 0 & 2\\0 & 2 & 1\\2 & 0 & 3\end{pmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$ **6**

19) 6

Using elementary transformation, find the inverse of the matrix $A = \begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ and use it to solve the following system of lines equations :

8x + 4y + 3z = 19

2x + y + z = 5

x + 2y + 2z = 7

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Section-A

1) $\begin{bmatrix} 3x & + & 4 \\ 2x & + & x \end{bmatrix} = \begin{bmatrix} 19 \\ 15 \end{bmatrix} \Rightarrow 3x = 15 \Rightarrow x = 5$ 1

2) Possible orders are : $1 \times 5, 5 \times 1$ 1

3) $x + y + z = 9, x + z = 5, y + z = 7 \Rightarrow z = 3, x = 2, y = 4; x - y + z = 2 - 4 + 3 = 1$ 1

4) We have $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & - & 6 & -6 & + & 12 \\ 5 & - & 14 & -15 & + & 28 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix} \Rightarrow \begin{bmatrix} -4 & 6 \\ -9 & 13 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix} \Rightarrow x = 13$ 1

5) 1

We have

$$\cos\theta\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix} = \begin{bmatrix} cos^2\theta & cos\theta sin\theta \\ -sin\theta cos\theta & cos^2\theta \end{bmatrix} + \begin{bmatrix} sin^2\theta & -sin\theta cos\theta \\ sin\theta cos\theta & sin^2\theta \end{bmatrix} = \begin{bmatrix} cos^2\theta + sin^2\theta & cos\theta sin\theta - sin\theta cos\theta \\ -sin\theta cos\theta + sin\theta cos\theta & cos^2\theta + sin^2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Section-B

6) We have, 2

$$\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} - 3\begin{bmatrix} x \\ 2y \end{bmatrix} = \begin{bmatrix} -2 \\ -9 \end{bmatrix}$$

⇒ $x^2 - 3x = -2$ and $y^2 - 6y = -9$

⇒ $x^2 - 3x + 2 = 0$ and $y^2 - 6y + 9 = 0$

⇒ $x^2 - 2x - x + 2 = 0$ and $y^2 - 3y - 3y + 9 = 0$

⇒ x(x - 2) - 1(x - 2) = 0 and y(y - 3) - 3(y - 3) = 0

⇒ (x - 2)(x - 1) = 0 and (y - 3)(y - 3) = 0

∴ x = 1, 2 and y = 3, 3

7) We have, 2

$$X + Y = \begin{bmatrix} 2 & 3 \\ 5 & 1 \end{bmatrix}, X - Y = \begin{bmatrix} 6 & 5 \\ 7 & 3 \end{bmatrix}$$

$$(X + Y) + (X - Y) = \begin{bmatrix} 2 & 3 \\ 5 & 1 \end{bmatrix} + \begin{bmatrix} 6 & 5 \\ 7 & 3 \end{bmatrix}$$

$$2X = \begin{bmatrix} 8 & 8 \\ 12 & 4 \end{bmatrix}$$

$$\Rightarrow \quad X = \begin{bmatrix} 4 & 4 \\ 6 & 2 \end{bmatrix}$$

$$\therefore \quad X = \begin{bmatrix} 4 & 4 \\ 6 & 2 \end{bmatrix}$$

and

$$Y = \begin{bmatrix} 2 & 3 \\ 5 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 4 \\ 6 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -1 \\ -1 & -1 \end{bmatrix}$$

8) We have, 2

$$A = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$$

$$A^T = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$$

$$\Rightarrow \quad A + A^T = \begin{bmatrix} 2cos\theta & 0 \\ 0 & cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad 2cos\theta = 1$$

$$\Rightarrow \quad cos\theta = \frac{1}{2} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\because \quad \theta = \frac{\pi}{3}$$

9) Let A be a skew-symmetric matrix. Then by definition $A' = -A$ 2

$\Rightarrow$ the (i, j)th element of A' = the (i, j)th element of (- A)

$\Rightarrow$ the (j, i)th element of A = - the (i, j)th element of A

For the diagonal elements i = j $\Rightarrow$ the (i, j)the element of A = - the (i, j)th element of A.

$\Rightarrow$ the (i, j)th element of A = 0

Hence the diagonal elements are all zero.

10) We shall prove the result by using principle of mathematical induction. 2

Let

$$P(n): A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$$

Now,

$$P(1): A^1 = \begin{bmatrix} 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

The result is true for n = 1.

Let the result be true for n = k.

So,

$$A^k = \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}$$

Now, we prove that P(k + 1) is true.

Now, $A^{k+1} = A. A^k$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}$$

$$= \begin{bmatrix} 3.3^{k-1} & 3.3^{k-1} & 3.3^{k-1} \\ 3.3^{k-1} & 3.3^{k-1} & 3.3^{k-1} \\ 3.3^{k-1} & 3.3^{k-1} & 3.3^{k-1} \end{bmatrix}$$

$$= \begin{bmatrix} 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \end{bmatrix}$$

$= A^{k+1}$

Hence, it is true n = k + 1.

Hence, by principle of mathematical induction P(n) is true for all $n \epsilon N$

11) We shall prove the result by mathematical induction on n. 2

Step 1 : When n = 1, by the definition or integral powers of a matrix, we have

$$A^1 = \begin{bmatrix} 1+2(1) & -4n \\ n & 1-2(1) \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

So, the result is true for n = 1.

Step 2 : Let the result be true for n = m. Then,

$$A^m = \begin{bmatrix} 1+2m & -4m \\ m & 1-2m \end{bmatrix}$$

Now, we will show that the result is true for n = m + 1, i.e.,

$$A^{m+1} = \begin{bmatrix} 1+2(m+1) & -4(m+1) \\ (m+1) & 1-2(m+1) \end{bmatrix}$$

By the definition of integral powers of a square matrix, we have

$A^{m+1} = A^m . A$

$$\Rightarrow \quad A^{m+1} = \begin{bmatrix} 1+2m & -4m \\ m & 1-2m \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

[by supposition (i)]

$$\Rightarrow \quad A^{m+1} = \begin{bmatrix} 3+6m-4m & -4-8m+4m \\ 3m+1-2m & -4m-4+2m \end{bmatrix}$$

$$= \begin{bmatrix} 1+2(m+1) & -4(m+1) \\ (m+1) & 1-2(m+1) \end{bmatrix}$$

This shows that the result is true for n = m + 1, whenever it is true for n = m.

Hence, by the principle of mathematical induction, the result is true for any positive integer n.

Section-C

12) We have: and 3

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 7 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}.$$

$$\therefore \quad 2A + 3B = 2\begin{bmatrix} 3 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 7 \end{bmatrix} + 3\begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & -10 & 0 \\ 0 & 0 & 14 \end{bmatrix} + \begin{bmatrix} -3 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 12 \end{bmatrix} = \begin{bmatrix} 6-3 & 0+0 & 0+0 \\ 0+0 & -10+6 & 0+0 \\ 0+0 & 0+0 & 14+12 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 26 \end{bmatrix}$$ =diag. $\begin{bmatrix} 3, & -4, & 26 \end{bmatrix}$.

13) We have: 3

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 0 & 1 & 2 \end{bmatrix}.$$

$$\therefore \quad A' = \begin{bmatrix} 3 & 0 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}. \therefore \quad AA' = \begin{bmatrix} 3 & 1 & -1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}. = \begin{bmatrix} 9+1+1 & 0+1-2 \\ 0+1-2 & 0+1+4 \end{bmatrix} = \begin{bmatrix} 11 & -1 \\ -1 & 5 \end{bmatrix}.$$

Which is a symmetric matrix.

14) 3

$$\left[\begin{pmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{pmatrix} + \begin{pmatrix} \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \\ \omega & \omega^2 & 1 \end{pmatrix}\right] \begin{bmatrix} 1 \\ \omega \\ \omega^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \left[\begin{pmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{pmatrix} + \begin{pmatrix} \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \\ \omega & \omega^2 & 1 \end{pmatrix}\right] \begin{bmatrix} 1 \\ \omega \\ \omega^2 \end{bmatrix} = \begin{pmatrix} 1+\omega & \omega+\omega^2 & \omega^2+1 \\ \omega+\omega^2 & \omega^2+1 & 1+\omega \\ \omega^2+\omega & 1+\omega^2 & \omega+1 \end{pmatrix} \begin{bmatrix} 1 \\ \omega \\ \omega^2 \end{bmatrix} = \begin{pmatrix} -\omega^2 & -1 & -\omega \\ -1 & -\omega & -\omega^2 \\ -1 & -\omega & -\omega^2 \end{pmatrix} \begin{bmatrix} 1 \\ \omega \\ \omega^2 \end{bmatrix} \ [$$

which is true.

$$= \begin{bmatrix} -0 \\ -0 \\ -0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

Section-D

15) 4

$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A');$ where $A = \frac{1}{2}(A + A')$ is symmetric and $\frac{1}{2}(A - A')$ is skew symmetric Proceed.

$$\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{5}{2} & -\frac{3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix}$$

16) $x = \frac{2 \pm \sqrt{4+36}}{2} = 1 \pm \sqrt{10}$ **4**

17) Matrix is $\begin{bmatrix} -1 & -4 & -7 \\ 1 & -2 & -5 \\ 3 & 0 & -3 \end{bmatrix}$ **4**

Section-E

18) $A^2 = \begin{pmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{pmatrix}$; $A^3 = \begin{pmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{pmatrix}$ **6**

L.H.S = $A^3 - 6A^2 + 7A + 2I$

$$= \begin{pmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{pmatrix} - \begin{pmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{pmatrix} + \begin{pmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{pmatrix} + \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = 0$$

= RHS

19) We know that $A = I_3A$

$$\begin{bmatrix}8&4&3\\2&1&1\\1&2&2\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}A$$

Applying $R_3 \to 2R_3$

$$\begin{bmatrix}8&4&3\\2&1&1\\2&4&4\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&2\end{bmatrix}A$$

Applying $R_3 \to R_3 - R_2$

$$\begin{bmatrix}8&4&3\\2&1&1\\0&3&3\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&-1&2\end{bmatrix}A$$

Applying $R_2 \to 4R_2$

$$\begin{bmatrix}8&4&3\\8&4&4\\0&3&3\end{bmatrix}=\begin{bmatrix}1&0&0\\0&4&0\\0&-1&2\end{bmatrix}A$$

Applying $R_2 \to R_2 - R_1$

$$\begin{bmatrix}8&4&3\\0&0&1\\0&3&3\end{bmatrix}=\begin{bmatrix}1&0&0\\-1&4&0\\0&-1&2\end{bmatrix}A$$

Applying $R_2 \to R_2 + R_3$

$$\begin{bmatrix}8&4&3\\0&3&4\\0&3&3\end{bmatrix}=\begin{bmatrix}1&0&0\\-1&3&2\\0&-1&2\end{bmatrix}A$$

Applying $R_3 \to R_3 - R_2$

$$\begin{bmatrix}8&4&3\\0&3&4\\0&0&-1\end{bmatrix}=\begin{bmatrix}1&0&0\\-1&3&2\\1&-4&0\end{bmatrix}A$$

Applying $R_1 \to R_1 + 3R_3$

$$\begin{bmatrix}8&4&0\\0&3&4\\0&0&-1\end{bmatrix}=\begin{bmatrix}4&-12&0\\-1&3&2\\1&-4&0\end{bmatrix}A$$

Applying $R_2 \to R_2 + 4R_3$

$$\begin{bmatrix}8&4&0\\0&3&0\\0&0&-1\end{bmatrix}=\begin{bmatrix}4&-12&0\\3&-13&2\\1&-4&0\end{bmatrix}A$$

Applying $R_1 \to R_1 - \frac{4}{3}R_2$

$$\begin{bmatrix}8&0&0\\0&3&0\\0&0&-1\end{bmatrix}=\begin{bmatrix}0&\frac{16}{3}&-\frac{8}{3}\\3&-13&2\\1&-4&0\end{bmatrix}$$

Applying $R_1 \to \frac{1}{8}R_1, R_2 \to \frac{1}{3}R_2$ and $R_3 = -R_3$

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}0&\frac{2}{3}&-\frac{1}{3}\\3&-\frac{13}{3}&\frac{2}{3}\\-1&4&0\end{bmatrix}A$$

$$A^{-1}=\begin{bmatrix}0&\frac{2}{3}&-\frac{1}{3}\\3&-\frac{13}{3}&\frac{2}{3}\\-1&4&0\end{bmatrix}$$

Matrix representation of given linear equation is

$$\begin{bmatrix}8&4&3\\2&1&1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}19\\5\end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 2 \end{bmatrix}\begin{bmatrix} z \end{bmatrix} \quad \begin{bmatrix} 7 \end{bmatrix}$$

$\Rightarrow$ AX = B

$\Rightarrow$ X = $A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & \frac{2}{3} & -\frac{1}{3} \\ 1 & -\frac{13}{3} & \frac{2}{3} \\ -1 & 4 & 0 \end{bmatrix}\begin{bmatrix} 19 \\ 5 \\ 7 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 + \frac{10}{3} - \frac{7}{3} \\ 19 - \frac{65}{3} + \frac{14}{3} \\ -19 + 20 + 0 \end{bmatrix}$$

$$\therefore \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$\therefore$ x = 1, y = 2, z = 1