QB365

Important Questions - Probability

12th Standard CBSE

Maths

Reg.No. :

Time : 01:00:00 Hrs

Total Marks : 50

Section-A

1) Given P(A)=$\frac{1}{2}$,P(B)=$\frac{1}{3}$ and $P(A \cap B) = \frac{1}{6}$ Are the events A and B independent? **1**

2) If P(A)=0.4, P(B)=p and $P(A \cup B) = 0.7$ find the value of p, if A and B are independent events. **1**

3) Given P(A)=0.2, P(B)=0.3 and $P(A \cap B) = 0.3$ Find P(A/B) **1**

4) Given P(A)=0.4, P(B)=0.7 and P(B/A)=0.6, Find $P(A \cup B)$ **1**

5) Events E and F are given to be independent. Find P(F) if it is given that P(E)=0.60 and P(E ∩ F)=0.35 **1**

Section-B

6) If P(E) =$\frac{6}{11}$, P(F) =$\frac{5}{11}$ and P(E ∪ F)=$\frac{7}{11}$ then find (a) P(E/F), (b) P(F/E) **2**

7) If P(F) = 0.35 and P(E ∪ F) = 0.85 and E and F are independent events. Find P(E). **2**

8) If P(F) =$\frac{1}{2}$$and \quad P(F) = \frac{1}{5}$ find $P(E \,\bar{\cup}\, F)$ If E and F are independent events. **2**

9) If E and F are independent events, then show that **2**

(a) E and $\bar{F}$

(b) $\bar{E}$ and F are also independent.

10) A couple has 2 children. Find the probability that both are boys, if it is known that (a) one of them is a boy (b) the older child is boys. **2**

11) If each element of a second order determinant is either 0 or 1, what is the probability that the value of determinant is positive? (Assume that the individual entries of the determinant are chosen independently, each value assumed with probability $\frac{1}{2}$ **2**

Section-C

12) A die is thrown twice and the sum of the numbers appearing is observed to be 6.What is the conditional probability that the number 4 has appeared atlaest once? **3**

13) If A and B are two independent events,then the probability of occurrence of at least one of A and B is given by: **3**

[1-$P(A^{'})$]$P(B^{'})$

14) Assume that each born child is equally likely to be a boy or a girl.IF a family has two children, What is the probability that both are girls given that (i) the youngest is a girl (ii) at least one is a girl? **3**

Section-D

15) There are 2000 scooter drivers, 4000 car drivers and 6000 truck drivers all insured. The probabilities of an accident involving a scooter, a car, a truck are 0.01,0.03,0.15 respectively. One of the insured drivers meets with an accident. What is the probability that he is a scooter driver? **4**

16) A and B throw a pair of die turn by turn. The first to throw 9 is awarded a prize. If A starts the game, show that the probability of A getting the prize is $\frac{9}{17}$ **4**

17) A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn at random and are found to both diamonds. Find the probability of the lost card being a diamond. **4**

Section-E

18) There are three coins. First is a biased that comes up tails 60% of the times, second is also a biased coin that comes up heads 75% of the times and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the first coin? **6**

19) A card from a pack of 52 cards is lost.From the remaining cards of the pack, two cards at random and are found to be hearts.Find the probability of the missing card to be a heart **6**

****

Section-A

1) Yes **1**

2) p=0.5 **1**

3) 1/3 **1**

4) 0.86 **1**

5) 0.58 **1**

Section-B

6) P(E ∩ F) = P(E) + P(F) - P(E ∪ F) **2**

$= \frac{6}{11} + \frac{5}{11} - \frac{7}{11}$

$= \frac{4}{11}$

(a) $P(E/F) = \frac{P(E \cap F)}{P(F)}$

$= \frac{\frac{4}{11}}{\frac{5}{11}} = \frac{4}{5}$

(b) P(E/E)=$\frac{P(E \cap F)}{P(E)}$

$= \frac{\frac{4}{11}}{\frac{6}{11}} = \frac{4}{6}$

$= \frac{2}{3}$

7) **2**

$P(E \cap F) = P(E) \times P(F) P(E \cup F) = P(E) + P(F) - P(E \cap F) \Rightarrow \quad P(E \cup F) = P(E) + P(F) - P(E) \times P(F) \Rightarrow \quad 0.85 = P(E) + 0.35 - P(E) \times 0.35 \Rightarrow \quad 0.85 = P(E)(1 - 0.35) + 0.35 \Rightarrow$

$\Rightarrow \quad P(E) = \frac{50}{65} = \frac{10}{13} \therefore \quad P(F) = 1 - \frac{10}{13} = \frac{3}{13}$

8) $P(E \cup F) = P(E) + P(F) - P(E \cap F)$ **2**

If E and F are independent, then

$= P(E \cap F) = P(E) \times P(F) P(E \cup F) = P(E) + P(F) - P(E) \times P(F) P(E \cup F) = \frac{1}{2} + \frac{1}{5} - \frac{1}{2} \times \frac{1}{5} = \frac{1}{2} + \frac{1}{5} - \frac{1}{10} - \frac{5+2-1}{10} = \frac{6}{10} = \frac{3}{5} P(\overline{E \cup F}) = 1 - P(E \cup F) = 1 - \frac{3}{5} = \frac{2}{5}$

9) If E and F are independent, then **2**

$P(E \cap F) = P(E) \times P(E)$

(a) $P(E \cap \bar{F}) = P(E) - P(E \cap F)$

$= P(E) - P(E) \times P(E) \quad by \quad eqn. \ldots (i)$

$P(E \cap \bar{F}) = P(E)[1 - P(F)]$

$\Rightarrow \quad P(E \cap \bar{F}) = P(E) \times P(\bar{F})$

$\therefore \quad E \quad and \quad \bar{F}$ are independent events.

(b) $P(\bar{E} \cap F) = P(F) - P(E \cap F)$

$= P(F) - P(E) \times P(F)$

$= P(F)[1 - P(E)]$

$= P(E) \times P(\bar{F})$

$= PF \times P\bar{E}$

$\therefore \quad \bar{E} and \quad F$ are independent.

10) Sample space ={B_1B_2, B_1G_2, G_1B_2, G_1G_2}, B_1 and G_1 are the older boy and girl respectively. **2**

Let E_1 = both the children are boys;

E_2 = one of the children are boys;

E_3 = the older child is a boy

Then, (a) $P(E_1/E_2)$= $P\left(\frac{E_1 \cap E_2}{E_2}\right) = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$

(b) $P(E_1/E_3)$ = $P\left(\frac{E_1 \cap E_3}{E_3}\right) = \frac{\frac{1}{4}}{\frac{2}{4}} = \frac{1}{2}$

11) There are four entries determinant of 2 x 2 order.Each entry may be filled up in two ways with 0 or 1.Therefore, number of determinants that can be formed **2**

= 2^4 = 16

The value of determinant is positive in the following cases

$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$

i.e, 3 determinants

Thus, the probability that the determinants is positive $= \frac{3}{16}$

Section-C

12) Let the events be as below: **3**

E: Number 4 appears atleast once

F:Sum of the numbers appearing is 6.

$\therefore$ E={(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),(1,4),(2,4),(3,4),(4,4),(5,4),(6,4)}

F={(1,5),(5,1),(2,4),(4,2),(3,3)}

and $E \cap F$={(6,5,4)}

$\therefore \ P(E) = \frac{11}{36}, P(F) = \frac{5}{36}, P(E \cap F) = \frac{2}{36}$

Hence the required probability=$P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{2/36}{5/36} = \frac{2}{5}$

13) P(at least one of A and B)=$P(A \cup B)$ 3

= P(A)+P(B)-$P(A \cup B)$

=P(A)+P(B)-P(A)P(B)

=P(A)+P(B)(1-P(A))

=P(A)+P(B).$P(A')$

= 1-$P(A')$+P(B)$P(A')$

=1-$P(A')$[1-P(B)]

=[1-$P(A')$]$P(B')$,which is true

14) Let B_1, B_2 and G_1, G_2 be first, second boy and first, second girl respectively. 3

∴ Sample space, S={(G_1, G_2), (G_1,B_2), (B_1, G_2), (B_1, B_2)}.

Let E: Both children are girls = {(G_1, G_2)}

F: Youngest child is a girl = {(G_1, G_2), (B_1, G_2)} and G: at least one is a girl ={(G_1, G_2), (G_1,B_2), (B_1, G_2)}.

∴ $E \cap F$={(G_1, G_2), $E \cap G$={G_1, G_2}.

∴ $P(E \cap F)=\frac{1}{4}$, $P(E \cap G) = \frac{1}{4}$,P(G)=$\frac{2}{4}$, P(G)=$\frac{3}{4}$.

(i) P(E/F) = $\frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{4}}{\frac{2}{4}} = \frac{1}{2}$.

(ii) P(E/G)= $\frac{P(E \cap G)}{P(G)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$.

Section-D

15) $\frac{1}{52}$ 4

16) $\frac{9}{17}$ 4

17) Let the events E_1 and E_2 be the events when lost card is a diamond and not a diamond respectively. 4

$P(E_1) = \frac{13}{52} = \frac{1}{4}$and

$P(E_2) = \frac{39}{52} = \frac{3}{4}$

Let A be the event:

"two cards drawn from the remaining pack are diamonds"

$P(A/E_1) = \frac{12 \times 11}{51 \times 50}$

$P(A/E_2) = \frac{13 \times 12}{51 \times 50}$

By Bayes' Theorem

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)}$$

$$= \frac{\left(\frac{1}{4}\right)\left(\frac{12 \times 11}{51 \times 50}\right)}{\left(\frac{1}{4}\right)\left(\frac{12 \times 11}{51 \times 50}\right)+\left(\frac{3}{4}\right)\left(\frac{13 \times 12}{51 \times 50}\right)}$$

$$= \frac{12 \times 11}{12 \times 11 + 3 \times 13 \times 12}$$

$$= \frac{132}{132+468} = \frac{132}{600} = \frac{11}{50}$$

Section-E

18) Let the events be: 6

E_1= Choosing 1st coin

E_2= Choosing 2nd coin

E_3=Choosing 3rd coin

A: Getting Heads

$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

$P(A/E_1) = \frac{40}{100}, P(A/E_2) = \frac{75}{100},$

$P(A/E_3) = \frac{1}{2}$

$P(E_1/A)$

$$= \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$$

$$= \frac{\frac{1}{3} \cdot \frac{40}{100}}{\frac{1}{3} \cdot \frac{40}{100} + \frac{1}{3} \cdot \frac{75}{100} + \frac{1}{3} \cdot \frac{1}{2}} = \frac{8}{33}$$

19) Let C_1, C_2, C_3, C_4 be the events that the lost card is of heart, spades, diamond or club respectively.

Obviously $P(C_1) = P(C_2) = P(C_3) = P(C_4)$

$= \frac{13}{52} = \frac{1}{4}$

Let S be the event of drawing two cards of heart from the remaining 51 cards.We wish to find $P\left(\frac{C_1}{S}\right)$

Now $P\left(\frac{C_1}{S}\right)$ is the probability of drawing two heart cards from 51 cards given that one heart card is lost

$= \frac{{}^{12}C_2}{{}^{51}C_2} = \frac{12 \times 11}{1 \times 2} \times \frac{1 \times 2}{51 \times 50} = \frac{22}{425}$

$P\left(\frac{S}{C_3}\right) = P\left(\frac{S}{C_3}\right) = P\left(\frac{S}{C_4}\right) = \frac{{}^{13}C_2}{{}^{51}C_2}$

$= \frac{13 \times 12}{1 \times 2} \times \frac{1 \times 2}{51 \times 50} = \frac{26}{425}$

By Bayes' Theorem

$$P\left(\frac{C_1}{S}\right) = \frac{P(C_1)\,.\,P\left(\frac{S}{C_1}\right)}{\sum P(C_1)\,.\,P\left(\frac{S}{C_1}\right)}$$

$$= \frac{\frac{1}{4} \times \frac{22}{425}}{\frac{1}{4} \times \frac{22}{425} + \frac{1}{4} \times \frac{26}{425} + \frac{1}{4} \times \frac{26}{425}}$$

$$= \frac{22}{22+26+26+26}$$

$$= \frac{11}{50}$$