QB365

Important Questions - Relations and Functions

12th Standard CBSE

Maths Reg.No. :

Time : 01:00:00 Hrs

Total Marks : 50

Section-A

1) If f(X)=X+7 and g(X)=X-7, $X \in R$, find fog(7). ? **1**

2) If the binary operation * on the set of integers Z is defined by a*b=a+3b^2 then find the value of 2*4. **1**

3) Let * be a binary operation on N given by a*b=HCF(a,b), $a, b \in N$. Write the value of 22*4. **1**

4) Let R be a relation in the set of natural numbers N defined by R={(a,b) $\in$ NXN;a **1**

5) Let $f: N \rightarrow N$ be defined by f(x)=3x. Show that f is not onto function. **1**

Section-B

6) Define Reflexive.Give one example. **2**

7) Define symmetric Relation.Give one example **2**

8) Define Transitive Relation. Give one example. **2**

9) Let f:$X \rightarrow Y$ be a function Define a relation R on X given be R=[(a,b) ; (f(b)] Show that R is an equivalence relation ? **2**

10) $f(x) = x^2, x \in R$ Find $\frac{f(1.1) - f(1)}{1.1 - 1}$ **2**

11) Draw graphs of function $f(x) = ax^2, x \in R$ **2**

Section-C

12) Are f and g both necessarily onto, if g of is onto? **3**

13) Show that subtraction and division are not binary operations on N. **3**

14) Show that $+ : R \times R \rightarrow R$ and $\times : R \times R \rightarrow R$ are commutative binary operations but $- : R \times R \rightarrow R$ and $\div : R \times R \rightarrow R$ are not commutative. **3**

Section-D

15) Let T be the set of all triangles in a plane with R a relation in T given by R={(T_1,T_2):T_1 is conguruent to T_2 and T_1 ,T_2 T}. Show that R is an equivalence relation. **4**

16) Show that the relation R defined by (a,b) R (c,d) $\Rightarrow$ a+d=b+c on the set N X N is an equivalence relation. **4**

17) Show that the relation S in the set R of real numbers, defined as S={(a,b): a,b $\in$ R and a $\leq$ b^3} is neither reflexive, nor symmetric nor transitive. **4**

Section-E

18) Show that the relation R in the Set A = {I, 2, 3, 4, 5} given by R = {(a, b) : Ia - b I is divisible by 2} is an equivalence relation. Write all the equivalence classes of R. **6**

19) Show that the function \(f:R[x\in R:1 defined by $f(x) = \frac{x}{1+|x|}, x \in R$ is one-one and onto function Hence find $f^{-1}(x)$ **6**

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Section-A

1) $fog = f(g(7)) = f(7-7) = f(0) = 0 + 7 = 7$ **1**

2) 2 * 4 = 2+3$(4)^2$ = 50 **1**

3) 22 * 4 = HCF(22,4) = 2 **1**

4) Given $R = \{(a, b) \in N \times N: a < b\}$.Not reflexive as for $(a, a) \in R, a < a$, not true **1**

5) **1**

$f: N \to N$defined by f(x) = 3x Let for $y \in N$(co-domain), threre exists, $x \in N$ of domain such that $f(x) = y \Rightarrow 3x = y \Rightarrow x = \frac{y}{3}$ which may not be a natural number.Hence, not onto.

Section-B

6) **2**

Reflexive Relation : A relation R on a set A is called reflexive relation if aRa for every $a \in A$; if (a,a) $\in$ R, for every $a \in A$

Example let

A=[1,2,3]

AxA =(1,1) (1,2)(1,3) (2,1) (2,2) (2,3) (3,1)(3,2) (3,3) $\in R$

Since (a,a) $\in R$ for every $a \in A$

7) **2**

Symmetric Relation : A relation R on a set A is called symmetric relation if aRb implies bRa, for every a,b $\in a$ i.,e if (a,b) $\in R$ $\Rightarrow$ (b,a) $\in R$ For every a,b $\in A$

Example

A=(1,2,3)

AxA =(1,2) (2,1) (1,1) (2,2) (3,3) (1,3) (2,3) (3,1) (3,2) $\in R$

since (a,b) $\in R$ (b,a) $\in R$ for every a,b $\in A$

Relation is said to be symmetric

8) **2**

A relation R on a non-empty set A is called a transitive relation if (a, b), (b, c) $\in R$ then (a, c) $\in R$, i.e., aRb, bRc implies aRc.

Thus a relation R on a non empty set A is said to be transitive if there exist a, b, c $\in A$ such that (a, b)(b, c) $\in R$ implies (a, c) . $\in R$

Example

Let A = (1,2, 3, 6)

R = (3,6)(6,1)(3,1)

., 3 R 6 and 6 R 1 $\Rightarrow$ (3,1) $\in R$

:. A is transitive.

9) (i) Let a $\in x$ then $f(a) = f(a), \Rightarrow (a, a) \Rightarrow R$ Relation is reflexive **2**

(ii) Let (a,b) $\in R$ then $f(a) = f(b) \Rightarrow f(b) = f(a), \Rightarrow (b, a) \in R$Relation is symetric

(iii) Let (a,b) (b,c) $\in R$ then $f(a) = f(b), f(b) = f(c) \Rightarrow f(a) = f(c)$ $(a, c) \in R$Relation is transitive.

All the three relation are satisfied the relation is equivalence.

10) $f(x) = x^2, x \in Ra$ **2**

$f(1.1) = (1.1)^2 z$ $= 1.21 f(1) = (1)^2 = 1 \frac{f(1.1) - f(1)}{1.1 - 1} = \frac{1.21}{1.1 - 1} = \frac{0.21}{0.1} = 2.1$

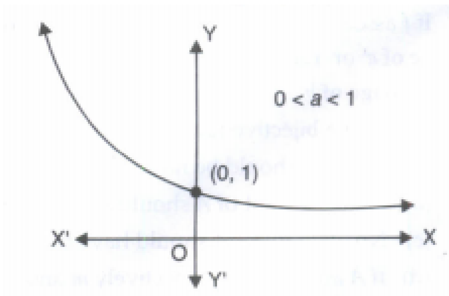
11) Case 1: **2**

$f(x) = a^x$

$0\, x < 0 = a^x > 1$

$x = 0 \Rightarrow a^x = 1$

$x > 0 \Rightarrow a^x < 1$



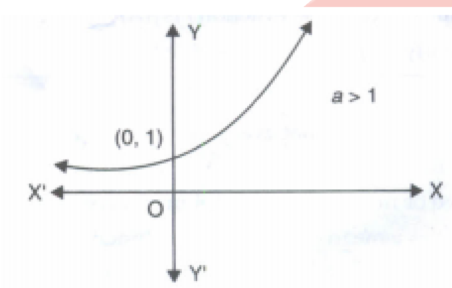
Case II

a>1

\(x<0=0

$x = 0 \Rightarrow a^x = 1$

$x > 0 \Rightarrow a^x < 1$



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Section-C

12) Consider $f: \{1, 2, 3, 4\} \rightarrow \{1, 2, 3, 4\}$ **3**

and $g: \{1, 2, 3, 4\} \rightarrow \{1, 2, 3\}$ defined by:

$f(1) = 1, f(2) = 2, f(3) = f(4) = 3$

$g(1) = 1, g(2) = 2, g(3) = g(4) = 3.$

Clearly g of is onto but f is not onto.

13) (i) $N \rightarrow N$ is given by: **3**

$(x, y) \rightarrow x - y$, which is not binary operation.

$$\left[\because Image \quad of \quad (4, 6) \quad under' - is \quad 4 - 6 = 2 \notin N \right]$$

(ii) $\div : N \rightarrow N;$ is given by:

$(x, y) \rightarrow x \div y$, which is not a binary operation.

$$\left[\because Image \quad of \quad (4, 6) \quad under' - is \quad 4 \div 6 = \frac{2}{3} \notin N \right]$$

14) (i) For all $a, b \in R$, 3

a+b=b+a and $a \times b = b \times a$.

Hence, '+' and ' $\times$ ' are commutative binary operations.

(i) For all $a, b \in R$,

$a - b \neq b - a$ $[For \quad ex. 4 - 5 \neq 5 - 4]$

and

$a \div b \neq b \div a$ $\left[For \quad ex. \frac{4}{5} \neq \frac{5}{4}\right]$

Hence, '-' and ' $\div$ ' are not commutative binary operations.

Section-D

15) Since R is reflexive, symmetric and transitive. Hence R is an equivalence relation. 4

16) R is an equivalence relation. 4

17) For reflexive: Not reflexive 4

For symmetric: Not symmetric

For transitive: true in both case

Hence, not transitive

Section-E

18) Given R = {(a, b) : Ia - b I is divisible by 2} 6

and A = {I,2,3,4,5}

R = {(I, I), (2, 2), (3,3), (4, 4), (5, 5) (1,3), (1, 5), (2, 4), (3, 5), (3, 1), (5,1), (4, 2), (5, 3)}

(i) $\forall a \in A, (a, a) \in R$

:. R is reflexive.

[As {(I, I), (2,2), (3, 3), (4,4) (5,5)} $\in R$

(ii) $\forall a \in A, (a, a) \in R$

R is symmetric

[As{(I, 3), (1, 5), (2,4), (3, 5) (3, 1), (5, 1), (4,2), (5, 3)} $\in R$]

(iii) $\forall (a, b)(b, c) \in R, (a, c) \in R$

R is transitive

[As{(I, 3), (3, 1) E R ~ (1, 1) E R and similarly others]

:. R is an equivalence relation.

Equivalence classes are

[1]= {I, 3,5}

and [2] = {2, 4}

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19) One-One : Let $x_1, x_2 \in R$

Such that $f(x_1)f(x_2)$

$\Rightarrow \frac{x}{1+|x_1|} = \frac{x_2}{1+|x_2|}$ Case (i) : If $x_1, x_2 > 0$ then

$\frac{x_1}{1+x_1} = \frac{x_2}{1+x_2}$

$\Rightarrow x_1 + x_1x_2 = x_2 + x_1x_2$

$\Rightarrow x_1, x_2 > 0$

$\frac{x_1}{1-x_1} = \frac{x_2}{1-x_2}$

$x_1 - x_1x_2 = x_2 - x_1x_2$

$\Rightarrow x_1 = x_2$

Case (iii) If $x_1 > 0, x_2 < 0 similar \quad for \quad x_1 < 0, x_2 < 0)$

$x_1 \neq x_2$

$\Rightarrow \frac{x_1}{1+x_1} \neq \frac{x_1}{1-x_1}$

$\Rightarrow f(x_1) \neq f(x_2)$

from (i),(ii),(iii) f is a one-one function

Onto : Let any

\(y\in [x\in R;-1

\((-1

such that y=f(x)

$\Rightarrow y = \frac{x}{1+x}$

$\Rightarrow y = \frac{x}{1 \pm x} \Rightarrow x = \frac{y}{1 \pm y}$

$As \quad y \neq -1, y \neq 1$

$x = \frac{y}{1 \pm y} \in R$

f is a onto function

$$f^{-1}(x) = \begin{cases} \frac{x}{1+x}, ifx < 0 \\ \frac{x}{1-x}, ifx \geq 0 \end{cases}$$