QB365

Important Questions - Three Dimensional Geometry

12th Standard CBSE

Maths Reg.No. : | | | | | | |

Time : 01:00:00 Hrs

Total Marks : 50

Section-A

1) Write the vector equation of a line whose Cartesian equation is $\frac{x+3}{2}=\frac{y-1}{4}=\frac{z+1}{5}$. **1**

2) Write the Cartesian equation of a line whose vector equation is $\vec{r}=(3\hat{i}+2\hat{j}-5\hat{k})+\lambda(-2\hat{i}+\hat{j}+3\hat{k})$. **1**

3) Find the angle between the planes $\vec{r}.(\hat{i}-2\hat{j}-2\hat{k})=1$ and $\vec{r}.(3\hat{i}-6\hat{j}+2\hat{k})=0$. **1**

4) Find the direction ratios of a line, normal to the plane 7x+y-2z=1. **1**

5) Find the coordinates of a point, where the line $\frac{x+2}{1}=\frac{y-5}{3}=\frac{z+1}{5}$ cuts yz-plane. **1**

Section-B

6) If a lines makes angle 60° and 45° with the positive directions of x-axis and z-axis respectively, then find the angle that it makes with the y-axis. **2**

7) If a line makes angle α,β,γ with the coordinates axis ,then find the value of cos $cos2\alpha+cos2\beta+cos2\gamma$ **2**

8) If the equation of a line is x=ay+b z=cy+d, then find direction ratios of the line and a point on the line. **2**

9) Find the distance between the point (5, ,4, - 6) and its image in xy-plane. **2**

10) Show that the line through the points (0, 3, 2), (3, 5, 6) is perpendicular of the line through the points (1, - 1, 2) and (3,4, - 2). **2**

11) Find the cartesian and vector equation of the line which passes through the point (- 2, 4, - 5) and parallel to the line given by $\frac{x+3}{3}=\frac{y-4}{5}=\frac{x-z}{-6}$ **2**

Section-C

12) Show that points: **3**

A(2,3,-4),B(1,-2,3) and C(3,8,-11) are collinear.

13) Find the angle between the pair of lines **3**

$\frac{x+3}{3}=\frac{y-1}{5}=\frac{z+3}{4}$ and

$\frac{x+1}{1}=\frac{y-4}{1}=\frac{z-5}{2}$.

14) Find the equation of the plane with intercept 2,3 and 4 on the x,y and z-axis respectively. **3**

Section-D

15) Find the distance of the point (1,-2,3) from the plane x-y+z=5 measured parallel to the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$ **4**

16) Find the shortest distance between the following two lines: $\vec{r}=(1+\lambda)\hat{i}+(2-\lambda)\hat{j}+(\lambda+1)\hat{k}$ **4**

$\vec{r}=(2\hat{i}-\hat{j}-\hat{k})+\mu(2\hat{i}+\hat{j}+2\hat{k})$

17) Find the equation of the perpendicular drawn from the point (1,-2,3) to the plane 2x-3y+4z+9=0. Also find the coordinates of the foot of the perpendicular. **4**

Section-E

18) Find the vector and cartesian forms of the equation of the plane passing through the point (1, 2, - 4) and parallel to the lines $\vec{r} = \left(\hat{i} + 2\hat{j} - 4\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$ and $\vec{r} = \left(\hat{i} - 3\hat{j} + 5\hat{k}\right) + \mu\left(\hat{i} + \hat{j} - \hat{k}\right)$ **6**

Also, find the distance of the point (9, -8, -10) from the plane thus obtained.

19) Find the distance of the point (2, 12, 5) from the point of intersection of the lines $\vec{r} = \left(2\hat{i} - 4\hat{j} + 2\hat{k}\right) + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$ and the plane . **6**

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Section-A

1) **1**

Position vector of point on line $\vec{r} = (2\lambda - 3)\hat{i} + (4\lambda + 1)\hat{j} + (5\lambda - 1)\hat{k}$ $\Rightarrow$ Line is $\vec{r} = (-3\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} + 4\hat{j} + 5\hat{k})$.

2) **1**

General point on line is $\vec{r} = (3 - 2\lambda)\hat{i} + (2 + \lambda)\hat{j} + (-5 + 3\lambda)\hat{k}$

$\Rightarrow 3 - 2\lambda = x, 2 + \lambda = y, -5 + 3\lambda = z$ $\Rightarrow$ Line is $\frac{x-3}{-2} = \frac{y-2}{1} = \frac{z+5}{3}$

3) $\theta = cos^{-1}\left(\frac{11}{21}\right)$ **1**

4) Coefficients of x,y,z are direction ratios of normal to the plane. Hence direction ratios are 7,1,-2. **1**

5) Point of intersection is (0,6+ 5,10-1), i.e.)0,11,9. **1**

Section-B

6) $\alpha = 60^\circ, \beta = ?, \gamma = 45^\circ$ **2**

Let α makes with x-axis, y makes with z-axis and β makes with y-axis

$\alpha = cos \quad 60^\circ, m = cos \quad \beta, y = cos 45^\circ$

$\alpha^2 + \beta^2 + \gamma^2 = 1$

$\Rightarrow cos^2 60^\circ + cos^2\beta + cos^2 45^\circ$

$\Rightarrow \frac{1}{4} + cos^2\beta + \frac{1}{2} = 1$

$\Rightarrow cos^2\beta = 1 - \frac{1}{4} - \frac{1}{2}$

$= \frac{4-1-2}{4}$

$= \frac{1}{4}$

$\Rightarrow cos \quad \beta = \pm\frac{1}{2}$

$\beta = \frac{\pi}{3} or \quad \frac{2\pi}{3}$

But angle between two lines in the interval $\left(0, \frac{\pi}{2}\right)$

Hence required angles is $\frac{\pi}{3}$

7) $I = cos\alpha, m = cos\beta, n = cos\gamma$ **2**

We know that

$i^2 + m^2 + n^2 = 1$

$\Rightarrow cos^2\alpha + cos^2\beta + cos^2\gamma = 1$

as we know that ,cos 2x=2 cos^2 x-1

$\Rightarrow \left(\frac{1+cos2\alpha}{2}\right) + \left(\frac{1+cos2\beta}{2}\right)\left(\frac{1+cos2\gamma}{2}\right) = 1$

$\Rightarrow 1 + cos2\alpha + 1 + cos2\beta + 1 + cos2\gamma = 2$

$\Rightarrow cos \quad 2\alpha + cos2\beta + 1 + cos2\gamma = 2$

$cos \quad 2\alpha + cos2\beta + cos \quad 2\gamma = -1$

8) Given equation **2**

x = ay + b, Z = cy + d

$\frac{x-b}{a} = y, \frac{x-d}{c} = y$

$\Rightarrow \frac{x-b}{z} = \frac{y-0}{1} = \frac{z-d}{c}$

Direction ratios are (a, 1, c) and a point on the given line is (b, 0, d).

9) Let A be the point (5, 4, - 6) **2**

Image A' be the point (5, 4, - 6)

∴ A'(5, 4, - 6)

Distance between AA'

=$\sqrt{(5-5)^2 + (4-4)^2 + (6+6)^2}$

$= \sqrt{0 + 0 + 12^2}$

=123 units

10) Let A(O, 3, 2), B(3, 5, 6) **2**

Direction ratios of AB(a, b, c) are (3 - 0), (5 - 3), (6 - 2)

(a_1, b_1, c_1) = (3, 2, 4)

Let C(1, - 1, 2), 0(3, 4, - 2)

Direction ratios of CD (a_2, b_2, c_2) are (3 - 1), (4 + 1) (-2,2) (a_2, b_2, c_2) are (2,56,-4)

When two lines are perpendicular if

$a_1, a_2 + b_1, b_2, +c_1, c_2 = 0$

$\Rightarrow 3 \times 2 + 2 \times 5 + 4 \times -4 = 0$

$\Rightarrow 6 + 10 - 16 = 0$

$\Rightarrow 16 - 16 = 0$

$\therefore AB \perp to \quad CD$

11) Here $\frac{x+3}{3} = \frac{y-4}{5} = \frac{x-z}{-6}$ **2**

is same as $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z-8}{-6}$

Cartesian equation of the line

$\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{-6}$

Vector equation of the line is

$\vec{r} = (-2\vec{i} + 4\vec{j} - 5\vec{k}) + \lambda(3\vec{i} + 5\vec{j} + 6\vec{k})$

Section-C

12) The direction-ratios of the line joining A and B are **3**

<1,-2,-2-3,3-(-4)> i.e. <-1,-5,7>.

The direction ratios of the joining B and C are:

<3-1,8-(-2),-11-3> i.e.<2,10,-14>

Clearly the above direction-ratios are proportional

=> AB is parallel to BC.

But B is the common point.

Hence, the points A, B and C are collinear.

13) The direction-ratios of the first line are <3,5,4> **3**

The direction-ratios of the second line are<1,1,2>.

If θ be the required, angle, then:

$$cos\theta = \left|\frac{(3)(1)+(5)(1)+(4)(2)}{\sqrt{9+25+16}\sqrt{1+1+4}}\right|$$

$$= \left|\frac{3+5+8}{\sqrt{50}\sqrt{6}}\right| = \frac{16}{5\sqrt{2}\sqrt{6}} = \frac{8\sqrt{3}}{15}$$

$$Hence, \theta - \cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)$$

14) The equation of the plane is **3**

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

$$\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1 \Rightarrow 6x + 4y + 3z = 12$$

Section-D

15) Let A(1,-2,3) be the given point. **4**

The given plane is x-y+z=5 .....(1)

and the given line is $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$ $\quad ....(2)$

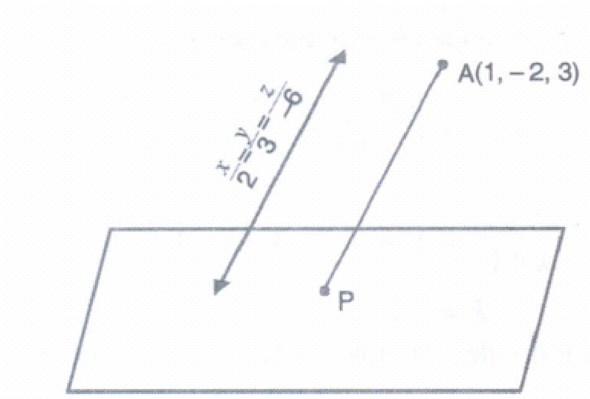
The equations of the line through A and parallel to (2) are:

$\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{-6}$

Any point on it is (2k+1,3k-2,-6k+3)

This is P if it lies on (1) if (2k+)-(3k-2)+(-6k+3)=5

if -7k+6=5 if -7k=-1 if k=1/7



... The point P is

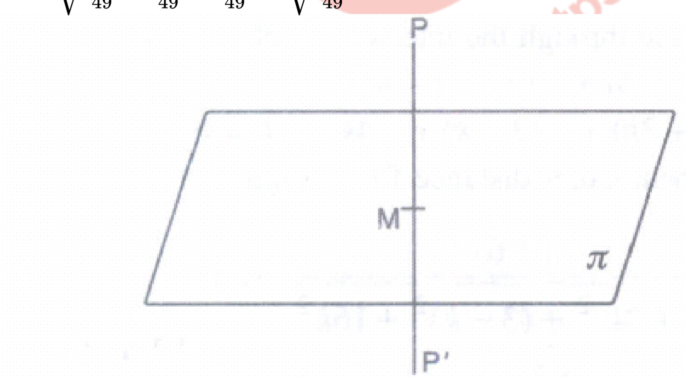
$\left(\frac{2}{7}+1, \frac{3}{7}-2, -\frac{6}{7}+3\right)$

$i.e.\quad \left(\frac{9}{7}, \frac{-11}{7}, \frac{15}{7}\right)$

:. Reqd. distance = IAPI

$\sqrt{\left(\frac{9}{7}-1\right)^2+\left(\frac{-11}{7}+2\right)^2+\left(\frac{15}{7}-3\right)^2}$

$=\sqrt{\frac{4}{49}+\frac{9}{49}+\frac{36}{49}}=\sqrt{\frac{49}{49}}=1\quad unit.$



16) Shortest distance=$\left|\frac{(\acute{i}-3\acute{j}-2\acute{k}).(-3\acute{i}+3\acute{k})}{3\sqrt{2}}\right|=\left|\frac{-3-6}{3\sqrt{2}}\right|=\frac{3}{\sqrt{2}}=\frac{3\sqrt{2}}{2}$ units **4**

17) **4**

$2(2\lambda+1)-3(-3\lambda-2)+4(4\lambda+3)+9=0 \Rightarrow 29\lambda=-29 \Rightarrow \lambda=-1$Substituting in (i), we get foot of the perpendicular as B(-1,1,-1).

Section-E

18) Let equation of plane through (1, 2, -4) be a(x-1) + b(y-2) + c(z+4) = 0. **6**

The plane is parallel to the given lines $\therefore$ 2a + 3b + 6c = 0; a + b - c = 0

Solving: $\frac{a}{-9} = \frac{b}{8} = \frac{c}{-1} = k\,(say)$

$\therefore$ a = -9k, b = 8k, c = -k

From (i), -9k(x-1) + 8k(y-2) - k(z+4) = 0

$\therefore$ Equation of plane in cartesian form is 9x - 8y + z+11 = 0

Vector form of plane is: $\Rightarrow \vec{r}.\left(9\hat{i} - 8\hat{j} + \hat{k}\right) = -11$

Distance of (9, -8, -10) from the plane = $\left|\frac{9.9-8(-8)+1(-10)+11}{\sqrt{81+64+1}}\right| = \sqrt{146}$

19) **6**

Any point on the line $\vec{r} = \left(2\hat{i} - 4\hat{j} + 2\hat{k}\right) + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$ is

$(2+3\lambda)\,\hat{i} + (-4+4\lambda)\,\hat{j} + (2+2\lambda)\,\hat{k}$. For the line to intersect the plane, the above point must satisfy the equation of plane, for some value of λ.

$\therefore \quad \left\{(2+3\lambda)\,\hat{i} + (-4+4\lambda)\,\hat{j} + (3+2\lambda)\,\hat{k}\right\}.\left(\hat{i} - 2\hat{j} + \hat{k}\right) = 0$

$\Rightarrow \quad 2 + 3\lambda + 8 - 8\lambda + 2 + 2\lambda = 0 \Rightarrow \lambda = 4$

$\therefore$ The point of intersection is $14\hat{i} + 12\hat{j} + 10\hat{k}$. Required distance = $\sqrt{12^2 + 0^2 + 5^2}$ = 13 units.