QB365

Important Questions - Vector Algebra

12th Standard CBSE

Maths Reg.No. :

Time : 01:00:00 Hrs

Total Marks : 50

Section-A

1) For what value of λ are the vectors $\vec{a}=2\vec{i}+\lambda\vec{j}+\vec{k}$ and $\vec{b}=\vec{i}-2\vec{j}+3\vec{k}$ perpendicular to each other? 1

2) If p(1,5,4) and Q(4,1,-2) find the direction ratios of $\overrightarrow{PQ}$ 1

3) For what value of p, is $(\hat{i}+\hat{j}+\hat{k})$p a unit vector? 1

4) Find λ when the projection of $\vec{a}=\lambda\hat{i}+\hat{j}+4\hat{k} \quad on \quad \vec{b}=2\hat{i}+6\hat{j}+3\hat{k}$ is 4 units. 1

5) If $\vec{a}=x\hat{i}+2\hat{j}-z\hat{k} \quad and \quad \vec{b}=3\hat{i}-y\hat{j}+\hat{k}$ are two equal vectors then write the value of x+y+z. 1

Section-B

6) If a unit vector $\vec{a}$ makes angle $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and an acute angle θ with $\hat{k}$ then find the value of θ and hence component of $\vec{a}$. 2

7) Find the unit vector in the direction of $\vec{a}+\vec{b}$if $\vec{a}=2\hat{i}+\hat{j}+3\hat{k}$, and $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$ 2

8) Find the position vector of c which divides the line segment joining A & B whose position vectors are $3\vec{a}+\vec{b}$and $\vec{a}-3\vec{b}$ internally in the ratio 2:3. 2

9) Find the direction cosines of the vector joining the points A(l, 2, - 3) and B(- 1, - 2, 1) directed from B to A. 2

10) Find $\left|\vec{a}\times\vec{b}\right|$ if $|\vec{a}|=10, \left|\vec{b}\right|=2$ and $\vec{a}.\vec{b}=12$. 2

11) Find $|\vec{a}|$ and $\left|\vec{b}\right|$ if $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})=8$ and $|\vec{a}|=8\left|\vec{b}\right|$. 2

Section-C

12) Write two Different vectors having the same direction. 3

13) Find the unit vector in the direction of the vector: $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$ 3

14) Find out the vector in the direction of vector $\overrightarrow{PQ}$ where P and Q are the points(1, 2, 3) and (4, 5, 6) respectively. 3

15) Find a vecto in a direction of vector $5\hat{i}-\hat{j}-2\hat{k}$,which has magnitude 8 units. 3

16) Find the direction-consines of the vector $\hat{i}+2\hat{j}+3\hat{k}$. 3

17) Find the direction -consines of the vector joining the points A(1, 2, -3) and (-1, -2, 1) directed from A to B. 3

Section-D

18) If $\vec{a}=\hat{i}+\hat{j}+\hat{k} \quad and \quad \vec{b}=\hat{j}-\hat{k}$ find a vector $\vec{c}$ such that $\vec{a}\times\vec{c}=\vec{b} \quad and \quad \vec{a}.\vec{c}=3$ 4

19) The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with the unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k} \quad and \quad \lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ 4

20) If $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=4\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{c}=\hat{i}-2\hat{j}+\hat{k}$ find a vector of a magnitude 6 units which is parallel to the vector $2\vec{a}-\vec{b}+3\vec{c}$ 4

21) If vectors $\vec{a}=2\hat{i}+2\hat{j}+3\hat{k}, \vec{b}=-\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}$ are such that $\vec{a}+\lambda\vec{b}$ is perpendicullar to $\vec{c}$, then find the value of λ **4**

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Section-A

1) $\lambda=\frac{5}{2}$ **1**

2) 3,-4 and -6 **1**

3) P=$\pm\frac{1}{\sqrt{3}}$ **1**

4) λ=5 **1**

5) 0 **1**

Section-B

6) **2**

(i) Here cos2 $\frac{\pi}{3}$ + cos2 $\frac{\pi}{4}$ + ocs2 θ= 1 $\Rightarrow \left(\frac{1}{2}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2+$ ocs2 θ= 1 $\Rightarrow \frac{1}{4}+\frac{1}{2}+$ ocs2 θ= 1$\Rightarrow ocs2\frac{1}{4}$

$\Rightarrow ocs2\frac{1}{4}$ (Take+ve sign) θ=$\frac{\pi}{3}$ (ii) Component of $\vec{a}$ are: $cos\frac{\pi}{3}$, $cos\frac{\pi}{4}, cos\frac{\pi}{3}$ i.e.,$\frac{1}{2}$, $\frac{1}{\sqrt{2}}$, $\frac{1}{2}$.

7) $\vec{c}=\vec{a}+\vec{b}$ **2**

$=(2\hat{i}+\hat{j}+3\hat{k})+(\hat{i}+2\hat{j}-\hat{k})$

$=3\hat{i}+3\hat{j}+2\hat{k}$

$\hat{c}=\frac{\vec{c}}{|c|}$

$=\frac{2i+3j+2k}{\sqrt{9+9+4}}$

$=\frac{3}{\sqrt{22}}\hat{i}+\frac{3}{\sqrt{22}}\hat{j}+\frac{2}{\sqrt{22}}\hat{k}$

8) Positive vector of c$=\frac{2(\vec{a}-3\vec{b})+3(2\vec{a}+\vec{b})}{2+3}$ **2**

$=\frac{2\vec{a}-6\vec{b}+6\vec{a}+3\vec{b}}{5}$

Position vector of c=$\frac{8\vec{a}-3\vec{b}}{5}$

9) $\overrightarrow{BA}$=Position vector of A- Position vector of B **2**

$\overrightarrow{BA}=(\hat{i}+2\hat{j}-3\hat{k})-(-\hat{i}-2\hat{j}+\hat{k})$

$\overrightarrow{BA}=2\hat{i}+4\hat{j}-\hat{k}$

$\left|\overrightarrow{BA}\right|=\sqrt{2^2+4^2+(-4)^2}$

$=\sqrt{4+16+16}$

$=\sqrt{36}=6$

Direction cosines of the vector $\overrightarrow{BA}$ are$\left(\frac{2}{6},\frac{4}{6},-\frac{4}{6}\right)$

$=\left(\frac{1}{3},\frac{2}{3},-\frac{2}{3}\right)$

10) $\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} = \frac{12}{10\times 2} = \frac{3}{5}$ **2**

$\cos\theta = \frac{3}{5}$

$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{9}{25}}$

$\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$

$\Rightarrow |\vec{a}\times\vec{b}| = |\vec{a}|\,|\vec{b}|\sin\theta$

$\Rightarrow |\vec{a}\times\vec{b}| = 10\times 2\times\frac{4}{5} = 16$

11) $(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = \vec{a}.\vec{a}+\vec{a}.\vec{b}-\vec{b}.\vec{a}-\vec{b}.\vec{b}$ **2**

$= |\vec{a}|^2 - |\vec{b}|^2$

$(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = (8|\vec{b}|)^2 - |\vec{b}|^2$

$= 64(|\vec{b}|)^2 - |\vec{b}|^2$

$= 63|\vec{b}|^2$

$(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = 8$

$\Rightarrow 63|\vec{b}|^2 = 8$

$\Rightarrow |\vec{b}| = \sqrt{\frac{8}{63}} = \frac{2\sqrt{2}}{3\sqrt{7}}$

$\therefore |\vec{a}| = \frac{8\times 2\sqrt{7}}{3\sqrt{7}} = \frac{16\sqrt{2}}{3\sqrt{7}}$

Section-C

12) **3**

Let $\vec{a} = \hat{i}+\hat{i}+\hat{k}$ $\vec{b} = 2\hat{i}+2\hat{i}+2\hat{k}$ Direction-cosines of $\vec{a}$ are $< \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} >$ Direction-cosines of $\vec{b}$ are $< \frac{2}{\sqrt{12}}, \frac{2}{\sqrt{12}}, \frac{2}{\sqrt{12}} >$ i.e. $< \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} >$ hence $\vec{a}\neq\vec{b}$ *but* $\vec{a} = \vec{b}$ have same direction.

13) **3**

We have:

$\vec{a} = \hat{i}+\hat{j}+2\hat{k}$ $\therefore$ $|\vec{a}| = \sqrt{(1)^2+(1)^2+(2)^2}$ $= \sqrt{1+1+4} = \sqrt{6}$ $\therefore$ unit vector in the direction of vector $\vec{a}$ $= \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i}+\hat{j}+2\hat{k}}{\sqrt{6}}$ $= \frac{1}{\sqrt{6}}\hat{i}+\frac{1}{\sqrt{6}}\hat{j}+\frac{2}{\sqrt{6}}\hat{k}$

14) **3**

$\overrightarrow{PQ}$ = P>V. of Q - P>V of P $= \left(4\hat{i}+5\hat{j}+6\hat{k}\right) - \left(\hat{i}+2\hat{j}+3\hat{k}\right) = \left(3\hat{i}+3\hat{j}+3\hat{k}\right)$

$\therefore |PQ| = \sqrt{3^2+3^2+3^2}$ $= \sqrt{9+9+9} = \sqrt{27} = 3\sqrt{3}$ $\therefore$ Unit vector in the direction of $\overrightarrow{PQ}$ $= \frac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|}$ $= \frac{3\hat{i}+3\hat{j}+3\hat{k}}{3\sqrt{3}}$ $= \frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\frac{2}{\sqrt{3}}\hat{k}$

15) **3**

The given vector is:

$\vec{a} = 5\hat{i} - \hat{j} - 2\hat{k} \;\therefore\; |\vec{a}| = \sqrt{5^2 + (-2)^2 + 2^2} \quad = \sqrt{25+1+4} = \sqrt{30} \;\therefore$ unit vector of given vector $\vec{a}$ $\quad = \frac{\vec{a}}{a} = \frac{1}{\sqrt{30}}\left(5\hat{i} - \hat{j} + 2\hat{k}\right) \;\therefore$ Vector of magnitude 8 in the direction of division of vector $\vec{a}$

$= 8\frac{\vec{a}}{a} = 8.\frac{1}{\sqrt{30}}\left(5\hat{i} - \hat{j} + 2\hat{k}\right) \;= \frac{40}{\sqrt{30}}\hat{i} - \frac{8}{\sqrt{30}}\hat{j} + \frac{16}{\sqrt{30}}\hat{k}$

16) **3**

We know that direction-consines of the vector $x\hat{i} + y\hat{j} + z\hat{k} \;< \frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{x^2+y^2+z^2}}, \frac{z}{\sqrt{x^2+y^2+z^2}} >$

Here $\vec{a}$ = $\hat{i} + 2\hat{j} + 3\hat{k} \;\therefore$ x = 1, y = 2, z = 3 $\therefore$ Direction consines of $\vec{a}$ are:

$< \frac{1}{\sqrt{1+4+9}}, \frac{2}{\sqrt{1+4+9}}, \frac{3}{\sqrt{1+4+9}} >$ i.e.$< \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} >$

17) **3**

Vector joining the point A and B = $\overrightarrow{AB}$ = P.V. of B - P.V of A

= $\left(-\hat{i} - 2\hat{j} + \hat{k}\right) - \left(\hat{i} + 2\hat{j} - 3\hat{k}\right)$ = $2\hat{i} - 4\hat{j} + 4\hat{k} \;\therefore$ Direction-cosines of $\overrightarrow{AB}$ are:

$< \frac{-2}{\sqrt{4+16+16}}, \frac{-4}{\sqrt{4+16+16}}, \frac{4}{\sqrt{4+16+16}} >$ i.e. $< \frac{-2}{6}, \frac{-4}{6}, \frac{4}{6} >$ $\quad i.e < \frac{-1}{3}, \frac{-2}{3}, \frac{2}{3} >$

Section-D

18) $\vec{c} = \frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$ **4**

19) **4**

Let $\hat{a} = \hat{i} + \hat{j} + \hat{k}$ and $\hat{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ now the unit vector along $\hat{b} + \hat{c}$

$\hat{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$

$= \frac{(\lambda+2)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(\lambda+2)^2+36+4}}$ By the question $(\hat{i} + \hat{j} + \hat{k}).= \frac{(\lambda+2)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(\lambda+2)^2+40}}$ =1 $\quad = \frac{1}{\sqrt{(\lambda+2)^2+40}}(\lambda+2+6) \quad = \quad 6$

$\Rightarrow \lambda + 6 = \sqrt{(\lambda+2)^2 + 40}$

Squaring, $\lambda^2 + 12\lambda + 36 \quad = \lambda^2 + 4\lambda + 4 + 40$

$\Rightarrow 12\lambda + 36 = 4\lambda + 44$

$\Rightarrow 8\lambda = 8$

$Hence \quad \lambda = 1$

20) $2\vec{i} - 4\vec{j} + 4\vec{k}$ **4**

21) λ=8 **4**