

Time : 02:00:00 Hrs

Total Marks : 100

Section-A

- 1) If $f(x)=x+7$ and $g(x)=x-7, x \in R$, find $f \circ g(7)$.? 1
- 2) If the binary operation $*$ on the set of integers Z is defined by $a*b=a+3b^2$ then find the value of $2*4$. 1
- 3) Let $*$ be a binary operation, on the set of all non-zero real numbers given by $a*b=\frac{ab}{5}$ for all $a, b \in R - \{0\}$ Find the value of x , given that $2*(x*5)=10$ 1
- 4) Using principal value, evaluate the following: $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ 1
- 5) Solve for X , $\tan^{-1}\frac{1-X}{1+X} = \frac{1}{2}\tan^{-1}X, X > 0$. 1
- 6) Write the principal values of the following: $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ 1
- 7) Write the value of $x+y+z$ from the following equation : $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$ 1
- 8) If A is a square matrix of order 3 and $|3A|=k|A|$, then write the value of k . 1
- 9) If $A = \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$, then write A^{-1} 1
- 10) If A square matrix of order 3 such that $|\text{adj}A|=225$, find $|A|$. 1

Section-B

- 11) Define Reflexive. Give one example. 2
- 12) Write in the simplest form : $\tan^{-1}\left[\frac{\cos x}{1+\sin x}\right], x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ 2
- 13) If $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} -1 & 4 \\ 0 & 2 \end{bmatrix}$, show that $AB \neq BA$. 2
- 14) If $A = \begin{bmatrix} 0 & x & -4 \\ -2 & 0 & -1 \\ y & -1 & 0 \end{bmatrix}$ is skew symmetric matrix, find the values of x and y . 2
- 15) Prove the following by the principle of mathematical induction : 2
if $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ for every positive integer n .
- 16) If $A = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 1 & 0 \\ 3 & 3 & 2 \end{bmatrix}$, find $M_{12} \times M_{21} + C_{21} \times C_{12}$ 2
when M_{ij} called minor and C_{ij} called co-factors of A .

Section-C

- 17) Show that the relation R in the set Z of integers given by: 3
 $R = \{(a, b) : 2 \text{ divides } a-b\}$ is an equivalence relation
- 18) Let $*$ be the binary operation on the set $\{1, 2, 3, 4, 5\}$ defined by: 3
 $a*b = \text{H.C.F. of } a \text{ and } b$.
Is the operation $*$ same as the operation $*$ defined in above? Justify your answer.
- 19) Show that : 3
 $\sin^{-1}(2x\sqrt{1-x^2}) = 2\cos^{-1}x, \frac{1}{\sqrt{2}} \leq x \leq 1$.
- 20) Find the value of $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$ 3
- 21) Solve the equation for x, y, z and t , if $2 \begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3 \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3 \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$. 3
- 22) Find the minor of the element '6' in the determinant $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$. 3

Section-D

- 23) Let Z be the set of all integers and R be the relation on Z defined as $R = \{(a,b): a,b \in Z, \text{ and } (a-b) \text{ is divisible by } 5\}$. Prove that R is an equivalence relation. 4
- 24) Consider the binary operation * on the set $\{1,2,3,4,5\}$ defined by $a*b = \min\{a,b\}$. Write the operation table of the operation *. 4
- 25) A binary operation on the set $\{0,1,2,3,4,5\}$ is defined as: $a * b = \begin{cases} a + b, & \text{if } a + b < 6 \\ a + b - 6, & \text{if } a + b \geq 6 \end{cases}$ Show that zero is the identity for this operation and each element a. of the set is invertible with 6-a, being the inverse of a. 4
- 26) Let $Y = \{n^2: n \in N\} \subset N$. Consider $f: N \rightarrow Y$ as $f(n) = n^2$ Show that f is invertible. Find the inverse of f. 4
- 27) Let * be a binary operation defined on Q. find which of the binary operations are associative. (i) $a*b = a-b$ (ii) $a*b = \frac{ab}{4}$ (iii) $a*b = a-b+ab$ (iv) $a*b = ab^2$ 4
- 28) Let $Y = \{n^2, n \in N\} \subset N$ $N \rightarrow Y$ $f(n) = n^2$ Consider $f: N \rightarrow Y$ as $f(n) = n^2$. Show that f is invertible. Also, find the inverse of 4

Section-E

- 29) Consider $f: R_+ \rightarrow [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$ Show that f is invertible find $f^{-1}(x)$ where R_+ is the set of all non-negative real numbers. 6
- 30) Let $A = R \times R$ and * be a binary operation on A defined by $(a,b)*(c,d) = (a+c, b+c)$ Show that * is commutative and associative. Find the identity element for * on A. Also find the inverse of every element $(a, b) \in A^*$. 6
- 31) Determine whether the operation * define below on Q is binary operation or not. 6
 $a*b = ab + 1$
 If yes, check the commutative and the associative properties. Also check the existence of identity element and the inverse of all elements is Q.
- 32) If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$ 6
- 33) Using properties of determinants, show that triangle ABC is isosceles if: $\begin{vmatrix} 1 & 1 & 1 \\ 1 + \cos A & 1 + \cos B & 1 + \cos C \\ \cos^2 A + \cos A & \cos^2 B + \cos B & \cos^2 C + \cos C \end{vmatrix} = 0$ 6
- 34) If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $(AB)^{-1}$. 6

Section-A

- 1) $f \circ g = f(g(7)) = f(7-7) = f(0) = 0 + 7 = 7$ 1
- 2) $2 * 4 = 2 + 3(4) = 14$ 1
- 3) $\Rightarrow 2 * (x * 5) = 10 \Rightarrow 2 * \left(\frac{5x}{5}\right) = 10 \Rightarrow 2 * x = 10 \Rightarrow \frac{2x}{5} = 10 \Rightarrow x = \frac{50}{2} = 25$ 1
- 4) π 1
- 5) $\frac{1}{\sqrt{3}}$ 1
- 6) $\frac{5\pi}{6}$ 1
- 7) $x + y + z = 9, x + z = 5, y + z = 7 \Rightarrow z = 3, x = 2, y = 4; x - y + z = 2 - 4 + 3 = 1$ 1
- 8) 27 1
- 9) $A^{-1} = \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$ **Alternative Method:** $A = \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix} A^{-1} = \frac{1}{|A|} (adj A) |A| = \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix} = 21 - 20 = 1 adj A = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$ 1
- 10) $|A^{-1}| = \pm 15$ **Alternative Method:** $|adj A| = |A|^{n-1}$, where n is the order of the matrix. $|A|_2 = 15 \Rightarrow \pm 15 \Rightarrow |A^{-1}| = \pm 15$ 1

Section-B

- 11) Reflexive Relation : A relation R on a set A is called reflexive relation if aRa for every $a \in A$; if $(a,a) \in R$, for every $a \in A$ 2
 Example let
 $A = \{1,2,3\}$
 $A \times A = \{(1,1) (1,2)(1,3) (2,1) (2,2) (2,3) (3,1)(3,2) (3,3) \in R$
 Since $(a,a) \in R$ for every $a \in A$

12)

2

$$\tan^{-1} \left[\frac{\cos \frac{x}{2}}{1 + \sin \frac{x}{2}} \right] \because \begin{cases} \cos x = \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} \\ \text{and } 1 + \sin x = \left(\cos \frac{x}{2} + \sin \frac{x}{2} \right)^2 \end{cases}$$

$$= \tan^{-1} \left[\frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{\left(\cos \frac{x}{2} + \sin \frac{x}{2} \right)^2} \right]$$

$$= \tan^{-1} \left[\frac{\left(\cos \frac{x}{2} + \sin \frac{x}{2} \right) \left(\cos \frac{x}{2} - \sin \frac{x}{2} \right)}{\left(\cos \frac{x}{2} + \sin \frac{x}{2} \right)^2} \right]$$

$$= \tan^{-1} \left[\frac{\cos \frac{x}{2} - \sin \frac{x}{2}}{\cos \frac{x}{2} + \sin \frac{x}{2}} \right] \text{ Divide by } \cos \frac{x}{2}, \text{ we get}$$

$$= \tan^{-1} \left[\frac{1 - \tan \frac{x}{2}}{1 + \tan \frac{x}{2}} \right]$$

$$= \tan^{-1} \left[\tan \left(\frac{\pi}{4} - \frac{x}{2} \right) \right] = \frac{\pi}{4} - \frac{x}{2}$$

13) We have,

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 4 \\ 0 & 2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} -1 & 4 \\ 0 & 2 \end{bmatrix}$$

$$AB = \begin{bmatrix} -2+0 & 8+2 \\ -3+0 & 12+8 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 10 \\ -3 & 20 \end{bmatrix}$$

and

$$BA = \begin{bmatrix} -1 & 4 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 15 \\ 6 & 8 \end{bmatrix}$$

$$\therefore AB \neq BA$$

14)

2

$$A = \begin{bmatrix} 0 & x & -4 \\ -2 & 0 & -1 \\ y & -1 & 0 \end{bmatrix}$$

$$A' = \begin{bmatrix} 0 & -2 & y \\ x & 3 & -1 \\ -4 & -1 & 0 \end{bmatrix}$$

For skew symmetric

$$A = -A'$$

$$\Rightarrow \begin{bmatrix} 0 & x & -4 \\ -2 & 0 & -1 \\ y & -1 & 0 \end{bmatrix} = - \begin{bmatrix} 0 & -2 & y \\ x & 3 & -1 \\ -4 & -1 & 0 \end{bmatrix}$$

$$x=2, y=4$$



15) We shall prove the result by mathematical induction on n.

2

Step 1 : When n = 1, by the definition or integral powers of a matrix, we have

$$A^1 = \begin{bmatrix} 1+2(1) & -4n \\ n & 1-2(1) \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

So, the result is true for n = 1.

Step 2 : Let the result be true for n = m. Then,

$$A^m = \begin{bmatrix} 1+2m & -4m \\ m & 1-2m \end{bmatrix}$$

Now, we will show that the result is true for n = m + 1, i.e.,

$$A^{m+1} = \begin{bmatrix} 1+2(m+1) & -4(m+1) \\ (m+1) & 1-2(m+1) \end{bmatrix}$$

By the definition of integral powers of a square matrix, we have

$$A^{m+1} = A^m \cdot A$$

$$\Rightarrow A^{m+1} = \begin{bmatrix} 1+2m & -4m \\ m & 1-2m \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

[by supposition (i)]

$$\Rightarrow A^{m+1} = \begin{bmatrix} 3+6m-4m & -4-8m+4m \\ 3m+1-2m & -4m-4+2m \end{bmatrix}$$

$$= \begin{bmatrix} 1+2(m+1) & -4(m+1) \\ (m+1) & 1-2(m+1) \end{bmatrix}$$

This shows that the result is true for n = m + 1, whenever it is true for n = m.

Hence, by the principle of mathematical induction, the result is true for any positive integer n.

16)

2

We have,

$$A = \begin{vmatrix} 2 & 3 & -1 \\ 4 & 1 & 0 \\ 3 & 3 & 2 \end{vmatrix}$$

$$M_{12} \begin{vmatrix} 4 & 0 \\ 3 & 2 \end{vmatrix} = 8 - 0 = 8M_{21} \begin{vmatrix} 3 & -1 \\ 3 & 2 \end{vmatrix} = 6 + 3 = 9C_{21} = - \begin{vmatrix} 3 & -1 \\ 3 & 2 \end{vmatrix} = -(6+3) = -9C_{12} = - \begin{vmatrix} 4 & 0 \\ 3 & 2 \end{vmatrix} = -(8-0) = -8M_{12} \times M_{21} + C_{21} \times C_{12} \times (9) + (-9)(-8) = 72 + 72 = 1.$$

Section-C

17) Here R = {(a,b):divides a-b}.

3

R is reflexive [$\because (a, a) \in R$ as 2 divides a-a=0]

R is symmetric [$\because (a, b) \in R \Rightarrow 2$ divides a-b $\Rightarrow 2$ divides b-a $\Rightarrow (b, a) \in R$]

R is transitive [$\because (a, b) \in R$ and $(b, c) \in R \Rightarrow 2$ divides a-b and 2 divides b-c $\Rightarrow 2$ divides (a-b)+(b-c) i.e (a-c) $\Rightarrow (a, c) \in R$]

Hence, R is an equivalence relation.

18) We have a*b=H.C.F. of a and b.

3

The composition table is as below:

1	2	3	4	5
1	1	1	1	1
2	1	2	1	2
3	1	1	3	1
4	1	2	1	4
5	1	1	1	5

19) Let $\cos^{-1} x = \theta$ so that $x = \cos \theta$

3

Since $\sin 2\theta = 2 \sin \theta \cos \theta$

$$\therefore \sin 2\theta = 2\sqrt{\cos^2 \theta} \cos \theta = 2\sqrt{1-x^2} \cdot x = 2x\sqrt{1-x^2} \Rightarrow 2\theta = \sin^{-1}(2x\sqrt{1-x^2})$$

Hence, $2\cos^{-1} x = \sin^{-1}(2x\sqrt{1-x^2})$

20)

3

$$\sin^{-1} \left(\sin \frac{3\pi}{5} \right) = \sin^{-1} \left(\sin \left(\pi - \frac{2\pi}{5} \right) \right) = \sin^{-1} \left(\sin \frac{2\pi}{5} \right) \left[\because \frac{2\pi}{5} \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right) \right] = \frac{2\pi}{5}$$

21)

3

$$\text{We have: } 2 \begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3 \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3 \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x & 2z \\ 2y & 2t \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x+3 & 2z-3 \\ 2y & 2t+6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

Equating corresponding elements:

$$2x+3=9 \Rightarrow 2x=9-3=6 \Rightarrow x=3$$

$$2y=12 \Rightarrow y=6$$

$$2z-3=15 \Rightarrow 2z=3+15=18 \Rightarrow z=9$$

$$\text{and } 2t+6=18 \Rightarrow 2t=18-6=12 \Rightarrow t=6.$$

Hence, $x=3, y=6, z=9$ and $t=6$.

22)

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & [6] \\ 7 & 8 & 9 \end{vmatrix}$$

Minor of 6 = Minor of a_{23}

$$\begin{vmatrix} 1 & 2 \\ 7 & 8 \end{vmatrix} = 8 - 14 = -16.$$

Section-D

23) For $a \in Z, a - a = 0$, which is divisible by 5.

$$\therefore (a, a) \in R \forall a \in Z.$$

Thus R is reflexive.

Now let $(a, b) \in R \Rightarrow a - b$ is divisible by 5

$$\Rightarrow b - a \text{ is divisible by } 5 \Rightarrow (b, a) \in R.$$

Thus R is symmetric.

Again let $(a, b) \in R, (b, c) \in R$

$$\Rightarrow a - b \text{ and } b - c \text{ are divisible by } 5$$

$$\Rightarrow (a - b) + (b - c) = a - c \text{ is divisible by } 5$$

$$\Rightarrow (a, c) \in R.$$

Thus R is transitive.

Hence, R is an equivalence relation.

24) Given $a * b = \min \{a, b\}$

Operation table for * is

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	2	3	3	3
4	1	2	3	4	4
5	1	2	3	4	5

25) The binary operation "*" on the set $\{0,1,2,3,4,5\}$ is defined as:

$$a * b = \begin{cases} a + b, & \text{if } a + b < 6 \\ a + b - 6, & \text{if } a + b \geq 6. \end{cases}$$

Thus we have the operation table:

Operation Table

*	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

$$[\because 0 * 0 = 0, \dots, 5 * 5 = 5; 0 * 1 = 1, \dots, 5 * 1 = 5 + 1 - 6 = 0; \text{etc.}]$$

Existence of Identity:

$$a * 0 = a = a + 0 \text{ for each } a \in \{0, 1, 2, 3, 4, 5\}$$

$$[\because 0 + 0 = 0, 1 + 0 = 1, \dots, 5 + 0 = 5]$$

Hence, '0' is identity for the given operation.

Existence of Inverse:

If 'b' be the inverse of 'a', then:

$$a * b = 0 = b * a.$$

$$\text{Now } a * b = 0 \Rightarrow a + b - 6 = 0 \Rightarrow b = 6 - a.$$

\therefore For each element 'a', '6-a' is the inverse of a.

When $a=1$, then $b=6-1=5$.

When $a=2$, then $b=6-2=4$.

When $a=3$, then $b=6-3=3$.

When $a=4$, then $b=6-4=2$.

When $a=5$, then $b=6-5=1$.

Hence, the inverse of 1,2,3,4 and 5 is 5,4,3,2 and 1 respectively



26) Let $y \in Y$, where y is arbitrary.

4

Here y is of the form n^2 , for $n \in N$

$$\Rightarrow \sqrt[4]{y}$$

This motivates a function:

$$g: Y \rightarrow N, \text{ defined by } g(y) = \sqrt[4]{y}.$$

$$\text{Now } g \circ f(n) = g(f(n)) = g(n^2) = \sqrt[4]{n^2} = n$$

$$\text{and } f \circ g(y) = f(g(y)) = f(\sqrt[4]{y}) = (\sqrt[4]{y})^2 = y$$

Thus $g \circ f = I_N$ and $f \circ g = I_Y$.

Hence, f is invertible with $f^{-1} = g$.

27) (i) Not associative (ii) Associative (iii) Not associative (iv) Not associative

4

28) $f^{-1} = g$, where $g(y) = \sqrt[4]{y}$

4

Section-E

29) $\forall x \in [0, \infty), y = 9x^2 + 6x - 5$

6

$$= (3x+1)^2 - 6 \geq -5 <$$

$$f = [-5, \infty)$$

Co-domain f , hence f is not onto and hence not invertible

Let us take the modified co-domain

$$f = [-5, \infty)$$

Let us now check whether f is one-one

$$\text{Let } x_1, x_2 \in [0, \infty) / f(x_1) = f(x_2) \Rightarrow (3x_1+1)^2 - 6 = (3x_2+1)^2 - 6 \Rightarrow 3x_1+1 = 3x_2+1$$

$$\Rightarrow x_1 = x_2$$

Hence f is one-one

Since with the modified co-domain = the range f , f is both one-one and onto hence invertible

From (i) above for any

$$y \in [-5, \infty)$$

$$x = \frac{\sqrt{y+6}-1}{3}$$

$$f^{-1}[-5, \infty) \rightarrow [0, \infty), f^{-1}(y)$$

$$= \frac{\sqrt{y+6}-1}{3}$$

30) Proving $*$ is commutative

6

Proving $*$ is associative

Getting identity element as $(0,0)$

Getting inverse of (a, b) as $(-a, -b)$

31) Given $*$ on Q , defined by $a * b = ab + 1$

6

Let, $a \in Q, b \in Q$ then

$$ab \in Q,$$

$$\text{and } (ab + 1) \in Q,$$

$$\Rightarrow a * b = ab + 1 \text{ is defined on } Q$$

$\therefore *$ is a binary operation on Q .

Commutative:

$$a * b = ab + 1$$

$$= ab + 1$$

$$= a * b = b * a$$

So $*$ is commutative on Q .

Associative:

$$(a * b) * c = (ab + 1) * c = (ab + 1)c + 1$$

$$= abc + c + 1$$

$$a * (b * c) = a * (bc + 1)$$

$$= a(bc + 1) + 1$$

$$= abc + a + 1$$

So $*$ is associative on Q .

Identity Element: Let $e \in Q$ be the identity element,

then for every a

$$a * e = a \text{ and } e * a = a$$

$$ac + 1 = a \text{ and } ca + 1 = a$$

$$\Rightarrow e = \frac{a-1}{a}, \frac{a-1}{a}$$

e is not unique as it depends on 'a', hence identity element does not exist for $*$.

Inverse: since there is no identity element, hence there is no inverse.

$$32) A = \begin{pmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{pmatrix}; A = \begin{pmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{pmatrix}$$

$$\text{L.H.S} = A^3 - 6A^2 + 7A + 2I$$

$$= \begin{pmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{pmatrix} - \begin{pmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{pmatrix} + \begin{pmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{pmatrix} + \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = 0$$

= RHS

$$33) \text{ Applying } C_2 \rightarrow C_2 - C_1 \text{ and } C_3 \rightarrow C_3 - C_1$$

$$\begin{vmatrix} 1 & 0 & 0 \\ 1 + \cos A & \cos B - \cos A \cos^2 B + \cos B & \cos C - \cos A \cos^2 C - \cos C \\ \cos^2 A + \cos A & \cos^2 A - \cos A & -\cos^2 A - \cos A \end{vmatrix} = 0$$

$$R_3 \rightarrow R_3 - R_2$$

$$\begin{vmatrix} 1 & 0 & 0 \\ 1 + \cos A & \cos B - \cos A & \cos C - \cos A \\ \cos^2 A - 1 & \cos^2 B - \cos^2 A & \cos^2 C - \cos A \end{vmatrix} = 0$$

$$C_3 \rightarrow C_3 - C_2$$

$$(\cos B - \cos A) \times (\cos C - \cos B)$$

$$\begin{vmatrix} 1 & 0 & 0 \\ 1 + \cos A & 1 & 0 \\ \cos^2 A - 1 & \cos B + \cos A & \cos C - \cos A \end{vmatrix} = 0$$

$$\therefore (\cos B - \cos A) \times (\cos C - \cos A) \times (\cos C - \cos B)$$

$$[1-0]=0 \text{ (Expanding along } C_3)$$

$$\therefore -\cos B = \cos A \text{ or } \cos C = \cos A \cos C = \cos B$$

$$\Rightarrow \cos B = \cos A \cos C = \cos A \cos C = \cos B$$

$$\Rightarrow \angle B = \angle A \text{ or } \angle C = \angle A \text{ or } \angle C = \angle B$$

$$\Rightarrow \triangle ABC \text{ is an isosceles triangle.}$$



QUESTION BANK 365

<https://www.qb365.in>

34)

$$|B| = \begin{vmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{vmatrix}$$

$$= 1(3 \cdot 0) - 2(-1 \cdot 0) - 2(2 \cdot 0)$$

$$= 3 + 2 - 4 \neq 0$$

i.e., B is invertible matrix.

$\Rightarrow B^{-1}$ exists.

Now

$$C_{11} = (-1)^{1+1} \begin{vmatrix} 3 & 0 \\ -2 & 1 \end{vmatrix} = 3 - 0 = 3$$

$$C_{12} = (-1)^{1+2} \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} = -(-1 - 0) = 1$$

$$C_{13} = (-1)^{1+3} \begin{vmatrix} -1 & 3 \\ 0 & -2 \end{vmatrix} = 2 - 0 = 2$$

$$C_{21} = (-1)^{2+1} \begin{vmatrix} 2 & -2 \\ -2 & 1 \end{vmatrix} = -(2 - 4) = 2$$

$$C_{22} = (-1)^{2+2} \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$$

$$C_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ 0 & -2 \end{vmatrix} = -(2 - 0) = 2$$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} 2 & -2 \\ 3 & 0 \end{vmatrix} = 0 + 6 = 6$$

$$C_{32} = (-1)^{3+2} \begin{vmatrix} 1 & -2 \\ -1 & 0 \end{vmatrix} = -(0 - 2) = 2$$

$$C_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ -1 & 3 \end{vmatrix} = (3 + 2) = 5$$

$$= \frac{1}{1} \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

Now $(AB)^{-1} = B^{-1} \cdot A^{-1}$

$$= \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9 - 30 + 30 & -3 + 12 - 12 & 3 - 10 + 12 \\ 3 - 15 + 10 & -1 + 6 - 4 & 1 - 5 + 4 \\ 6 - 30 + 25 & -2 + 12 - 10 & 2 - 10 + 10 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

