QB365

Model Question Paper 2

12th Standard CBSE

Maths

Reg.No. :

Time : 02:00:00 Hrs

Total Marks : 100

Section-A

1) Examine the continuity of the function f (x) = x^2+5 at x=-1 **1**

2) Verify MVT for the following :f (x) =e^x in [0, 1]. **1**

3) Verify MVT for the following : f (x) = $(x-1)^{2/3}$, in [0, 2]. **1**

4) The amount of pollution content added in air in a city due to X diesel vehicles is given by P(x)=$0.005x^3+0.02x^2+30x$.Find the marginal increase in pollution content when 3 diesel vehicles are added and write which value is indicated in the above question? **1**

5) The money to be spent for the welfare of the employees of a firm is proportional to the rate of change of its total revenue(marginal revenue).If the total revenue(in rupees)received from the sale of x units of a product is given by R9x)=$3x^2+36+5$,find the marginal revenue,when=5,and write which value does the question indicate? **1**

6) The total cost C(x) associated with provision of free mid-day meals to x students of a school in primary classes is given by C(x)=$0.005x^3-0.02x^2+30x+50$ If the marginal cost is given by rate of change $\frac{dC}{dX}$of total cost,write the marginal cost of food for 300students.What value is shown here? **1**

7) Evaluate the integral: $\int \frac{x^2}{1+x^3}dx.$ **1**

8) Evaluate the integral: $\int \frac{2cos\,x}{3sin^2}dx.$ **1**

9) Write the anti-derivative of $\left(\sqrt[3]{x}+\frac{1}{\sqrt{x}}\right)$ **1**

10) Find the area of the region by the curve $y=\frac{1}{x}$, X-axis and between X = 1, X = 4. **1**

Section-B

11) if y = $f(e^{sin^{-1}}2x)$, find dy/dx. **2**

12) If x = $\theta sin\theta$, y = $\theta cos\theta$ find dy/dx at $\theta=\pi/4$ **2**

13) The length x of a rectangle in decreasing at the rate of 5 cm/min and the width y increasing at the rate of 4 cm/min. find the rate of change its area when x = 5 cm and y = 8 cm. **2**

14) Find the value of a if tangent to curve y =x^2-ax+7 is parallel to the line 2x - y + 9 = 0 at (- 1, 1). **2**

15) $\int sin^{-1}(cosx)dx$ **2**

16) $\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx$ **2**

Section-C

17) For what values of 'a' and 'b', the function 'f' is defined as: **3**

$f(x)=\begin{cases}3ax+b & if\quad x<1\\ 11 \quad if \quad x=1 \\ 5ax-2b & if \quad x>1\end{cases}$ is continuous at x=1.

18) Find the values of 'p' and 'q', for which: **3**

$$\begin{cases}\frac{1-sin^3x}{3cos^2x}, & if\quad x<\frac{\pi}{2}\\ p, & if\quad x=\frac{\pi}{2}\\ \frac{q(1-sinx)}{(\pi-2x)^2}, & if\quad x>\frac{\pi}{2}\end{cases}$$

is continuous at x=2.

19) If xsin(a+y)+sin a cos(a+y)=0, then prove that: **3**

$\frac{dy}{dx}=\frac{sin^2(a+y)}{sin\quad a}$

20) Differentiate $tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t.x. **3**

21) Find $\frac{dy}{dx}$ if $y=sin^{-1}\left[\frac{6x-4\sqrt{1-4x^2}}{5}\right]$ **3**

22) Differentiate $log(x^x+cosec^2x)$ w.r.t. x. **3**

QB365 QUESTION BANK 365 https://www.qb365.in

Section-D

23) Evaluate: $\int \frac{\sin\ (x-a)}{\sin\ (x+a)}dx$ **4**

24) Find: $\int \frac{\sqrt{x}}{\sqrt{a^3-x^3}}dx$ **4**

25) Evaluate : $\int_0^{2\pi} \frac{1}{1+e^{\sin x}}dx$ **4**

26) $\int_0^1 \log\left(\frac{1}{x} - 1\right)dx = 0.$ **4**

27) $\int_2^5 f(x)dx,$ when: $f(x) = |x-2| + |x-3| + |x-5|.$ **4**

28) Prove that: $\int_0^{\pi/4}(\sqrt{\tan x} + \sqrt{\cot x})dx = \sqrt{2}\frac{\pi}{2}.$ **4**

Section-E

29) Find the point P on the curve $y^2 = 4ax$ which is nearest to the point (11a, 0). **6**

30) Evaluate: $\int_1^3 (2x^2 + 5x)dx$ as a limit of a sums. **6**

31) Using integration, find the area of the region bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$. **6**

32) Using integration find the area of the region given by $\{(x,y):(x^2 \le y \le |x|)\}$. **6**

33) Using the method of integration, find the area of the region bounded by the lines: **6**
3x - y - 3 = 0, 2x + y - 12 = 0 and x - 2y -1 = 0

34) Find the area of the smaller region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and the straight line 8x + 3y = 12. **6**

****

Section-A

1) Hence, continuous at x =-1. **1**

2) MTV verified as e^x is continuous and differentiable in [0, 1]. Ans. c = log (e-1) **1**

3) Not verified as function f (x) = $(x - 1)^{2/3}$ is not differentiable at x = 1 [0, 2]. **1**

4) 30.255 Concern for environment;Responsibility for pollution free envirnment **1**

5) R'(5)=66 Value indicated is concern for other,respect,manual labour. **1**

6) 1368 Concern for children health and nutrient food for every child. **1**

7) $= \frac{1}{3}log\,|\,1 + x^3\,| + c.$ **1**

8) $= -\frac{2}{3}cosec\quad x + C$ **1**

9) $\left(\sqrt[3]{x} + \frac{1}{\sqrt{x}}\right) = \int \sqrt[3]{x}dx + \int \frac{1}{\sqrt{x}} = 3\left[\frac{x^{3/2}}{3/2}\right] + \left[\frac{x^{1/2}}{1/2}\right] = 2x^{3/2} + 2x^{1/2} + C = 2\left[\sqrt{x} + \sqrt[x]{x}\right] + C = 2\sqrt{x}(x+1) + C$ **1**

10) log 4 sq units. **1**

Section-B

11) We have y = $f(e^{sin^{-1}}2x)$ **2**

dy/dx = f'$(e^{sin^{-1}}2x)$ x d/dx $(e^{sin^{-1}}2x)$

=f'$(e^{sin^{-1}}2x)$x$(e^{sin^{-1}}2x)$xd/dx($sin^{-1}2x$)

=f'$(e^{sin^{-1}}2x)$x$(e^{sin^{-1}}2x)$x$\frac{1}{\sqrt{1-4x^2}} \times 2$

=$\frac{2e^{sin-1}2x}{\sqrt{1-4x^2}}f'(e^{sin-1}2x)$

12) $\frac{dx}{d\theta} = \theta cos\theta + sin\theta$ **2**

$\frac{dy}{d\theta} = -\theta sin\theta + cos\theta$

$\frac{dy}{dx} = \frac{cos\theta - \theta sin\theta}{\theta cos\theta + sin\theta}$

dy/dx at $\theta = \pi/4$

$= \frac{cos\frac{\pi}{4} - \frac{\pi}{4}sin\frac{\pi}{4}}{\frac{\pi}{4}cos\frac{\pi}{4} + sin\frac{\pi}{4}}$

$= \frac{\frac{1}{\sqrt{2}} - \frac{\pi}{4} \times \frac{1}{\sqrt{2}}}{\frac{\pi}{4} \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}}$

$= \frac{1 - \frac{\pi}{4}}{\frac{\pi}{4} + 1}$

$\Rightarrow \frac{dy}{dx} = \frac{4-\pi}{4+\pi}$

QB 365 QUESTION BANK 365 https://www.qb365.in

13) Let A denote the area of rectangle at instant t **2**

$\therefore$ A=xy (area of rectangle),

$\frac{dx}{dt} = -5cm/min$

$\frac{dx}{dt} = 4cm/min$

$\frac{dx}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt}$

$\Rightarrow \quad \frac{dA}{dt} = 5 \times 4 + 8 \times -5$

$\Rightarrow \quad \frac{dA}{dt} = 20 - 40$

$\Rightarrow \quad \frac{dA}{dt} = -20\ cm^2/min$

(-) ve sign shows that area is decreasing at the rate of 20cm^2/min.

14) Given, $y = x^2 - ax + 7$ **2**

$\Rightarrow \quad \frac{dy}{dx} = 2x - a$

$m_1 = 2x - a$

Line $2x - y + 9 = 0$

$\Rightarrow \quad 2 - \frac{dy}{dx} = 0$

$\Rightarrow \quad \frac{dy}{dx} = 2 = m_2$

for parallel, $m_1 = m_2$

$\therefore \quad 2x - a = 2$

$\Rightarrow \quad 2(-1) - a = 2$

$\Rightarrow \quad -2 - 2 = a$

$\Rightarrow \quad a = -4$

15) **2**

$\int sin^{-1}(cosx)dx = \int sin^{-1}\left[sin\left(\frac{\pi}{2} - x\right)\right]dx$

$= \int\left(\frac{\pi}{2} - x\right)dx$

$= \frac{\pi}{2}\int dx - \int xdx$

$= \frac{\pi}{2}x - \frac{x^2}{2} + C$

16) $\int\frac{e^x}{x}dx - \int\frac{e^x}{x^2}dx$ **2**

$= \frac{1}{x}\int e^x - \int\left(\frac{d}{dx}\left(\frac{1}{x}\right)\int e^x dx\right)dx - \int\frac{e^x}{x^2}$

$= \frac{e^x}{x} - \int -\frac{1}{x^2}e^x dx - \int\frac{e^x}{x^2}dx$

$= \frac{e^x}{x} + \int\frac{e^x}{x^2}dx - \int\frac{e^x}{x^2}dx$

$= \frac{e^x}{x} + C$

Section-C

17) $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-}(3ax + b)$ **3**

$= \lim_{h \to 0}[3a(1 - h) + b]$

$= 3a(1 - 0) + b$

$= 3a + b$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+}(5ax - 2b)$

$= \lim_{h \to 0}[5a(1 + 0) - 2b]$

$= 5a - 2b$

$f(1) = 11$

Also

Since'f' is continuous at x=1

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$

From first and third 3a+b=11 ....(1)

From last two 5a-2b=11 .....(2)

Multiplying (1) by 2, 6a+2b=22 .....(3)

Adding (2) and (3), 11a=33=a=3

Putting in (1), 3(3)+b=11

b=11-9=2

Hence a=3 and b=2.

18) $\lim_{x\to\frac{\pi}{2}^-}f(x) = \lim_{x\to\frac{\pi}{2}^-}\frac{1-\sin^3x}{3\cos^2x}$ **3**

$= \lim_{h\to 0}\frac{1-\sin^3\left(\frac{\pi}{2}-h\right)}{3\cos^2\left(\frac{\pi}{2}-h\right)}$

$= \lim_{h\to 0}\frac{1-\cos^3h}{3\sin^2h}$

$= \lim_{h\to 0}\frac{(1-\cos h)(1+\cos^2h+\cos h)}{3(1-\cos h)(1+\cos h)}$

$= \lim_{h\to 0}\frac{1+\cos^2h+\cos h}{3(1+\cos h)}$

$= \frac{1+1+1}{3(1+1)} = \frac{1}{2}$

$\lim_{x\to\frac{\pi}{2}^+}f(x) = \lim_{x\to\frac{\pi}{2}^-}\frac{q(1-\sin x)}{(\pi-2x)^2}$

$= \lim_{h\to 0}\frac{q\left[1-\sin\left(\frac{\pi}{2}+h\right)\right]}{\left[\pi-2\left(\frac{\pi}{2}+h\right)\right]^2} = \lim_{h\to 0}\frac{q(1-\cos h)}{(\pi-\pi-2h)^2}$

$= \lim_{h\to 0}\frac{q(1-\cos h)}{4h^2} = \lim_{h\to 0}\frac{q.2\sin^2\frac{h}{2}}{4h^2}$

$= \lim_{h\to 0}\frac{q}{8}\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)^2 = \frac{q}{8}(1)^2 = \frac{q}{8}$

Also
$f\left(\frac{\pi}{2}\right) = p$

For continuity, $\lim_{x\to\frac{\pi}{2}^-}f(x) = \lim_{x\to\frac{\pi}{2}^+}f(x)$

$= f\left(\frac{\pi}{2}\right) = \frac{1}{2} = \frac{q}{8} = p$

Hence, $p = \frac{1}{2}and \quad q = 4$

19) **3**

We have:xsin(a+y)+sin a cos(a+y)=0 ......(1)

$x = -\frac{\sin\ a\ \cos(a+y)}{\sin\ (a+y)}...(2)$

Diff.w.r.t.x,

$x\ \ \cos(a+y)\left(0+\frac{dy}{dx}\right)+\sin(a+y).1+\sin\ \ a(-\sin(a+y))\left(0+\frac{dy}{dx}\right) = 0[x\ \ \cos(a+y)-\sin\ \ a\ \ \sin(a+y)]\frac{dy}{dx} = -\sin(a+y) = \left[-\frac{\sin\ a\ \cos(a+y)}{\sin\ (a+y)}\cos(a+y)-\sin a\sin(a+y)\right]\frac{dy}{dx} =$

Hence, $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin\ a}$ which is true.

20) Let y= **3**

$\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

Put $x = \tan\theta$

$y = \tan^{-1}\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta} = \tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right) = \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right) = \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right) = \tan^{-1}\left(\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right) = \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{\theta}{2} = \frac{1}{2}\tan^{-1}x$

Hence $\frac{dy}{dx} = \frac{1}{2}\frac{1}{1+x^2} = \frac{1}{2(1+x^2)}$

21) We have: **3**

$y = \sin^{-1}\left[\frac{6x-4\sqrt{1-4x^2}}{5}\right]$

$= \sin^{-1}\left(\frac{6x}{5}-\frac{4}{5}\sqrt{1-4x^2}\right) = \sin^{-1}\left((2x)\frac{3}{5}-\frac{4}{5}\sqrt{1-4x^2}\right) = \sin^{-1}\left((2x)\sqrt{1-\left(\frac{4}{5}\right)^2}-\left(\frac{4}{5}\right)\sqrt{1-(2x)^2}\right) = \sin^{-1}(2x)-\sin^{-1}\frac{4}{5}$

Hence, $\frac{dy}{dx} = \frac{1}{\sqrt{1-(4x)^2}}.(2)-0 = \frac{2}{\sqrt{1-4x^2}}$

22) **3**

Let $y = \log(x^x+\mathrm{cosec}^2x)\frac{dy}{dx} = \frac{1}{x^x+\mathrm{cosec}^2x}\frac{d}{dx}(x^x+\mathrm{cosec}^2x) = \frac{1}{x^x+\mathrm{cosec}^2x}\left[x^x(1+\log\ \ x)+2\mathrm{cosec}\ \ x(-\mathrm{cosec}\ \ x\ \ \cot\ \ x)\right] = \frac{1}{x^x+\mathrm{cosec}^2x}\left[x^x(1+\log\ \ x)-2\cot\ \ x.\mathrm{cosec}^2x\right]$

Section-D

23) **4**

$\int\frac{\sin\ (x-a)}{\sin\ (x+a)}dx = \int\frac{\sin\ [x+a)-2a]}{\sin\ (x+a)}dx = \int\frac{\sin\ (x+a)\cos\ 2a-\cos\ (x+a)\sin\ 2a}{\sin\ (x+a)}dx[\because \sin(A-B) = \sin A\cos B-\cos B\sin A] = \int[\cos 2a-\cot(x+a)\sin 2a]dx = \cos 2a\int 1.dx-\sin 2a\int\cot(x+a)dx = \cos 2$

24) 4

Let $I = \frac{\sqrt{x}}{\sqrt{a^3-x^3}}dx$ Put $x^{3/2} = t$ so that $\frac{3}{2}x^{1/2}dx = dt$ i.e. $\sqrt{x}dx = \frac{2}{3}dt.$

∴ $I = \int \frac{\frac{2}{3}dt}{\sqrt{a^3-t^2}} = \frac{2}{3}\int \frac{dt}{\sqrt{(a^{3/2})^2-t^2}}$ | From: $\int \frac{dx}{\sqrt{a^3-x^2}} = \frac{2}{3}\sin^{-1}\frac{t}{a^{3/2}} + c = \frac{2}{3}\left[\sin^{-1}\left(\frac{x^{3/2}}{a^{3/2}}\right)\right] + c = \frac{2}{3}\sin^{-1}\left(\frac{x}{a}\right)^{3/2} + c.$

25) 4

Let $I = \int_0^{2\pi}\frac{1}{1+e^{\sin x}}dx$ ....(1) ∴ $I = \int_0^{2\pi}\frac{1}{1+e^{\sin(2\pi-x)}}dx = \int_0^{2\pi}\frac{1}{1+e^{-\sin x}}$ .....(2) Adding (1) and (2), $2I = \int_0^{2\pi}\left(\frac{1}{1+e^{\sin x}}+\frac{1}{1+e^{-\sin x}}\right)dx$

$= \int_0^{2\pi}\left(\frac{1}{1+e^{\sin x}}+\frac{e^{\sin x}}{1+e^{\sin x}}\right)dx = \int_0^{2\pi}\left(\frac{1+e^{\sin x}}{1+e^{\sin x}}\right)dx = \int_0^{2\pi}1.dx = [x]_0^{2\pi} = 2\pi - 0 = 2\pi.$ Hence $I = \pi$

26) Let $I = \int_0^1 \log\frac{1-x}{x}dx = \int_0^1\log\left(\frac{1-(1-x)}{1-x}\right)dx = \int_0^1 \log\left(\frac{x}{1-x}\right)dx = -\int_0^1\log\left(\frac{1-x}{x}\right)dx = -1 \Rightarrow 2I = 0 \Rightarrow I = 0.$ 4

27) 4

$\int_2^5(|x-2|+|x-3|+|x-5|)dx,$ ......(1) Now $\int_2^5|x-2|dx = \int_2^5(x-2)dx = \left[\frac{x^2}{2}-2x\right]_2^5 = \left(\frac{25}{2}-10\right)-\left(\frac{4}{2}-4\right) = \frac{5}{2}+2=\frac{9}{2}.$ And

$\int_2^5|x-3|dx = \int_2^3|x-3|dx+\int_3^5|x-3|dx$

$= \int_2^3 -(x-3)dx + \int_3^5(x-3)dx = \left[\frac{x^2}{2}-3x\right]_2^3 + \left[\frac{x^2}{2}-3x\right]_3^5 = -\left[\left(\frac{9}{2}-9\right)-\left(\frac{4}{2}-6\right)\right]+\left[\left(\frac{25}{2}-15\right)-\left(\frac{9}{2}-9\right)\right] = -\left[-\frac{9}{2}+4\right]+\left[-\frac{5}{2}+\frac{9}{2}\right] = \frac{1}{2}+2=\frac{5}{2}$ Finally

$\int_2^5|x-5|dx = \int_2^5 -(x-5)dx = -\left[\frac{x^2}{2}-5x\right]_2^3 = -\left[\left(\frac{9}{2}-15\right)-\left(\frac{4}{2}-10\right)\right] = -\left[-\left(\frac{21}{2}+8\right)\right] = \frac{5}{2}.$ ∴ From (1), $\int_2^5[|x-2|+|x-3|+|x-5|]dx = \frac{9}{2}+\frac{5}{2}+\frac{5}{2}\frac{19}{2}.$

28) 4

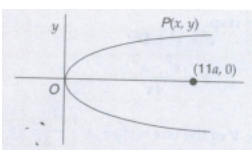
Let $\int_0^{\pi/4}(\sqrt{\tan x}+\sqrt{\cot x})dx = \int_0^{\pi/4}\frac{\tan x+1}{\sqrt{\tan x}}dx$ Put $\sqrt{\tan x} = t$ i.e. $\tan x x = t^2$ so that $\sec^2 x dx = 2t\ dt \Rightarrow dx = \frac{2t}{1+t^4}dt.$ When $x = 0,\ t = 0.$ When

$x = \frac{\pi}{4},\ t = 1.$ ∴ $I = \int_0^1\frac{t^2+1}{t}.\frac{2t}{1+t^4}dt = 2\int_0^1\frac{t^2+1}{t^4+1}dx = 2\int_0^1\frac{1+1/t^2}{t^2+1/t^2}dt$ Put $t-\frac{1}{t} = y$ so that $\left(1+\frac{1}{t^2}\right)dt = dy.$ Also $t^2-2+\frac{1}{t^2} = y^2 \Rightarrow t^2+\frac{1}{t^2} = y^2+2$

∴ $I = 2\int_{t=0}^1\frac{dy}{y^2+2} = \frac{2}{\sqrt{2}}\left[\tan^{-1}\frac{y}{\sqrt{2}}\right]_{t=0}^1 = \sqrt{2}\left[\tan^{-1}\frac{1}{\sqrt{2}}\left(t-\frac{1}{t}\right)\right]_0^1 = \sqrt{2}\left[\tan^{-1}(0)-\tan^{-1}(-\infty)\right] = \sqrt{2}\left[\tan^{-1}(-\infty)\right] = \sqrt{2}.\frac{\pi}{2}.$

Section-E

29) 6



Let P(x,y) be the nearest point

∴ $D = \sqrt{(x-11a)^2+y^2}$

$S = (x-11a)^2+y^2 = (x-11a)^2+4ax$

$\frac{dS}{dx} = 2(x-11a)+4a$

$\frac{dS}{dx} = 0 \Rightarrow x = 9a$

∴ $y = \pm 6a$

$\Rightarrow \frac{d^2S}{dx^2} = 2 > 0$

For minimum distance, coordinates are $p(9a, \pm 6a)$

QB365 QUESTION BANK 365 https://www.qb365.in

30) $I = \int_1^3 (2x^2 + 5x)dx$ **6**

$\lim_{h \to 0} h[f(1) + f(1 + h) + f(1 + 2h) + \ldots\ldots\ldots + f(1 + (n - 1)h)]$

$where \quad f(x) = 2x^2 + 5x \quad and \quad h = \frac{2}{n} or \quad nh = 2$

$f(1) = 7$

$f(1 + h) = 2(1 + h)^2 + 5(1 + h)$

$= 7 + 9h + 2h^2$

$f(1 + 2h) = 2(1 + 2h)^2 + 5(1 + 2h)$

$= 7 + 18h + 2.2^2h^2$

$f(1 + 3h) = 2(1 + 3h)^2 + 5(1 + 3h)$

$= 7 + 27h + 2.3^2h^2$

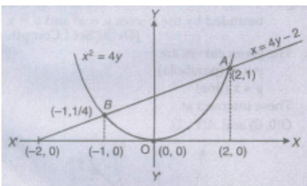
$= 7 + 27h + 2.3^2h^2$

$I = \lim_{h \to 0} h[7n + 9h\frac{n(n-1)}{2} + 2h^2. \frac{n(n-1)(2n-1)}{6}$

$= \lim_{h \to 0} h\left[7nh + \frac{9}{2}nh(nh - h) + \frac{1}{3}nh(nh - h)(2nh - h)\right]$

$= 14 + 18 + \frac{16}{3} = \frac{112}{3}$

31) $\therefore$ Point of intersection are (2, 1) and (-1. 1/4). **6**



Required Area of region (△ OABO)

$= \int_{-1}^{2} \frac{x+2}{4}dx - \int_{-1}^{2} \frac{x^2}{4}dx$

$= \frac{1}{4}\left[\frac{(x+2)^2}{2} - \frac{x^3}{3}\right]_{-1}^{2}$

$= \frac{1}{4}\left[\frac{16}{3} - \frac{5}{6}\right] = \frac{27}{24} sq.\ units$

$= \frac{9}{8} sq.\ units$

32) Given, $x^2 \le y$ ....(i) **6**

and $y \le |x|$ ....(ii)

Clearly curve (i) is parabola directed upward with vertex at (0, 0) and symmetrical about y-axis.

Also, $y = |x| = \begin{cases} x \ , & if \quad x \ge 0 \\ -x, & if \quad x < 0 \end{cases}$

The lines y=x and y=-x both passes through origin and have slope of +1 & -1 respectively.

$\Rightarrow$ Required Area = 2 Standard Area on a side

$= -\left[\left(\frac{1}{2} - 1\right) - \left(\frac{16}{2} - 4\right)\right]$

$= 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$

$= 2\left[\frac{1}{2} - \frac{1}{3}\right] = 2 \times \frac{1}{6}$

$= \frac{1}{3} sq.\ units$

QB365
QUESTION BANK 365
https://www.qb365.in

33) Let given equation of lines are 6

AB : 3x - y - 3 = 0 ....(i)

BC : 2x + y - 12 = 0 ....(ii)

CA : x - 2y - 1 = 0 ....(iii)

Solving eqns. (i) and (ii), we get x = 3, y = 6 ⇒ B(3, 6)

Solving eqns. (ii) and (iii), we get x = 5, y = 2 ⇒ C(5, 2)

Solving eqns. (i) and (iii), we get x = 1, y = 0 ⇒ A(1, 0)

Required area of △ABC - Area of △ABP + Area of trapezium BCQP-Area of △ACQ.

$= \int_{AB} y\,dx + \int_{BC} y\,dx - \int_{AC} y\,dx$

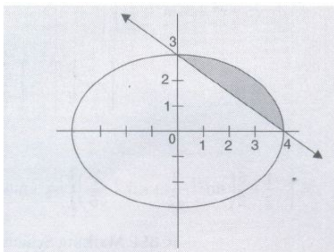
$= \int_1^3 (3x-3)dx + \int_3^5 (12-2x)dx - \int_1^5 \frac{1}{2}(x-1)dx$

$= 3\left[\frac{x^2}{2} - x\right]_1^3 + \left[12x - x^2\right]_3^5 - \frac{1}{2}\left[\frac{x^2}{2} - x\right]_1^5$

$= 3\left[\left(\frac{9}{2} - 3\right) - \left(\frac{1}{2} - 1\right)\right] + [(60-25) - (36-9)] - \frac{1}{2}\left[\left(\frac{25}{2} - 5\right) - \left(\frac{1}{2} - 1\right)\right]$

$= 3[2] + [8] - \frac{1}{2}[8] = 6 + 8 - 4 = 10 sq.\ units$

34) 6



Getting the points of intersection as (4, 0), (0, 3).

∴ Required area

$= \int_0^4 \frac{3}{4}\sqrt{16 - x^2}dx - \frac{1}{4}\int_0^4 (12 - 3x)dx$

$= \left[\frac{3}{4}\left[\frac{x}{2}\sqrt{16 - x^2} + 8sin^{-1}\frac{x}{4}\right] - \frac{1}{4}\left(12x - \frac{3x^2}{2}\right)\right]$

$= \left(\frac{3}{4}.8.\ \frac{\pi}{2} - 6\right) = (3\pi - 6)\ sq.\ units$