QB365

Model Question Paper 3

12th Standard CBSE

Maths

Reg.No. :

Time : 02:00:00 Hrs

Total Marks : 100

Section-A

1) Find the angle between two vectors $\vec{a}$ and $\vec{b}$ having the same length $\sqrt{2}$ and their vector product is $-\hat{i}-\hat{j}+\hat{k}$. **1**

2) Find λ and μ if $(\hat{i}+3\hat{j}+9\hat{k})\times(3\hat{i}-\lambda\hat{j}+\mu\hat{k})=0$ **1**

3) Find the distance of the plane 3x-4y+12z=3 from the origin. **1**

4) Write the direction cosines of a line parallel to the z-axis. **1**

5) Find the Cartesian equation of the line which passes through the point (-2,4,-5) and is parallel to the line $\frac{x+3}{3}=\frac{4-y}{5}=\frac{z+8}{6}$ **1**

6) Write the vector equation of a line passing through the point (1,-1,2) and parallel to the line whose equations are ,$\frac{x-3}{1}=\frac{y-1}{2}=\frac{z+1}{-2}$ **1**

7) Given P(A)=$\frac{1}{2}$,P(B)=$\frac{1}{3}$ and $P(A\cap B)=\frac{1}{6}$ Are the events A and B independent? **1**

8) If P(A)=0.4, P(B)=p and $P(A\cup B)=0.7$ find the value of p, if A and B are independent events. **1**

9) Given P(A)=0.2, P(B)=0.3 and $P(A\cap B)=0.3$ Find P(A/B) **1**

10) Given P(A)=0.4, P(B)=0.7 and P(B/A)=0.6, Find $P(A\cup B)$ **1**

Section-B

11) Obtain the differential equation of the family of circles passing through the points (a,0) and (- a, 0). **2**

12) Find $|\vec{a}|$ and $|\vec{b}|$if $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})=8$and $|\vec{a}|=8|\vec{b}|$. **2**

13) If $\vec{a}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, then show that $\vec{a}\neq\vec{d}$ and $\vec{b}\neq\vec{c}$ **2**

14) If a line makes angle α,β,γ with the coordinates axis ,then find the value of cos $cos2\alpha+cos2\beta+cos2\gamma$ **2**

15) If E and F are two events such that $P(E)=\frac{1}{4}$, $P(E)=\frac{1}{2}$ and $P(E\cap F)=\frac{1}{8}$, **2**

find

(a) P(E or F)

(b) P(not E and not F).

16) A die is rolled. If E = (1,3,5), F(2,3) and G{2, 3,4,5}, find **2**

(a) P(E ∪ F)/G

(b) [(E ∩ F)/G].

Section-C

17) Verify that the function y = a cos x + b sin x, where a, b ϵ R is a solution of differential equation $\frac{d^2y}{dx^2}+y=0$. **3**

18) Find the general solution of the differential equation: $\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$. **3**

19) Write two different having same magnitude. **3**

20) Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$ **3**

21) Find the angle between the line: **3**

$\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane 10x + 2y - 11z = 3.

22) Show that the points: **3**

(2,3,4);(-1,-2,1);(5,8,7) are collinear.

Section-D

23) Find the equation of the plane passing through the point (1,1,1) and containing the line **4**

$\vec{r} = \left(-3\hat{i} + \hat{j} + 5\hat{k}\right) + \lambda\left(3\hat{i} - \hat{j} + 5\hat{k}\right)$ .Also show that the plane

$\vec{r} = \left(-\hat{i} + 2\hat{j} + 5\hat{k}\right) + \lambda\left(\hat{i} - 2\hat{j} - 5\hat{k}\right)$

24) Find the shortest distance between the following pair of skew lines : **4**

$\frac{x-1}{2} = \frac{2-y}{3} = \frac{x+2}{-1} = \frac{y-3}{2} = \frac{z}{3}$

25) Find the shortest distance between the lines: **4**

$\vec{r} = (t+1)\hat{i} + (2-t)\hat{j} + (1+t)3\hat{k}$

$\vec{r} = (2s+2)\hat{i} - (1-s)\hat{j} + (2s-1)3\hat{k}$

26) A producer has 30 and 17 units of labour and capital respectively, which he can use to produce two types of goods X and Y. To produce one unit of X, 2 units of labour and 3 units of capital are required. Similarly 3 units of labour and 1 unit of capital is required to produce one unit of Y. If X and Y are priced at Rs.100 and Rs.120 per unit respectively, how should the producer use his resources to maximise the total revenue. Solve the problem graphically. **4**

27) Two cards are drawn simultaneously (without replacement) from a well-shuffled pack of 52 cards. Find the mean and variance of the number of red cards. **4**

28) An unbiased coin is tossed 4 times. Find the mean and variance of number of heads obtained. **4**

Section-E

29) Solve the following differential equation : **6**

$\left(1 + y^2\right)dx = \left(\tan^{-1}y - x\right)dy$

30) Find the particular solution of the differential equation ($\tan^{-1}$ y - x) dy = (1 + y^2) dx, given that when x = 0, y = 0. **6**

31) Find the distance of the point$3\hat{i} - 2\hat{j} + \hat{k}$from the plane 3x+y-z+2=0 measured parallel to the line **6**

$\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z-1}{1}$. Also, find the foot of the . Also, find the foot of the perpendicular from the given point upon the given plane.

32) Find the co-ordinates of the foot of the .⊥ and the length of the .L drawn from the point P(5, 4, 2) to the line **6**

$\vec{r} = -\hat{i} + 3\hat{j} + \hat{k} + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$

Also find the image of P in this line.

33) A retired person wants to invest an amount of Rs 50,000. His broker recommends investing in two type of bonds 'A' and 'B' yielding 10% and 9% return respectively on the invested amount. He decides to invest at least Rs 20,000 in bond 'A' and at least Rs 10,000 in bond 'B'. He also wants invest at least as much in boud 'A' as in bond ·'B'. Solve this linear programming problem graphically to maximise his returns. **6**

34) A card from a pack of 52 cards is lost.From the remaining cards of the pack, two cards at random and are found to be hearts.Find the probability of the missing card to be a heart **6**

****

Section-A

1) or **1**

$\sin\theta = \frac{\left|1-\hat{i}-\hat{j}+\hat{k}\right|}{\sqrt{2}.\sqrt{2}}$ $\sin\theta = \frac{\sqrt{3}}{2}$ $\theta = 60^\circ$ $\theta = \frac{\pi}{3}$

2) **1**

Getting and Alternative Method :

$\lambda = -9$ $\mu = 27$ $(\hat{i} + 3\hat{j} + 9\hat{k}) \times (3\hat{i} - \lambda\hat{j} + \mu\hat{k}) = 0 \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 9 \\ 3 & -\lambda & \mu \end{vmatrix}$

$\Rightarrow \hat{i}(3\mu + 9\lambda) - \hat{j}(\mu - 27) + \hat{k}(-\lambda - 9) = 0 \Rightarrow 3\mu + 9\lambda = 0....(i) \Rightarrow \mu - 27 = 0...(ii) \Rightarrow -\lambda - 9 = 0...(iii)$

from eqn.(ii) and (iii), $\mu = 27$ and $\lambda = -9$

3) Distance = $\left|\frac{0-0+0-3}{\sqrt{9+16+144}}\right| = \frac{3}{13}$ units **1**

4) DC's are 0,0,1 **1**

5) Cartesian equation of the line is $\frac{x+3}{3} = \frac{y-4}{-5} = \frac{z+5}{6}$ **1**

6) **1**

Here, D.R's or required line are (1, 2, - 2) Passing through (1, - 1, 2) Therefore, vector equation of line will be

$\vec{r}(\hat{i} - \hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} - 2\hat{k}) \Rightarrow \frac{x-1}{1} = \frac{y+1}{2} = \frac{z-2}{-2}$

7) Yes **1**

8) p=0.5 **1**

9) 1/3 **1**

10) 0.86 **1**

Section-B

11) $x^2 + (y-b)^2 = a^2 + b^2$ **2**

or $x^2 + y^2 - 2by = a^2$ ... (i)

$\Rightarrow 2x + 2y\frac{dy}{dx} - 2b\frac{dy}{dx} = 2b$

$\Rightarrow 2b = \frac{2x+2y\frac{dy}{dx}}{\frac{dy}{dx}}$

Substituting the value in eqn. (i),

$\left(x^2 - y^2 - a^2\right)\frac{dy}{dx} - 2xy = 0$

12) $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})$ **2**

$\vec{a}.\vec{a}+\vec{a}.\vec{b}-\vec{b}.\vec{a}-\vec{b}.\vec{b} = |\vec{a}|^2-|\vec{b}|^2$

$(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = 8|\vec{b}|^2-|\vec{b}|^2$

$= 64|\vec{b}|^2-|\vec{b}|^2$

$= 63|\vec{b}|^2$

$(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = 8$

$63|\vec{b}|^2 = 8$

$|\vec{b}| = \sqrt{\frac{8}{63}} = \frac{2\sqrt{7}}{3\sqrt{7}}$

$|\vec{a}| = \frac{8\times 2\sqrt{2}}{3\sqrt{7}} = \frac{16\sqrt{2}}{3\sqrt{7}}$

13) Given, **2**

$\vec{a}\times\vec{b} = \vec{a}\times\vec{b}$ and $\vec{a}\times\vec{c} = \vec{b}\times\vec{d}$

$\Rightarrow \vec{a}\times\vec{b}-\vec{a}\times\vec{c} = \vec{c}\times\vec{d}-\vec{b}\times\vec{d}$

$\Rightarrow \vec{a}\times\vec{d}-\vec{a}\times\vec{c}+\vec{b}\times\vec{b}-\vec{c}\times\vec{d} = \vec{0}$

$\Rightarrow \vec{a}\times(\vec{b}-\vec{c})+(\vec{b}-\vec{c})\times\vec{d} = \vec{0}$ [By left and right distributive law]

$\Rightarrow \vec{a}\times(\vec{b}-\vec{c})-\vec{b}\times(\vec{b}-\vec{c}) = \vec{0}$ [$\because \vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$]

$\Rightarrow (\vec{a}-\vec{d})\times(\vec{b}-\vec{c}) = \vec{0}$

$\Rightarrow (\vec{a}-\vec{d}) \parallel (\vec{b}-\vec{c})$

14) $I = cos\alpha, m = cos\beta, n = cos\gamma$ **2**

We know that

$i^2+m^2+n^2 = 1$

$\Rightarrow cos^2\alpha+cos^2\beta+cos^2\gamma = 1$

as we know that ,cos 2x=2 $\cos^2$ x-1

$\Rightarrow \left(\frac{1+cos2\alpha}{2}\right)+\left(\frac{1+cos2\beta}{2}\right)\left(\frac{1+cos2\gamma}{2}\right) = 1$

$\Rightarrow 1+cos2\alpha+1+cos2\beta+1+cos2\gamma = 2$

$\Rightarrow cos \quad 2\alpha+cos2\beta+1+cos2\gamma = 2$

$cos \quad 2\alpha+cos2\beta+cos \quad 2\gamma = -1$

15) Given, $P(E) = \frac{1}{4}$ **2**

$P(F) = \frac{1}{2}$

and $P(E \cap F) = \frac{1}{8}$

(a) P(E or F) = (E ∪ F) = P(E) + P(F) - P(E ∩ F)

$= \frac{1}{4} + \frac{1}{2} - \frac{1}{8}$

$= \frac{2+4-1}{8}$

$= \frac{5}{8}$

(b) P(not E and not F) = $P(\bar{E} \cap \bar{F}) = P(\overline{E \cup F})$

= 1 - P(E ∪ F)

=1-$\frac{5}{8} = \frac{3}{8}$

16) Given, E = (1,3, 5), F(2, 3), G(2, 3, 4, 5) **2**

$P(E) = \frac{3}{6}$

$P(F) = \frac{2}{6}$

$P(G) = \frac{4}{6}$

$P(G) = \frac{2}{3}$

$P(E \cup F) = \frac{2}{3}$

$P(E \cap F) = \frac{1}{6}$

(a)$(E \cup F) \cap G = (2, 3, 5)$

$P[(E \cup F) \cap G] = \frac{3}{6}$

$P[(E \cup F/G] = \frac{P[(E \cup F) \cap G]}{P(G)}$

$= \frac{\frac{3}{6}}{\frac{4}{6}} = \frac{3}{4}$

(b) $(E \cap F) \cap G = 3$

$P[(E \cup F) \cap G] = \frac{1}{6}$

$P[(E \cup F)/G] = \frac{P[(E \cup F) \cap G]}{P(G)}$

$= \frac{\frac{1}{6}}{\frac{4}{6}} = \frac{1}{4}$

Section-C

17) **3**

We have: y = a cos x + b sin x ....(1) Differntiating w.r.t. x, $\frac{dy}{dx} = -asinx + bcosx$ ....(2)

Again Differentiating w.r.t. x, $\frac{d^2y}{dx^2} = -acosx - bsinx$ $= -y$ [Using (1)]

⇒ $\frac{d^2y}{dx} + y = 0.$ Hence, the verification.

18) 3

We have; $\frac{dy}{dx}=\frac{1+y^2}{1+x^2}.$ $\Rightarrow$ $\frac{dy}{1+y^2}=\frac{dx}{1+x^2}$ **| Variables Separable**

Integrating, $\int\frac{dy}{1+y^2}=\int\frac{dx}{1+x^2}+c$ $\Rightarrow$ tan^{-1}y= tan^{-1}x+c, Which is the reqd. general

solution.

19) 3

let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}+2\hat{j}+2\hat{k}$ Here $|\vec{a}|=\sqrt{(1)^2+(2)^2+(3)^2}$ $=\sqrt{1+4+9}=\sqrt{14}$

and b $|\vec{b}|=\sqrt{(2)^2+(3)^2+(1)^2}$ $=\sqrt{4+9+1}=\sqrt{14}$ Here, $\vec{a}\neq\vec{b}$ but $|\vec{a}|$ $=$ $|\vec{b}|$

20) 3

Let $\vec{a}=\hat{i}+3\hat{j}+7\hat{k}$

$\vec{b}=7\hat{i}-\hat{j}+8\hat{k}$ $\therefore$ $|\vec{b}|=\sqrt{7^2+(-1)^2+8^2}$ $=\sqrt{49+1+64}=\sqrt{144}$ and $\vec{a}$ $\vec{b}$

$=\left(\hat{i}+2\hat{j}+3\hat{k}\right),\left(7\hat{i}-\hat{j}+8\hat{k}\right)$

= (1) (7) + (3) (-1) + (7) (8)

= 7 - 3 + 56 = 60. $\therefore$ Projection of $\vec{a}$ on $\vec{b}=\frac{\vec{a}.\vec{b}}{|\vec{b}|}=\frac{60}{\sqrt{140}}$

21) 3

The given line is $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$ ...(1)

and the given plane is 10x+2y-11z=3 ...(2)

If the line (1) makes an angle 'θ' with the plane (2), then the line (1) will make angle $(90°-\theta)$ with the normal to plane (2).

Now direction-ratios of line (1) are <2,3,6>

and direction-ratios of normal to plane (2) are <10,2, -11>.

$cos(90°-\theta)=\left|\frac{(2)(10)+(3)(2)+(6)(-11)}{\sqrt{4+9+39}\sqrt{1+4+121}}\right|$ => $sin\theta=\left|\frac{20+6-66}{(7)(15)}\right|=\frac{40}{105}=\frac{8}{21}$ $Hence,$ $\theta=\sin^{-1}\left(\frac{8}{21}\right)$

22) Let A(2,3,4);B(-1,-2,1) and C(5,8,7) be the given points 3

Direction ratios of AB are:

(-1-2,-2-3,1-4)

i.e. (-3,-5,-3) i.e. (3,5,3)

Direction-ratios of BC are:

(5-(-1),8-(-2(,7-1)

i.e. (6,10,6) i.e. (3,5,3)

$\Rightarrow$ AB and BC have same direction ratios

$\Rightarrow$ AB||BC

But B is a common point.Hence A,B,C are collinear.

Section-D

23) **4**

Let A(1,1,1) be the given point.

Let P(-3,1,5) be the point on the given line:

$\vec{r} = \left(-3\hat{i} + \hat{j} + 5\hat{k}\right) + \lambda\left(3\hat{i} - \hat{j} + 5\hat{k}\right)$

$\vec{A}P = (P.V.of P) - (P.V.of \quad A) \qquad = \left(-3\hat{i} + \hat{j} + 5\hat{k}\right) - \left(\hat{i} + \hat{j} + \hat{k}\right) = -4\hat{i} + 4\hat{k}$

Since the plane contains(1),

vector perp. to the plane is:

$$(-4\hat{i} + 4\hat{k}) \times \left(3\hat{i} - \hat{j} - 5\hat{k}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & 0 & 4 \\ 3 & -1 & -5 \end{vmatrix} = \hat{i}(-0+4) - \hat{j}(20-12) + \hat{k}(4-0) = -4\hat{i} - 8\hat{j} + 4\hat{k} \quad i.e. \quad \hat{i} - 2\hat{j} + \hat{k}$$

The equation of the plane is:

$\vec{r}.\left(\hat{i} - 2\hat{j} + \hat{k}\right) = (\hat{i} + \hat{j} + \hat{k}).(\hat{i} - 2\hat{j} + \hat{k}) = 0 \Rightarrow \qquad x - 2y + z = 0 \qquad .....(2)$

Now since

$\left(\hat{i} - 2\hat{j} - 5\hat{k}\right).\left(\hat{i} - 2\hat{j} + \hat{k}\right) = 1 + 4 - 5 = 0$

the line $\left(-\hat{i} + 2\hat{j} + 5\hat{k}\right) + \lambda\left(\hat{i} - 2\hat{j} + 5\hat{k}\right)$ is

parallel to the plane.

Further the point(-1,2,5) satisfies(2). [-1-4+5=0]

Hence the plane contains the line.

24) $\vec{r}_1 = \vec{i} + 2\vec{j} - \vec{k} + \lambda(2\vec{i} - 3\vec{j} + 4\vec{k})$ **4**

$\vec{r}_2 = \overrightarrow{2i} + 3\vec{j} + \mu(-\vec{i} - 2\vec{i} - 3\vec{j} + 4\vec{k})$

$\vec{a}_1 = \vec{i} + 2\vec{j} - \vec{k}\overrightarrow{b_1} = 2\vec{i} - 3\vec{j} + 3\vec{k}$

$\vec{a}_2 = \overrightarrow{2i} + 3\vec{j} - \overrightarrow{b_2} = -\vec{i} + 2\vec{j} + 3\vec{k}$

$\vec{a}_2 = 2\vec{i} + 3\vec{j}, \overrightarrow{b_2} \quad = -\vec{i} + 2\vec{j} + 3\vec{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 4 \\ -1 & 2 & 3 \end{matrix}$$

$= -17\hat{i} - 10\hat{j} + \hat{k}$

The required shortest distance

$$\frac{(\vec{a}_2 - \vec{a}_1)(\vec{b}_1 - \vec{b}_2)}{\vec{b}_1 \times \vec{b}_2}$$

$$= \frac{(-3\hat{i} + \hat{j} + \hat{k})(-17\hat{i} - 10\hat{j} + \hat{k})}{\sqrt{(-17)^2 + (-10)^2 + (1)^2}}$$

$$= \frac{42}{\sqrt{390}} units$$

25) Equations of lines can be written as : **4**

$\vec{r} = (t+1)\hat{i} + (2-t)\hat{j} + (1+t)\hat{k}$

$\vec{r} = (2s+2)\hat{i} - (1-s)\hat{j} + (2s-1)2\hat{k}$ Let , $\hat{a}_1 = 2\hat{i} + \hat{j} + \hat{k}$

$\vec{b}_2 = c2\hat{i} - \hat{j} - 2\hat{k}$

Then $\vec{a}_2 - \vec{a}_1 = \hat{i} - 3\hat{j} - 2\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$

$= -3\hat{i} + 3\hat{k}$

:. Shortest distance

$$\left| \frac{(\vec{b}_1 \times \vec{b}_2)(\vec{a}_1 \times \vec{a}_2)}{\vec{b}_1 \times \vec{b}_2} \right|$$

$= \frac{-3-6}{\sqrt{9+9}}$

$= \frac{3}{\sqrt{2}} or \quad \frac{3\sqrt{2}}{2}$

26) 3 units of X, 8 units of Y and maximum revenue Rs.1260. **4**

27) Number of red cards = 26 **4**

Let X be a random variable which can take values

0,1,2, where X is the no. of red cards selected

$\therefore$ X = 0 means 0 red cards

$P(X=0) = \frac{26_{C_2}}{52_{C_2}} = \frac{26\times25}{52\times51} = \frac{25}{102}$

$P(X=1) = \frac{26_{C_1}\times26_{C_1}}{52_{C_2}} = \frac{26\times26\times2}{52\times51} = \frac{52}{102}$

$P(X=2) = \frac{26_{C_2}}{52_{C_2}} = \frac{26\times25}{52\times51} = \frac{25}{102}$

Probability distribution of random variable X is

X	0	1	2
P(X)	$\frac{25}{102}$	$\frac{52}{102}$	$\frac{25}{102}$

$Mean = \sum xP(X) = \frac{52+50}{102} = 1$

$Variance = \sum x^2 P(X) - \{xP(X)\}^2$

$= \frac{152}{105} - 1$

$= \frac{50}{102} or \frac{25}{51}$

28) 4

Given n = 4

Getting $p = \frac{1}{2} \quad and \quad q = \frac{1}{2}$

No. of Successes(x)	0	1	2	3	4
P(x)	$4_{C_0}\left(\frac{1}{2}\right)^2 = \frac{1}{16}$	$4_{C_1}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right) = \frac{4}{16}$	$4_{C_2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 = \frac{6}{16}$	$4_{C_3}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^3 = \frac{4}{16}$	$4_{C_1}\left(\frac{1}{2}\right)^4 = \frac{1}{16}$
xP(x)	0	$\frac{4}{16}$	$\frac{12}{16}$	$\frac{12}{16}$	$\frac{4}{16}$
x^2p(x)	0	$\frac{4}{16}$	$\frac{24}{16}$	$\frac{36}{16}$	$\frac{16}{16}$

Mean= $\sum xp(x) = \frac{32}{16} = 2$

$Variance = \sum x^2 p(x) - \{\sum P(x)\}^2$

$= \frac{80}{16} - (2)^2 = 5 - 4 = 1$

Section-E

29) Writing the given differential equation as 6

$\frac{dx}{dy} = \frac{\tan^{-1} y - x}{1+y^2}$

or $\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1} y}{1+y^2}$

The above equation is linear in 'x'.

Getting $I.F. e^{\int \frac{1}{1+y^2} dy}$

$= e^{\tan^{-1} y}$

Multiplying both sides by I.F.and integrating

we get $xe^{\tan^{-1} y} = \int e^{\tan^{-1} y} \frac{\tan^{-1} y}{1+y^2} dy$

$\Rightarrow \quad xe^{\tan^{-1} y} = \int e^t . tdt$

$\left(Assuming \quad \tan^{-1} y = t \Rightarrow \frac{1}{1+y^2} dy = dt\right)$

$\Rightarrow \quad xe^{\tan^{-1} y} = te^t - e^t + C$

$\Rightarrow \quad xe^{\tan^{-1} y} = \left(\tan^{-1} y\right)\left(e^{\tan^{-1} y}\right) - e^{\tan^{-1} y} + C$

$\Rightarrow \quad x = \tan^{-1} y - 1 + Ce^{-\tan^{-1} y}$

30) Given $\left(\tan^{-1}y - x\right)dy = \left(1 + y^2\right)dx$ **6**

$\Rightarrow \quad \frac{dx}{dy} = \frac{\tan^{-1}y - x}{1+y^2}$

$\Rightarrow \quad \frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$ ...(i)

This is a linear differential equation with :

$P = \frac{1}{1+y^2}$ and $Q = \frac{\tan^{-1}y}{1+y^2}$

$\Rightarrow \quad I.F. = e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$

Multiplying both sides of eqn. (i) by

$I.F. = e^{\tan^{-1}y}$, we get

$x.I.F. = \int Q.I.F.dy$

$xe^{\tan^{-1}y} = \int \frac{\tan^{-1}y}{1+y^2}e^{\tan^{-1}y}dy + C$

$\Rightarrow xe^{\tan^{-1}y} = \int te^t dt + C$, where $t = \tan^{-1}y$

$\Rightarrow \quad xe^{\tan^{-1}y} = e^t(t-1) + C$

$\Rightarrow \quad xe^{\tan^{-1}y} = e^{\tan^{-1}y}\left(\tan^{-1}y - 1\right) + c$ ... (ii)

It is given that y(0) = 0 i.e., y = 0 when x = 0

Putting x = 0, y = 0 in eqn. (ii), we get

$0 = e^0(0-1) + c \Rightarrow c = 1$

Putting c = 1 in eq (ii), we get

$xe^{\tan^{-1}y} = e^{\tan^{-1}y}\left(e^{\tan^{-1}y} - 1\right) + 1$

$\Rightarrow \quad \left(x - \tan^{-1}y + 1\right)e^{\tan^{-1}y} = 1$

31) **6**

The equation of the line passing through point (3, -2, 1) and parallel to the given line is

$\frac{x-3}{2} = \frac{y+2}{-3} = \frac{z-1}{1} = \lambda$

Any point on this line, we have $(2\lambda+3, -3\lambda-2, \lambda+1)$

If it lies on the plane, we have $3(2\lambda+3)-3\lambda-2-\lambda-1+2=0$

$\Rightarrow \quad \lambda = -4$

Hence, the point common to the plane and the line is (-5, 10, -3).

Hence the required distance = $\sqrt{(3+5)^2 + (-2-10)^2 + (1+3)^2} = 4\sqrt{14}units$

The equation of the line passing through (3, -2, 1) and perpendicular to the plane is $\frac{x-3}{3} = \frac{y+2}{1} = \frac{z-1}{-1} = \mu$

Any point on it is $(3\mu+3, \mu-2, -\mu+1)$

If it lies on the plane, we get $3(3\mu+3)+\mu-2+\mu-1+2 = 0$

$\Rightarrow \quad \mu = \frac{8}{11}$

The required foot of the perpendicular is $\left(\frac{9}{11}, \frac{-30}{11}, \frac{19}{11}\right)$.

32) Converting the given equation of the line into cartesian form, we have

$\frac{x+1}{2}=\frac{y-3}{3}=\frac{z-1}{-1}=\lambda$

Any point on the line (i) is :

$Q(2\lambda-1, 3\lambda+3, -\lambda+1)$

For some values of A, let Q be perpendicular from P on the line (i).

Direction ratio's of PQ are:

$2\lambda-6, 3\lambda-1, -\lambda-1$

Also, d.r.'s of line (i) are 2,3,-1.

Since AM .Lt line (i), then

$2(2\lambda-6)+3(3\lambda-1)-1(-\lambda-1)=0$

$\Rightarrow 14\lambda-14=0$

$\Rightarrow \lambda=1$

:. From (ii), coordinates of foot of perpendicular Q are (10; 6, 0).

Now ⊥ distance PQ

$=\sqrt{(1-5)^2+(6-4)^2+(0-2)^2}$

$=\sqrt{16+4+4}=\sqrt{24}=2\sqrt{6}$

Again, let the image of P in the line (i) is $p'(x_1, y_1, z_1)$ then is the mid-point of PP'.

$\therefore \frac{5+x_1}{2}, =1$

$\frac{4+y_1}{2}=6$

$\frac{2+z_1}{2}=0$

$\therefore \quad x_1=-3, y_1=8, z_1=-2$

∴ ImageofP in the line (i)is P' (-3,S,-2).

33)

Let the investment in bond A be Rs x and in bond B Rs y.

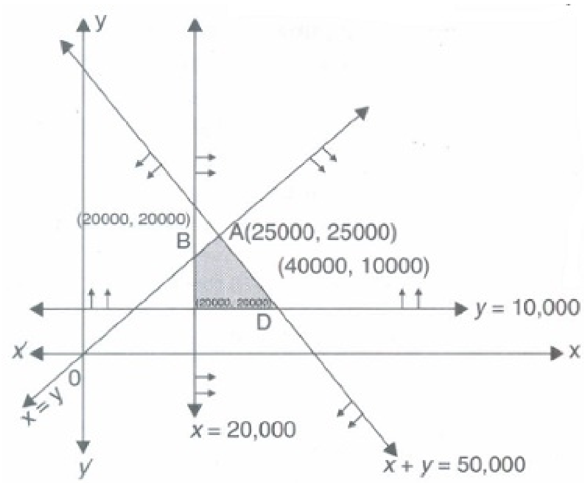
Obective function is

$Z = \frac{x}{10} + \frac{9}{100}y$

Subject to constraints

x + y $\leq$ 50,000; x $\geq$ 20,000;

y $\geq$ 10,000, x $\geq$ y(*)



Vertices of feasible region are A, B, C and D.

Corner Points	$Z = \frac{x}{10} + \frac{9}{100}y$	Value
A(25,000, 25,000)	2,500 + 2,250	4,750
B(20,000, 20,000)	2,000 + 18,00	3,800
C(20,000, 10,000)	2,000 + 900	2,900
D(40,000, 10,000)	4,000 + 900	4,900

Return is maximum when Rs 40,000 are invested in Bond A and Rs 10,000 in Bond B maximum return is Rs 4,900.

Since there are more than 3 constraints, student may be given full 6 marks even if reaches upto (*).

34) Let C_1, C_2, C_3, C_4 be the events that the lost card is of heart, spades, diamond or club respectively.

Obviously $P(C_1) = P(C_2) = P(C_3) = P(C_4)$

$$= \frac{13}{52} = \frac{1}{4}$$

Let S be the event of drawing two cards of heart from the remaining 51 cards.We wish to find $P\left(\frac{C_1}{S}\right)$

Now $P\left(\frac{C_1}{S}\right)$ is the probability of drawing two heart cards from 51 cards given that one heart card is lost

$$= \frac{{}^{12}C_2}{{}^{51}C_2} = \frac{12 \times 11}{1 \times 2} \times \frac{1 \times 2}{51 \times 50} = \frac{22}{425}$$

$$P\left(\frac{S}{C_3}\right) = P\left(\frac{S}{C_3}\right) = P\left(\frac{S}{C_4}\right) = \frac{{}^{13}C_2}{{}^{51}C_2}$$

$$= \frac{13 \times 12}{1 \times 2} \times \frac{1 \times 2}{51 \times 50} = \frac{26}{425}$$

By Bayes' Theorem

$$P\left(\frac{C_1}{S}\right) = \frac{P(C_1)\,.P\left(\frac{S}{C_1}\right)}{\sum P(C_1)\,.P\left(\frac{S}{C_1}\right)}$$

$$= \frac{\frac{1}{4} \times \frac{22}{425}}{\frac{1}{4} \times \frac{22}{425} + \frac{1}{4} \times \frac{26}{425} + \frac{1}{4} \times \frac{26}{425}}$$

$$= \frac{22}{22+26+26+26}$$

$$= \frac{11}{50}$$