QB365

## Important Questions - Mechanical Properties of Solids <br> 11th Standard CBSE

## Physics

Reg.No.


Time : 01:00:00 Hrs

## Section-A

1) Stress and pressure are both forces per unit area. Then, in what respect does stress differ from pressure ?
2) Which type of strain is there, when a spiral spring is stretched by a force ?
3) Is it possible to double the length of a metallic wire by applying a force over it ?
4) What is the Young's modulus for a perfect rigid body?
5) Awire increase by $10^{-3}$ of its lengyh when a stress of $10^{8} \mathrm{Nm}^{-2}$ is applied to it. What is the Young's modulus of the material of the wire?
6) The star Sirius has a mass of $7 \times 10^{30} \mathrm{~kg}$, its distance from the earth is $8 \times 10^{16 \mathrm{~m}}$ and the mass of the earth is $6 \times 10^{24} \mathrm{~kg}$. Calculate the cross-section of a steel cable that can withstand the gravitational pull between the Sirius and the earth. Given $\mathrm{G}=6.67 \times 10^{-11} \mathrm{Nm}^{2} \mathrm{~kg}^{-2}$ and breaking stress $=10^{10} \mathrm{Nm}^{-2}$
7) A wire of length 2.5 m has a percentage strain of $0.012 \%$ under a tensile force. Determine the extension in the wire.
8) A wire elongates by 8 mm when a load of 9 kg is suspended from it. What is the elongation when its radius is doubled, if other quantities are the same as before?
9) Find the change in volume which 1 cc of water at the surface will undergo, when it is taken to the bottom of the lake 100 m deep, given that volume elasticity is 22000 atmospheres.
10) A square lead slab of side 50 cm and thickness 10 cm is subjected to a shearing force (on its narrow edge) of $9.0 \times 10^{4} \mathrm{~N}$. The lower edge is reveted to the floor. How much will the upper edge be displaced? Modulus of rigidity of lead $=5.6 \times 10^{9} \mathrm{~N} / \mathrm{m}^{2}$

## Section-B

11) Two wires made of same material are subjected to forces in the ratio $1: 4$. Their lengths are in the ratio $2: 1$ and diameters in the ratio $1: 3$. What is the ratio of their extensions ?
12) Two wires $A$ and $B$ are of the same material. Their lengths are in the ratio $1: 2$ and the diameters in the ratio $2: 1$ If they are pulled by the same force, then what will be the ratio of their increase in lengths?
13) A solid sphere of radius $R$ made of a material of bulk modulus $B$ is surrounded by a liqiud in a cylindrical container. A massless piston of area A floats on the surface of the liqiud. When a mass $M$ is placed on the piston on the piston to compress the liqiud, find fractional change in the radius of the sphere?
14) The Marina trench is located in the Pacific ocean and at one place, it is nearly 11 km beneath the surface of water. The water pressure at the bottom of the trench is about $1.1 \times 10^{8} \mathrm{~Pa}$. A steel ball of initial volume $0.32 \mathrm{~m}^{3}$ is dropped into the ocean and falls to the bottom of trench. What is the change in the voulme of the ball when it reaches to the bottom if the Bulk modulus of steel is $1.6 \times 10^{11} \mathrm{~N} / \mathrm{m}^{2}$ ?
15) To what depth must a rubber ball be taken in deep sea so that its volume is decreased by $0.1 \%$ ?
16) Determine the fractional change in volume as the pressure of the atmosphere $1.0 \times 10^{5} \mathrm{~Pa}$ around a metal block is reduced to zero by placing the block in vacuum. The Bulk modulus for the block is $1.25 \times 10^{11} \mathrm{Nm}^{-2}$
17) A steel wire of length 4.7 m and cross-sectional area $3.0 \times 10^{5} \mathrm{~m}^{2}$ stretches by the same amount as a copper wire of length 3.5 m and cross-sectional area $4.0 \times 10^{-5} \mathrm{~m}^{2}$ under a given load. What is the ratio of the Young's modulus of stell to that of copper?
18) Two parallel steel wires $A$ and $B$ are fixed to rigid support at the upper ends and subjected to the same load at the lower ands. The lengths of wires are in the ratio $4: 5$ and their radii are in the ratio 4:3. The increase in the length of the wire $A$ is 1 mm . Calculate the increase in the length of the wire $B$.
19) Assume that if the shear stress in steel exceeds about $4 \times 10^{8} \mathrm{~N} / \mathrm{m}^{2}$, the steel ruptures. Determine the shearing force necessary to (a) shear a steel bolt 1.00 cm in diameter and (b) punch a 1 cm diameter hole in a steel plate 0.5000 cm thick.
20) A uniform heavy rod of weight $W$, cross-sectional area A and length I is hanging from a fixed , support. Young's modulus of the material of the rod is $Y$. Neglecting the lateral contraction, find the elongation produced in the rod.

## Section-C

21) Caluculate the percentage increase in length of a wire of diameter 2.5 mm stretched by a force of 100 kg weight. YOung's modulus of elasticity of wire is $12.5 \times 10^{11}$ dyne/sq cm. his mistake.
What values do you associate with Ajay?
22) The edge of an aluminium cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall.Amass of 100 kg is then attached to the opposite face of the cube.

The shear modulus of aluminium is 25 GPa . What is the vertical deflection of this face?
24) A14.5 kg mass, fastened to one end of a steel wire of unstretched length 1 m is whirled in a vertical circle with an angular frequency of 2 rev/s at the
bottom ofthe circle. The
cross-sectional area of the wire is $0.065 \mathrm{~cm}^{2}$.Calculate the elongation of the wire when the mass is at the lowest point of its path.

## 

## Section-A

1) 

Pressure is an external force per unit area, while stress is the internal restoring force which comes into play in a deformed body acting transversely per unit area of body.

Longitudinal strain and shear strain.
No, it is not possible because within elastic limit, strain is only order of $10^{-3}$, wires actually break much before it is stretched to double the length.
4)

Young's modulus $Y=\frac{F}{A} \times \frac{l}{\Delta l}$ For a perfectly rigid body, change in length $\Delta l=0$
$Y=\frac{F}{A} \times \frac{l}{0}=\infty$ Therefore, Young's modulus for a perfectly rigid bosy is $\infty$
5)

Given, $\Delta L=10^{-3} L$ with $L$ as the original length $\quad$ Strain $Y=\frac{\Delta L}{L}=10^{-3}$ and Stress $=\frac{F}{A}=10^{8} \mathrm{~N} / \mathrm{m}^{2}$
$Y=\frac{\text { Stress }}{\text { Strain }}=\frac{F / A}{\Delta L / L} Y=\frac{1 \times 10^{8}}{10^{-3}}=10^{11} \mathrm{~N} / \mathrm{m}^{2}$
6) $44 \mathrm{~m}^{2}$
7) Here, original length, $L=2.5 \mathrm{~m}$

$$
\begin{aligned}
\text { Strain } & =\frac{\Delta L}{L}=0.012 \%=\frac{0.012}{100} \\
\Delta L & =\text { Strain } \times \mathrm{L} \\
\Delta L & =\text { extension }=\frac{0.012}{100} \times L \\
& =\frac{0.012 \times 2.5}{100}=3 \times 10-4 \mathrm{~m}=0.3 \mathrm{~mm}
\end{aligned}
$$

8) 2 mm
9) 0.00044 cc
10) 0.16 mm

## Section-B

11) According to Hooke's law,

Modulus of elasticity, $\mathrm{E}=\frac{F}{\pi r^{2}} \times \frac{l}{\Delta l}$
or $\quad \Delta l=\frac{F l}{\pi r^{2} E}$
or $\quad \Delta l \propto \frac{F l}{r^{2}} \quad[\because \mathrm{E}$ is same for two wires $]$
$\therefore \quad \frac{\Delta l_{1}}{\Delta l_{2}}=\frac{F_{1}}{F_{2}} \times \frac{l_{1}}{l_{2}} \times \frac{r_{2}^{2}}{r_{1}^{2}}$

$$
=\frac{1}{4} \times \frac{2}{1} \times\left(\frac{3}{1}\right)^{2}=\frac{9}{2}
$$

So, $\quad \Delta l_{1}: \Delta l_{2}=9: 2$
12) We know $\Delta L=\frac{F L}{A Y}, \frac{L_{A}}{L_{B}}=\frac{1}{2}$ and $\frac{r_{A}}{r_{B}}=\frac{2}{1}$
[the wires $A$ and $B$ are pulled by the same force and they are made up of same material, hence, $F_{A}=F_{B}=F, Y_{A}=Y_{B}=Y$ ]

$$
\frac{\Delta L_{A}}{\Delta L_{B}}=\frac{L_{A}}{\pi r_{B}^{2}} \times \frac{\pi r_{B}^{2}}{L_{B}} \frac{\Delta L_{A}}{\Delta L_{B}}=\frac{L_{A}}{L_{B}} \times\left(\frac{r_{B}}{r_{A}}\right)^{2} \frac{\Delta L_{A}}{\Delta L_{B}}=\frac{1}{2} \times\left(\frac{1}{2}\right)^{2}=\frac{1}{8} \frac{\Delta L_{A}}{\Delta L_{B}}=\frac{1}{8}
$$

When mass $M$ is placed on the piston, the excess pressure, $p=M g / A$. As the pressure is equally applicable from all the direction on the sphere, hence there will be decrease in volume due to decrease in raius sphere. Volume of the sphere, $V=\frac{3}{4} \pi R^{3}$

Differentiating it, we get,
$\Delta V=\frac{4}{3} \pi\left(3 R^{2}\right) \Delta R=4 \pi R^{2} \Delta R \frac{\Delta V}{V}=\frac{4 \pi R^{2} \Delta R}{\frac{4}{3} \pi R^{3}}=\frac{3 \Delta R}{R}$
We know that, $B=\frac{P}{d V / V}=\frac{M g}{A} \quad \frac{3 \Delta R}{R}$
or $\quad \frac{\Delta R}{R}=\frac{M g}{3 B A}$
14) $\operatorname{Depth}(\mathrm{h})=11 \mathrm{~km}=11 \times 10^{3} \mathrm{~m}$

Pressure at the bottom of the trench $(\mathrm{p})=1.1 \times 10^{8} \mathrm{~Pa}$
Initial voulme of the ball $(\mathrm{V})=0.32 \mathrm{~m}^{3}$
Bulk modulus of stell $(\mathrm{B})=1.6 \times 10^{11} \mathrm{~N} / \mathrm{m}^{2}$
Bulk modulus of steel $(B)=\frac{p}{(\Delta V / V)}=\frac{p V}{\Delta V}$

$$
\Delta V=\frac{p V}{B}=\frac{1.1 \times 10^{8} \times 0.32}{1.6 \times 10^{11}} \Delta V=2.2 \times 10^{-4} \mathrm{~m}^{3}
$$

15) Bulk modulus of rubber (b) $=9.8 \times 10^{8} \mathrm{~N} / \mathrm{m}^{2}$

Density of seawater $(p)=10^{3} \mathrm{~kg} / \mathrm{m}^{3}$
Percentage decrease in volume

$$
\left(\frac{\Delta V}{V} \times 100\right)=0.1 \quad \text { or } \quad \frac{\Delta V}{V}=\frac{0.1}{100} \text { or } \quad \frac{\Delta V}{V}=\frac{1}{1000}
$$

Let the rubber ball be taken up to depth $h$.
Change in pressure ( p )=hpg
Bulk modulus $_{(B)}=\frac{p}{(\Delta V / V)}=\frac{h p g}{(\Delta V / V)}$
or $h=\frac{B \times(\Delta V / V)}{p g}=\frac{9.8 \times 10^{8} \times \frac{1}{1000}}{10^{3} \times 9.8}=100 \mathrm{~m}$
16) $8 \times 10^{-7}$
17) Given for steel wireLength $\left(l_{1}\right)=4.7 \mathrm{~m}$

Area of cross-section $\left(A_{1}\right)=3.0 \times 10^{-5} \mathrm{~m}^{2}$
For copper wire
Length $\left(l_{2}\right)=3.5 \mathrm{~m}$
Area of crosssection $\left(\mathrm{A}_{2}\right)=4.0 \times 10^{-5} \mathrm{~m}^{2}$
Let F be the given load under which steel and copper wires be streched by the same amount $\Delta l$
Young's modulus $(Y)=\frac{F / A}{\Delta l / l}=\frac{F \times l}{A \times \Delta l}$
For steel $\quad Y_{s}=\frac{F \times l_{1}}{A_{1} \times \Delta l}$
For copper $\quad Y_{c}=\frac{F \times l_{2}}{A_{2} \times \Delta l}$
Dividing Eq (i)by Eq(ii) we get

$$
\frac{Y_{s}}{Y_{c}}=\frac{F \times l_{1}}{A_{1} \times \Delta l} \times \frac{A_{2} \times \Delta l}{F \times l_{2}} \quad=\frac{l_{1}}{l_{2}} \times \frac{A_{2}}{A_{1}}=\frac{4.7}{3.5} \times \frac{4.0 \times 10^{-5}}{3.0 \times 10^{-5}} \frac{Y_{s}}{Y_{c}}=\frac{18.8}{10.5}=1.79=1.8
$$

18) 2.22 mm
19) $3.14 \times 10^{4} \mathrm{~N}, 6.28 \times 10^{4} \mathrm{~N}$


As shown in figure, consider a small element of thickness $d x$ at distance $x$ from the fixed support. Force acting on the element $d x$ is
$F=$ Weight of length $(1-x)$ of the rod
$=\frac{W}{l}(\mathrm{l}-\mathrm{x})$
Elongation of the element $=$ Original length $\times \frac{\text { Stress }}{Y}$

$$
=d x \times \frac{F / A}{Y}=\frac{W}{l A y}(l-x) d x
$$

Total elongation produced in the $\operatorname{rod}=$

$$
\frac{W}{l A y} \int_{0}^{l}(l-x) d x=\frac{W}{l A y}\left[l x-\frac{x^{2}}{2}\right]_{0}^{l}
$$

$$
\frac{W}{l A y}\left[l^{2}-\frac{l^{2}}{2}\right]=\frac{W}{2 A y}
$$

21) 

Here, $2 \mathrm{r}=2.5 \mathrm{~mm}=0.25 \mathrm{~cm}$
or $r=0.125 \mathrm{~cm}$

$$
a=\pi r^{2}=\frac{22}{7} \times(0.125)^{2} \text { sq. cm } \quad F=100 \quad \mathrm{~kg}=100 \times 1000 g \quad F=10 \times 1000 \times 980 \quad \text { dyne } \quad Y=12.5 \times 10^{11} \text { dyne } / \mathrm{sq} \cdot \mathrm{cmAs} \quad Y=\frac{F \times l}{a \times \Delta l} \quad \frac{\Delta l}{l}=\frac{F}{a Y}
$$

22) Ajay is naughty but when he is made to realise his mistake, he is ready to apologise
23) 



Given, side of a cube $(\mathrm{l})=10 \mathrm{~cm}=0.1 \mathrm{~m}$
Area of its each face $(A)=l^{2}=(0.1)^{2}=0.01 \mathrm{~m}^{2}$

$$
\operatorname{Load}(m)=100 \mathrm{~kg}
$$

Tangential force acting on one face of the cube, $\mathrm{F}=\mathrm{mg}=100 \times 9.8=980 \mathrm{~N}$
Shear stress acting on this face $=\frac{F}{A}=\frac{980}{0.01} \mathrm{~N} / \mathrm{m}^{2}$

$$
=9.8 \times 10^{4} \mathrm{~N} / \mathrm{m}^{2}
$$

Shear modulus of aluminium $(\eta)=25 \mathrm{GPa}$

$$
=25 \times 10^{9} \mathrm{~N} / \mathrm{m}^{2}
$$

Shear modulus $(\eta)=\frac{\text { Shearing stress }}{\text { Shearing strain }}$
or shearing strain $\left(\frac{\Delta L}{L}\right)=\frac{\text { Shearing stress }}{\text { Shear modulus }}$
or $\Delta L=\frac{\text { Shearing stress }}{\text { Shear modulus }} \times \mathrm{L}=\frac{9.8 \times 10^{4}}{25 \times 10^{9}} \times 0.1=0.0392 \times 10-5 \mathrm{~m}=3.92 \times 10-7 \mathrm{~m}$
24) Given, $\operatorname{mass}(\mathrm{m})=14.5 \mathrm{~kg}$

Length of wire $(\mathrm{I})=1 \mathrm{~m}$
Angular frequency $(\mathrm{v})=2$ revls
Angular velocity $(\omega)=2 \pi \mathrm{v}$
$=2 \pi \times 2 \mathrm{rad} / \mathrm{s}=4 \pi \mathrm{rad} / \mathrm{s}$


Area of cross-section of wire $(A)=0.065 \mathrm{~cm}^{2}$

$$
=6.5 \times 10^{-6} \mathrm{~m}^{2}
$$

Young's modulus for steel $(\mathrm{Y})=2 \times 10^{11} \mathrm{~N} / \mathrm{m}^{2}$.
At lowest point of the vertical circle, $\mathrm{T}-\mathrm{mg}=\mathrm{ml} \omega^{2}$
or $\mathrm{T}=\mathrm{mg}+\mathrm{m} \omega^{2}$
$=(14.5 \times 9.8)+14.5 \times 1 \times(4 \pi)^{2}$
$=14.5\left(9.8+16 \pi^{2}\right)$
$=14.5(9.8 \times 16 \times 9.87) \quad\left[\because \pi^{2}=9.87\right]$
$=14.5 \times 167.72 \mathrm{~N}=2431.94 \mathrm{~N}$
Young's modulus $(\mathrm{Y})=\frac{\text { Stress }}{\text { Strain }}=\frac{(T / A)}{\Delta l / l}=\frac{T l}{A \cdot \Delta l}$
$\therefore \quad \Delta l=\frac{T . l}{A . Y}=\frac{2431.94 \times 1}{6.5 \times 10^{-6} \times 2 \times 10^{11}}$ $=1.87 \times 10^{-3} \mathrm{~m}=1.87 \mathrm{~mm}$

