QB365

Important Questions - Complex Numbers and Quadratic Equations

11th Standard CBSE

Mathematics

Reg.No. :

Time : 01:00:00 Hrs

Total Marks : 50

Section-A

1) Find the modulus of the complex number 4 + 3i^7 **2**

2) Express $\frac{1}{1-cos\theta+2isin\theta}$ in the form a+ib. **2**

3) Simplify the following **2**

i^6+i^8

4) Express the equation in the form of a+ib $\left(\frac{1}{3}+3i\right)^3$ **2**

5) Express $\left[\left(\sqrt{5}+\frac{i}{2}\right)\left(\sqrt{5}-2i\right)\right]\div(6+5i)$ in the form of a+ib. **2**

6) Find the equation $\left(\frac{1}{1-4i}-\frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$ as a single complex number **2**

Section-B

7) Express the following in the form of a + ib. $\left[\left(\frac{1}{3}+\frac{7}{3}i\right)+\left(4+\frac{1}{3}i\right)\right]-\left(-\frac{4}{3}+i\right)$ **3**

8) Find the value of $\sqrt{-25}+3\sqrt{-4}+2\sqrt{-9}$ **3**

9) Express the following in the form of a + ib. **3**

$\left(\frac{1}{2}+\frac{5}{2}i\right)-\frac{3}{2}i+\left(-\frac{5}{2}-i\right)$

10) Find the conjugate of complex 3+i. **3**

11) Express $(-2-5i)\div(3-6i)$in the form a+ib **3**

12) A person is represented by a complex number z=x+iy. If a person is represented only by x, then he is not sensitive towards environment and if a person is represented only by y, then he is sensitive towards environment. **3**

If a person is related by the relation $\left|\frac{z-5i}{z+5i}\right|=1$do you think that the person is eco-friendly?

Section-C

13) If z_1 =3+5i and z_2 =-3i, then verify that $\overline{\left(\frac{z_1}{z_2}\right)}=\frac{\bar{z}_1}{\bar{z}_2}$ **4**

14) Convert the complex numbers in polar form. -1+i **4**

15) If f(z) = $\frac{7-z}{1-z^2}$ where z= 1 + 2i , then find $|f(z)|$ **4**

16) Express the complex number $\frac{2+6\sqrt{3}i}{5+\sqrt{3}i}$ into polar form. **4**

17) Find the value of x and y, if $\frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y+i}{3-i}=i$ **4**

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Section-A

1) z = 4 + 3i^4i^3 = 4 - 3i = 5 **2**

2) $z=\frac{1}{(1-cos\theta)+2-sin\theta}\times\frac{(1-cos\theta)-2isin\theta}{(1-cos\theta)-2isin\theta}$ **2**

$=\frac{(1-cos\theta)-2isin\theta}{(1-cos\theta)^2+4sin^2\theta}=\frac{(1-cos\theta)-2isin\theta}{1+cos^2\theta-2cos\theta-4sin^2\theta}$

Ans. $(\frac{1-cos\theta}{2-2cos\theta+3sin^2\theta})+i(\frac{-2sin\theta}{2-2cos\theta+3sin^2\theta})$

3) 0 **2**

4) $-\frac{242}{27}-26i$ **2**

5) $\frac{\left(\sqrt{5}+\frac{i}{2}\right)(\sqrt{5}-2i)}{6+5i}=\frac{5+1+\left(\frac{\sqrt{5}}{2}-2\sqrt{5}\right)i}{6+5i}$ **2**

$=\frac{6-\frac{3\sqrt{5i}}{2}}{6+5i}\times\frac{6-5i}{6-5i}=\frac{36-\frac{15\sqrt{5}}{2}+\left(\frac{-18\sqrt{5}}{2}-30\right)i}{36+25}$

$Ans.\ \frac{72-15\sqrt{5}}{122}-i\left(\frac{30+9\sqrt{5}}{61}\right)$

6) $\left(\frac{1}{1-4i}-\frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$ **2**

$=\frac{(-1+9i)(3-4i)}{(5-3i)(5+i)}=\frac{33+31i}{28-10i}\times\frac{28+10i}{28+10i}$

$=\frac{614+1198i}{784+100}=\frac{614+1198i}{884}$

$Ans.\ \frac{307}{442}+\frac{559}{442}i$

Section-B

7) Consider the given expression. 3

$\left[\left(\frac{1}{3}+\frac{7}{3}i\right)+\left(4+\frac{1}{3}i\right)\right]-\left(-\frac{4}{3}+i\right)$

$=\left[\left(\frac{1}{3}+4\right)+i\left(\frac{7}{3}+\frac{1}{3}\right)\right]-\left(-\frac{4}{3}+i\right)$

$=\left(\frac{13}{3}+\frac{8}{3}i\right)+\left(\frac{4}{3}-i\right)=\left(\frac{13}{3}+\frac{4}{3}\right)+i\left(\frac{8}{3}-1\right)$

$=\frac{17}{3}+\frac{5}{3}i$, which is in the form of a + ib.

8) We have, $\sqrt{-25}+3\sqrt{-4}+2\sqrt{-9}$ 3

= $\sqrt{25}\sqrt{-1}+3\sqrt{4}\sqrt{-1}+2\sqrt{9}\sqrt{-1}$

= 5 x i + 3 x 2 x i + 2 x 3 i

= 5i + 6i + 6i = 17i

9) $\left(\frac{1}{2}+\frac{5}{2}i\right)-\frac{3}{2}i+\left(-\frac{5}{2}-i\right)$ 3

$=\left(\frac{1}{2}-\frac{5}{2}\right)+i\left(\frac{5}{2}-\frac{3}{2}-1\right)$

= -2 + i0, which is in the form of a + ib.

10) Let z =3+i 3

$\therefore \bar{z}=3-i$

[Since, the conjugate of complex number z, is te complex number, obtained by changing the sign of imaginary part of z]

11) $(-2-5i)\div(3-6i)=\frac{-2-5i}{3-6i}$ 3

$=\frac{-(2+5i)}{3-6i}\times\frac{(3+6i)}{(3+6i)}$

[by rationalizing the denominator]

$=\quad-\frac{[6+12i+15i+30i^2]}{3^2-(6i)^2}\left[\because\quad(z_1+z_2)\quad(z_1-z_2)=z_1^2-z_2^2\right]$

$=\frac{-[6+27i-30]}{9+36}\qquad\left[\because\quad i^2=-1\right]$

$=\frac{-(-24+27i)}{45}=\frac{24}{45}-\frac{27}{45}i$

$=\frac{8}{15}-\frac{3}{5}i=\frac{8}{5}+i\left(\frac{-3}{5}\right)$

Which is in the form (a+ib)

12) $Let\quad z=x+iy$ 3

$Now, we\quad have,\quad\left|\frac{z-5i}{z+5i}\right|=1\Rightarrow\frac{|z-5i|}{|z+5i|}=1\qquad\left[\because\left|\frac{z_1}{z_2}\right|=\frac{|z_1|}{|z_2|}\right]$

$\Rightarrow\quad|z-5i|=|z+5i|$

$\Rightarrow\quad|x+(y-5)i|^2=|x+(y+5)i|^2\qquad[put\quad z=x+iy]$

$\Rightarrow\quad x^2+(y-5)^2=x^2+(y+5)^2$

$\Rightarrow\quad y^2+25-10y=y^2+25+10y\Rightarrow 20y=0\Rightarrow y=0$

$\therefore\quad z=x+iy=x+i0=x$

Which represent he is not sensitive towards environment.Hence, the person is not eco-friendly.He needs to be eco-friendly by taking remedial measures.

Section-C

13) 4

$Given,\qquad z_1=3+5i\quad and\quad z_2=2-3i$

$Now,\qquad\frac{z_1}{z_2}=\frac{3+5i}{2-3i}=\frac{3+5i}{2-3i}\times\frac{3+5i}{2-3i}$

$[by\quad rationalising\quad the\quad denominator]$

$=\frac{6+9i+10i+15i^2}{4-9i^2}=\frac{6+19i-15}{4+9}\qquad[\because i^2=-1]$

$=\frac{-9+19i}{13}=\frac{-9}{13}+\frac{9}{13}i\qquad.....(i)$

$\because LHS=\overline{\left(\frac{z_1}{z_2}\right)}=\overline{\left(\frac{-9}{13}+\frac{9}{13}i\right)}=\frac{-9}{13}-\frac{9}{13}i$

$Now,\quad consider\quad RHS\quad=\frac{\bar{z_1}}{\bar{z_2}}=\frac{\overline{3+5i}}{\overline{2-3i}}=\frac{3+5i}{2-3i}$

$=\frac{3+5i}{2-3i}\times\frac{2-3i}{2-3i}$

$[by$

$=\frac{6-9i-10i+15i^2}{4-9i^2}=\frac{6-19i-15}{4+9}\qquad[\because i^2=-$

$=\frac{-9-19i}{13}=\frac{-9}{13}-\frac{9}{13}i\qquad.....(ii)$

$From\quad Eqs.\quad(i)\quad and\quad(ii),\quad we\quad get$

$\overline{\left(\frac{z_1}{z_2}\right)}=\frac{\bar{z_1}}{\bar{z_2}}$

14) $z=\sqrt{2}\left[cos\frac{3\pi}{4}+isin\frac{3\pi}{4}\right]$ 4

15) $\frac{2-i}{2}$ 4

16) First convert the given complex in the standard form and then find its polar form 4

$2\left(cos\frac{\pi}{3}+isin\frac{\pi}{3}\right)$

17) Given $\frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y+i}{3-i}=i$ 4

$\Rightarrow\quad\frac{x+(x-2)i}{3+i}+\frac{2y+(1-3y)i}{3-i}=i$

$\Rightarrow\quad\frac{[x+(x-2)i](3-i)+[2y+(1-3y)i](3+i)}{(3+i)(3-i)}=i$

$\Rightarrow\quad(4x+9y-3)+i\,(2x-7y-3)=10i$

$\Rightarrow\quad 4x+9y-3=0\quad and\quad 2x-7y-3=10$

$Ans.\quad x=3\quad and\quad y=-1$