QB365

Important Questions - Conic Sections

11th Standard CBSE

Mathematics

Reg.No. :

Time : 01:00:00 Hrs

Total Marks : 50

Section-A

1) The focal distance of a point on the parabola $y^2 = 12x$ is 4. Find the abscissa of this point. **2**

2) Find the equation of ellipse, if foci are $(\pm 5, 0)$ and a=6. **2**

3) Find the equation of the hyperbola whose conjugate axis is 5 and the distance between the foci is 13. **2**

4) Find the area of the circle which touches the both axes in the first quadrant and whose radius is a **2**

5) A circle of radius 5 units touches the coordinate axes in the first quadrant.If the circle makes one complete roll on X-axis along the positive direction of X-axis, then find its equation in the new position **2**

6) Find the equation of a circle whose center is(2,0) and touches Y-axis. **2**

Section-B

7) Find the centre and radius of the circle given by the equation $2x^2 + 2y^2 + 3x + 4y + \frac{9}{8} = 0$ **3**

8) Find the equation of the ellipse, whose distance between directrices is 5 and distance between foci is 4. **3**

9) Prove that the line lx + my + n=0 will touch the parabola $y^2=4ax$, if $ln=am^2$ **3**

10) In each of the following questions, find the coordinates of the focus, axis of the parabola, the equation of the directrix and the length of the latusrectum..$y^2=12x$ **3**

11) If e and e' are the eccentricities of the hyperbola $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and its conjugate hyperbola, then prove that $\frac{1}{e^2} + \frac{1}{e'^2} = 1$ **3**

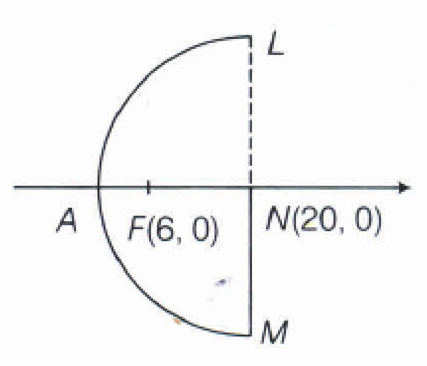
12) Find the equation of the ellipse referred to its axis as the axes of coordinates with latusrectum of length 4 and distance between foci $4\sqrt{2}$ **3**

Section-C

13) Find the equation of the circle whose center is (a,b) and passes through the origin **4**

14) Draw the shape of $\frac{x^2}{100} + \frac{y^2}{400} = 1$and find their vertices, major axis, minor axis, eccentricity, foci, and length of latusrectum. **4**

15) The focus of a parabolic mirror as shown In figure is at a distance 6cm from its vertex. If the mirror is 20cm deep, then find the distance LM. **4**



16) Draw the shape of ellipse $\frac{x^2}{49}+\frac{y^2}{16}=1$and find the foci. **4**

17) The cable of a uniformly loaded suspension bridge hangs in the form of a parabola. The roadway which is horizontal and 100 m long is supported by vertical wires attached to the cable, the longest wire being 30 m and the shortest being 6 m. Find the length of a supporting wire attached to the roadway 10 m from the middle. **4**

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Section-A

1) Given parbola is $y^2 = 12x$ **2**

Here, 4a= 12$\Rightarrow$ a=3

Focus: F= (3,0)

Let P(x, y) be any point on the parabola, then PF = 4

$\Rightarrow \quad (x-3)^2+(y-0)^2= \quad (4)^2$ **Ans** 1

$\Rightarrow \quad x^2+9-6x+12x=16 \qquad [y^2=12x]$

$\Rightarrow \quad x^2+6x-7= \quad 0$

$\Rightarrow \quad x= \quad 1 \qquad [x\neq -7]$

2) $\frac{x^2}{36}+\frac{y^2}{11}=1$ **2**

3) 2b=5 and 2c=13 **2**

$\Rightarrow b=\frac{5}{2} and \quad c=\frac{13}{2}$

$c^2=a^2+b^2 \Rightarrow \frac{169}{4}=\frac{25}{4}+b^2 \Rightarrow a^2=\frac{12}{2}$

$\frac{x^2}{144}-\frac{y^2}{25}=\frac{1}{4}$

4) Required equation of circle is $(x - a)^2 +(y - a)^2 = a^2$ **2**

$x^2+y^2 - 2 ax - 2ay +a^2 = 0$

5) Let C be the centre of the circle in its initial position and D be its centre in the new position, **2**

Then C $\equiv$ (5,5) and D = (5+ 10π , 5)

Now, centre of the circle in the new position is (5+ 10π, 5) and its radius is 5, therefore its equation will be

$(x - 5 -10 \pi)^2 + (y - 5)^2 = 5^2$

$x^2+ 25 + 100 \pi^2- 10x - 20\pi x + 100\pi + y^2+25 - 10y = 25$

$x^2+y^2 - 10 (2\pi+1) x -10y +100\pi^2+ 100+25 = 0$

6) Given, center (h,k)=(2,0)and circle touches Y-axis. **2**

Radius(r)=x-coordinate of center=2 so, the equation of circle is

$(x-2)^2+(y-0)^2 = 2^2$ $[(x-h)^2+(y-k)^2=r^2]$

$x^2+4-4x+y^2=4$ $[(A-B)^2=A^2+B^2-2AB]$

$x^2+y^2-4x+4=4$

$x^2+y^2-4x=0$

which is the required equation of circle

Section-B

7) Given equation is $2x^2 + 2y^2 + 3x + 4y + \frac{9}{8} = 0$ **3**

On dividing both sides by 2, we get

$$x^2 + y^2 + \frac{3}{2}x + 2y + \frac{9}{16} = 0 \qquad \dots.(i)$$

From Eq.(i) we have coefficient of x = $\frac{3}{2}$

and coefficient of y = 2

$$\alpha = \frac{-1}{2}\left(\frac{3}{2}\right) = -\frac{3}{4}$$

$$\beta = \frac{-1}{2}(2) = -1$$

$$centre = \left(-\frac{3}{4}, -1\right)$$

From Eq. (i) we have constant term = $\frac{9}{16}$ Radius = $\sqrt{\left(\frac{-3}{4}\right)^2 + (-1)^2 - \frac{9}{16}}$

$$= \sqrt{\frac{9}{16} + 1 - \frac{9}{16}} = 1$$

8) Let the equation of ellipse be **3**

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$$

Thus, equation of directrices are

$$x = \pm\frac{a^2}{c} = \pm\frac{a^2}{ae} = \pm\frac{a}{e}$$

$$i.e \quad x = \frac{a}{e} and \quad x = -\frac{a}{e}$$

$$Distance \quad between \quad directrices = \frac{2a}{e} = 5$$

$$e = \frac{2a}{5}$$

$$Also, \quad its \quad foci \quad are \quad (\pm c, 0) \quad or \quad (\pm ae, 1).$$

$$Distance \quad between \quad foci = \quad 2ae = 4 \Rightarrow \quad ae = 2$$

$$\Rightarrow a\left(\frac{2a}{5}\right) = 2 \Rightarrow a^2 = 5$$

$$e = \frac{2\sqrt{5}}{5} = \frac{2}{\sqrt{5}} and \quad b^2 = a^2(1 - e^2)$$

$$\Rightarrow b^2 = 5\left(1 - \frac{4}{5}\right) = 5 \times \frac{1}{5}$$

$$b^2 = 1$$

$$Hence, \quad the \quad equation \quad of \quad ellipse \quad is$$

$$\frac{x^2}{5} + \frac{y^2}{1} = 1$$

9) $Given \quad equation \quad of \quad line \quad is \quad lx + my + n = 0.$ **3**

$\Rightarrow \qquad y = \frac{-lx-n}{m}$ $\qquad .....(i)$

$and \quad equation \quad of \quad parabola \quad is \quad y^2 = 4ax$ $\qquad ......(ii)$

$From \quad Eqs. \quad (i) \quad and \quad (ii), \quad we \quad get$

$\left(\frac{-lx-n}{m}\right)^2 = 4ax$

$\Rightarrow \qquad l^2x^2 + 2lxn + n^2 = 4m^2ax$

$\Rightarrow \quad l^2x^2 + 2lxn - 4am^2x + n^2 = 0$

$\Rightarrow l^2x^2 = x(2ln - 4am^2) \quad = \quad n^2 = 0$

$Since, \quad the \quad line \quad lx + my = n \quad touches \quad the \quad parabola.$

$So, Eq.(iii) \quad have \quad equal \quad roots.$

$i.e \quad discriminant \quad (D) = 0 \quad \Rightarrow \quad B^2 - 4AC = 0$

$\Rightarrow \qquad (2ln - 4am^2)^2 - 4l^2n^2 = 0$

$\Rightarrow 4l^2n^2 - 16lnam^2 = 16a^2m^4 - 4l^2n^2 = 0$

$ln = am^2 \qquad Hence \quad Proved$

10) **3**

Given, equation of parabola is .y^2=12x, which is of the form .y^2=4ay i.e focus lies on the positive direction of x-axis

Here, 4a=12 ⇒ a=3

∴ Focus = (a, 0) =$(3, 0)$

Axis = X-axis

Directrix, x=-a ⇒ x=-3 and length of latusrectum = 4a =4×3=12

11) Given hyperbola is $\frac{1}{e^2} + \frac{1}{e'^2} = 1$ **3**

The eccentricity e of this hyperbola is

$e^2 = 1 + \frac{b^2}{a^2} \Rightarrow e^2 = \frac{a^2+b^2}{a^2}$

$\Rightarrow \frac{1}{e^2} = \frac{a^2}{a^2+b^2}$ ....(i)

The equation of the conjugate hyperbola is $\frac{y^2}{b^2} + \frac{x^2}{a^2} = 1$

The eccentricity e' of this hyperbola is

$e'^2 = 1 + \frac{a^2}{b^2} \Rightarrow e'^2 = \frac{b^2+a^2}{b^2}$

$\Rightarrow \frac{1}{e'^2} = \frac{b^2}{a^2+b^2}$ ....(ii)

On adding Eqs.(i) and (ii) we,get

$\frac{1}{e^2} + \frac{1}{e'^2} = \frac{a^2+b^2}{a^2+b^2} = 1$

$\frac{1}{e^2} + \frac{1}{e'^2} = 1$

Hence proved.

12) $Let \quad the \quad equation \quad of \quad ellipse \quad be \quad \frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$ **3**

$Length \quad of \quad latusrectum = \frac{2b^2}{a}=4$

$\Rightarrow b^2=2a$

$Distance \quad between \quad the \quad foci \quad = 2ae = 4\sqrt{2}$

$ae=2\sqrt{2}$

$b^2=a^2(1-e^2)$

$2a=a^2-a^2e^2$

$\Rightarrow 2a=a^2-(2\sqrt{2})^2$

$\Rightarrow 2a=a^2-8 \Rightarrow a^2-2a-8=0$

$\Rightarrow (a-4)(a+2)=0 \Rightarrow a=4 \quad or \quad -2$

$But \quad a \quad cannot \quad be \quad negative.$

$a=4$

$from \quad eqn(i) \quad we \quad get \quad b^2=2\times 4=8$

$Hence, \quad equation \quad of \quad ellipse \quad is$

$\frac{x^2}{16}+\frac{y^2}{8}=1 \Rightarrow x^2+2y^2=16$

Section-C

13) **4**

We know that,circle passes through the origin,so radius of circle will be equal to the distance between poin(a,b) and origin

$\therefore$ Radius of circle=Distancen between points(0,0) and (a,b)

$=\sqrt{(0-a)^2+(0-b)^2}=\sqrt{a^2+b^2}$ [by distance formula]

$\because$ Centre=(h,k)=(a,b)

On putting these values in equation of circle

$(x-h)^2+(y-k)^2=r^2$ we get

$(x-a)^2+(y-b)^2=\left(\sqrt{a^2+b^2}\right)^2$

$\Rightarrow x^2+a^2-2ax+y^2+b^2-2by=a^2+b^2$

$[\because (A-B)^2=A^2-2AB+B^2]$

$\Rightarrow x^2+y^2-2ax-2by=0$

Which is the required equation of circle

14) $Vertices=(0,\pm 20), \quad Major \quad axis=12 \quad ,$ **4**

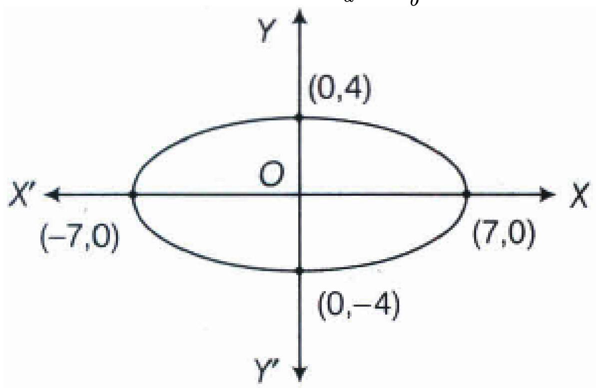
$Minor \quad axis=20, \quad Eccentricity=\frac{\sqrt{3}}{2}$

$Foci=(0,\pm 10\sqrt{3}), \quad Latusrectum=10$

15) $\sqrt[8]{30}\,cm$ **4**

16) $Given \quad equation \quad of \quad ellipse \quad is \quad \frac{x^2}{49}+\frac{y^2}{16}=1.$ **4**

$On \quad comparing \quad with \quad \frac{x^2}{a^2}+\frac{y^2}{b^2}=1, we \quad get \quad a=7, b=4$



Here, a>b, so major axis is along X-axis.

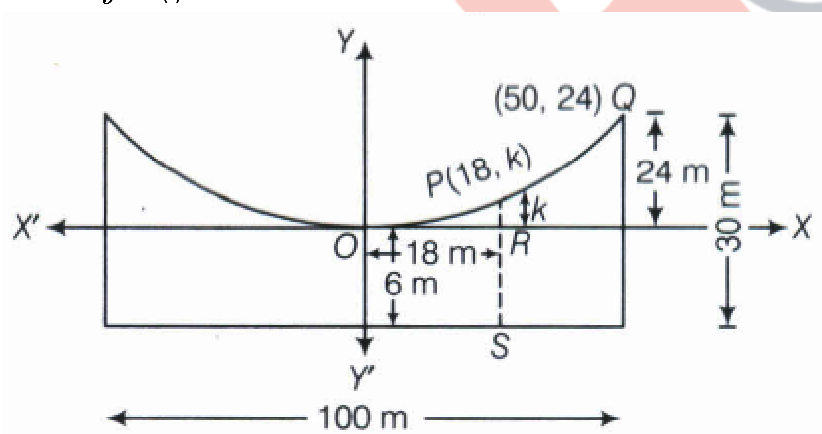
$Foci, (\pm c, 0)=(\pm\sqrt{33}, 0)$

17) **4**

Here, wire are vertical.

Let equation of the parabola be in the form

$x^2 = 4ay$ ...(i)



Focus is at the middle of the cable and shortest and longest vertical supports are 6 m and 30 m and roadway in 100 m long.

Clearly, the coordinates of Q(50, 24) will satisfy Eq.(i)

$\therefore \quad (50)^2=4a\times 24 \Rightarrow 2500=96a \Rightarrow a=\frac{2500}{96}$

$Hence, from \quad Eq.(i), \quad x^2=4\times\frac{2500}{96}y \Rightarrow x^2=\frac{2500}{24}y$

$Let \quad PR=km$

$Then, \quad point \quad P(18, k) \quad will \quad satisfy \quad the \quad equation \quad of \quad parabola.$

$\therefore \quad From \quad Eq.(i), \quad (18)^2=\frac{2500}{24}\times k$

$\Rightarrow \quad 324=\frac{2500}{24}k \Rightarrow k=\frac{324\times 24}{2500}=\frac{324\times 6}{625}=\frac{1944}{625}$

$\Rightarrow \quad k=3.11$

$\therefore \quad Required \quad length \quad = \quad 6+k=6+3.11=9.11m(approx.)$