QB365

Important Questions - Introduction to Three Dimensional Geometry

11th Standard CBSE

Mathematics

Reg.No. :

Time : 01:00:00 Hrs

Total Marks : 50

Section-A

1) Let L,M,N be the feet of the perpendiculars drawn from a point P(3,4,5) on the X,Y and Z-axes respectively.Find the coordinates of L, M and N. **2**

2) If A, B and C are the feet of perpendiculars from a point P on XY, YZ and ZX-planes respectively, then find the coordinates of A, B and C, where the point P is (-5,3,7) **2**

3) If the distance between the points (a,0,1) and (0,1,2) is $\sqrt{27}$,then find the value of a. **2**

4) By using distance formula, show that the points (2,-1,0), (-2,3,-2) and (0,3,0) are collinear **2**

5) A point is on the X-axis, What are its y and z-coordinates? **2**

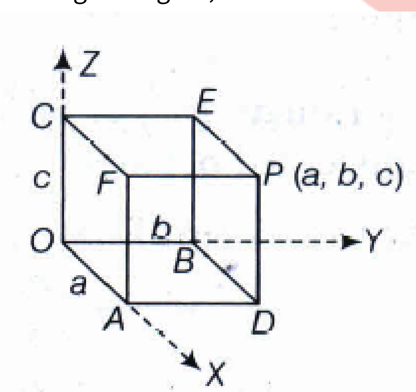
6) If the origin is the centroid of the $\triangle PQR$ with vertices P(2a, 2, 6), Q(-4, 3b, -10) and R(8, 14, 2c), then find the values of a, b and c. **2**

Section-B

7) Find the distance between the points A(2,3,1) and B(1,-2,0) **3**

8) Find the coordinates of a point equidistant from the four points O(0,0,0), A(l,0,0), B(0,m,0), and C(0,0,n) **3**

9) In the given figure, if the coordinates of point P are (a,b,c), then write the coordinates of A,D,B,C and E. **3**



10) What are the conditions of the vertices of a cube whose edge is 5 units, one of whose vertices coincides with the origin and threee edges passing through the origin coincides with the positive direction of the axes through the origin? **3**

11) If a parallelopiped is formed by planes drawn through the points (5,8,10) and (3,6,8) parallel to the coordinate planes, then the length of edges and diagonal of the parallelpiped by using distance formula. **3**

12) Show that the points (0,7,10), (-1,6,6) and (-4,9,6) are the vertices of a right angled isosceles triangle. **3**

Section-C

13) Show that the points (-2,6,-2), (0,4,-1),(-2,3,1) and (-4,5,0) are the vertices of a squre. **4**

14) Show that ΔABC with vertices A(0,4,1), B(2,3,-1), and C(4,5,0) is right angled. **4**

15) Show that the points P(0,7,10), Q(-1,6,6)and R(-4,9,6) form a right angled isosceles triangle. **4**

16) The mid-points of the sides of a triangle are (1,5,-1), (0,4,-2) and (2,3,4). Find its vertices **4**

17) Using the section formula, show that the points, (2, -3, 4), (-1, 2, 1) and (0, $\frac{1}{3}$, 2) are collinear. **4**

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Section-A

1) L(3,0,0), M(0,4,0) and N(0,0,5) **2**

2) A(-5,3,0), B(0,3,7), C(-5,0,7) **2**

3) $AB = \sqrt{(a-0)^2 + (0-1)^2 + (1-2)^2}$ **2**

$\Rightarrow \sqrt{27} = \sqrt{a^2 + 1 + 1}$

$Ans. a = \pm 5$

4) To prove, collinear, AB+BC=AC **2**

5) Coordinates of any point on the X-axis is (x,0,0). **2**

Because at X-axis, both y and z-coordinates are zero.

So, y and z-coordinates of points are zero.

6) Given vertices of $\triangle PQR$ are P(2a, 2, 6), Q(-4, 3b, -10) and R(8, 14, 2c). Centroid of $\triangle PQR$ is **2**

$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$

$= \left(\frac{2a-4+8}{3}, \frac{2+3b+14}{3}, \frac{6-10+2c}{3}\right) = \left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3}\right)$

Also, given that centroid of triangle is origin i.e. (0, 0, 0)

$\therefore \quad \frac{2a+4}{3} = 0 \Rightarrow 2a + 4 = 0 \quad \Rightarrow a = -2$

$\frac{3b+16}{3} = 0 \Rightarrow 3b + 16 = 0 \quad \Rightarrow b = \frac{-16}{3}$

$and \quad \frac{2c-4}{3} = 0 \Rightarrow 2c - 4 = 0 \quad \Rightarrow c = 2$

$Hence \quad a = -2, \quad b = \frac{-16}{3}, \quad c = 2$

Section-B

7) Given points are A(2,3,1) and B(-1,2,0) **3**

Here, x_1=2, y_1=3, z_1=1

and x_2=1, y_2=-2, z_2=0

$\therefore$The distance between the points A and B

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 + z_1)^2}$

$= \sqrt{(1-2)^2 + (-2-3)^2 + (0-1)^2}$

$= \sqrt{(-1)^2 + (-5)^2 + (-1)^2}$

$= \sqrt{1 + 25 + 1}$

$= \sqrt{27} = 3\sqrt{3} units$

8) $Let \quad P(x,y,z) \quad be \quad required \quad point.$ **3**

$Then, \quad OP = PA = PB = PC$

$Now, \quad OP = PA \Rightarrow OP^2 = PA^2$

$\Rightarrow (0-x)^2 + (0-y)^2 + (0-z)^2 = x - 1^2 + y - 0^2 + (z-0)^2$

$\Rightarrow x^2 + y^2 + z^2 = x^2 - 2lx + l^2 + y^2 + z^2$

$\Rightarrow \qquad 2lx = l^2 \Rightarrow x = \frac{1}{2}$

$Similarly, \quad OP = PB \Rightarrow y = \frac{m}{2} and \quad OP = PC \Rightarrow x = \frac{n}{2}$

$Hence, \quad the \quad coordinates \quad of \quad the \quad required \quad point \quad are \quad \left(\frac{1}{2}, \frac{m}{2}, \frac{n}{2}\right).$

9) **3**

Given, the coordinates of point P are (a,b,c).

Which shows that, OA=a, OB=b and OC=c.

Now, point A lies on X-axis, so its coordinates are (a,0,0). Point D lies in XY-plane, so its coordinates are (a,b,0). Point B lies on Y-axis, so its coordinates are (0,b,0).

Point C lies on Z - axis, so its coordinate are (0,0,c) and point E lies in YZ-plane, so its coordinate are (0,b,c).

Hence, the coordinates of required points are

A(a,0,0), D(a,b,0), B(0,b,0), C(0,0,c) and E(0,b,c).

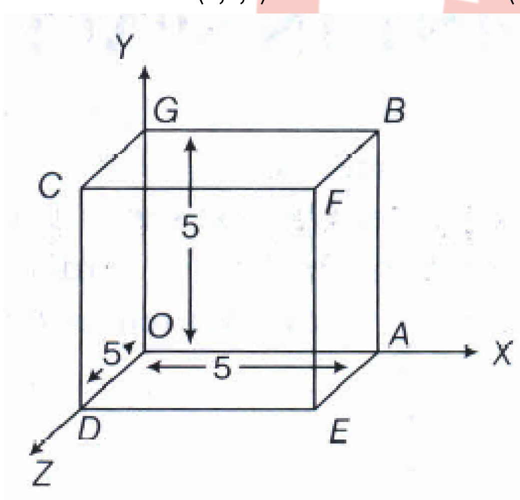
10) Given, edge of a cube is 5 unit. It is clear that **3**

Coordinate of O=(0,0,0) Coordinate of A=(5,0,0)

Coordinate of G=(0,5,0) Coordinate of D=(0,0,5)

Coordinate of B=(5,5,0) Coordinate of F=(5,5,5)

Coordinate of E=(5,0,5) Coordinate of C=(0,5,5)

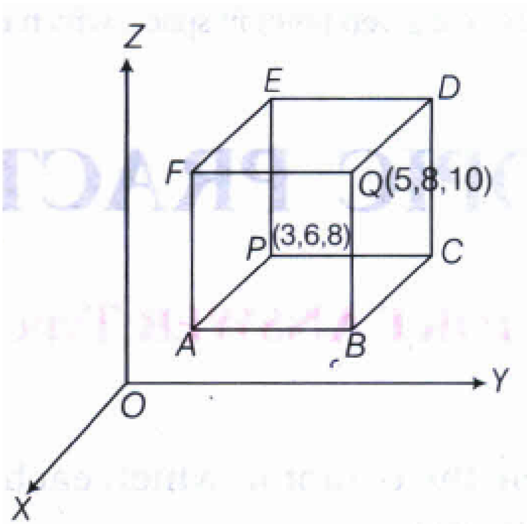


11) Let P=(3,6,8) and Q=(5,8,10) 3

Now, PE=Distance between parallel planes ABCP and FQDE

=|10-8| [therefore these planes are perpendicular to Z-axis]

=2 units



PA=Distance between parallel planes ABQF and PCDE

=|5-3| [because these planes are perpendicular to X-axis]

=2 units

PC=Distance between parallel planes APEF and BCDQ

=|8-6| [because these planes are perpendicular to Y-axis]

=2 unis

Therefore Length of diagonal = Distance between Pand Q

$= \sqrt{(5-3)^2 + (8-6)^2 + (10-8)^2}$ [using the distance formula]

$= \sqrt{2^2 + 2^2 + 2^2}$ units

$= \sqrt{4+4+4} = \sqrt{12} = 2\sqrt{3}$

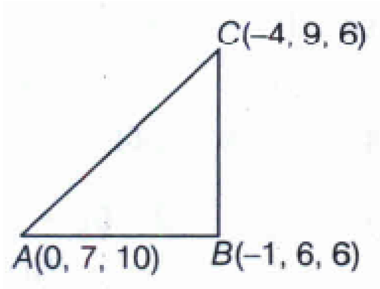
Hence the length of each edge of parallelopiped is 2 units and the length of its diagonal is $2\sqrt{3}$ units.

12) Let A(0,7,10), B(-1,6,6) and C(-4,9,6) be the given points.

Then AB= $= \sqrt{(-1-0)^2 + (6-7)^2 + (6-10)^2}$

[using the distance formula]

$= \sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2}$units



$BC = \sqrt{(-4+1)^2 + (9-6)^2 + (6-6)^2}$

$= \sqrt{9+9+0} = \sqrt{18} = 3\sqrt{2} units$

$and \quad AC = \sqrt{(-4-0)^2 + (9-7)^2 + (6-10)^2} = \sqrt{16+4+16}$

$\Rightarrow AC = \sqrt{36} = 6 \quad units \qquad \ldots.(i)$

$Now, \quad AB^2 + BC^2 = (3\sqrt{2})^2 + (3\sqrt{2})^2 = 18 + 18 = 36 \quad units$

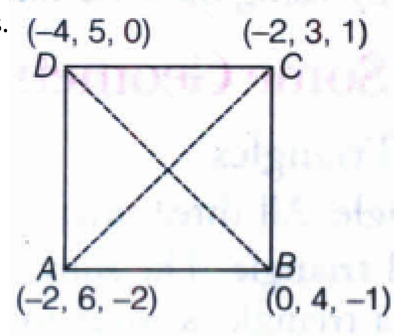
$\therefore AB^2 + BC^2 = AC^2$

$Also, \quad AB = BC^2 = AC^2$

Hence, ABC is a right angled isosceles triangle.

Section-C

13) Let A(-2,6,-2), B(0,4,-1), C(-2,3,1) and D (-4,5,0) be the given points. **4**



$\therefore B=\sqrt{(0+2)^2+(4-6)^2+(1+2)^2}$ [Using the distance formula]

$=\sqrt{4+4+1}=\sqrt{9}=3 \quad units$

$BC=\sqrt{(-2-0)^2+(3-4)^2+(1+1)^2}$

$\sqrt{4+1+4}=\sqrt{9}=3 \quad units$

$CD=\sqrt{(-4+2)^2+(5-3)^2+(0-1)^2}$

$=\sqrt{4+4+1}=\sqrt{9}=3 \quad units$

$AD=\sqrt{(-4+2)^2+(5-6)^2+(0+2)^2}$

$=\sqrt{4+1+4}=\sqrt{9}=3 \quad units$

Here, AB=BC=CD=DA

Now, $AC=\sqrt{(-2+2)^2+(3-6)^2+(1+2)^2}$

$[\because \quad distance=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

$=\sqrt{0+9+9=\sqrt{18} \quad units}$

$and \quad BD=\sqrt{(-4-0)^2+5-4)^2+(0-1)^2}$

$=\sqrt{16+1+1}=\sqrt{18}\, units$

Sine,diagonal AC=diagonal BD

Hence, ABCD is a square.

14) $Show \quad that \quad AB^2+BC^2=AC^2$ **4**

15) $PQ=QR \quad and \quad PQ^2+QR^2=PR^2$ **4**

16) (1, 2, 3), (3, 4, 5) and (-1, 6, -7) **4**

17) **4**

Let C (0, $\frac{1}{3}$, 2) divides the joint of A (2, -3) and B(-1, 2,1) in the ratio k:1

Then coordinates of C are

$\left(\frac{-k+2}{k+1}, \frac{2k-3}{k+1}, \frac{k+4}{k+1}\right)$

[using internal ratio formula]

But coordinates of C are (0, $\frac{1}{3}$, 2)

On comparing Eqs.(i) and (ii) we get

$\frac{-k+2}{k+1}=0 \Rightarrow -k+2=0 \Rightarrow 2$

$\frac{2k-3}{k+1}=\frac{1}{3} \Rightarrow 6k-9=k+1 \Rightarrow 5k=10 \Rightarrow k=2$

$and \qquad \frac{k+4}{k+1}=2 \Rightarrow k+4=2k+2 \Rightarrow k=2$

From each of these equations, we get k=2 Since, from each equation, we get the same value of k.

Therefore, the given points are collinear nad C divides AB internally in the ratio 2:1.