QB365

Important Questions - Limits and Derivatives

11th Standard CBSE

Mathematics

Reg.No. : | | | | | |

Time : 01:00:00 Hrs

Total Marks : 50

Section-A

1) Show that $\lim_{x\to 4}\frac{|x-4|}{x-4}$ does not exist. 2

2) Evaluate the following limits $lim_{x\to 2}\frac{x^2-3x+2}{x^2+x-6}$ 2

3) Evaluate the following limits $lim_{x\to 3}\frac{x^4-81}{2x^2-5x-3}$ 2

4) Evaluate the following limits $lim_{x\to -1}\frac{x^3+1}{x^5+1}$ 2

5) $lim_{x\to 0}\frac{sin^2 2x}{sin^2 4x}$ 2

6) Find the derivative of $x^n + ax^{n-1} + a^2x^{n-2} + \dots a^{n-1}x + a^n$ for some fixed real number a. 2

Section-B

7) Let us consider two functions $f(x) = x^2 + 4$and g(x) = x-3 such that f(x) and g(x) exist at x=5. Find the limit of the following fuctions at x=5. 3

(i) f(x) + g(f)

8) Find the derivative of $f(x)=x^n$, where n is positive integer, by first principle. 3

9) Find the derivative of (x-1)(x-2) from first principle. 3

10) Evaluate $\lim_{x\to 0}\frac{x\left(e^x-1\right)}{1-\cos x}$ 3

11) Find the derivative of the function log x, by using first principle. 3

12) Evaluate the following limits $\lim_{x\to 5}\frac{\log x-\log 5}{x-5}$ 3

Section-C

13) Let $f(x) = \{\begin{matrix}2x+3, if & x\le 0\\ 3(x+1), if & x>0\end{matrix}$, then 5

(i) evaluate $\lim_{x\to 0}f(x)$

14) e^{2x} 5

15) cosecx 5

16) If $y = \frac{sinx+cosx}{sinx-cosx}, then \quad find \quad \frac{dy}{dx} at \quad x = 0.$ 5

17) $\frac{sinx}{x}$ 5

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Section-A

1) Given $\lim_{x\to 4}\frac{|x-4|}{x-4}$ 2

$LHL = \lim_{X\to 4^1}\frac{-(x-4)}{x-4} = 1 \qquad [\because |x-4| = -(x-4), x<4]$

$RHL = \lim_{X\to 4^+}\frac{(x-4)}{x-4} = 1 \qquad [\because |x-4| = (x-4), x<4]$

2) $lim_{x\to 2}\frac{x^2-3x+2}{x^2+x-6} = lim_{x\to 2}\frac{(x-2)(x-1)}{(x-2)(x+3)} \quad \frac{1}{5}$ 2

3) $lim_{x\to 3}\frac{x^4-81}{2x^2-5x-3} = lim_{x\to 3}\frac{(x-3)(x+3)(x^2+9)}{(x-3)(2x+1)} \quad \frac{108}{7}$ 2

4) $lim_{x\to -1}\frac{x^3+1}{x^5+1} = lim_{x\to -1}\frac{\frac{x^3+1}{(x+1)}}{\frac{x^5+1}{x+1}} = lim_{x\to -1}\frac{x^3+1}{x+1} \div lim_{x\to -1}\frac{x^5+1}{x+1} \quad = lim_{x\to -1}\frac{x^3-(-1)^3}{x-(-1)} \div lim_{x\to -1}\frac{x^5-(-1)^5}{x-(-1)} \quad \frac{3}{5}$ 2

5) $lim_{x\to 0}\frac{sin^2 2x}{[sin \quad 2(2x)]^2} = lim_{x\to 0}\frac{sin^2 2x}{(2sin \quad 2x \quad cos \quad 2x)^2} = lim_{x\to 0}\frac{sin^2 2x}{4sin^2 2x \quad cos^2 2x} = lim_{x\to 0}\frac{1}{4cos^2 2x}$ 2

Ans. 1

6) $Let \quad f(x) = x^n + ax^{n-1} + a^2x^{n-2} + \dots a^{n-1}x + a^n Then, \quad f'(x) = \frac{d}{dx}(x^n) + \frac{d}{dx}(ax^{n-1}) + \frac{d}{dx}(a^2x^{n-2}) + \dots + \frac{d}{dx}(a^{n-1}x) + \frac{d}{dx}(a^n)$ 2

Ans : $\boldsymbol{nx^{n-1} + a(n-1)x^{n-2} + a^2(n-2)x^{n-3} + \dots + a^{n-1}}$

Section-B

7) Given functions are f(x) = $x^2 + 4$ and g(x) = x-3 **3**

Clearly, $\lim_{x \to 5} f(x) = \lim_{x \to 5} x^2 + 4 = (5)^2 + 4 = 25 + 4 = 29 \quad \ldots (i)$

and $\lim_{x \to 5} g(x) = \lim_{x \to 5}(x-3) = (5-3) = 2 \quad \ldots (ii)$

(i) $\lim_{x \to 5} [f(x) + g(x)] = \lim_{x \to 5} f(x) + \lim_{x \to 5} g(x)$

=29+2 [from Eqs.(i) and (ii)]

=31

8) **3**

By definition of first principle, we have

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h} \ldots (i)$$

By binomial theorem, we have

$$(x+h)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} h + {}^nC_2 x^{n-2} h^2 + \ldots + {}^nC_n h^n$$

$$\Rightarrow (x+h)^n = x^n + nhx^{n-1} + \frac{n(n-1)}{2} h^2 x^{n-2} + \ldots + h^n \Rightarrow (x+h)^n - x^n = nhx^{n-1} + \frac{n(n-1)}{2} h^2 x^{n-2} + \ldots h^n$$

On putting this value in Eq.(i), we get

$$f'(x) = \lim_{h \to 0} \frac{nhx^{n-1} + \frac{n(n-1)}{2} h^2 x^{n-2} + \ldots + h^{n-1}}{h} = \lim_{h \to 0} \frac{h\left(nx^{n-1} + \frac{n(n-1)}{2} hx^{n-2} + \ldots + h^n\right)}{h} = \lim_{n \to 0}\left[nx^{n-1} + \frac{n(n-1)}{2} hx^{n-2} + \ldots + h^{n-1}\right] = nx^{n-1} hence, \quad f'(x) or \frac{d}{dx} f(x)$$

9) **3**

Let f(x)=(x-1)(x-2)=x^2-3x+2

By first principle of derivative, we have

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{[(x+h)^2 - 3(x+h) + 2] - [x^2 - 3x + 2]}{h} = \lim_{h \to 0} \frac{[x^2 + h^2 + xh - 3x - 3h + 2] - [x^2 - 3x + 2]}{h} = \lim_{h \to 0} \frac{2hx + h^2 - 3h}{h} = \lim_{h \to 0} \frac{h(2x + h - 3)}{h} = 2$$

10) **3**

$$\lim_{x \to 0} \frac{x\left(e^x - 1\right)}{1 - \cos x} = \lim_{x \to 0} \frac{x\left(e^x - 1\right)}{2\sin^2 \frac{x}{2}} \left[\because \quad 1 - \cos x = 2\sin^2 \frac{x}{2}\right] = \frac{1}{2} \lim_{x \to 0} \frac{x\left(e^x - 1\right)}{\sin^2 \frac{x}{2}} \times \frac{x}{x}$$

[Multiplying numerator and denominator by x]

$$= \frac{1}{2} \lim_{x \to 0} \frac{\left(e^x - 1\right)}{x} \times \frac{1}{\frac{\sin^2 \frac{x}{2}}{x^2}} = \frac{1}{2} \lim_{x \to 0} \frac{\left(e^x - 1\right)}{x} \times \frac{1}{\frac{\sin^2 \frac{x}{2}}{\left(\frac{x}{2}\right)^2 \times 4}} = \frac{1}{2} \lim_{x \to 0} \frac{\left(e^x - 1\right)}{x} \times 4 \lim_{x \to 0} \frac{1}{\left(\frac{\sin \frac{x}{2}}{\frac{x}{2}}\right)^2} = \frac{1}{2} \times 1 \times 4 \times \frac{1}{(1)^2} \left[\because \lim_{\theta \to 0} \frac{e^\theta - 1}{\theta} = 1 \quad and \quad \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1\right] = \frac{4}{2} = 2$$

11) Let **3**

f(x) = log(x)

By using first principle of derivative, we have

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\log(x+h) - \log x}{h} = \lim_{h \to 0} \frac{\log\left(\frac{x+h}{x}\right)}{h} = \lim_{h \to 0} \frac{\log\left(1 + \frac{h}{x}\right)}{h} \times \frac{1}{x} = 1 \times \frac{1}{x} = \frac{1}{x} \quad \left[\because \lim_{h \to 0} \frac{e^y - 1}{y} = 1\right]$$

12) Put x-5 = h and as $x \to 5$, then $h \to 0$ **3**

$$\therefore \lim_{h \to 0} \frac{\log(h+5) - \log 5}{h} = \lim_{h \to 0} \frac{\log\left(1 + \frac{h}{5}\right)}{\frac{h}{5} \times 5} Ans. \frac{1}{5}$$

Section-C

13) (i) $LHL = \lim_{x \to 0^-}(2x = 3) = \lim_{h \to 0}[2(0-h) + 3] = 3 RHL = \lim_{x \to 0^+} 3(x+1) = \lim_{h \to 0}[3(0+h+1)] = 3$ **5**

Ans.3

14) $f'(x) = \lim_{h \to 0} \frac{e^{2x+2h} - e^{2x}}{h} = \lim_{h \to 0} \frac{e^{2x}(e^{2h} - 1)}{2h} x2 \quad = 2e^{2x}$ **5**

15) **5**

$$f'(x) = \lim_{h \to 0} \frac{cosec(x+h) - cosecx}{h} = \lim_{h \to 0} \frac{sinx - sin(x+h)}{hsinxsin(x+h)} = \lim_{h \to 0} \frac{2cos\left(x + \frac{h}{2}\right) sin\left(-\frac{h}{2}\right)}{sinxsin(x+h)h} = -cosecxcotx$$

16) $Given, \quad y = \frac{sinx + cosx}{sinx - cosx} \therefore \frac{[(sinx - cosx)(cosx - sinx) - (sin + cosx)(cosx + sinx)]}{(sinx - cosx)^2} = \frac{-(sinx - cosx)^2 - (sinx + cosx)^2}{(sinx - cosx)^2} = -2$ **5**

17) **5**

$$f'(x) = \lim_{h \to 0} \frac{\frac{sin(x+h)}{x+h} - \frac{sinx}{x}}{h} = \lim_{h \to 0} \frac{x[sin(x+h) - sinx] - hsinx}{h.x(x+h)} = \lim_{h \to 0} \frac{x\left[2.cos\left(\frac{x+h+x}{2}\right).sin\left(\frac{x+h-x}{2}\right)\right] - hsinx}{h.x(x+h)} = \lim_{h \to 0} \frac{x\left[2.sin\frac{h}{2}.cos\left(x + \frac{h}{2}\right)\right] - hsinx}{h.x(x+h)}$$