QB365

Important Questions - Linear Inequalities

11th Standard CBSE

Mathematics

Reg.No. :

Time : 01:00:00 Hrs

Total Marks : 50

Section-A

1) Find the conjugate and modulus of the complex number $\frac{2+3i}{3+2i}$ **2**

2) Simplify the following **2**

i^{-39}

3) Find the multiplicative inverse of 1+i. **2**

4) Express the equation in the form of a+ib $(1+i)^4$ **2**

5) Find the equation $\left(\frac{1}{1-4i}-\frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$ as a single complex number **2**

6) Find the real and imaginary parts of the following complex numbers **2**

7

Section-B

7) Find the real values of a and b, if **3**

(3a - 6) + 2ib = - 6b + (6 + a)i

Equate the real and imaginary parts to get the required result

8) Find the value of $\sqrt{-25}+3\sqrt{-4}+2\sqrt{-9}$ **3**

9) Express the following in the form of a + ib. **3**

$\left(\frac{1}{2}+\frac{5}{2}i\right)-\frac{3}{2}i+\left(-\frac{5}{2}-i\right)$

10) If x + iy = $=\sqrt{\frac{1+i}{1-i}}$, then prove that $x^2+y^2=1$ **3**

11) If (1+i)(1+2i)(1+3i)...(1+ni)=(x+iy), then show that 2.5.10....$(1+n^2)=x^2+y^2$ **3**

12) Solve that equation $|z|=z+1+2i$. **3**

Section-C

13) If $\left|z_1\right|=\left|z_2\right|=\ldots=\left|z_n\right|=1$then show that $\left|z_1+z_2+\ldots+z_n\right|=\left|\frac{1}{z_1}+\frac{1}{z_2}+\ldots+\frac{1}{z_n}\right|$ **4**

14) Find the principal argument of $(1+i\sqrt{3})^2$. **4**

15) Solve $5x^2-4ix+9=0$ **4**

16) Find the value of x and y, if $\frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y+i}{3-i}=i$ **4**

17) Find the value of $2x^4+5x^3+7x^2-x+41, when x=-2-\sqrt{3i}$ **4**

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Section-A

1) $z=\frac{2+3i}{3+2i}X\frac{3-2i}{3-2i}=\frac{12+5i}{13}\bar{z}=\frac{12}{13}-\frac{5}{13}i \quad and \quad |z|=1$ **2**

2) i **2**

3) $z=1+i \therefore \quad Multiplication \quad inverse=\frac{1}{z}=\frac{1}{1+i}\times\frac{1-i}{1-i}=\frac{1-i}{2}$ **2**

Ans.$\frac{1}{2}-\frac{i}{2}$

4) $(1+i)^4=(1+i)^2(1+i)^2=(1-1+2i)(1-1+2i)=(2i)(2i)-4+0i$ **2**

5) $\left(\frac{1}{1-4i}-\frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)=\frac{(-1+9i)(3-4i)}{(5-3i)(5+i)}=\frac{33+31i}{28-10i}\times\frac{28+10i}{28+10i}=\frac{614+1198i}{784+100}=\frac{614+1198i}{884}Ans.\ \frac{307}{442}+\frac{559}{442}i$ **2**

6) Let z = 7 = 7 + 0i **2**

Here, Re (z) = 7 and Im(z) = 0

Section-B

7) We have, (3a - 6)+ 2ib = - 6b + (6 +a)i **3**

On equating real and imaginary parts, we get

3a - 6 = - 6b ....(i)

and 2b = 6 + a ....(ii)

Above equations can be rewritten as

3a + 6b = 6 ....(iii)

and a - 2b = -6 ....(iv)

On multiplying Eq.(iv) by 3 and then adding with Eq.(iii), we get

3a + 6b + 3a - 6b = 6 - 18

$\Longrightarrow$ 6a = -12 $\Longrightarrow$ a = - 2

On substituting a = -2 in Eq.(iv), we get

- 2 - 2b = - 6

$\Longrightarrow$ - 2b = - 6 + 2

$\Longrightarrow$ $b = \frac{-4}{-2} = 2$

a = - 2 and b = 2

8) We have, $\sqrt{-25} + 3\sqrt{-4} + 2\sqrt{-9}$ **3**

$= \sqrt{25}\sqrt{-1} + 3\sqrt{4}\sqrt{-1} + 2\sqrt{9}\sqrt{-1}$

= 5 x i + 3 x 2 x i + 2 x 3 i

= 5i + 6i + 6i = 17i

9) $\left(\frac{1}{2} + \frac{5}{2}i\right) - \frac{3}{2}i + \left(-\frac{5}{2} - i\right) \quad = \left(\frac{1}{2} - \frac{5}{2}\right) + i\left(\frac{5}{2} - \frac{3}{2} - 1\right)$ **3**

= -2 + i0, which is in the form of a + ib.

10) **3**

$We \quad have, \quad x + iy = \sqrt{\frac{1+i}{1-i}} \Rightarrow x + iy = \sqrt{\frac{1+i}{1-i} \times \frac{1+i}{1+i}}$

[by rationalising the denominator]

$\Rightarrow \quad x + iy = \sqrt{\frac{(1+i)^2}{1-i^2}} \quad [\because (z_1 - z_2)(z_1 + z_2) = z_1^2 - z_2^2] \Rightarrow \quad x + iy \quad = \frac{1+i}{\sqrt{1+1}} = \frac{1+i}{\sqrt{2}} \quad [\because i^2 = -1] \quad = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$

Now, taking conjugate on both sides, we get

$\overline{x + iy} = \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right) \Rightarrow x - iy = \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} \quad \ldots.(ii)$

On multiplying Eqs. (i) and (ii), we get

$(x + iy)(x - iy) = \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right) \Rightarrow \quad x^2 - (iy)^2 = \left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{i}{\sqrt{2}}\right)^2 \quad [\because (z_1 + z_2)(z_1 - z_2) = z_1^2 - z_2^2] \Rightarrow \quad x^2 \quad - i^2y^2 = \frac{1}{2}$ ·

11) **3**

$|(1 + i)(1 + 2i)(1 + 3i)\ldots.(1 + ni)| = |x + iy| \Rightarrow |1 + i||1 + 2i||1 + 3i|\ldots.|1 + ni| = |x + iy| \Rightarrow \sqrt{1 + 1}\sqrt{1 + 4}\sqrt{1 + 9}\ldots.\sqrt{1 + n^2} = \sqrt{x^2 + y^2} \Rightarrow \sqrt{2}\sqrt{5}\sqrt{10}\ldots.\sqrt{1 + n^2} = \sqrt{x^2 + y^2}$

12) $Put \quad z = x + iy \therefore \quad |x + iy| = (x + iy) + 1 + 2i \Rightarrow \sqrt{x^2 + y^2} = (x + 1) + i(y + 2) \Rightarrow \sqrt{x^2 + y^2} = x + 1 \quad and \quad y + 2 = 0$ **3**

Ans. $\frac{3}{2} - 2i$

Section-C

13) $Given, \quad |z_1| = |z_2| = \ldots = |z_n| = 1 \Rightarrow |z_1|^2 = |z_2|^2 = \ldots = |z_n|^2 = 1 \therefore \quad z_1\bar{z}_1 = z_2\bar{z}_2 = \ldots = z_n\bar{z}_n = 1 \therefore \quad z_1 = \frac{1}{\bar{z}_1}, z_2 = \frac{1}{\bar{z}_2}, \ldots, z_n = \frac{1}{\bar{z}_n}$ **4**

$|z_1 + z_2 + \ldots + z_n| = |\overline{z_1 + z_2 + \ldots + z_n}| = |\bar{z}_1 + \bar{z}_2 + \ldots + \bar{z}_n| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \ldots + \frac{1}{z_n}\right|$

14) **4**

$z = (1 + i\sqrt{3})^2 = 1 - 3 + 2\sqrt{3}i = -2 + 2\sqrt{3}i tan\theta = \left|\frac{2\sqrt{3}}{-2}\right| = \sqrt{3}$

Ans. $\frac{2\pi}{3}$

15) **4**

$We \quad have, \quad 5x^2 - 4ix + 9 = 0 \quad a = 5, \quad b = -4i \quad and \quad c = 9 \quad \therefore \quad x = \frac{-(-4i) \pm \sqrt{(-4i)^2 - 4 \times 5 \times 9}}{2 \times 5} \quad = \frac{-4i \pm \sqrt{(-4i)^2 - 4 \times 5 \times 9}}{2 \times 5}$

16) **4**

Given $\frac{(1+i)x - 2i}{3+i} + \frac{(2-3i)y + i}{3-i} = i$

$\Rightarrow \quad \frac{x + (x-2)i}{3+i} + \frac{2y + (1-3y)i}{3-i} = i \Rightarrow \quad \frac{[x + (x-2)i](3-i) + [2y + (1-3y)i](3+i)}{(3+i)(3-i)} = i \Rightarrow \quad (4x + 9y - 3) + i(2x - 7y - 3) = 10i \Rightarrow \quad 4x + 9y - 3 = 0 \quad and \quad 2x - 7y - 3 = 10 Ans. \quad x = 3$

17)

We have, $x = -2 - \sqrt{3}i$ **[NCERT Exemplar; HOTS]**

$\Rightarrow \quad x + 2 = -\sqrt{3}i$

On squaring both sides, we get

$$(x+2)^2 = (-\sqrt{3}i)^2 \Rightarrow x^2 + 4 + 4x = 3i^2$$

$$[\because (z_1 + z_2)^2 = z_1^2 + z_2^2 + 2z_1z_2]$$

$\Rightarrow \quad x^2 + 4x + 4 = -3 \qquad [\because i^2 = -1]$

$\Rightarrow \quad x^2 + 4x + 7 = 0$

Now divide $2x^4 + 5x^3 + 7x^2 - x + 41$ by $x^2 + 4x + 7$.

$$\begin{array}{r l}
 & 2x^2 - 3x + 5 \\
x^2 + 4x + 7 \,\big)\!\! & 2x^4 + 5x^3 + 7x^2 - x + 41 \\
 & 2x^4 + 8x^3 + 14x^2 \\
 & - \quad - \quad - \\
\hline
 & -3x^3 - 7x^2 - x + 41 \\
 & -3x^3 - 12x^2 - 21x \\
 & + \quad + \quad + \\
\hline
 & 5x^2 + 20x + 41 \\
 & 5x^2 + 20x + 35 \\
 & - \quad - \quad - \\
\hline
 & 6
\end{array}$$

Thus,

$2x^4 + 5x^3 + 7x^2 - x + 41 = \left(x^2 + 4x + 7\right)(2x^2 - 3x + 5) + 6$ $[\because \textit{dividend} = \textit{quotient} \times \textit{divisor} + \textit{remainder}]$ $= 0 \times ((2x^2 - 3x + 5) + 6 = 6$ $[\because x^2 + 4x + 7 = 0]$