QB365

Important Questions - Statistics

11th Standard CBSE

Mathematics

Reg.No. :

Time : 01:00:00 Hrs

Total Marks : 50

Section-A

1) The marks obtained by 7 students are 8,9,11 ,13,14,15,21.Find the variance and standard deviation of these marks. **2**

2) Find the variance of the data 6,5,9,13,12,8 and 10. **2**

3) Find the standard deviation of first 10 natural numbers **2**

4) The mean of 100 observation is 50 and their standard deviation is 5.Find the sum of all squares of all the observations. **2**

5) Find the variance and standard deviation for the following data, 6,7,10,12,13,4,8,12. **2**

6) If $\bar{x}$ is the mean and σ^2 is the variance of n observations,$x_1 + x_2 + x_3 + . . . + x_n$ then prove that the mean and variance of the observations $ax_1, ax_2, . . . , ax_n$are a $\bar{x}$ and a^2 σ^2, respectively (where,$a \neq 0$) **2**

Section-B

7) Find the variance and standard deviation for the following data. **3**

45,60,62,50,65,58,68,44,48

8) Find the mean deviation about the mean for the following data. **3**

38,70,48,40,42,55,63,46,54,44

9) Find the mean deviation from the median for the following data. **3**

x_i	15	21	27	30	35
f_i	3	5	6	7	8

10) Find the mean deviation about the mean for the following data. **3**

Marks obtained	10 - 20	23 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
Number of students	2	3	8	14	8	3	2

11) If each of the observation x_1, x_2 ,....,x is increased by a, where a is a negative or positive number, then show that the variance remains unchanged. **3**

12) Find the mean deviation from the mean for the following data **3**

6, 5, 5.25, 5.5, 4.75, 4.5, 6.25, 7.75, 9.

Section-C

13) The scores of a batsman in 10 innings are 48, 80, 58, 44, 52, 65, 73, 56, 64, 54. Find the mean deviation from the median. **4**

14) Find the mean deviation from the median for the following data. **4**

x_i	15	21	27	30	35
f_i	3	5	6	7	8

15) Find the mean and standard deviation for the following data. **4**

class interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	2	4	6	5	5	5	2	8	5

16) Calculate the mean deviation from the median for the following data. **4**

Wages Per day	Number of workers
20-30	3
30-40	8
40-50	12
50-60	9
60-70	8

17) Find the standard deviation and variance of the following data. **4**

x_i	140	145	150	155	160	165	170	175
f_i	4	6	15	30	36	24	8	2

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Section-A

1) Here, $\bar{x} = \frac{8+9+11+13+14+15+21}{7} = \frac{91}{7} = 13 marks$ **2**

We make the table from the given data

Marks(x_i)^2	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
8	-5	25
9	-4	16
11	-2	4
13	0	0
14	1	1
15	2	4
21	8	64
Total		114

Here, n=7, $\sum (x_i - \bar{x})^2 = 114$

$\therefore \quad \sigma^2 = \frac{1}{n}\sum (x_i - \bar{x})^2 = \frac{114}{7} = 16.29$

Also, standard deviation of marks, $\sigma = \sqrt{16.29} = 4.04$

Hence, variance is 16.29 and standard deviation is 4.04

2) $\frac{52}{7}$ **2**

3) 2.87 **2**

4) Given, $\bar{x}$=50, n=100 and σ=55 **2**

We know that, $\sigma^2 = \frac{\sum x_i^2}{n} - \left(\bar{x}\right)^2 \Rightarrow \frac{\sum x_i^2}{n} = \sigma^2 + \left(\bar{x}\right)^2$

$\Rightarrow \sum x_i^2 = n\left[\sigma^2 + \left(\bar{x}\right)^2\right] = 100\left[5^2 + (50)^2\right]$

=100(25+2500)=252500

Hence, the sum of all squares of all the observation is 252500

5) Given observations are 6,7,10,12,13,4,8,12 **2**

Number of observations =8

$\therefore \quad Mean(\bar{x}) = \frac{6+7+10+12+13+4+8+12}{8} = \frac{72}{8} = 9$

Now, let us make the following table for deviation.

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
6	-3	9	13	4	16
7	-2	4	4	-5	25
10	1	1	8	-1	1
12	3	9	12	3	9
Total		74	Total		74

$\therefore$ Sum of squares of deviations = $\sum_{i=1}^{8}(x_i - \bar{x})^2 = 74$

Hence, variance, $\sigma^2 = \frac{\sum_{i=1}^{8}(x_i-\bar{x})^2}{n} = \frac{74}{8}$=9.25

and standard deviation =$\sqrt{\sigma} = \sqrt{9.25}$=3.04

6) **2**

We have ,mean

$(\bar{x}) = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$ $Now, \quad mean \quad of \quad ax_1, ax_2, \ldots, ax_n = \frac{ax_1 + ax_2 + \ldots + ax_n}{n} = \frac{a(x_1 + x_2 + x_3 + \ldots + x_n}{n} = a\bar{x} \quad [using \quad eq.(i)] Also, \quad we$

$\therefore Variance \quad of \quad ax_1, ax_2, ax_3, \ldots, ax_n = \frac{\sum (ax_1 - a\bar{x})^2}{n} = \frac{a^2(x_1-\bar{x})^2 + a^2(x_2-\bar{x})^2 + \ldots + a^2(x_n-\bar{x})^2}{n} = \frac{a^2\sum (x_1 - \bar{x})^2}{n} = a^2\sigma^2$

Section-B

7) Let $\bar{x}$ be the mean of the given set of observations. 3

Number of observations = 10

$\therefore \quad \bar{x} = \frac{[45+60+62+50+65+58+68+44+48]}{10} = \frac{560}{10} = 56$

Make a table from the given data.

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
45	45-56=-11	121
60	60-56=4	16
62	62-56=6	36
60	60-56=4	16
50	50-56=-6	36
65	65-56=9	81
58	58-56=2	4
68	68-56=12	144
44	44-56=-12	144
48	48-56=-8	64
Total		662

We have, n=10 and $\sum(x_i - \bar{x})^2$ =662

$\therefore \quad Variance, \sigma^2 = \frac{1}{n}\sum(x_i - \bar{x})^2 = \frac{662}{10} = 66.2 and \quad standard \quad deviation = \sqrt{\sigma} = \sqrt{66.2} = 8.136$

8) Given observations are 38,70,48,40,42,55,63,46,54,44 3

Here, number of observations, n = 10

$\therefore$ Mean,

$\bar{x} = \frac{(38+70+48+40+42+55+63+46+54+44)}{10} = \frac{500}{10} = 50$

Let us make the table for deviation and absolute deviation

x_i	$x_i - \bar{x}$	$\left\lvert x_i - \bar{x} \right\rvert$
38	38-50=-12	12
70	70-50=20	20
48	48-50=-2	2
40	40-50=-10	10
42	42-50=-8	8
55	55-50=5	5
63	63-50=13	13
46	46-50=-4	4
54	54-50=4	4
44	44-50=-6	6
Total		$\sum_{i-1}^{10}\left\lvert x_i - \bar{x}\right\rvert = 84$

Now,

$MD = \frac{\sum_{i-1}^{10}\left\lvert x_i - \bar{x}\right\rvert}{10} = \frac{84}{10} = 8.4$

9) The given observations are already in ascending order. Now, let us make the following table from the given data. 3

x_i	f_i	cf	$\lvert x_i - M \rvert$	$f_i \lvert x_i - M \rvert$
15	3	3	15	45
21	5	8	9	45
27	6	14	3	18
30	7	21	0	0
35	8	29	5	40
Total	29		32	148

Here, $N = \Sigma f_i = 29$, which is odd.

So, the median $\left(\frac{N+1}{2}\right)$ th.e. $\left(\frac{29+1}{2}\right)$ = 15th observation, which is equal to 30.

observation

[15th observation lie in the cumulative frequency 21 and its corresponding observation is 30.]

Thus, the median (M) = 30.

Hence, mean deviation from the median.

$$= \frac{\Sigma \lvert x_i - 30 \rvert}{\Sigma f_i} = \frac{148}{29} = 5.1$$

10) Let us make the following table from the given data. 3

Mark obtained	Number of students (f_i)	Mid-point (x_i)	$f_i x_i$	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
10-20	2	15	30	30	60
20-30	3	25	75	20	60
30-40	8	35	280	10	80
40-50	14	45	630	0	0
50-60	8	55	440	10	80
60-70	3	65	195	20	60
70-80	2	75	150	30	60
Total	40		1800		400

Here, $N = \sum_{i=1}^{7} f_i = 40$ and $\sum_{i=1}^{7} f_i x_i = 1800$

Therefore, mean

$$(\bar{x}) = \frac{1}{N}\sum_{i=1}^{7} f_i x_i = 1800 = 45$$

Now, mean deviation,

MD

$$(\bar{x}) = \frac{1}{N}\sum_{i=1}^{7} f_i \lvert x_i - \bar{x} \rvert \quad \left[\sum_{i=1}^{7} f_i \lvert x_i - \bar{x} \rvert = 400\right]$$

$$= \frac{1}{40} \times 400 = 10$$

Hence, the mean deviation about mean is 10.

11) Let $\bar{x}$ be the mean of $x_1, x_2, ..., x_n$. 3

Then, the variance is given by $\sigma_1^2 = \frac{1}{n}\Sigma_{i=1}^{n}(x_i - \bar{x})^2$ . . . . (i)

If a is added to each observation, then the new observation will be $y_i = x_i + a$ ....(ii)

Let the mean of the new observation be $\bar{y}$. Then, $\bar{y} = \frac{1}{n}\Sigma_{i=1}^{n} y_i \Sigma_{i=1}^{n}(x_i + a) = \frac{1}{n}\left[\Sigma_{i=1}^{n} x_i + \Sigma_{i=1}^{n} a\right] = \frac{1}{n}\left[\Sigma_{i=1}^{n} x_i + na\right] = \bar{x} + a$ i.e. $\bar{y} = \bar{x} + a$ . . . (iii)

Now, new variance σ_2^2 is given by

$$\sigma_2^2 = \frac{1}{n}\Sigma_{i=1}^{n}(y_i - \bar{y}) = \frac{1}{n}\Sigma_{i=1}^{n}(x_i + a - \bar{x} - a)^2$$

[using Eqs.(ii) and (iii)]

$$= \frac{1}{n}\Sigma_{i=1}^{n}(x_i - \bar{x} -)^2 = \sigma_1^2$$

Hence, variance remains unchanged

12) Let $\bar{x}$ be the mean of given data. **3**

Then, $\bar{x} = \frac{6+5+5.25+5.5+4.75+4.5+6.25+7.75+9}{9} = \frac{54}{9} = 6$

We make the table from the given data.

x_i	$x_i - \bar{x} = x_i - 6$	$\|x_i - \bar{x}\|$
6	0	0
5.0	-1	1.00
5.25	-0.75	0.75
5.5	-0.5	0.50
4.75	-1.25	1.25
4.5	-1.50	1.50
6.25	0.25	0.25
7.75	1.75	1.75
9	3	3
Total		10.00

∴ Mean deviation from mean,

$MD(\bar{x}) = \frac{\sum |x_i - \bar{x}|}{n} = \frac{10}{9} = 1.1$

Hence, mean deviation form mean is 1.1.

Section-C

13) Arranging the data in ascending order, we have 44, 48, 52, 54, 56, 58, 64, 65, 73, 80 **4**

Here, n=10. So, median is the mean of 5th and 6th terms.

∴ Median (M) = $\left(\frac{56+58}{2}\right) = 57$

We make the table from the given data.

$Scores(x_i)$	Deviation from median $x_i - M$	$\|x_i - M\|$
44	44-57=-13	13
48	48-57=-9	9
52	52-57=-5	5
54	54-57=-3	3
56	56-57=-1	1
58	58-57=1	1
64	64-57=7	7
65	65-57=8	8
73	73-57=16	16
80	80-57=23	23
Total		**86**

∴ Mean deviation = $\frac{\sum |x_i - M|}{n} = \frac{86}{10} = 8.6$

Hence, the mean deviation from the median is 8.6.

14) **4**

Find the median and then calculate the mean deviation about the median by using the formula $\frac{\sum f_i |x_i - M|}{\sum f_i}$.

Here, $N = \sum f_i = 29$, which is odd. So, the median is the $\left(\frac{n+1}{2}\right)th$ observation i.e., $\frac{29+1}{2} = 15th$ observation, which is equal to 30. Thus, median is 30. We make the table from the given data.

x_i	f_i	cf	$\|x_{i-M}\|$	$f_i\|x_i-M\|$
15	3	3	15	45
21	5	8	9	45
27	6	14	3	18
30	7	21	0	0
35	8	29	5	40
Total	29			148

∴ Mean deviation from the median = $\frac{\sum f_i |x_i - 30|}{\sum f_i}$

$= \frac{148}{29} = 5.1$

15) Here, h=10 and a=55 **4**

Let us make the following table from the given data.

Class interval	f_i	Mid-value(x_i)	$u_i = \frac{x_i - 55}{10}$	f_iu_i	u_i^2	$f_iu_i^2$
0-10	3	5	-5	-15	25	75
10-20	2	15	-4	-8	16	32
20-30	4	25	-3	-12	9	36
30-40	6	35	-2	-12	4	24
40-50	5	45	-1	-5	1	5
50-60	5	55	0	0	0	0
60-70	5	65	1	5	1	5
70-80	2	75	2	4	4	8
80-90	8	85	3	24	9	72
90-100	5	95	4	20	16	80
	$N = \sum f_i = 45$			$\sum f_iu_i = 1$		$\sum f_iu_i^2 = 337$

Here,

$$N = \sum f_i = 63, \quad \sum f_iu_i = 1, \sum f_iu_i^2 = 337 \quad \therefore \quad Mean(\bar{x}) = a + \left(\frac{1}{N}\sum f_iu_i\right) \times h and \quad standard \quad deviation, \sigma = \sqrt{h^2}\sqrt{\left\{h^2\frac{1}{N}\sum f_iu_i^2 - \left(\frac{1}{N}\sum f_iu_i\right)^2\right\}}$$

Ans. 55.22,27.36

16) Let us make the following table from the given data. **4**

Wages per day(in Rs)	**Mid value**	**f_i**	**cf**	$\lvert x_i - M\rvert = \lvert x_i - 47.5\rvert$	$f_i\lvert x_i - M\rvert$
20-30	25	3	3	22.5	67.5
30-40	35	8	11	12.5	100.0
40-50	45	12	23	2.5	30.0
50-60	55	9	32	7.5	67.5
60-70	65	8	40	17.5	140.0
Total		40			405.0

Here, $\frac{N}{2} = \frac{40}{2} = 20$. The cumulative frequency just greater than 20 is 23, so the median class is 40-50.

So, we have l=40, f=12, cf=11, h=10 and N=40

Now, Median(M)= $l + \frac{\frac{N}{2} - cf}{f} \times h = 40 + \frac{20-11}{12} \times 10 = 40 + \frac{90}{12} = 40 + 7.5 = 47.5$

and Mean deviation from the median = $\frac{\sum f_i\lvert x_i - M\rvert}{N} = \frac{405}{40} = 10.125$

17) **4**

Let us take assumed mean,a=155

We make the table from the given data.

x_i	f_i	$u_i = \frac{x_i - 155}{5}$	u_i^2	f_iu_i	f_iu_i2
140	4	-3	9	-12	36
145	6	-2	4	-12	24
150	15	-1	1	-15	15
155	30	0	0	0	0
160	36	1	1	36	36
165	24	2	4	48	96
170	8	3	9	24	72
175	2	4	16	8	32
Total	125			77	311

$$\therefore \quad Variance, \sigma^2 = \left[\frac{\sum f_iu_i^2}{N} - \left(\frac{\sum f_iu_i}{N}\right)^2\right] \times h^2 = \left[\frac{311}{125} - \left(\frac{77}{125}\right)^2\right] \times 25 = 25 \times \frac{311}{125} - 25 \times \frac{77 \times 77}{125 \times 125} = \frac{7775}{125} - \frac{148225}{15625} = 62.200 - 9.4864 = 52.7136 \therefore \quad standard \quad deviation = \sqrt{Variai}$$