QB365

Important Questions - Trigonometric Functions

11th Standard CBSE

Mathematics Reg.No. :

Time : 01:00:00 Hrs

Total Marks : 50

Section-A

1) Prove that $cos\left(\frac{3\pi}{4}+x\right)-cos\left(\frac{3\pi}{4}-x\right)=-\sqrt{2}sinx.$ **2**

2) Prove that $cos\quad 2\alpha cos\quad 2\beta+sin^2\left(\alpha-\beta\right)-sin^2\left(\alpha+\beta\right)\quad=\quad cos\quad 2(\alpha+\beta)$ **2**

3) Find the length of arc of circle of radius 5cm, subtending a central angle measuring 15^o . **2**

4) A horse is tied to a post by a rope. If the horse moves along circular path always keeping the rope tight and describe 70 m when it has traced out 80^o at the centre, find the length of the rope. **2**

5) Prove that sin 50^0 cos 85^0 = $\frac{1-\sqrt{2}sin\quad 35^0}{2\sqrt{2}}$ **2**

6) Prove that **2**

$$\sqrt{\frac{1-sin\theta}{1+sin\theta}}=sec\theta-tan\theta,if-\frac{\pi}{2}<\theta<\frac{\pi}{2}$$

$$-sec\theta+tan\theta,if\frac{\pi}{2}<\theta<\frac{3\pi}{2}$$

Section-B

7) $If\quad sin\quad A=\frac{3}{5},0<A<\frac{\pi}{2}and\quad cosB=-\frac{12}{13},\pi<B<\frac{3\pi}{2},find\quad tan)(A-B)$ **3**

8) Convert the following into radians. **3**

240^o

9) Find the values of sin 75^0 **3**

10) Evaluate **3**

sin78^0 cos18^0-cos78^0 sin18^0

11) In any ΔABC , prove that **3**

$\frac{b^2-c^2}{\cos B+\cos C}+\frac{c^2-a^2}{\cos C+A}+\frac{a^2-b^2}{\cos A+\cos B}=0$

12) If $\frac{sin(x+y)}{sin(x-y)}=\frac{a+b}{a-b}$ then show that $\frac{sin(x+y)}{sin(x-y)}=\frac{a+b}{a-b}$ **3**

Section-C

13) Prove that cos 20^0 cos 40^0 cos 60^0 cos 80^0=$\frac{1}{16}$ **4**

14) Solve the following trigonometric equations. **4**

(ii) $secx-tanx=\sqrt{3}$

15) If $\lim_{x\longrightarrow(-a)}\frac{x^7-(-a)^7}{x-(-a)}=7$,then find the value of a. **4**

16) If $\lim_{x\longrightarrow b}\frac{x^3-b^3}{x-b}=\lim_{x\longrightarrow 1}\frac{x^4-1}{x1}$Find all possible value of b. **4**

17) A tree stands vertically on a hill side which makes an angle of 15^o with the horizontal. From a point on the ground 35m down the hill from the base of the tree, the angle of elevation of the hop of tree is 60^o. Find the height of the tree. **4**

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Section-A

1) $cos\left(\frac{3\pi}{4}+x\right)-cos\left(\frac{3\pi}{4}-x\right)$ **2**

$=\left(-\frac{1}{\sqrt{2}}cos\quad x\quad-\quad\frac{1}{\sqrt{2}}sin\quad x\right)-\left(-\frac{1}{\sqrt{2}}cos\quad x\quad+\quad\frac{1}{\sqrt{2}}sin\quad x\right)$

$=-\sqrt{2}sinx.$

2) $cos\quad 2\alpha\quad cos\quad 2\beta\quad+\quad sin\quad(\alpha-\beta+\alpha+\beta)\,sin\quad[\alpha-\beta-(\alpha+\beta)]$ **2**

$=\quad cos\quad 2\alpha\quad cos\quad 2\beta+sin\quad 2\alpha sin\quad(-2\beta)$

$=\quad cos\quad 2\alpha\quad cos\quad 2\beta\quad-\quad sin2\alpha sin\quad 2\beta$

3) $l=r\theta=5\times\frac{\pi}{12}=5\times\frac{22/7}{12}=1.30cm$ **2**

4) $\theta=80^o=\left(80\times\frac{\pi}{180}\right)^c=\frac{4\pi}{9};l=70m$ **2**

$\therefore\qquad\theta=\frac{l}{r}\Rightarrow\frac{4\pi}{9}=\frac{70}{r};r=50.11m$

5) LHS=$\frac{1}{2}$[2sin 50^0 cos 85^0] = $\frac{1}{2}$[sin135^0-sin35^0] **2**

= $\frac{1}{2}$[sin(90^0+45^0)-sin 35^0] = $\frac{1}{2}$[cos 45^0-sin35^0

6) $\sqrt{\frac{1-sin\theta}{1+sin\theta}}=\sqrt{\frac{(1-sin\theta)(1-sin\theta)}{(1+sin\theta)(1-sin\theta}}$ **2**

$=\sqrt{\frac{(1-sin\theta)^2}{1-sin^2}}=\frac{1-sin\theta}{\sqrt{cos^2\theta}}$

$=\frac{1-sin\theta}{|cos\theta|}$

$=\left\{\frac{1-sin\theta}{cos\theta},if\quad-\frac{\pi}{2}<\theta<\frac{\pi}{2}\right.$

$\left.\frac{1-sin\theta}{-cos\theta},if\quad\frac{\pi}{2}<\theta<\frac{3\pi}{2}\right\backslash$

Section-B

7) $Cos \quad A=\sqrt{1-sin^2}A$ **3**

$$=\sqrt{1-\frac{9}{25}=\sqrt{\frac{16}{25}}=\frac{4}{5}}$$

$$\Rightarrow tanA=\frac{3}{4}$$

$$Also, sinB=-\sqrt{1-cos^2}B$$

$$=-\sqrt{1-\frac{144}{169}=-\sqrt{\frac{25}{169}}=-\frac{5}{13}}$$

$$=tan \quad B=\frac{5}{12}$$

$$Now, \quad tan(A-B)=\frac{tanA-tanB}{1+tanAtanB}=\frac{16}{63}$$

$$and \quad cot(A+B)=1$$

$$\Rightarrow \frac{cotAcotB-1}{cotB+cotA}=1$$

$$\Rightarrow cotAcotB-=cotA+cotB$$
$$\Rightarrow cotAcotB-cot \quad A-cotB=1$$
$$\Rightarrow cotAcotB-cotA-cotB+1=2$$
$$\Rightarrow cotA(cotB-1)-(cotB-1)=2$$
$$\Rightarrow (cotB-1)(cotA-1)=2$$

8) $240^o=\frac{\pi}{180}\times 240rad=\frac{4\pi}{3}rad$ **3**

9) $sin75^0=sin\left(45^0 \quad +30^0\right)$ **3**

$$=sin45^0 \quad cos30^0+cos45^0 \quad sin30^0$$
$$[sin(A+B)=sinA \quad cosB+cosA \quad sinB]$$
$$=\frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}.\frac{1}{2}$$
$$\left[sin30^0=\frac{1}{2}, \quad cos30^0=\frac{\sqrt{3}}{2}and \quad sin45^0=cos45^0=\frac{1}{\sqrt{2}}\right]$$
$$=\frac{\sqrt{3}}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}=\frac{\sqrt{3}+1}{2\sqrt{2}}$$

10) $sin78^0 \quad cos18^0-cos78^0 \quad sin18^0$ **3**

$$sin78^0 \quad cos18^0-cos78^0 \quad sin18^0=sin\left(78^0-18^0\right)$$
$$[sinA \quad cosB-cosA \quad sinB=sin(A-B)]$$
$$=sin60^0=\frac{\sqrt{3}}{2}$$

11) Now, $\frac{b^2-c^2}{\cos B+\cos C} = \frac{K^2\sin^2 B-K^2\sin^2 C}{\cos B+\cos C}$ **3**

$= \frac{K^2(\sin^2 B-\sin^2 C)}{\cos B+\cos C}$

$= \frac{K^2(1-\cos^2 B-1+\cos^2 C)}{\cos B+\cos C}$

$= \frac{K^2(\cos^2 -\cos^2 B)}{\cos B+\cos C}$

$= \frac{K^2(\cos B+\cos C)(\cos C-\cos B)}{\cos B+\cos C}$

$= K^2[\cos C - \cos B]$

Similarly, $\frac{c^2-a^2}{\cos C+\cos A} = K^2[\cos A-\cos C]$

and $\frac{a^2-b^2}{\cos A+\sin B} = K^2[\cos B-\cos A]$

$LHS = K^2[\cos C-\cos B+\cos A-\cos C+\cos B-\cos A]$

$= K^2\times 0 = 0$

12) given $\frac{sin(x+y)}{sin(x-y)} = \frac{a+b}{a-b}$ **3**

using componendo and dividendo rule,we get

$\frac{sin(x+y)+sin(x-y)}{sin(x+y)-sin(x-y)} = \frac{a+b+a-b}{a+b-a-b}$

$\Rightarrow \quad \frac{2sin\left(\frac{x+y+x-y}{2}\right).cos\left(\frac{x+y-x+y}{2}\right)}{2cos\left(\frac{x+y+x-y}{2}\right).sin\left(\frac{x+y-x+y}{2}\right)} = \frac{2a}{2b}$

$\left[sinA+sinB = 2sin\frac{A+B}{2}.cos\frac{A-B}{2} and \quad sinA-sinB = 2cos\frac{A+B}{2}.sin\frac{A-b}{2}\right]$

$\Rightarrow \quad \frac{sin \quad x.cos \quad y}{cos \quad x.sin \quad y} = \frac{a}{b} \Rightarrow \quad \frac{tan \quad x}{tan \quad y} = \frac{a}{b} \quad Hence \quad Proved$

Section-C

13) LHS=$\frac{1}{2}$ $\cos 60^0 \cos 20^0(2\cos 40^0\cos 80^0)$ **4**

$=\frac{1}{2}X\frac{1}{2}\cos 20^0(\cos 120^0+\cos 40^0)$

$= \frac{1}{4}\left[\frac{cos20^0}{2} + \frac{1}{2}(cos60^0+cos20^0\right]$

$= \frac{1}{4}\left[\frac{1}{2}\times\frac{1}{2}\right] = \frac{1}{16}$

14) Given equation can be rewritten as **4**

$\sqrt{3}cosx + sinx = 1$

$\Rightarrow \frac{\sqrt{3}}{2}cosx + \frac{1}{2}sinx = \frac{1}{2} \Rightarrow cos(x-\frac{\pi}{6}) = cos\frac{\pi}{3}$

$\Rightarrow x - \frac{\pi}{6} = 2n\pi \pm \frac{\pi}{3}, n\in Z \Rightarrow x = 2n\pi \pm \frac{\pi}{3} + \frac{\pi}{6}, n \in Z$

15) $\lim_{x\longrightarrow(-a)} \frac{x^7-(-a)^7}{x-(-a)} = 7 \Rightarrow 7(-a)^{7-1} = 7$ **4**

$a = \pm 1$

16) $\lim_{x\longrightarrow b}\frac{x^3-b^3}{x-b} = \lim_{x\longrightarrow 1}\frac{x^4-1}{x1}$ **4**

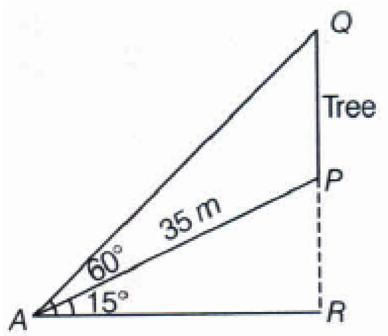
17) According to given information, we have the following figure. **4**

In $\triangle ARQ$, we have

$\angle RAQ = 60^{\circ} and \quad \angle ARQ = 90^{\circ}$

$\therefore \angle AQR = 30^{\circ}$

Now, in $\triangle AQP$, we have $\angle PAQ = 45^{\circ}$ and $\angle AQP = 30^{\circ}$



Using sine rule in $\triangle APQ$, we get

$\frac{AP}{\sin \angle AQP} = \frac{PQ}{\sin \angle PAQ} \Rightarrow PQ = 35\sqrt{2}m.$