QB365

Model Question Paper 1

11th Standard CBSE

Mathematics Reg.No. :

Time : 02:00:00 Hrs

Total Marks : 100

Section-A

1) In a group of 400 people, 250 can speak Hindi and 200 can speak English.How many can speak both Hindi and English? **2**

2) Let n(U) = 700, n(A) = 200, n(B) = 300 and $n(A \cap B)$ = 100 .Find $n(A^{'} \cap B^{'})$. **2**

3) If A and B are two sets such that A ⊂ B, then show that A ⊂ B = B. **2**

4) Let A ={x :x ia natural number } and B ={x :x is an even number}. Find A∩B. **2**

5) Write the range of $y = \frac{|x-1|}{x-1}$. **2**

6) If A= {1,2,3), B={1,2,3,4}, C= {5,6} and D={5,6,7,8}, then verify that $(A \times C) \subset (B \times D)$ **2**

7) In △ABC, if cos A=$\frac{sin \quad B}{2 \quad sin \quad C}$, show that the triangle is isosceles. **2**

8) Prove that $cos \quad 2\alpha cos \quad 2\beta + sin^2(\alpha - \beta) - sin^2(\alpha + \beta) \quad = \quad cos \quad 2(\alpha + \beta)$ **2**

9) Find the length of arc of circle of radius 5cm, subtending a central angle measuring 15^o . **2**

10) A horse is tied to a post by a rope. If the horse moves along circular path always keeping the rope tight and describe 70 m when it has traced out 80^o at the centre, find the length of the rope. **2**

11) Prove that **2**

$$2sec^2\theta - sec^4\theta - 2cosec^2\theta + cosec^4\theta = \frac{1 - tan^8\theta}{tan^4\theta}$$

12) Prove that 2+6+18+.....2.3^{n-1} = (3^n - 1) for all $n \in N$. **2**

13) Using principle of mathematical induction, prove that **2**

$$\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \ldots + \frac{1}{(3n-1)(3n+2)} = \frac{n}{6n+4}$$

14) Find the conjugate of the complex number $\frac{1-i}{1+i}$ **2**

15) Find the equation $\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$ as a single complex number **2**

Section-B

16) Which of the following sets are finite and which are infinite? **3**

{ xϵR:0<x<1 }

17) From the following sets given below, pair the equivalent sets. **3**

A= {1, 2, 3}, B= {t, p, q, r, s}, C= {α, β, γ} and D= {a, e, i, o, u}

18) Show that n {P [P (P (ϕ))]}=4 **3**

19) Let A and B be two sets. Prove that $(A - B) \cup B = A$ if and only if $B \subset A$ **3**

20) Which of the following relations are functions? **3**

$\{(2,0),(4,8),(2,1),(3,6)\}$

21) If n(A)=3 and B={2,3,4,6,7,8}, then find the number of relations from A to B. **3**

22) Let f be the exponential function and g be the logarithmic function. Find **3**

(f+g) (1)

23) Prove that **3**

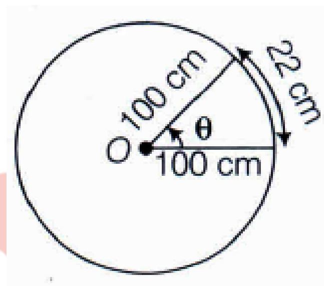
$tan^2 \quad \theta - sin^2\theta = tan^2\theta sin^2\theta.$

24) Differentiate the following functions **3**

(iii) $log_e e^{cos \quad x}$

25) Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length **3**

22 cm as shown in figure.[use π = 22/7]



Section-C

26) If A={4^n-3n-1, nϵN} and **4**

B={9(n-1):nϵN}, show that A$\subset$B.

27) There are 200 individuals with a skin disorder, 120 has been exposed to chemical C_1, 50 to chemical C_2 and 30 to both the chemicals C_1, C_2 . FInd the number of individuals exposed to chemical C_1 but not chemical C_2. **4**

28) Let f and g be real functions defined by **4**

f(x)=2x+1 and g(x)=4x-7.

For what real numbers x, f(x)<g(x)?

29) Determine the domain and range of the relation R, where R = {(2x+3, x^3)}:x is a prime number less than 10}. **4**

30) Find the domain of the function f defined by **4**

$f(x) = \sqrt{4-x} + \frac{1}{\sqrt{x^2-1}}.$

31) Let A={-1,3,4} and B={2,3}.Represent the product $A \times B$ graphically **4**

32) Prove that **4**

(ii) $sin\frac{\theta}{2}sin\frac{7\theta}{2} + 2sin\frac{3\theta}{2}sin\frac{11\theta}{2}$ =$sin2\theta sin5\theta$

33) If $\lim_{x \longrightarrow b} \frac{x^3-b^3}{x-b} = \lim_{x \longrightarrow 1} \frac{x^4-1}{x1}$ Find all possible value of b. **4**

34) If a + ib = $\frac{(x^2+1)}{2x^2+1}$, prove that $a^2+b^2 = \frac{(x^2+1)^2}{(2x+1)^2}$ **4**

35) Find the value of x and y, if $\frac{(1+i)x-2i}{3+i} + \frac{(2-3i)y+i}{3-i} = i$ **4**

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Section-A

1) n(H) = 250, n(E) = 200 and $n(H\cup E)$= 400 **2**

$\therefore n(H\cup E)=n(H)+n(E)-n(H\cap E)$

2) $n(A^{'}\cap B^{'})=n(U)-n(A\cup B)$ **2**

Ans.300

3) **2**

A⊂B means that, all the elements of set A are in Set B and also there are some other elements in B. We known that, A ∪B contains all the elements either in A or in B or in both A and B. Thus, A∪B = B

4) **2**

Given, A ={x :x is a natural number} ={ 1, 2,3, 4.....} and B ={x : x is an even naturai number} = 2, 4, 6,.......}

We observe that 2, 4, 6...... are the elements which are common to both the sets A and B.

A∩B={2, 4, 6, ...}=B

5) {-1,1} **2**

6) **2**

$A\times C$={(1,5),(1,6),(2,5),(2,6),(3,5),(3,6)}

$B\times D$={(1,5),(1,6),(1,7),(1,8),(2,5),(2,6),(2,7),(2,8),(3,5),(3,6),(3,7),(3,8),(4,5),(4,6),(4,7),(4,8)}

Here, elements (1,5),(1,6),(2,5),(2,6),(3,5),(3,6) $\in A\times C$ are also belongs to $B\times D$, therefore $(A\times C)\subset(B\times D)$

7) $cos\quad A=\frac{sin\quad B}{2sin\quad C}\Rightarrow\frac{b^2+c^2-a^2}{2bc}=\frac{kb}{2kc}$ **2**

$\Rightarrow b^2+c^2-a^2=b^2\Rightarrow c^2=a^2\Rightarrow c=a$

8) $cos\quad 2\alpha\quad cos\quad 2\beta\quad+\quad sin\quad(\alpha-\beta+\alpha+\beta)\,sin\quad[\alpha-\beta-(\alpha+\beta)]$ **2**

$=\quad cos\quad 2\alpha\quad cos\quad 2\beta+sin\quad 2\alpha sin\quad(-2\beta)$

$=\quad cos\quad 2\alpha\quad cos\quad 2\beta\quad-\quad sin2\alpha sin\quad 2\beta$

9) $l=r\theta=5\times\frac{\pi}{12}=5\times\frac{22/7}{12}=1.30cm$ **2**

10) $\theta=80^o=\left(80\times\frac{\pi}{180}\right)^c=\frac{4\pi}{9};l=70m$ **2**

$\therefore\qquad\theta=\frac{l}{r}\Rightarrow\frac{4\pi}{9}=\frac{70}{r};r=50.11m$

11) $LHS=2sec^2\theta-sec^4\theta-2cos\quad ec^2\theta+cos\quad ec^4\theta$ **2**

$=2(1+tan^2\theta)-(1+tan^2\theta)^2-2(1+cot^2\theta+cot\theta)-(1+2tan^2\theta+tan^4\theta)$

$=2(tan^2\theta-cot^2\theta)+(2tan^2\theta-2tan^2\theta)+(cot^4\theta-tan^4\theta)$

$=\frac{1}{tan^4\theta}-tan^4\theta=\frac{1-tan^8\theta}{tan^4\theta}=RHS$

12) Consider P(k):2+2.3+2.3^2 + ...+2.3^{k-1} = (3^k-1) **2**

Now,P(k+1):2+2.3+2.3^2 + ...+2.3^{k-1} +2.3^k

=(3^K-1)+2.3^k =3.3^k -1 =3^{k+1}-1

13) $\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+\dots+\frac{1}{(3k-1)(3k+2)}=\frac{k}{6k+4}$ **2**

Now P(k+1): $\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+\dots+\frac{1}{(3k-1)(3k+2)}+\frac{1}{[3(k+1)-1][3(k+1)+2]}$

$=\frac{k}{2(3k+2)}+\frac{1}{(3k+2)(3k+5)}$

$=\frac{3k^2+5k+2}{2(3k+2)(3k+5)}+\frac{(3k+2)(k+1)}{2(3k+2)(3k+5)}=\frac{k+1}{2(3k+5)}$

$=\frac{k+1}{6(k+1)+4}$

14) $z=\frac{1-i}{1+i} X \frac{1-i}{1-i}=\frac{1-1-2i}{1+1}=-i$ = i **2**

15) $\left(\frac{1}{1-4i}-\frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$ **2**

$=\frac{(-1+9i)(3-4i)}{(5-3i)(5+i)}=\frac{33+31i}{28-10i}\times\frac{28+10i}{28+10i}$

$=\frac{614+1198i}{784+100}=\frac{614+1198i}{884}$

$Ans. \frac{307}{442}+\frac{559}{442}i$

Section-B

16) Given, { xϵR:0<x<1 } Here, 0 We know that between any two real numbers, there are infinite real numbers. **3**

∴ The set { xϵR:0<x<1 } is an infinite set.

17) Given, A= {1, 2, 3} ⇒ n(A) = 3 **3**

B= {t, p, q, r, s} ⇒ n(B) = 5

C= $\{\alpha, \beta, \gamma\}$ ⇒ n(C) = 3

D= {a, e, i, o, u} ⇒ n(D) = 5

Here n(A) = n(C) = 3 and n(B) = n(D) = 5

∴ The sets A, C and B, D are equivalent sets.

18) We have, P (ϕ)={ϕ } **3**

∴ P (P(ϕ))={ϕ ,{ϕ }}

⇒ P[P(Pϕ))]={ϕ },{ϕ },{{ϕ}},{ϕ ,{ϕ }}}

Hence, number of elements in P [P(P (ϕ))] is 4

i.e. n {P [P(P (ϕ))]}=4

19) Let $(A-B)\cup B=A$ **3**

$\Rightarrow \quad (A\cap B')\cup B=A \quad [\because A-B=A\cap B']$

$\Rightarrow \quad (A\cup B)\cap(B'UB)=A \quad [by \quad distribute \quad Law]$

$\Rightarrow \quad (A\cup B)\cap=A \quad [\because B'\cup B=U]$

$\Rightarrow \quad A\cup B=A$

$\Rightarrow \quad B\subset A$

Conversely, let $B\subset A$

$\because \quad (A-B)\cup B=(A\cap B')\cup B$

$=(A\cup B)\cap(B'\cup B) \quad [bu \quad distribute \quad law]$

$=(A\cup B)\cap U \quad [\because B'\cup B=U]$

$=A\cup B=A \quad [\because B\subset A]$

20) $\{(2,0),(4,8),(2,1),(3,6)\}$ **3**

It is not a function because first elements of (2,0) and (2,1) are same.

21) Given, n(A) = 3 **3**

and B = {2,3,4,6,7,8}

⇒ n(B) = 6

∴ Number of relations from A to B $=2^{n(A)\times n(B)}$

$=2^{3\times 6} \quad =2^{18}$

22) We have, f: R---> given by f(x)=e^x **3**

and g: R----> R given by g(x)=$log_e x$

Since, domain (f)∩ domain(g)(x)=$R \cap R^+ = R^+$

∴ f+g: R^+ ---> R is defined as

(f+g)(x)=f(x)+g(x)=e^x+$log_e x \forall x \epsilon R^+$

Clearley,1ϵR^+

∴ (f=g)(1)=f(1)+g(1)=e^1+$log_e 1$=e+0=e [∴log1=0]

23) LHS= **3**

$$= tan^2\theta - sin^2\theta$$
$$= \frac{sin^2\theta}{cos^2\theta} - sin^2\theta \qquad \left[\because tan\theta = \frac{sin\theta}{cos\theta}\right]$$
$$= \frac{sin^2\theta - sin^2\theta cos^2\theta}{cos^2\theta} = \frac{sin^2\theta(1-cos^2\theta)}{cos^2\theta} \qquad [\because 1 - cos^2\theta = sin^2\theta]$$
$$= \frac{sin^2\theta . sin^2\theta}{cos^2\theta} = tan^2\theta . sin^2\theta \qquad [\because \frac{sin\theta}{cos\theta} = tan\theta]$$

=LHS Hence Proved.

24) Let y = $log_e e^{cos \quad x}$ ⇒ y = cos x $[\because logm^n = nlogmand \quad log_e e = 1$ **3**

$\because log_e e^{cos \quad x} = cosxloglog_e e = cosx]$

On differentiating both sides w.r.t.x,we get

$$\frac{dy}{dx} = \frac{d}{dx}(cosx) = -sinx$$

25) Given, radius, r = 100 cm and arc length, l = 22 cm **3**

We know that, l = rθ

$\therefore \theta = \frac{1}{r} = \frac{Arc \quad length}{Radius} = \frac{22}{100} = 0.22 \quad rad$

$= 0.22 \times \frac{180}{\pi} degree$

$= \left(0.22 \times \frac{180 \times 7}{22}\right)^o = \left(\frac{22}{100} \times \frac{180 \times 7}{22}\right)^0$

$= \left(\frac{126}{10}\right)^0 = \left(12\frac{6}{10}\right)^0 = 12^0 + \frac{6}{10} \times 60' \, [\because 1^0 = 60']$

$= 12^0 + 36' = 12^0 \quad 36'$

Hence, the degree measure of the required angle is $12^0 \quad 36'$

Section-C

26) Now, 4^n-3n-1=$(3+1)^n$-3n-1 **4**

$$= 1 + 3n + \frac{n(n-1)}{2}.(3)^2 + ... - 3n - 1$$
$$= 9(\frac{n(n-1)}{2} + ...)$$

and B={9(n-1):nϵN}

27) Required number of individuals=$n(C_1 \cap C_2^{'})$ **4**

=$n(C_1) - n(C_1 \cap C_2)$ = 120-30= 90

28) $\because \quad f(x) < g(x)$ **4**

$\therefore \quad 2x + 1 < 4x - 7 \quad -2x + 1 < -7$

$\Rightarrow \quad -2x < -8 \Rightarrow x > 4$

29) Domain = {7, 9, 13, 17} Range = {8, 27, 125, 343} **4**

30) Domain = $(-\infty, -1) \cup (1, 4)$ **4**

31) **4**

Given sets are A={-1,3,4} and B={2,3} which are in tabular form

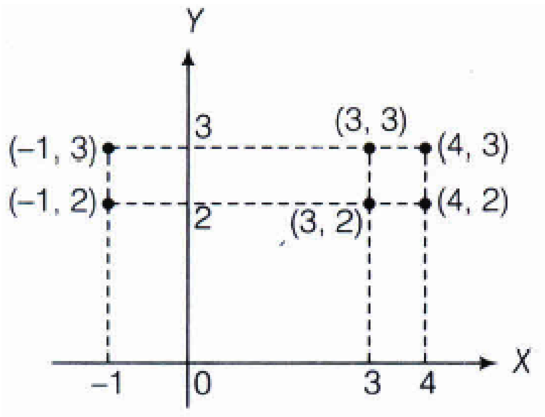
Now,

$A \times B = \{(-1,2),(-1,3),(3,2),(3,3),(4,2),(4,3)\}$

Let us draw two perpendicluar lines OX and OY intersecting at O.Represent the elements of set A i.e -1,3,4 on horizontal line OX and elements of set B i.e 2,3 on vertical line OY,by taking 1cm=1unit

Draw vertical dotted lines through the poits -1,3 and 4 OX and horizontal dotted lines through the poits 2 and 3 of OY.points of intersection of the se lines are(-1,2),(-1,3),(3,3),(3,2),(4,3)and (4,2)

The poits so obtained represent the cartesian product of sets A and B on graph



32) $LHS \quad = \frac{1}{2}\left(2sin\frac{\theta}{2}sin\frac{7\theta}{2}+2sin\frac{3\theta}{2}sin\frac{11\theta}{2}\right)$ **4**

$= \frac{1}{2}(cos3\theta - cos4\theta + cos4\theta - cos7\theta)$

$= \frac{1}{2}(cos3\theta - cos7\theta) = sin5\theta sin2\theta$

33) $\lim\limits_{x \longrightarrow b} \frac{x^3-b^3}{x-b} = \lim\limits_{x \longrightarrow 1} \frac{x^4-1}{x1}$ **4**

34) We have, a + ib = $\frac{(x^2+1)}{2x^2+1}$ ......(i) **4**

Take modulus both sides of E_q . (i) and then solve it.

35) Given $\frac{(1+i)x-2i}{3+i} + \frac{(2-3i)y+i}{3-i} = i$ **4**

$\Rightarrow \quad \frac{x+(x-2)i}{3+i} + \frac{2y+(1-3y)i}{3-i} = i$

$\Rightarrow \quad \frac{[x+(x-2)i](3-i)+[2y+(1-3y)i](3+i)}{(3+i)(3-i)} = i$

$\Rightarrow \quad (4x+9y-3)+i\,(2x-7y-3) = 10i$

$\Rightarrow \quad 4x+9y-3=0 \quad and \quad 2x-7y-3=10$

$Ans. \quad x=3 \quad and \quad y=-1$