QB365

Model Question Paper 2

11th Standard CBSE

Mathematics

Reg.No. : | | | | | | |

Time : 02:00:00 Hrs

Total Marks : 100

Section-A

1) Solve 3x + 8> 2, when
(i) x is an integer. **2**

2) In drilling world's deepest hole it was found that the temperature T in degree Celcius x km below the Earth's surface was given by
T (x) = 30 + 25(x - 3), where $3 \leq x \leq 15$. At what depth will the temperature be between 155^{o} C and 205^{o} C? **2**

3) Solve $12 + 1\frac{5}{6}x \leq 5 + 3x$, When (i) x ϵ N **2**

4) Anushu obtained 73, 67, 72 marks in the Mathematics test. How many should he get in his fourth test, so as to have an average of at least 75? **2**

5) In how many ways can the letters of the word 'MATHEMATICS' be arranged so that vowels are always together? **2**

6) Find n, if (n=2)!=2550 **2**

7) How many words, with or without meaning can be formed using all the letters of the word EQUATION, using each letter exactly once? **2**

8) How many different signals can be made by 5 flags from 8 flags of different colours? **2**

9) Simplify the following expression **2**
$\left(y + \frac{1}{y}\right)^6 - \left(y - \frac{1}{y}\right)^6$

10) Find the coefficient of **2**
x^3 in the expansion of $\left(3x - \frac{1}{x}\right)^7$

11) Find the coefficient of x in the expansion of $(1-3x+7x^2)(1-x)^{16}$ **2**

12) The gate receipts at the show of ' Comedy Nights ' 9500 on the first night and showed a drop of every succeeding night. If the operational expenses of the show are 2000 a day , then find on which night, the show ceases to be a profitable? **2**

13) Using the distance formula show that the points A(3,-2), B(5,2) and C(8,8) are collinear. **2**

14) Find the equation of the line passing through (1,2) and parallel to the line y=3x-1 **2**

15) Find the values of θ and p, if the equation xcos θ +ysin θ =p is the normal form of the line $\sqrt{3x}$ +y+2=0 **2**

Section-B

16) Solve the linear inequality 3x - 5 < x + 7, when
(i) x is a natural number **3**

17) Solve the linear inequality 3x - 5 < x + 7, when
(ii) x is a whole number **3**

18) In a monthly test, the teacher decides that there will be three questions, one from each of the Exercise 7, 8 and 9 of textbook. If there are 11 questions in Exercise 7, 15 in Exercise 8 and 10 in Exercise 9, in how many ways can three questions be selected ? **3**

19) Find the number of different words can be formed from the letters of the word "TRIANGLE", so that **3**
(i) all vowels occur together.
(ii) all vowels do not occur together.

20) Evaluate $\frac{n!}{(n-r)!}$ when r =3. **3**

21) The English alphabet has 5 vowels and 21 consonants. How many worlds with 2 different vowels and 2 different consonants can be formed from the alphabet ? **3**

22) Find the number of terms in the expansions of following expressions. **3**
$(x+3y)^2$

23) Find the number of terms in the expansions of following expressions. **3**
$(1-z)^4$

24) If the third term in the expansion of $\left(\frac{1}{x} + x^{log10^x}\right)^5$ is 1000, then find x. **3**

25) Find the equation of hyperbola, the length of whse latusrectum is 8, eccentricity is $\frac{3}{\sqrt{5}}$ and whose transverse and conjugate axes are along the X and Y-axes respectively. **3**

Section-C

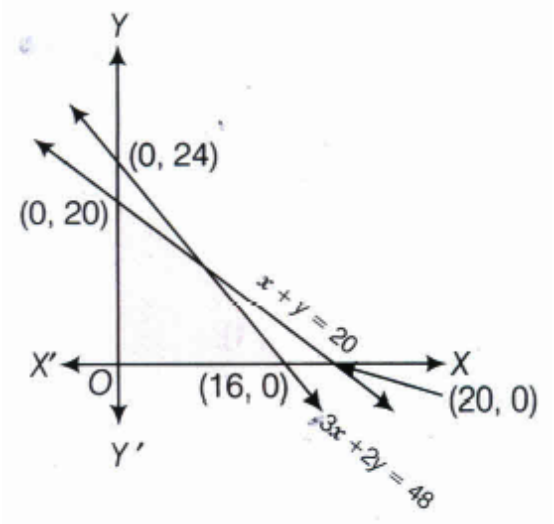
26) Find all pairs of consecutive odd natural numbers, both of which are larger than 10, such that their sum is less than 40. **4**

27) A solution is to be kept between 86° F and 95° F What is the range of temperature in degree Celsius, if the Celsius (C)/ Fahrenheit (F) **4**
Conversion formula is given by $F = \frac{9}{5} C + 32$.

28) The cost and revenue functions of products given by C(X)=20+4000 and R(X)=60+2000 respectively, where x is the number of items produced and sold. How many items must be sold to realise some profit? **4**

29) A man wants to cut three lengths from a single piece of board of length 91 cm, the second length is to be 3 cm longer than the shortest and third length is to be twice as long as the shortest. What are the possible lengths for the shortest board, if third piece is to be at least 5cm longer than the second? **4**

30) Find the linear inequalities for which the shaded region on the given figure is the solution set. **4**



31) Evaluate $^{20}C_5 + \sum_{r=1}^{5} {}^{25-r}C_4$ **4**

32) Find how many arrangement can be made with the letter of the word 'MATHEMATICS'? **4**

33) Using binomial theorem, evaluate each of the following **4**

$(12)^5 + (8)^5$

34) Evaluate $(1.025)^{-1/3}$ correct to three places of decimal. **4**

35) If A is the arithmetic mean and G_1 , G_2 be two geometeric means between any two numbers, then prove that **4**

$\frac{G1^2}{G_2} + \frac{G2^2}{G_1}$ =2A

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Section-A

1) (i) {-1, 0, 1, 2, 3,...} **2**

2) Let at s km below the Earth's surface, the temperature is between 155°C and 205°C. **2**

$\therefore$ 155 < T(s) < 205

$\Rightarrow$ 155 < 30 + 25(s - 3)< 205

8<s<10.

3) $12 + \frac{11x}{6} \leq 5 + 3x \Rightarrow \frac{7x}{6} \geq 7 \Rightarrow \quad x \geq 6$ **2**

(i) {6, 7, 8 , 9,...}

4) $\frac{73+67+72+x}{4} \geq 75$ **2**

88 marks

5) **Ans**. 120960 **2**

6) n=49 **2**

7) Their are 8 distinct letters in a given word. So, total number of words can be formed is 8. **2**

Ans. 40320

8) Required number of signals is same as number of permutatons of 8 different things taken 5 at a time. **2**

Ans. 6720

9) $\left(y + \frac{1}{y}\right)^6 - \left(y - \frac{1}{y}\right)^6 = 2$ **2**

$\left[{}^6C_1 y^{6-1}\left(\frac{1}{y}\right)^1 + {}^6C_3 y^{6-3}\left(\frac{1}{y}\right)^3 + {}^6C_5 y^{6-5}\left(\frac{1}{y}\right)^5\right]$

$= 2\left[6y^5.\frac{1}{y} + 20y^3.\frac{1}{y^3} + 6y.\frac{1}{y^5}\right] = 12\left(y^4 + \frac{1}{y^4} + \frac{10}{3}\right)$

10) $T_{r+1} = {}^7C_r \quad 3^{7-r} \times \quad x^{7-2r} \quad (-1)^r$ **2**

For coefficient of x^3, put 7-2r=3 $\Rightarrow$ r=2

$T_3 = T_{2+1} = {}^7C_r \quad 3^{7-2} \quad x^{7-4} \quad = 5103$

11) $(1-3x+7x^2)(1-x)^{16}$ **2**

$=(1-3x+7x^2)({}^{16}C_0-{}^{16}C_1x+{}^{16}C_2x^2+\ldots)$

$=({}^{16}C_0-{}^{16}C_1x+{}^{16}C_2x^2 \quad -\ldots.)-3x({}^{16}C_0-{}^{16}C_1x+\ldots)+7x^2({}^{16}C_0-{}^{16}C_1x+\ldots)$

Here, the term containing x is

${}^{16}C_1x-3{}^{16}C_0x=-16x-3x$

$\therefore$ Coefficient of x=-16-3=-19

Ans: -19

12) 1400 **2**

13) **2**

14) 3x-y-1=0 **2**

15) Given equation of line is $\sqrt{3x}$ + y+ 2 =0 **2**

$-\sqrt{3x}$ - y=2

on dividing both sides of Eq (i) by

$\sqrt{(-\sqrt{3})^2+(-1)^2}=2$ we get

$\frac{-\sqrt{3}}{2}x-\frac{1}{2}y=1 \quad or \quad xcos210^0+ysin210^0=1$

$\theta=210^0 \quad and \quad p=1$

Section-B

16) We have, 3x - 5 < x + 7 **3**

$\Rightarrow$ 3x -5 + 5 < x + 7 + 5 [adding 5 on both sides]

$\Rightarrow$ 3x < x + 12

$\Rightarrow$ 3x -x < x + 12 - x

[Subtracting x from both sides]

$\Rightarrow$ 2x < 12

$\Rightarrow$ $\frac{2x}{2}<\frac{12}{2}$

$\Rightarrow$ x < 6

Now if

(i) x is natural number, then the solution set is

{1, 2, 3, 4, 5}

17) We have, 3x - 5 < x + 7 **3**

$\Rightarrow$ 3x -5 + 5 < x + 7 + 5 [adding 5 on both sides]

$\Rightarrow$ 3x < x + 12

$\Rightarrow$ 3x -x < x + 12 - x

[Subtracting x from both sides]

$\Rightarrow$ 2x < 12

$\Rightarrow$ $\frac{2x}{2}<\frac{12}{2}$

$\Rightarrow$ x < 6

Now if

(ii) x is a whole number, then the solution set

{0, 1, 2, 3, 4, 5}

18) 3

Clearly, from Excercise 7, one question can be selected in 11 ways.

From Exercise 8, one question can be selected in 15 ways.

and from Exercise 9, one question can be selected in 10 ways.

Since, each operation is performed after performing the previous operation, so we use fundamendal principle of multiplication.

∴ Number of ways of selecting three questions

$= 11 \times 15 \times 10 = 1650$

19) 3

There are 8 distinct letters in the word TRIANGLE, out of which 3 are vowels namely A, E, I and 5 are consonants, namely T, R, N, G, L.

(i) Since the vowels have to occur together with 5 remaining letters will be counted as 6 objects and these can be arranged in ${}^{6}P_{6} = 6!$ ways.

Corresponding to each of these permutations, we have 3! = 6 permutations of the three vowels A, E and I taken all at a time.

Hence, by fundamental principle of multiplication, the required number of words = Number of all possible arrangements of 8 letters taken all at a time - Number of permutation in which the vowels are always together

$= {}^{6}P_{6} - 6! \times 3! = 8! - 6! \times 3!$

$= 8 \times 7 \times 6! - 6! \times 3! = 6!(56-6)$

$= 720 \times 50 = 36000$

20) We have, $\frac{n!}{(n-r)!} = \frac{n!}{(n-3)!}$ $[\because r = 3, \text{given}]$ 3

$= \frac{n(n-1)(n-2)(n-3)!}{(n-3)!}$

$[\because n! = n(n-1)(n-2)(n-3)!]$

$= n(n-1)(n-2)$

21) 3

Here we have 5 vowels and 2 consonants. We have to select 2 vowels out of 5 vowels and 2 consonants out of 21consonants Also, we have to arrange these 4 letters which can be in 4! ways ∵ requored number of words = ${}^{5}C_{2} \times {}^{21}C_{2} \times 4!$

$= \frac{5\times4}{2} \frac{21\times20}{2} \times 24$

=10 x 210 x 24 =50400

Hence, the total number of words having 2 consonants and 2 vowels is 50400

22) Given expression is $(x+3y)^2$. Here, n = 2 3

∴ The number of terms in expansions is (n+1)

i.e 2+1 = 3

23) Given expression is $(1-z)^{4.}$ Here, n= 4 3

∴ The number of terms in the expansion is (n+1)

i.e. 4+1 = 5

24) Given, T_3=1000 **3**

$\Rightarrow {}^5C_2\left(\frac{1}{x}\right)^{5-2}(x^{\log 10 \quad x})^2 = 1000$

$\Rightarrow 10(x^{\log 10 \quad x})^2 \times x^{-3} = 1000 \Rightarrow x^{2log10 \quad x} \times x^{-3} = 100$

$\Rightarrow x^{2 \quad log10 \quad x-3} = 10^2$

$\Rightarrow 2log_{10 \quad} x - 3 = log_x 10^2$

$\Rightarrow 2log_{10 \quad} x - 3 = \frac{2}{log_{10} x}$

$\Rightarrow 2y - 3 = \frac{2}{y}, where \quad y = log_{10} x$

$\Rightarrow 2y^2 - 3y - 2 = 0 \Rightarrow (2y+1)(y-2) = 0$

$\Rightarrow y = 2 \quad or \quad y = -\frac{1}{2} \Rightarrow log_{10} x = 2 \quad or \quad log_{10} x = -\frac{1}{2}$

$\Rightarrow x = 10^2 or \quad x = 10^{-1/2}$

$\therefore$ x= 100 or $x = \frac{1}{\sqrt{10}}$

25) We have, transverse axis is along the X-axis. So, the equation of hyperbola is **3**

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ ---- (i)

Length of its latusrectum = $\frac{2b^2}{a}$

$\Rightarrow \frac{2b^2}{a} = 8 \quad \Rightarrow b^2 = 4a$

$Now, \quad e = \sqrt{1 + \frac{b^2}{a^2}} \Rightarrow \frac{3}{\sqrt{5}} = \sqrt{1 + \frac{4a}{a^2}}$

$[\because e = 3\sqrt{5} and \quad b^2 = 4a]$

$\Rightarrow \frac{9}{5} = 1 + \frac{4}{a} \Rightarrow \frac{4}{a} = \frac{4}{5} \Rightarrow a = 5$

$Thus, \quad a^2 = 25 \quad and \quad b^2 = 4a = 4(5) = 20$

Hence, the equation of hyperbola is $\frac{x^2}{25} - \frac{y^2}{20} = 1$

Section-C

26) (11, 13), (13, 15), (15, 17), (17, 19) **4**

27) Between 30°C and 35°C **4**

28) We know that, Profit=Revenue- Cost **4**

In order to realise some profit, revenue should be greater than the cost

Thus, we should have R(x)>C(x)

$\Rightarrow 60x + 2000 > 20x + 4000$

$\Rightarrow 60x + 2000 - 20x + 4000 - 20x \quad [subtracting \quad 20x \quad from \quad both \quad sides]$

$\Rightarrow 40x + 2000 > 4000$

$\Rightarrow 40x + 2000 - 2000 > 4000 - 2000 [subtracting \quad 2000 \quad from \quad both \quad sides]$

$\Rightarrow 40x > 2000 \Rightarrow \frac{40x}{40} > \frac{2000}{40} [dividing \quad both \quad sides \quad by \quad 40]$

$\Rightarrow x > 50$

$Hence, \quad the \quad manufacturer \quad must \quad sell \quad more \quad than$

$50 \quad times \quad to \quad realise \quad some \quad profit$

29) 4

Let the length of the shortest piece be x cm, so that the lengths of second and third piece are(x+3) cm and 2x cm respectively.Then x+ (x+3)+2x$\leq$91

and 2x$\geq$(x+3)+5

From inequality (i), we get

$4x + 3 \leq 91$

$\Rightarrow 4x + 3 - 3 \leq 91 - 3[subtarcting \quad 3 \quad from \quad both \quad sides]$

$\Rightarrow 4x \leq 88[dividing \quad both \quad sides \quad by \quad 4]$

$\Rightarrow \frac{4x}{4} \leq \frac{88}{4}$

$\Rightarrow x \leq 22$

From inequality (ii), we get

$2x \geq x + 8$

$\Rightarrow 2x - x \geq x + 8 - x[subtracting \quad x \quad from \quad both \quad sides]$

$\Rightarrow x \geq 8$

From inequality (iii) and (iv), we get

$8 \leq x \leq 22$

Hence, the shortest piece must be atleast 8 cm long but not more than 22cm long

30) 4

For the equation x+y=20, the shaded area and origin both lies on the same side of the live, therefore the compounding inequality is $x + y \leq 20$.

Similarly, $3x + 2y \leq 48$

Also,the shaded portion lines in Ist quadrant.

$\therefore \quad x \geq 0 \quad and \quad y \geq 0$

Ans. $x + y \leq 20, 3x + 2y \leq 48, x \geq 0, y \geq 0$

31) 42504 4

32) The letters in the word 'MATHEMATICS' are (MM, AA, TT, H, E, I, C, S). $\because$Total arragements = $\frac{11!}{2!2!2!}$ 4

33) $(12)^5 + (8)^5 = (10 + 2)^5 + (10 - 2)^5$ 4

$(10 + 2)^5 = {}^5C_0 \times 10^5 + {}^5C_1 \times 10^4 \times 2 + {}^5C_2 \times 10^3 \times 2^2 + \ldots\ldots\ldots (i)$

$(10 - 2)^5 = {}^{10}C_5 \times 5 - {}^5C_1 \times 10^4 \times 2 + {}^5C_2 \times 10^3 \times 2^2 + \ldots\ldots\ldots (ii)$

$Now \quad adding \quad equ(i) \quad and \quad (ii)$

$(12)^5 + (8)^5 = 2\left[{}^5C_0 10^5 + {}^5C_2 (10)^3 (2)^2 + {}^5C_4 (10)^1 (2)^4\right]$

$= 281600$

34) $(1.025)^{-1/3} = (1+0.025)^{-1/3}$ 4

$= 1 - \left(\frac{1}{3}\right)(0.025) + \frac{\left(\frac{1}{3}\right)\left(\frac{1}{3}+1\right)}{2}(0.025)^2$

= 1 - 0.0083 + 0.000139 - ...

= 0.991

35) A = $\frac{a+b}{2}$ and G_1 c ar, $G_2 = ar^2$ 4

Where, $r = \left(\frac{b}{a}\right)^{\frac{1}{2+1}} = \left(\frac{b}{a}\right)^{1/3}$ .....(i)

$\therefore \quad \frac{G1^2}{G_2} + \frac{G2^2}{G_1} = \frac{a^2r^2}{ar^2} + \frac{a^2r^4}{ar}$