Unit 1 to 5 Two Marks Questions With Answer

12th Standard CBSE

Maths

25 x 2 = 25

1) Define Reflexive.Give one example.

Answer :

Reflexive Relation : A relation R on a set A is called reflexive relation if aRa for every $a \in A$; if (a,a) ∈ R, for every $a \in A$

Example let

A=[1,2,3]

AxA =(1,1) (1,2)(1,3) (2,1) (2,2) (2,3) (3,1)(3,2) (3,3) $\in R$

Since (a,a) $\in R$ for every $a \in A$

2) Define Transitive Relation. Give one example.

Answer : A relation R on a non-empty set A is called a transitive relation if (a, b), (b, c) $\in R$ then (a, c) $\in R$, i.e., aRb, bR

Thus a relation R on a non empty set A is said to be transitive if there exist a, b, c $\in A$ such that (a, b)(b, c) $\in R$ implies (

Example

Let A = (1,2, 3, 6)

R = (3,6)(6,1)(3,1)

., 3 R 6 and 6 R 1 ⇒(3,1) $\in R$

:. A is transitive.

3) Let f:$X \rightarrow Y$ be a function Define a relation R on X given be R=[(a,b) ; (f(b)] Show that R is an equivalence relation ?

Answer : (i) Let a $\in x$ then $f(a) = f(a), \Rightarrow (a,a) \Rightarrow R$ Relation is reflexive

(ii) Let (a,b) $\in R$ then $f(a) = f(b) \Rightarrow f(b) = f(a), \Rightarrow (b,a) \in R$ Relation is symetric

(iii) Let (a,b) (b,c) $\in R$ then $f(a) = f(b), f(b) = f(c) \Rightarrow f(a) = f(c) \quad (a,c) \in R$ Relation is transitive.

All the three relation are satisfied the relation is equivalence.

4) Let A be the set of all human beings in a town at a particular time. Determine whether the relation R = {(x, y) : x is wife of y ; x, Y∈A} is reflexive, symmetric and transitive.

Answer :

Given A = Set of all human beings in a town at a particular time and R = {(x, y); x is a wife of y; x, Y ∈ A}

(i) Since x is a wife of x, is not true (x, x) ∉ R

So, R is not reflexive.

(ii) x is a wife of y, but y not wife of (y, x) ∉ : R. So, R is not symmetric.

(iii) x is a wife of y, and y is wife of z But this situation does not exist. So, R is not transitive.

5) What is meant by one-one function?

Answer : A function $f: A \rightarrow B$ is said to be one-one if

$a \neq b$

$\Rightarrow f(a) \neq f(b) for \quad all \quad a,b, \in A$

or f(a)=f(b)

a=b for all a,b,A

In other words, is one-one, if no two elements of A have same image, i.e., no two elements of A are mapped to same eleme

6) $f(x) = x^2, x \in R$ Find $\frac{f(1.1)-f(1)}{1.1-1}$

Answer : $f(x) = x^2, x \in Ra$

$f(1.1) = (1.1)^2 z$

$= 1.21$

$f(1) = (1)^2 = 1$

$\frac{f(1.1)-f(1)}{1.1-1} = \frac{1.21}{1.1-1} = \frac{0.21}{0.1}$

$= 2.1$

7) Let A = (a, b, e) and B = (I, 2, 3). Find r of the following function f from A to B, if it exists. (i)f = {(a, 3)(b,2)(e, I)} (ii) {(a, 2)(b, 1)(e, I)}.

Answer : we have f = {(a, 3) (b, 2) (e, I)}, f is a one-one function.

$f^{-1}, f^0 = [(3,a),(2,b)(1,c)]$

(ii) f = {(a, 2), (b, l)(e, I)}. f is not one-one, f is not onto because 3 E B as it has no Pre-image.

8) If the binary operation * on the set of integers Z is defined by a*b=3a+b^2 then find the value of (i) 4*3 (ii) 5*2

Answer : (i) Given a*b=3a+b^2

$\Rightarrow 4*3=3(4)+(3)^2$

=12+9

4*3=21

(ii) Given a*b=3a+b^2

$\Rightarrow 5*2=3(5)+2^2$

$\Rightarrow 5*2=15$

$\therefore 5*2=19$

9) Show that : $tan^{-1}\frac{3}{4}+tan^{-1}\frac{3}{5}-tan^{-1}\frac{8}{19}=\frac{\pi}{4}$

Answer : $tan^{-1}\frac{3}{4}+tan^{-1}\frac{3}{5}-tan^{-1}\frac{8}{19}$

$=tan^{-1}\left(\frac{\frac{15+12}{20}}{\frac{20-9}{20}}\right)-tan^{-1}\frac{8}{19}$

$\left[\because tan^{-1}x+tan^{-1}y=tan^{-1}\left(\frac{x+y}{1+xy}\right)\right]$

$=tan^{-1}\frac{27}{11}-tan^{-1}\frac{8}{19}$

$\left[\because tan^{-1}x-tan^{-1}y=tan^{-1}\left(\frac{x-y}{1+xy}\right)\right]$

$=tan^{-1}\left(\frac{\frac{27}{11}-\frac{8}{19}}{1+\frac{27\times8}{11\times19}}\right)$

$=tan^{-1}\left(\frac{\frac{513-88}{209}}{\frac{209+216}{209}}\right)$

$=tan^{-1}\left(\frac{425}{425}\right)$

$=tan^{-1}1=\frac{\pi}{4}$

10) Show that : $tan^{-1}\frac{2}{3}=\frac{1}{2}tan^{-1}\frac{12}{5}$

Answer : $L.H.S.=tan^{-1}\frac{2}{3}=\frac{1}{2}\left(2tan^{-1}\frac{2}{3}\right)$

$=\frac{1}{2}\left[tan^{-1}\left(\frac{2\times\frac{2}{3}}{1-\frac{4}{9}}\right)\right]$

$\left[\because 2tan^{-1}x=tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$

$=\frac{1}{2}tan^{-1}\left[\frac{\frac{4}{3}}{\frac{9-4}{9}}\right]$

$=\frac{1}{2}tan^{-1}\left(\frac{4\times9}{3\times5}\right)$

$=\frac{1}{2}tan^{-1}\left(\frac{12}{5}\right)=R.H.S.$

11) Prove that : $sin^{-1}x+cos^{-1}x=\frac{\pi}{2};if\in x[-1,\quad 1]$

Answer : We have $x\in[-1,\quad 1]$

Let x = sin θ $\quad\therefore\quad \theta\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$\Rightarrow\quad \theta=sin^{-1}x$

$\Rightarrow\quad -\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$

$\Rightarrow\quad \frac{\pi}{2}\ge-\theta\ge-\frac{\pi}{2}$

$\Rightarrow\quad \frac{\pi}{2}+\frac{\pi}{2}\ge\frac{\pi}{2}-\theta\ge-\frac{\pi}{2}+\frac{\pi}{2}$

$\Rightarrow\quad \pi\ge\frac{\pi}{2}-\theta\ge0$

$\Rightarrow\quad cos\left(\frac{\pi}{2}-\theta\right)=sin\quad\theta=x$

$\Rightarrow\quad \frac{\pi}{2}-\theta=cos^{-1}x$

$\Rightarrow\quad cos^{-1}x=\frac{\pi}{2}-sin^{-1}x$

$\Rightarrow\quad sin^{-1}x+cos^{-1}x=\frac{\pi}{2}if\quad x\in[-1,\quad 1]$

12) Write in the simplest form : $sin\left[2tan^{-1}\sqrt{\frac{1-x}{1+x}}\right]$

Answer : Let x = cos 2θ

$=sin\left[2tan^{-1}\sqrt{\frac{1-cos2\theta}{1+cos2\theta}}\right]$

$=sin\left[2tan^{-1}\sqrt{\frac{2sin^2\theta}{2cos^2\theta}}\right]$

$\left[\because cos2\theta=1-2sin^2\theta\right.$

$\left.and\quad cos\quad 2\theta=2cos^2\theta-1\right]$

$=sin\left[2tan^{-1}\left(tan\quad\theta\right)\right]$

$=sin(2\theta)=\sqrt{1-cos^2 2\theta}$

$=sin\quad 2\theta=\sqrt{1-x^2}$

13) Solve the equation $tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}tan^{-1}x$

Answer : $tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}tan^{-1}x$ [Given]

Put, x = tan θ $\Rightarrow$ $\tan^{-1}$ x=θ

$\Rightarrow$ $tan^{-1}\left[\frac{1-tan\theta}{1+tan\theta}\right]=\frac{1}{2}\theta$

$\Rightarrow$ $tan^{-1}\left[tan\left(\frac{\pi}{4}-\theta\right)\right]=\frac{\theta}{2}$

$\Rightarrow$ $\frac{\pi}{4}-\theta=\frac{\theta}{2}$

$\Rightarrow$ $\frac{\pi}{4}=\frac{\theta}{2}+\theta$

$\Rightarrow$ $\frac{\pi}{4}=\frac{3\theta}{2}\Rightarrow\theta=\frac{\pi}{6}$

$\Rightarrow$ $tan^{-1}x=\frac{\pi}{6}$

$\Rightarrow$ $x=tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}$

14) Solve the matrix equation $\begin{bmatrix}x^2\\y^2\end{bmatrix}-3\begin{bmatrix}x\\2y\end{bmatrix}=\begin{bmatrix}-2\\-9\end{bmatrix}$

Answer : We have, $\begin{bmatrix}x^2\\y^2\end{bmatrix}-3\begin{bmatrix}x\\2y\end{bmatrix}=\begin{bmatrix}-2\\-9\end{bmatrix}$

$\Rightarrow$ x^2 - 3x = - 2 and y^2 - 6y = - 9

$\Rightarrow$ x^2 - 3x + 2 = 0 and y^2 - 6y + 9 = 0

$\Rightarrow$ x^2 - 2x - x + 2 = 0 and y^2 - 3y - 3y + 9 = 0

$\Rightarrow$ x(x - 2) - 1(x - 2) = 0 and y(y - 3) - 3(y - 3) = 0

$\Rightarrow$ (x - 2)(x - 1) = 0 and (y - 3)(y - 3) = 0

$\therefore$ x = 1, 2 and y = 3, 3

15) Find the value of X and Y if $X+Y=\begin{bmatrix}2&3\\5&1\end{bmatrix},X-Y=\begin{bmatrix}6&5\\7&3\end{bmatrix}$

Answer : We have, $X+Y=\begin{bmatrix}2&3\\5&1\end{bmatrix},X-Y=\begin{bmatrix}6&5\\7&3\end{bmatrix}$

$(X+Y)+(X-Y)=\begin{bmatrix}2&3\\5&1\end{bmatrix}+\begin{bmatrix}6&5\\7&3\end{bmatrix}$

$2X=\begin{bmatrix}8&8\\12&4\end{bmatrix}$

$\Rightarrow$ $X=\begin{bmatrix}4&4\\6&2\end{bmatrix}$

$\therefore$ $X=\begin{bmatrix}4&4\\6&2\end{bmatrix}$

and $Y=\begin{bmatrix}2&3\\5&1\end{bmatrix}-\begin{bmatrix}4&4\\6&2\end{bmatrix}$

$=\begin{bmatrix}-2&-1\\-1&-1\end{bmatrix}$

16) If is $A=\begin{bmatrix}0&b&-2\\3&1&3\\2a&3&-1\end{bmatrix}$ skew symmetric matrix, find the values of a and b.

Answer : If A is symmetric matrix then

$A=A'$

$\Rightarrow$ $\begin{bmatrix}0&b&-2\\3&1&3\\2a&3&-1\end{bmatrix}=\begin{bmatrix}0&3&2a\\b&1&3\\-2&3&-1\end{bmatrix}$

$\therefore$ By equality of matrices,

b = 3 and a = - 1

17) If $A=\begin{bmatrix}0 & x & -4\\ -2 & 0 & -1\\ y & -1 & 0\end{bmatrix}$ is skew symmetric matrix, find the values of x and y.

Answer : $A=\begin{bmatrix}0 & x & -4\\ -2 & 0 & -1\\ y & -1 & 0\end{bmatrix}$

$A'=\begin{bmatrix}0 & -2 & y\\ x & 3 & -1\\ -4 & -1 & 0\end{bmatrix}$

For skew symmetric

$A=-A'$

$\Rightarrow \begin{bmatrix}0 & x & -4\\ -2 & 0 & -1\\ y & -1 & 0\end{bmatrix}=-\begin{bmatrix}0 & -2 & y\\ x & 3 & -1\\ -4 & -1 & 0\end{bmatrix}$

x = 2, y = 4

18) If $\begin{bmatrix}cos\theta & -sin\theta\\ sin\theta & cos\theta\end{bmatrix}$, find the value of θ satisfying the equation $A + A^T = I_2$, where $0\leq 0\leq \frac{\pi}{2}$.

Answer : We have, $A=\begin{bmatrix}cos\theta & -sin\theta\\ sin\theta & cos\theta\end{bmatrix}$

$A^T=\begin{bmatrix}cos\theta & sin\theta\\ -sin\theta & cos\theta\end{bmatrix}$

$\Rightarrow \quad A+A^T=\begin{bmatrix}2cos\theta & 0\\ 0 & cos\theta\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\Rightarrow \quad 2cos\theta=1$

$\Rightarrow \quad cos\theta=\frac{1}{2}\Rightarrow\theta=\cos^{-1}\left(\frac{1}{2}\right)$

$\because \quad \theta=\frac{\pi}{3}$

19) Prove the following by the principle of mathematical induction : if

$A=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$, then $A^n=\begin{bmatrix}1+2n & -4n\\ n & 1-2n\end{bmatrix}$ for every positive integer n.

Answer : We shall prove the result by mathematical induction on n.

Step 1 : When n = 1, by the definition or integral powers of a matrix, we have

$A^1=\begin{bmatrix}1+2(1) & -4n\\ n & 1-2(1)\end{bmatrix}=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$

So, the result is true for n = 1.

Step 2 : Let the result be true for n = m. Then,

$A^m=\begin{bmatrix}1+2m & -4m\\ m & 1-2m\end{bmatrix}$

Now, we will show that the result is true for n = m + 1, i.e.,

$A^{m+1}=\begin{bmatrix}1+2(m+1) & -4(m+1)\\ (m+1) & 1-2(m+1)\end{bmatrix}$

By the definition of integral powers of a square matrix, we have

$A^{m+1}=A^m.A$

$\Rightarrow \quad A^{m+1}=\begin{bmatrix}1+2m & -4m\\ m & 1-2m\end{bmatrix}\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$

[by supposition (i)]

$\Rightarrow \quad A^{m+1}=\begin{bmatrix}3+6m-4m & -4-8m+4m\\ 3m+1-2m & -4m-4+2m\end{bmatrix}$

$=\begin{bmatrix}1+2(m+1) & -4(m+1)\\ (m+1) & 1-2(m+1)\end{bmatrix}$

This shows that the result is true for n = m + 1, whenever it is true for n = m.

Hence, by the principle of mathematical induction, the result is true for any positive integer n.

20) If $A=\begin{vmatrix} 6 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix}$, then show that |2A|=8|A|:

Answer : We have, $A=\begin{vmatrix} 6 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix}$

|A|=6(4-0)-0(0-0)+1(0-0)

$|2A|=\begin{vmatrix} 12 & 0 & 2 \\ 0 & 2 & 4 \\ 0 & 0 & 8 \end{vmatrix}$

=12(16-0)-0(0-0)+2(0-0)

=192

|2A|=8x24=8|A|

21) if y = $f(e^{sin^{-1}}2x)$, find dy/dx.

Answer : We have y = $f(e^{sin^{-1}}2x)$

dy/dx = f$(e^{sin^{-1}}2x)$ x d/dx $(e^{sin^{-1}}2x)$

=f$(e^{sin^{-1}}2x)$x$(e^{sin^{-1}}2x)$xd/dx(sin-12x)

=f$(e^{sin^{-1}}2x)$x$(e^{sin^{-1}}2x)$x$\frac{1}{\sqrt{1-4x^2}}\times 2$

=$\frac{2e^{sin-1}2x}{\sqrt{1-4x^2}}f'\left(e^{sin-1}2x\right)$

22) If x = θsinθ, y = θcosθ find dy/dx at θ = π/4

Answer : $\frac{dx}{d\theta}=\theta cos\theta+sin\theta$

$\frac{dy}{d\theta}=-\theta sin\theta+cos\theta$

$\frac{dy}{dx}=\frac{cos\theta-\theta sin\theta}{\theta cos\theta+sin\theta}$

dy/dx at θ= π/4

$=\frac{cos\frac{\pi}{4}-\frac{\pi}{4}sin\frac{\pi}{4}}{\frac{\pi}{4}cos\frac{\pi}{4}+sin\frac{\pi}{4}}$

$=\frac{\frac{1}{\sqrt{2}}-\frac{\pi}{4}\times\frac{1}{\sqrt{2}}}{\frac{\pi}{4}\times\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}}$

=$\frac{1-\frac{\pi}{4}}{\frac{\pi}{4}+1}$

$\Rightarrow\frac{dy}{dx}=\frac{4-\pi}{4+\pi}$

23) If y = tan$^{-1}\sqrt{\frac{sinx}{1+cosx}}$,$find\frac{dy}{dx}$

Answer : Given, y = tan-1$\sqrt{\frac{sinx}{1+cosx}}$

y = tan$_{-1}\sqrt{\frac{2sin\frac{x}{2}cos\frac{x}{2}}{2cos^2\frac{x}{2}}}$

y = tan-1$\left(\sqrt{tan\frac{x}{2}}\right)$

y = tan^{-1}

24) If y = $log\left(tanx\frac{x}{2}\right)\ find\frac{dy}{dx}$

Answer : We have, y = $log\left(tanx\frac{x}{2}\right)$

$\frac{dy}{dx}=\frac{1}{\frac{tanx}{2}}\times sec^2\frac{x}{2}\times\frac{1}{2}$

$\Rightarrow\frac{dy}{dx}=\frac{\frac{cosx}{2}}{\frac{sinx}{2}}\times\frac{1}{cos^2\frac{x}{2}}\times\frac{1}{2}$

$=\frac{1}{2sin\frac{x}{2}cos\frac{x}{2}}=\frac{1}{sinx}$

$\Rightarrow\frac{dy}{dx}=cosecx$

25) $y=tan^{-1}\frac{5x}{1-6x^2}$,\(-\frac { 1 }{ \sqrt { 6 } } . then prove that $\frac{dy}{dx}=\frac{2}{1+4x^2}+\frac{3}{1+9x^3}$.

Answer : $y=tan^{-1}\frac{3x+2x}{1-3x2x}$

=$tan^{-1}3x+tan^{-1}2x$

$\Rightarrow\frac{dy}{dx}=\frac{3}{1+9x^3}+\frac{2}{1+4x^2}$