Class XII

Mathematics

Chapter:1

Relations and Functions

Points to Remember

Key Concepts

1. A relation R between two non empty sets A and B is a subset of their Cartesian Product $A \times B$. If A = B then relation R on A is a subset of $A \times A$

2. If (a, b) belongs to R, then a is related to b, and written as a R b If (a, b) does not belongs to R then $a \not R b$.

3. Let R be a relation from A to B.
Then Domain of $R \subset A$ and Range of $R \subset B$ co domain is either set B or any of its superset or subset containing range of R

4. A relation R in a set A is called **empty** relation, if no element of A is related to any element of A, i.e., $R = \phi \subset A \times A$.

5. A relation R in a set A is called **universal** relation, if each element of A is related to every element of A, i.e., $R = A \times A$.

6. A relation R in a set A is called

   a. **Reflexive**, if $(a, a) \in R$, for every $a \in A$,

   b. **Symmetric**, if $(a_1, a_2) \in R$ implies that $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$.

   c. **Transitive**, if $(a_1, a_2) \in R$ and $(a_2, a_3) \in R$ implies that $(a_1, a_3) \in R$, or all $a_1, a_2, a_3 \in A$.

7. A relation R in a set A is said to be an **equivalence relation** if R is reflexive, symmetric and transitive.

8. The empty relation R on a non-empty set *X* (i.e. a R b is never true) is not an equivalence relation, because although it is vacuously symmetric and transitive, it is not reflexive (except when *X* is also empty)

9. Given an arbitrary equivalence relation R in a set X, R divides X into mutually disjoint subsets S_i called partitions or subdivisions of X satisfying:

- All elements of S_i are related to each other, for all i
- No element of S_i is related to S_j ,if i$\neq$j
- $\bigcup_{i=1}^{n} S_j$ =X and $S_i \cap S_j$ =ϕ, if i$\neq$j
- The subsets S_j are called Equivalence classes.

10. A function from a non empty set A to another non empty set B is a correspondence or a rule which associates every element of A to a unique element of B written as
f:A$\rightarrow$B s.t f(x) = y for all x$\in$A, y$\in$B. All functions are relations but converse is not true.

11. If f: A$\rightarrow$B is a function then set A is the domain, set B is co-domain and set {f(x):x $\in$ A } is the range of f. Range is a subset of codomain.

12. f: A $\rightarrow$ B is one-to-one if

For all x, y$\in$ A f(x) = f(y) $\Rightarrow$ x = y or x $\neq$ y $\Rightarrow$ f(x) $\neq$ f(y)

A one- one function is known as injection or an Injective Function.

Otherwise, f is called many-one.

13. f: A $\rightarrow$ B is an onto function ,if for each b $\in$ B there is atleast one a $\in$ A such that f(a) = b

i.e if every element in B is the image of some element in A, f is onto.

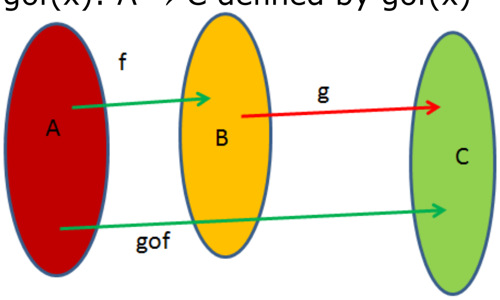
14. A function which is both one-one and onto is called a bijective function or a bijection.

15. For an onto function range = co-domain.

16. A one – one function defined from a finite set to itself is always onto but if the set is infinite then it is not the case.

17. Let f : $A \rightarrow B$ and g : $B \rightarrow C$ be two functions. Then the composition of f and g, denoted by gof is defined as the function gof: A $\rightarrow$ C given by

gof(x): A $\rightarrow$ C defined by gof(x) = g(f(x)) $\forall$ x$\in$ A



Composition of f and g is written as gof and not fog
gof is defined if the range of f $\subseteq$ domain of f and fog is defined if range of g $\subseteq$ domain of f

18. Composition of functions is not commutative in general
fog(x) ≠ gof(x).Composition is associative
If f: X$\rightarrow$Y, g: Y$\rightarrow$Z and h: Z$\rightarrow$S are functions then
ho(g o f)=(h o g)of

19. A function f: X $\rightarrow$ Y is defined to be invertible, if there exists a function g : Y $\rightarrow$ X such that gof = I_X and fog = I_Y. The function g is called the inverse of f and is denoted by f^{-1}

20. If f is invertible, then f must be one-one and onto and conversely, if f is one- one and onto, then f must be invertible.

21. If f:A $\rightarrow$ B and g: B $\rightarrow$C are one-one and onto then gof: A $\rightarrow$ C is also one-one and onto. But If g o f is one –one then only f is one –one g may or may not be one-one. If g o f is onto then g is onto f may or may not be onto.

22. Let f: X $\rightarrow$ Y and g: Y $\rightarrow$ Z be two invertible functions. Then gof is also Invertible with $(gof)^{-1} = f^{-1}o\, g^{-1}$.

23. If f: R$\rightarrow$ R is invertible,
f(x)=y, then $f^{-1}(y)$=x and $(f^{-1})^{-1}$ is the function f itself.

24. A binary operation * on a set A is a function from A X A to A.

25. Addition, subtraction and multiplication are binary operations on R, the set of real numbers. Division is not binary on R, however, division is a binary operation on R-{0}, the set of non-zero real numbers

26. A binary operation $*$ on the set X is called commutative, if $a*b = b*a$, for every a,b$\in$X

27. A binary operation $*$ on the set X is called associative, if $a*(b*c) = (a*b)*c$, for every a, b, c$\in$X

28. An element $e \in A$ is called an **identity** of A with respect to *, if for each $a \in A$, $a * e = a = e * a$.
The identity element of (A, *) if it exists, is **unique**.

29. Given a binary operation $*$ from $A\times A \rightarrow A$, with the identity element e in A, an element $a \in A$ is said to be invertible with respect to the operation $*$, if there exists an element b in A such that $a * b = e = b * a$, then b is called the inverse of a and is denoted by a^{-1}.

30. If the operation table is symmetric about the diagonal line then, the operation is commutative.

*	a	b	c	d
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

The operation * is commutative.

31. Addition '+' and multiplication '·' on N, the set of natural numbers are binary operations But subtraction '–' and division are not since (4, 5) = 4 - 5 = -1 $\notin N$ and 4/5 = .8 $\notin N$

Class XII

Mathematics

Chapter:2

Inverse Trigonometric Functions

Points to Remember

Key Concepts

1. Inverse trigonometric functions map real numbers back to angles.
2. Inverse of sine function denoted by $\sin^{-1}$ or arc sin(x) is defined on [-1,1] and range could be any of the intervals $\left[\frac{-3\pi}{2},\frac{-\pi}{2}\right],\left[\frac{-\pi}{2},\frac{\pi}{2}\right],\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$.
3. The branch of $\sin^{-1}$ function with range $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ is the principal branch.

   So $\sin^{-1}$: $[-1,1]\rightarrow\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$
4. The graph of $\sin^{-1} x$ is obtained from the graph of sine x by interchanging the x and y axes
5. Graph of the inverse function is the mirror image (i.e reflection) of the original function along the line y = x.
6. Inverse of cosine function denoted by $\cos^{-1}$ or arc cos(x) is defined in [-1,1] and range could be any of the intervals $[-\pi,0]$, $[0,\pi]$, $[\pi,2\pi]$. So, $\cos^{-1}$: $[-1,1]\rightarrow [0,\pi]$.

7. The branch of $\tan^{-1}$ function with range $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$ is the principal branch. So $\tan^{-1}$: R $\rightarrow \left(\frac{-\pi}{2},\frac{\pi}{2}\right)$.

8. The principal branch of cosec^{-1} x is $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$-{0}.

cosec^{-1} x :R-(-1,1) $\rightarrow \left[\frac{-\pi}{2},\frac{\pi}{2}\right]$-{0}.

9. The principal branch of $\sec^{-1}$ x is $[0,\pi]-\{\frac{\pi}{2}\}$.

$\sec^{-1}$ x :R-(-1,1) $\rightarrow [0,\pi]-\{\frac{\pi}{2}\}$.

10. $\cot^{-1}$ is defined as a function with domain R and range as any of the intervals $(-\pi,0)$, $(0,\pi)$,$(\pi,2\pi)$. The principal branch is $(0,\pi)$

So $\cot^{-1}$: R $\rightarrow (0,\pi)$

11. The value of an inverse trigonometric function which lies in the range of principal branch is called the principal value of the inverse trigonometric functions.

12. Inverse of a function is not equal to the reciprocal of the function.

13. Properties of inverse trigonometric functions are valid only on the principal value branches of corresponding inverse functions or wherever the functions are defined.

Key Formulae

1.Domain and range of Various inverse trigonometric Functions

Functions	Domain	Range (Principal Value Branches)
$y = \sin^{-1} x$	[−1, 1]	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1} x$	[−1, 1]	$[0, \pi]$
$y = \text{cosec}^{-1} x$	R − (−1,1)	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$y = \sec^{-1} x$	R − (−1, 1)	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$y = \tan^{-1} x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$y = \cot^{-1} x$	R	$(0, \pi)$

2. **Self Adjusting property**

$\sin(\sin^{-1}x)=x$; $\sin^{-1}(\sin x) = x$

$\cos(\cos^{-1} x)=x; \cos^{-1}(\cos x)=x$

$\tan(\tan^{-1} x)=x; \tan^{-1}(\tan x)=x$

Holds for all other five trigonometric ratios as well.

3. **Reciprocal Relations**

$\sin^{-1}\left(\frac{1}{x}\right)=\text{cosec}^{-1} x,\ x \geq 1 \text{ or } x \leq -1$

$\cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1} x, x \geq 1 \text{ or } x \leq -1$

$\tan^{-1}\left(\frac{1}{x}\right) = \cot^{-1} x, x > 0$

4. Even and Odd Functions

(i) $\sin^{-1}(-x)$ = -sin (x), x $[-1,1$

(ii) $\tan^{-1}(-x)$ = -$\tan^{-1}(x)$, $x \in R$

(iii) $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x, |x| \geq 1$

(iv) $\cos^{-}(-x) = \pi - \cos^{1} x, x \in [-1,$

(v) $\sec^{-1}(-x) = \pi - \sec^{-1} x, |x| \geq 1$

(vi) $\cot^{-1}(-x) = \pi - \cot^{-1} x, x \in R$

5. Complementary Relations

(i) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}, x \in [-1,1]$

(ii) $\tan^{-1} x + \cot^{-} x = \frac{\pi}{2}, x \in R$

(iii) $\operatorname{cosec}^{-1}x + \sec^{-1} x = \frac{\pi}{2}, |x| \geq 1$

6. Sum and Difference Formuale

(i) $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right), xy < 1$

(ii) $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right), xy > -1$

(iii) $\sin^{-1}x + \sin^{-1}y = \sin^{-1}[x\sqrt{1-y^2} + y\sqrt{1-x^2}]$

(iv) $\sin^{-1}x - \sin^{-1}y = \sin^{-1}[x\sqrt{1-y^2} - y\sqrt{1-x^2}]$

(v) $\cos^{-1}x + \cos^{-1}y = \cos^{-1}[xy - \sqrt{1-x^2}\sqrt{1-y^2}]$

(vi) $\cos^{-1}x - \cos^{-1}y = \cos^{-1}[xy + \sqrt{1-x^2}\sqrt{1-y^2}]$

(vii) $\cot^{-1}x + \cot^{-1}y = \cot^{-1}\left(\frac{xy-1}{x+y}\right)$

(viii) $\cot^{-1}x - \cot^{-1}y = \cot^{-1}\left(\frac{xy+1}{y-x}\right)$

7. Double Angle Formuale

(i) $2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2}, |x| \le 1$

(ii) $2\tan^{-1}x = \cos^{-1}\frac{1-x^2}{1+x^2}, x \ge 0$

(iii) $2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}, -1 < x < 1$

(iv) $2\sin^{-1}x = \sin^{-1}(2x\sqrt{1-x^2}), \frac{-1}{\sqrt{}} \le x \le \frac{1}{\sqrt{2}}$

(v) $2\cos^{-1}x = \sin^{-1}(2x\sqrt{1-x^2}), \frac{1}{\sqrt{2}} \le x \le 1$

8. Conversion Properties

(i) $\sin^{-1}x = \cos^{-1}\sqrt{1-x^2}$

$= \tan^{-1}\frac{x}{x\sqrt{1-x^2}} = \cot^{-1}\frac{\sqrt{1-x^2}}{x}$

(ii) $\cos^{-1}x = \sin^{-1}\sqrt{1-x^2}$

$= \tan^{-1}\frac{\sqrt{1-x^2}}{x} = \cot^{-1}\frac{x}{\sqrt{1-x^2}}$

(iii) $\tan^{-1}x = \sin^{-1} = \frac{x}{\sqrt{1-x^2}}$

$= \cos^{-1}\frac{x}{\sqrt{1+x^2}} = \sec^2\sqrt{1+x^2}$

θ

$$= \operatorname{cosec}^{-1}\frac{\sqrt{1+x^2}}{x}$$

Properties are valid only on the values of x for which the inverse functions are defined.

Class XII: Maths

Chapter 3: Matrices

Chapter Notes

Top Definitions

1. Matrix is an ordered rectangular array of numbers (real or complex) or functions or names or any type of data. The numbers or functions are called the elements or the entries of the matrix.

2. The horizontal lines of elements are said to constitute, rows of the matrix and the vertical lines of elements are said to constitute, columns of the matrix.

3. A matrix is said to be a column matrix if it has only one column.

$A = [a_{ij}]_{m \times 1}$ is a column matrix of order m x 1

4. A matrix is said to be a row matrix if it has only one row.

$B = [b_{ij}]_{1 \times n}$ is a row matrix of order 1 x n.

5. A matrix in which the number of rows is equal to the number of columns is said to be a square matrix. A matrix of order "m × n" is said to be a square matrix if m = n and is known as a square matrix of order 'n'.

$A = [a_{ij}]_{m \times n}$ is a square matrix of order m.

6. If $A = [a_{ij}]$ is a square matrix of order n, then elements $a_{11}, a_{22}, \ldots, a_{nn}$ are said to constitute the diagonal, of the matrix A

7. A square matrix $B = [b_{ij}]_{m \times m}$ is said to be a diagonal matrix if all its non diagonal elements are zero, that is a matrix $B = [b_{ij}]_{m \times m}$ is said to be a diagonal matrix if $b_{ij} = 0$, when $i \neq j$.

8. A diagonal matrix is said to be a scalar matrix if its diagonal elements are equal, that is, a square matrix $B = [b_{ij}]_{n \times n}$ is said to be a scalar matrix if

$b_{ij} = 0$, when $i \neq j$

$b_{ij} = k$, when $i = j$, for some constant k.

9. A square matrix in which elements in the diagonal are all 1 and rest are all zero is called an identity matrix. A square matrix

$A = [a_{ij}]_{n \times n}$ is an identity matrix, if $a_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$

10. A matrix is said to be zero matrix or null matrix if all its elements are zero.

11. Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are said to be equal if

(i) They are of the same order

(ii) Each elements of A is equal to the corresponding element of B, that is $a_{ij} = b_{ij}$ for all i and j.

12. If $A = [a_{ij}]$ be an m x n matrix, then the matrix obtained by interchanging the rows and columns of A is called the transpose of A. Transpose of the matrix A is denoted by A′ or (A^T).

13. For any square matrix A with real number entries, A + A′ is a symmetric matrix and A – A′ is a skew symmetric matrix.

14. If $A = \left[a_{ij}\right]_{nxn}$ is an $n \times n$ matrix such that A^T=A, then A is called symmetric matrix. In a symmetric matrix, $a_{ij} = a_{ji}$ for all i and j

15. If $A = \left[a_{ij}\right]_{nxn}$ is an $n \times n$ matrix such that A^T=−A, then A is called skew

symmetric matrix. In a skew symmetric matrix, $a_{ij} = -a_{ji}$

If i=j, then $a_{ii} = -a_{ii} \Rightarrow a_{ii} = 0$

16. Let A and B be two square matrices of order n such that AB=BA=I. Then A is called inverse of B and is denoted by B=A^{-1}. If B is the inverse of A , then A is also the inverse of B.

17. If A and B are two invertible matrices of same order, then $(AB)^{-1} = B^{-1} A^{-1}$

Top Concepts

1. Order of a matrix gives the number of rows and columns present in the matrix.

2. If the matrix A has m rows and n columns then it is denoted by $A = [a_{ij}]_{m \times n}$. a_{ij} is i-j th or $(i, j)^{th}$ element of the matrix.

3. The simplest classification of matrices is based on the order of the matrix.

4. In case of a square matrix, the collection of elements a_{11} , $a_{22,}$ and so on constitute the **Principal Diagonal** or simply the diagonal of the matrix Diagonal is defined only in case of square matrices.

$$\begin{bmatrix} a_{11} & a_{12} & --- & --- & a_{1n} \\ a_{21} & a_{22} & --- & --- & a_{2n} \\ --- & --- & --- & --- & --- \\ a_{i1} & a_{i2} & --- & --- & a_{in} \\ --- & --- & --- & --- & --- \\ a_{m1} & a_{m2} & --- & --- & a_{mn} \end{bmatrix}_{m \times n}$$

5. Two matrices of same order are comparable matrices.

6. If $A = \left[a_{ij}\right]_{m \times n}$ and $B = \left[b_{ij}\right]_{m \times n}$ are two matrices of order m × n, their sum is defined as a matrix $C = \left[c_{ij}\right]_{m \times n}$ where

$c_{ij} = a_{ij} + b_{ij}$ for $1 \le i \le m, 1 \le j \le n$

7.Two matrices can be added9or subtracted) if they are of same order. For multiplying two matrices A and B number of columns in A must be equal to the number of rows in B.

8. $A = [a_{ij}]_{m \times n}$ is a matrix and k is a scalar, then kA is another matrix which is obtained by multiplying each element of A by the scalar k. Hence $kA = [ka_{ij}]_{m \times n}$

9. If $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$ are two matrices, their difference is represented as $A - B = A + (-1)B$.

10. Properties of matrix addition

(i) Matrix addition is commutative i.e A+B = B+A.

(ii) Matrix addition is associative i.e (A + B) + C = A + (B + C).

(iii) Existence of additive identity: Null matrix is the identity w.r.t addition of matrices

Given a matrix $A = [a_{ij}]_{m \times n}$, there will be a corresponding null matrix O of same order such that A+O=O+A=A

(iv) The existence of additive inverse Let A = $[a_{ij}]_{m \times n}$ be any matrix, then there exists another matrix –A = -$[a_{ij}]_{m \times n}$ such that

A + (–A) = (–A) + A = O.

11. Properties of scalar multiplication of the matrices:

If $A = [a_{ij}], B = [b_{ij}]$ are two matrices, and k,L are real numbers then

(i) k(A +B) = k A + kB, (ii) (k + I)A = k A + I A

(ii) k (A + B) = k($[a_{ij}]$+$[b_{ij}]$) = k$[a_{ij}]$+k$[b_{ij}]$ = kA+kB

(iii) $(k+L)A = (k+L)[a_{ij}] = [(k+L)a_{ij}]$ = k $[a_{ij}]$ + L $[a_{ij}]$ = k A + L A

12. If $A = [a_{ij}]_{mxp}, B = [b_{ij}]_{pxn}$ are two matrices, their product AB, is given by $C = [c_{ij}]_{mxn}$ such that

$$c_{ij} = \sum_{k=1}^{p} a_{ik}b_{kj} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{ip}b_{pj}.$$

In order to multiply two matrices A and B the number of columns in A = number of rows in B.

13. **Properties of Matrix Multiplication**
Commutative law does not hold in matrices, whereas the associative and distributive laws hold for matrix multiplication

(i) In general $AB \neq BA$

(ii) Matrix multiplication is associative $A(BC)=(AB)C$

(iii) Distributive laws:
$A(B+C)=AB+BC$;
$(A+B)C=AC+BC$

14. The multiplication of two non zero matrices can result in a null matrix.

15. Properties of transpose of matrices
(i) If A is a matrix, then $(A^T)^T=A$
(ii) $(A + B)^T = A^T + B^T$,
(iii) $(kB)^T = kB^T$, where k is any constant.

16. If A and B are two matrices such that AB exists then $(AB)^T = B^T A^T$

17. Every square matrix can be expressed as the sum of a symmetric and skew symmetric matrix i.e $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$ for any square matrix A .

18. A square matrix A is called an orthogonal matrix when $AA^T=A^TA=I$.

19. A null matrix is both symmetric as well as skew symmetric.

20. Multiplication of diagonal matrices of same order will be commutative.

21. There are 6 elementary operations on matrices.Three on rows and 3 on columns. First operation is interchanging the two rows i.e $R_i \leftrightarrow R_j$ implies the i^{th} row is interchanged with j^{th} row. The two rows are interchanged with one another the rest of the matrix remains same.

22. Second operation on matrices is to multiply a row with a scalar or a real number i.e $R_i \to kR_i$ that i^{th} row of a matrix A is multiplied by k.

23. Third operation is the addition to the elements of any row, the corresponding elements of any other row multiplied by any non zero number

i.e $R_i \to R_i + kR_j$ k multiples of jth row elements are added to ith row elements

24. Column operation on matrices are
(i) Interchanging the two columns: $C_r \leftrightarrow C_k$ indicates that rth column is interchanged with kth column

(ii) Multiply a column with a non zero constant i.e $C_i \to kC_i$
(iii) Addition of scalar multiple of any column to another column i.e $C_i \to C_i + kC_j$

25. Elementary operations helps in transforming a square matrix to identity matrix

26. Inverse of a square matrix, if it exists is unique.

27. Inverse of a matrix can be obtained by applying elementary row operations on the matrix A= IA. In order to use column operations write A=AI

28.Either of the two operations namely row or column operations can be applied. Both cannot be applied simultaneously

Top Formulae

1. An m x n matrix is a square matrix if m = n.
2. A = $[a_{ij}]$ = $[b_{ij}]$ = B if (i) A and B are of same order, (ii) $a_{ij} = b_{ij}$ for all possible values of i and j.
3. kA = $k[a_{ij}]_{m \times n} = [k(a_{ij})]_{m \times n}$.
4. - A = (-1) A
5. A – B = A + (-1) B

6. If $A = [a_{ij}]_{m \times n}$ and $B = [b_{ik}]_{n \times p}$, then $AB = C = [c_{ik}]_{m \times p}$, where $c_{ik} = \sum_{j=1}^{n} a_{ij} b_{ij}$

7. Elementary operations of a matrix are as follow:

i. $R_i \leftrightarrow R_j$ or $C_i \leftrightarrow C_j$

ii. $R_i \to kR_i$ or $C_i \to kC_i$

iii. $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$

Class XII: Mathematics

Chapter 4: Determinants

Chapter Notes

Top Definitions

1. To every square matrix A $=[a_{ij}]$ a unique number (real or complex) called determinant of the square matrix A can be associated. Determinant of matrix A is denoted by det(A) or |A| or Δ.
2. A determinant can be thought of as a function which associates each square matrix to a unique number (real or complex).

   $f:M \to K$ is defined by $f(A) = k$ where $A \in M$ the set of square matrices and $k \in K$ set of numbers(real or complex)
3. Let A = [a] be the matrix of order 1, then determinant of A is defined to be equal to a.
4. Determinant of order 2

   $$\text{If } A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{then, } |A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$
5. Determinant of order 3

$$\text{If } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \text{ then}$$

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

6. Minor of an element a_{ij} of a determinant is the determinant obtained by deleting its ith row and jth column in which element a_{ij} lies. Minor of an element a_{ij} is denoted by M_{ij}.

7. Cofactor of an element a_{ij}, denoted by A_{ij} is defined by
$A_{ij} = (-1)^{i+j} M_{ij}$ where M_{ij} is the minor of a_{ij}.

8. The adjoint of a square matrix $A=[a_{ij}]$ is the transpose of the cofactor matrix $[Aij]_{n\times n}$.
9. A square matrix A is said to be singular if $|A| = 0$

10. A square matrix A is said to be non – singular if $|A| \neq 0$

11. If A and B are nonsingular matrices of the same order, then AB and BA are also nonsingular matrices of the same order.

12. The determinant of the product of matrices is equal t product of the respective determinants, that is, $|AB| = |A|\,|B|$, where A and B are square matrices of the same order.
13. A square matrix A is invertible i.e its inverse exists if and only if A is nonsingular matrix. Inverse of matrix A if exists is given by

$$A^{-1} = \frac{1}{|A|}(\operatorname{adj} A)$$

14. A system of equations is said to be consistent if its solution (one or more) exists.
15. A system of equations is said to be inconsistent if its solution does not exist.

Top Concepts

1. A determinant can be expanded along any of its row (or column). For easier calculations it must be expanded along the row (or column) containing maximum zeros.

2. If A=kB where A and B are square matrices of order n, then $|A|=k^n|B|$ where n =1,2,3.

Properties of Determinants

3. **Property 1** Value of the determinant remains unchanged if its row and columns are interchanged. If A is a square matrix, the det (A) = det (A'), where A' = transpose of A.
4. **Property 2** If two rows or columns of a determinant are interchanged, then the sign of the determinant is changed. Interchange of rows and columns is written as $R_i \leftrightarrow R_j$ or $C_i \leftrightarrow C_j$
5. **Property 3:** If any two rows (or columns) of a determinant are identical, then value of determinant is zero.
6. **Property 4**: If each element of a row (or a column) of a determinant is multiplied by a constant k, then its value get multiplied by k. If Δ_1 is the determinant obtained by applying $R_i \rightarrow kR_i$ or $C_i \rightarrow kC_i$ to the determinant Δ, then $\Delta_1 = k\Delta$. So .if A is a square matrix of order n and k is a scalar, then $|kA|=k^n|A|$. This property enables taking out of common factors from a given row or column.
7. **Property 5:** If in a determinant, the elements in two rows or columns are proportional, then the value of the determinant is zero. For example.

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ ka_1 & ka_2 & ka_3 \end{vmatrix} = 0 \text{ (rows } R_1 \text{ and } R_3 \text{ are proportional)}$$

8. **Property 6:** If the elements of a row (or column) of a determinant are expressed as sum of two terms, then the determinant can be expressed as sum of two determinants.

9. **Property 7:** If to any row or column of a determinant, a multiple of another row or column is added, the value of the determinant remains the same i.e the value of the determinant remains same on applying the operation $R_i \to R_i + kR_j$ or $C_i \to C_i + k\,C_j$

10. If more than one operation like $R_i \to R_i + kR_j$ is done in one step, care should be taken to see that a row that is affected in one operation should not be used in another operation. A similar remark applies to column operations.

11. Since area is a positive quantity, so the absolute value of the determinant is taken in case of finding the area of the triangle.

12. If area is given, then both positive and negative values of the determinant are used for calculation.

13. The area of the triangle formed by three collinear points is zero.

14. Minor of an element of a determinant of order $n(n \geq 2)$ is a determinant of order $n - 1$.

15. Value of determinant of a matrix A is obtained by sum of product of elements of a row (or a column) with corresponding cofactors. For example $|A| = a_{11}A_{11}+a_{12}A_{12}+a_{13}A_{13}$

16. If elements of a row (or column) are multiplied with cofactors of any other row (or column), then their sum is zero. For example $a_{11}A_{21}+a_{12}A_{22}+a_{13}A_{23} = 0$

17. If A is a nonsingular matrix of order n then $|adj.A|=|A|^{n-1}$

18. Determinants can be used to find the area of triangles if its vertices are given

19. Determinants and matrices can also be used to solve the system of linear equations in two or three variables.

20. System of equations
$$\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned}$$

can be written as A X = B, where

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{and} B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

Then matrix $X = A^{-1}B$ gives the unique solution of the system of equations if |A| is non zero and A^{-1} exists.

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Top Formulae

1. Area of a Triangle with vertices (x_1, y_1), (x_2, y_2) & (x_3, y_3) is

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

2. Determinant of a matrix $A = [a_{ij}]_{1 \times 1}$ is given by $|a_{11}| = a_{11}$

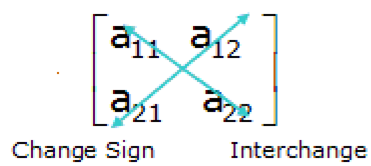
3. If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ then, $|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$

If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

4. Cofactor of a_{ij} is $A_{ij} = (-1)^{i+j} \cdot M_i$

5. If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ then adj.A =



If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, then $\text{adj} A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$

where, A_{ij} are cofactors of a_{ij}

6. $|AB| = |A|\,|B|$,

7. $A^{-1} = \frac{1}{|A|}(\text{adj} A)$ where $|A| \neq 0$.

8. $|A^{-1}| = \frac{1}{|A|}$ and $(A^{-1})^{-1} = A$

9. Unique solution of equation AX = B is given by $X = A^{-1}B$, where $|A| \neq 0$.

10. For a square matrix A in matrix equation AX = B,

i. $|A| \neq 0$, there exists unique solution.

ii. $|A| = 0$ and (adj A) B $\neq 0$, then there exists no solution.

iii. $|A| = 0$, and (adj A) B = 0, then system may or may not be consistent.

Class XII

Mathematics

Chapter:5

Continuity and Differentiability

Chapter Notes

Key Definitions

1. A function f(x) is said to be continuous at a point c if, $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = f(c)$
2. A real function f is said to be continuous if it is continuous at every point in the domain of f.
3. If f and g are real valued functions such that (f o g) is defined at c. If g is continuous at c and if f is continuous at g(c), then (f o g) is continuous at c.
4. A function f is differentiable at a point c if LHD=RHD

   i.e $\lim_{h\to 0^-} \frac{f(c+h)-f(c)}{h} = \lim_{h\to 0^+} \frac{f(c+h)-f(c)}{h}$
5. Chain Rule of Differentiation: If f is a composite function of two functions u and v such that f = vou and t =u(x)

   if both $\frac{dt}{dx}$ and $\frac{dv}{dx}$, exists then, $\frac{df}{dx} = \frac{dv}{dt} \cdot \frac{dt}{dx}$
6. Logarithm of a to base b is xi.e $\log_b a = x$ if $b^x = a$ where b > 1 be a real number. Logarithm of a to base b is denoted by $\log_b a$.

7. Functions of the form x = f(t) and y = g(t) are parametric functions.
8. **Rolle's Theorem**: If f : [a, b] → **R** is continuous on [a, b] and differentiable on (a, b) such that f (a) = f (b), then there exists some c in (a, b) such that f′(c) = 0

9. **Mean Value Theorem**: If f :[a, b] $\to$ **R** is continuous on [a, b] & differentiable on (a, b). Then there exists some c in (a, b) such that

$$f'(c) = \lim_{h\to 0} \frac{f(b) - f(a)}{b - a}$$

Key Concepts

1. A function is continuous at x = c if the function is defined at x = c and the value of the function at x = c equals the limit of the function at x = c.
2. If function f is not continuous at c, then f is discontinuous at c and c is called the point of discontinuity of f.
3. Every polynomial function is continuous.
4. Greatest integer function, [x] is not continuous at the integral values of x.
5. Every rational function is continuous.
6. Algebra of Continuous Functions

   Let f and g be two real functions continuous at a real number c, then

   (1) f + g is continuous at x = c

   (2) f – g is continuous at x = c

   (3) f. g is continuous at x = c

   (4) $\left(\frac{f}{g}\right)$ is continuous at x = c, (provided g(c) ≠ 0).
7. Derivative of a function f with respect to x is f′(x) which is given by

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

8. If a function f is differentiable at a point c, then it is also continuous at that point.
9. Every differentiable function is continuous but converse is not true.
10. Chain Rule is used to differentiate composites of functions.
11. **Algebra of Derivatives:**

If u & v are two functions which are differentiable, then

(i) $(u \pm v)' = u' \pm v'$ (Sum and Difference Formula)

(ii) $(uv)' = u'v + uv'$ (Product rule)

(iii) $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ (Quotient rule)

12. **Implicit Functions**

If it is not possible to "separate" the variables x & y then function f is known as implicit function.

13. **Exponential function:** A function of the form $y = f(x) = b^x$ where base $b > 1$

(1) Domain of the exponential function is R, the set of all real numbers.

(2) The point (0, 1) is always on the graph of the exponential function

(3) Exponential function is ever increasing

14. Properties of Logarithmic functions

(i) Domain of log function is R^+.

(ii) The log function is ever increasing

(iii) For x very near to zero, the value of log x can be made lesser than any given real number.

15. Logarithmic differentiation is a powerful technique to differentiate functions of the form $f(x) = [u(x)]^{v(x)}$. Here both f(x) and u(x) need to be positive.

16. **Logarithmic Differentiation**

$y = a^x$

Taking logarithm on both sides

$\log y = \log a^x$.

Using property of logarithms

$\log y = x \log a$

Now differentiating the implicit function

$$\frac{1}{y} \cdot \frac{dy}{dx} = \log a$$

$$\frac{dy}{dx} = y \log a = a^x \log a$$

.

17. A relation between variables x and y expressed in the form x=f(t) and y=g(t) is the parametric form with t as the parameter .Parametric equation of parabola $y^2=4ax$ is $x=at^2, y=2at$

18.**Parametric Differentiation:**

Differentiation of the functions of the form x = f(t) and y = g(t)

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$$

19.If y =f(x) and $\frac{dy}{dx}$ =f′(x) and if f′(x) is differentiable then

$\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d^2y}{dx^2}$ or f′′ (x) is the second order derivative of y w.r.t x

Top Formulae

1. Derivative of a function at a point

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

2. Properties of Logarithms

$$\log(xy)=\log x+\log y$$

$$\log\left(\frac{x}{y}\right)=\log x-\log y$$

$$\log\left(x^y\right)=y\log x$$

$$\log_a x=\frac{\log_b x}{\log_b a}$$

3.Derivatives of Functions

- $\frac{d}{dx}x^n = nx^{n-1}$
- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$
- $\frac{d}{dx}(\sec x) = \sec x \tan x$
- $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$
- $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
- $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$
- $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$
- $\frac{d}{dx}(\operatorname{cosec}^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$
- $\frac{d}{dx}(e^x) = e^x$
- $\frac{d}{dx}(\log x) = \frac{1}{x}$

Class XII: Mathematics
Chapter 6: Application of Derivatives

Chapter Notes

Key Concepts

1. Derivatives can be used to (i) determine rate of change of quantities(ii)to find the equation of tangent and normal(iii)to find turning points on the graph of a function(iv) calculate n^{th} root of a rational number (v) errors in calculations using differentials.

2. Whenever one quantity y varies with another x satisfying some rule y =f(x), then $\frac{dy}{dx}$ or f′(x) represents the rate of change of y with respect to x.
3. $\frac{dy}{dx}$ is positive if y and x increases together and it is negative if y decreases as x increases.
4. The equation of the tangent at (x_0, y_0) to the curve y = f (x) is:
   $y - y_0 = f'(x_0)(x - x_0)$
   Slope of a tangent = $\frac{dy}{dx} = \tan\theta$
5. The equation of the normal to the curve y = f (x) at (x_0, y_0) is:
   $(y-y_0)f'(x_0)+(x-x_0)= 0$
   Slope of Normal = $\frac{-1}{\text{slope of the tangent}}$

6. The angle of intersection of two curves is defined to be the angle between the tangents to the two curves at their point of intersection.

7. Let I be an open interval contained in domain of a real valued function f. Then f is said to be:

i. Increasing on I if $x_1 < x_2$ in I
$\Rightarrow f(x_1) \le f(x_2)$ for all $x_1, x_2 \in I$
ii. Strictly increasing on I if $x_1 < x_2$ in I
$\Rightarrow f(x_1) < f(x_2)$ for all $x_1, x_2 \in I$
iii. Decreasing on I if $x_1 < x_2$ in I
$\Rightarrow f(x_1) \ge f(x_2)$ for all $x_1, x_2 \in I$
iv. Strictly decreasing on I if $x_1 < x_2$ in I
$\Rightarrow f(x_1) > f(x_2)$ for all $x_1, x_2 \in I$

8. Let f be a continuous function on [a,b] and differentiable on (a,b).Then
(a)f is increasing in[a,b] if $f'(x)>0$for each $x\in(a,b)$
(b) f is decreasing in[a,b] if $f'(x)<0$ for each $x\in(a,b)$
(c) f is constant in[a,b] if $f'(x)=0$ for each $x\in(a,b)$

9. Let f be a continuous function on [a,b] and differentiable on (a,b).Then
(a) f is strictly increasing in (a,b) if $f'(x)>0$for each $x\in(a,b)$
(b) f is strictly decreasing in (a,b) if $f'(x)<0$ for each $x\in(a,b)$
(c) f is constant in (a,b) if $f'(x)=0$ for each $x\in(a,b)$

10. A function which is either increasing or decreasing is called a monotonic function

11. Let f be a function defined on I.Then

a f is said to have a maximum value in I, if there exists a point c in I such that $f(c) > f(x)$, for all $x \in I$.

The number f (c) is called the maximum value of f in I and the point c is called a point of maximum value of f in I.

b. f is said to have a minimum value in I, if there exists a point c in I such that $f(c) < f(x)$, for all $x \in I$.

The number f (c), in this case, is called the minimum value of f in I and the point c, in this case, is called a point of minimum value of f in I.

c. f is said to have an extreme value in I if there exists a point c in I such that f (c) is either a maximum value or a minimum value of f in I.

The number f (c) , in this case, is called an extreme value of f in I and the point c, is called an extreme point.

12. Every monotonic function assumes its maximum/ minimum value at the end points of the domain of definition of the function.

13. Every continuous function on a closed interval has a maximum and a minimum value

14. Derivative of a function at the point *c* represents the slope of tangent to the given curve at a point $x=c$.

15.If $f'(c)=0$ i.e. derivative at a point $x=c$ vanishes, which means slope of the tangent at $x=c$ is zero. Geometrically, this will imply that this tangent is parallel to x axis so $x=c$ will come out to be a turning point of the curve. Such points where graph takes a turn are called extreme points.

16.Let f be a real valued function and let c be an interior point in the domain of f. Then

a. c is called a point of local maxima if there is $h > 0$ such that f (c) > f (x), for all x in (c – h, c + h)

The value f (c) is called the local maximum value of f.

b. c is called point of local minima if there is an h > 0 such that

f (c) < f (x), for all x in (c – h, c + h)

The value f (c) is called the local minimum value of f.

17. Let f be a function defined on an open interval I. Suppose c ∈ I be any point. If f has a local maxima or a local minima at x = c, then either f′ (c) = 0 or f is not differentiable at c.

18. **I Derivative Test:** Let f be a function defined on an open interval I. Let f be continuous at a critical point c in I. Then

i. If f ′(x) > 0 at every point sufficiently close to and to the left of c & f ′(x) < 0 at every point sufficiently close to and to the right of c, then c is a point of local maxima.
ii. If f ′(x) < 0 at every point sufficiently close to and to the left of c, f ′(x) > 0 at every point sufficiently close to and to the right of c, then c is a point of local minima.
iii. If f ′(x) does not change sign as x increases through c, then point c is called point of inflexion.

19. **II Derivative Test:** Let f be a function defined on an interval I & c ∈ I. Let f be twice differentiable at c. Then

i. x = c is a point of local maxima if f ′(c) = 0 & f ″(c) < 0.
ii. x = c is a point of local minima if f′(c) = 0 and f ″(c) > 0
iii. The test fails if f ′(c) = 0 & f ″(c) = 0.
By first derivative test, find whether c is a point of maxima, minima or a point of inflexion.

20. Working Rule to find the intervals in which the function f(x) increases or decreases

a) Differentiate f(x) first i.e. find f′(x)
b) Simplify f′(x) and factorise it if possible in case of polynomial functions.
c) Equate f′(x) to zero to obtain the zeroes of the polynomial in case of polynomial functions and angles in the given interval in case of trigonometric functions.
d) Divide the given interval or the real line into disjoint subintervals and then find the sign f′(x) in each interval to check whether f(x) is increasing or decreasing in a particular interval.

21. Let f be a continuous function on an interval I = [a, b]. Then f has the absolute maximum attains it at least once in I. Also, f has the absolute minimum value and attains of a function it at least once in I.

22. Let f be a differentiable function on a closed interval I and let c be any interior point of I. Then

a. f′ (c) = 0 if f attains its absolute maximum value at c.

b. f′ (c) = 0 if f attains its absolute minimum value at c.

23. Working Rule for finding the absolute maximum and minimum values in the interval [a,b]

Step 1: Find all critical points of f in the interval, i.e., find points x where either

f′ (x) = 0 or f is not differentiable

Step 2: Take the end points of the interval.

Step 3: At all these points (listed in Step 1 and 2), calculate the values of f.

Step 4: Identify, the maximum and minimum values of f out of the values calculated in Step 3. This maximum and minimum value will be the absolute maximum (greatest) value f and the minimum value will be the absolute minimum (least) value of f.

24. Let y =f(x), Δx be small increments in x and Δy be small increments in y corresponding to the increment in x, i.e., $\Delta y = f(x+\Delta x)-f(x)$.
Then

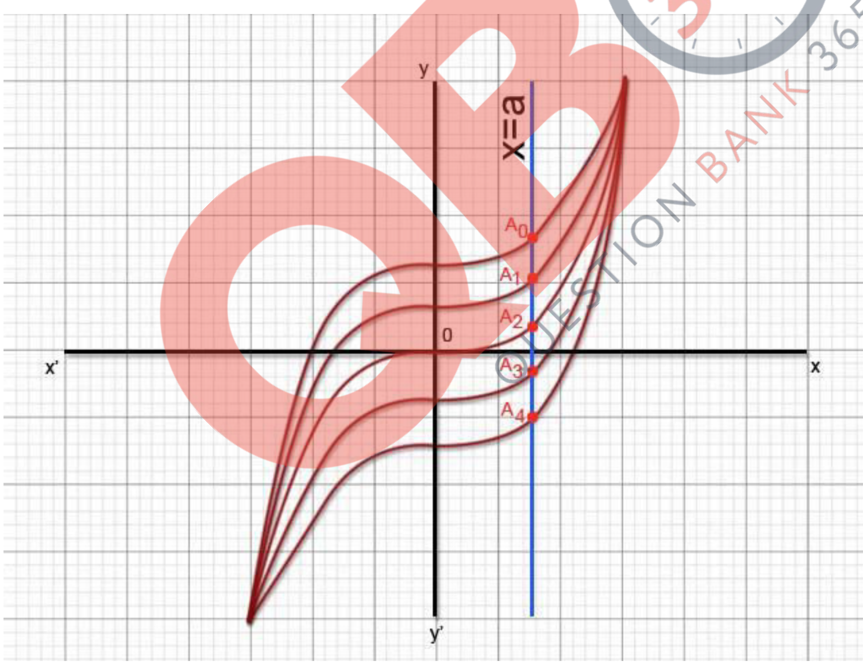
$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x \text{ or } dy = \left(\frac{dy}{dx}\right)\Delta x \quad \Delta y \approx dy \text{ and } \Delta x \approx dx$$

Class XII: Mathematics
Chapter 7: Integrals

Chapter Notes

Key Concepts

1. Integration is the inverse process of differentiation. The process of finding the function from its primitive is known as integration or anti differentiation.
2. Indefinite Integral $\int f(x)dx = F(x) + C$ where F(x) is the antiderivative of f(x).
3. $\int f(x)dx$ means integral of f w.r.t x ,f(x) is the integrand, x is the variable of integration, C is the constant of integration.
4. Geometrically indefinite integral is the collection of family of curves, each of which can be obtained by translating one of the curves parallel to itself.
   Family of Curves representing the integral of $3x^2$



$\int f(x)dx = F(x) + C$ represents a family of curves where different values of C correspond to different members of the family, and these members are obtained by shifting any one of the curves parallel to itself.

5. **Properties of antiderivatives:**

- $\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$

- $\int kf(x)dx = k\int f(x)dx$ for any real number k
- $\int [k_1f_1(x) + k_2f_2(x) + ... + k_nf_n(x)]dx = k_1\int f_1(x)dx + k_2\int f_2(x)dx + ... + k_n\int f_n(x)dx$
  where, $k_1,k_2...k_n$ are real numbers & $f_1,f_2,..f_n$ are real functions

6.By knowing one anti-derivative of function f infinite number of anti derivatives can be obtained.

7.Integration can be done using many methods prominent among them are
(i)Integration by substitution
(ii)Integration using Partial Fractions
(iii)Integration by Parts
(iv) Integration using trigonometric identities

8. A change in the variable of integration often reduces an integral to one of the fundamental integrals. Some standard substitutions are
x^2+a^2 substitute x = a tan θ
$\sqrt{x^2-a^2}$ substitute x = a sec θ
$\sqrt{a^2 - x^2}$ substitute x = a sin θ or a cos θ

9. A function of the form $\frac{P(x)}{Q(x)}$ is known as rational function. Rational functions can be integrated using Partial fractions.

10. **Partial fraction** decomposition or **partial fraction expansion** is used to reduce the degree of *either* the numerator or the denominator of a rational function.

11. **Integration using Partial Fractions**
A rational function $\frac{P(x)}{Q(x)}$ can be expressed as sum of partial fractions if $\frac{P(x)}{Q(x)}$ this takes any of the forms.

- $\frac{px+q}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$, a≠b
- $\frac{px+q}{(x-a)^2} = \frac{A}{x-a} + \frac{B}{(x-a)^2}$
- $\frac{px^2+qx+r}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$
- $\frac{px^2+qx+r}{(x-a)^2(x-b)} = \frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$
- $\frac{px^2+qx+r}{(x-a)(x^2+bx+c)} = \frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c}$

where $x^2 + bx + c$ cannot be factorised further.

12.To find the integral of the function $\int \frac{dx}{ax^2 + bx + c}$ or $\int \frac{dx}{\sqrt{ax^2 + bx + c}}$

ax^2+bx+c must be expressed as $a\left[\left(x + \frac{b}{2a}\right)^2 + \left(\frac{c}{a} - \frac{b^2}{4a^2}\right)\right]$

13. To find the integral of the function $\int \frac{(px+q)dx}{ax^2 + bx + c}$ or $\int \frac{(px+q)dx}{\sqrt{ax^2 + bx + c}}$; px+q

$= A.\ \frac{d}{dx}(ax^2+bx+c)+B = A(2ax+b)+B$

14.To find the integral of the product of two functions integration by parts is used.I and II functions are chosen using ILATE rule

I- inverse trigonometric

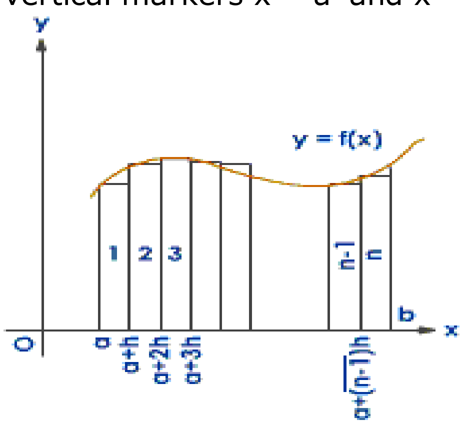
L- logarithmic A-algebra T-Trigonometric E-exponential , is used to identify the first function

14. Integration by parts:

The integral of the product of two functions = (first function) × (integral of the second function) – Integral of [(differential coefficient of the first function)× (integral of the second function)]

$\int f_1(x).f_2(x)dx = f_1(x)\int f_2(x)dx - \int\left[\frac{d}{dx}f_1(x).\int f_2(x)dx\right]dx$ where f_1 & f_2 are functions of x.

15.Definite integral $\int_a^b f(x)dx$ of the function f(x) from limits a to b represents the area enclosed by the graph of the function f(x) the x axis, and the vertical markers x= 'a' and x = 'b'



16. **Definite integral as limit of sum:** The process of evaluating a definite integral by using the definition is called integration as limit of a sum or integration from first principles.

17. Method of evaluating $\int_a^b f(x)dx$

(i) Calculate anti derivative F(x)
(ii) calculate F(3) – F(1)

18. Area function

$A(x) = \int_a^x f(x)dx$, if x is a point in [a,b]

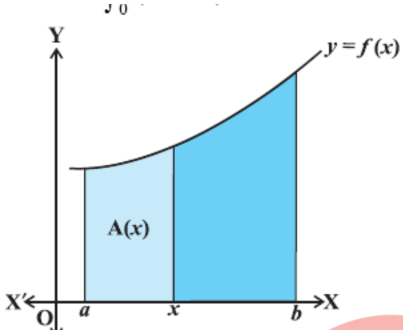


Fig 7.3

19. **Fundamental Theorem of Integral Calculus**

- **First Fundamental theorem** of integral calculus: If Area function,

  $A(x)= \int_a^x f(x)dx$ for all x≥a, & f is continuous on [a,b].Then A′(x)= f (x)

  for all $x \in [a, b]$.
- **Second Fundamental theorem** of integral calculus: Let *f* be a continuous function of x in the closed interval [*a*, *b*] and let F be antiderivative of $\frac{d}{dx}F(x) = f(x)$ for all x in domain of f, then

  $\int_a^b f(x)dx = [F(x)+C]_a^b = F(b) - F(a)$

Key Formulae

1.Some Standard Integrals

- $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$

- $\int dx = x + C$
- $\int \cos x\, dx = \sin x + C$
- $\int \sin x\, dx = -\cos x + C$
- $\int \sec^2 x\, dx = \tan x + C$
- $\int \operatorname{cosec}^2 x\, dx = -\cot x + C$
- $\int \sec x \tan x\, dx = \sec x + C$
- $\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + C$
- $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$
- $\int \frac{dx}{\sqrt{1-x^2}} = -\cos^{-1} x + C$
- $\int \frac{dx}{1+x^2} = \tan^{-1} x + C$
- $\int \frac{dx}{1+x^2} = \cot^{-1} x + C$
- $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + C$
- $\int \frac{dx}{x\sqrt{x^2-1}} = -\operatorname{cosec}^{-1} x + C$
- $\int e^x dx = e^x + C$
- $\int a^x dx = \frac{a^x}{\log a} + C$
- $\int \frac{1}{x} dx = \log|x| + C$
- $\int \tan x\, dx = \log|\sec x| + C$
- $\int \cot x\, dx = \log|\sin x| + C$
- $\int \sec x\, dx = \log|\sec x + \tan x| + C$
- $\int \operatorname{cosec} x\, dx = \log|\operatorname{cosec} x - \cot x| + C$

2.Integral of some special functions

- $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + C$
- $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + C$
- $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$

- $\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x+\sqrt{x^2-a^2}\right| + C$
- $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C$
- $\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left|x+\sqrt{x^2+a^2}\right| + C$
- **Error!** = **Error!**
- **Error!** = **Error!**
- **Error!** = **Error!**

3. Integration by parts

(i) $\int f_1(x).f_2(x)dx = f_1(x)\int f_2(x)dx - \int\left[\frac{d}{dx}f_1(x).\int f_2(x)dx\right]dx$ where f_1 & f_2 are functions of x

(ii) $\int e^x\left(f(x)+f'(x)\right)dx = e^x f(x) + C$

4. Integral as a limit of sums:

$$\int_a^b f(x)dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+....+f(a+(n-1)h)\right] \text{ where } h=\frac{b-a}{n}$$

5. Properties of Definite Integrals

- $\int_a^b f(x)dx = \int_a^b f(t)dt$
- $\int_a^b f(x)dx = -\int_b^a f(x)dx$

  In particular, $\int_a^a f(x)dx = 0$
- $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$
- $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$
- $\int_0^a f(x)dx = \int_0^a f(a-x)dx$
- $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$

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- $$\int_0^{2a} f(x)dx = 2\int_0^{a} f(x)dx, \text{if } f(2a-x) = f(x)$$
$$= 0 \qquad , \text{if } f(2a-x) = -f(x)$$

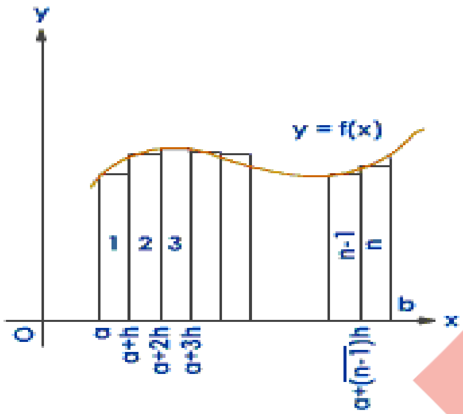
$$\int_{-a}^{a} f(x)dx = 2\int_0^{a} f(x)dx, \text{if } f(-x) = f(x)$$
$$= 0 \qquad , \text{if } f(-x) = -f(x)$$

Class XII: Mathematics
Chapter 8: Applications of Integrals

Chapter Notes

Key Concepts

1. Definite integral $\int_a^b f(x)dx$ of the function f(x) from limits a to b represents the area enclosed by the graph of the function f(x) the x axis, and the vertical lines x= 'a' and x = 'b'



2. Area function is given by

$A(x) = \int_a^x f(x)dx$, where x is a point in [a, b]

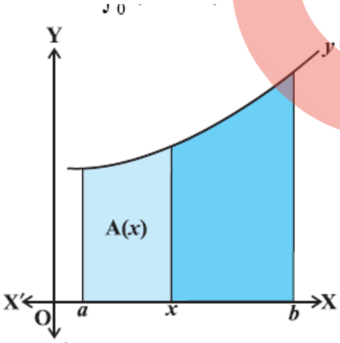
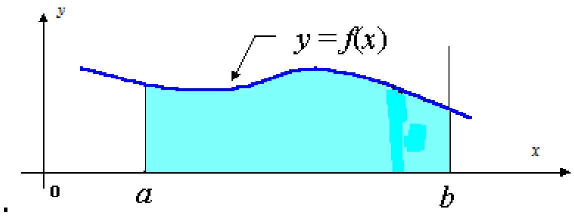


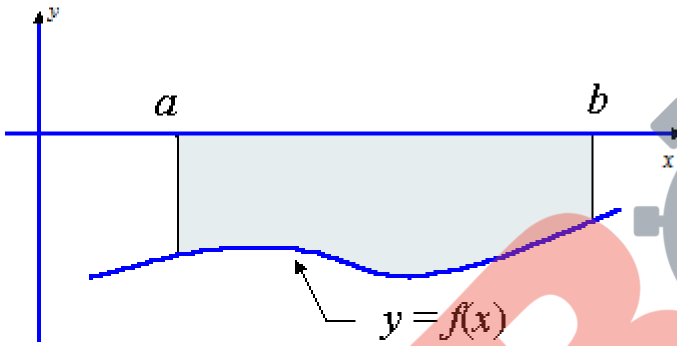
Fig 7.3

3. Area bounded by a curve, x-axis and two ordinates
Case 1: when curve lies above axis as shown below



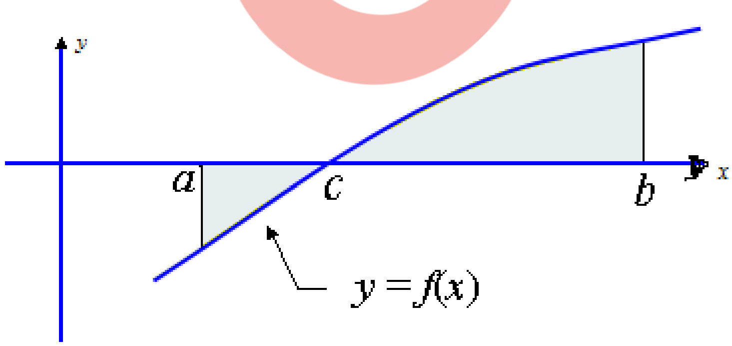
$$\text{Area} = \int_a^b f(x)dx$$

Case 2: Curves which are entirely below the x-axis as shown below



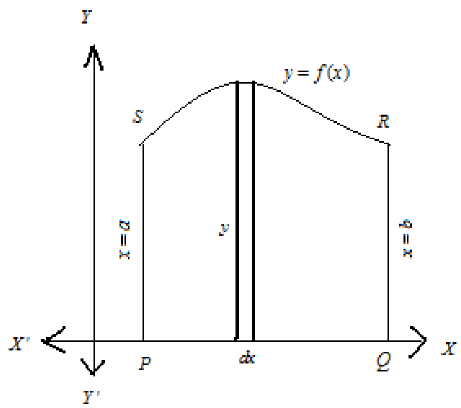
$$\text{Area} = \left|\int_a^b f(x)dx\right|$$

Case 3: Part of the curve is below the *x*-axis and part of the curve is above the *x*-axis.



$$\text{Area} = \left|\int_a^c f(x)dx\right| + \int_c^b f(x)dx$$

4, area bounded by the curve y=f(x), the x-axis and the ordinates x=a and x=b using elementary strip method is computed as follows

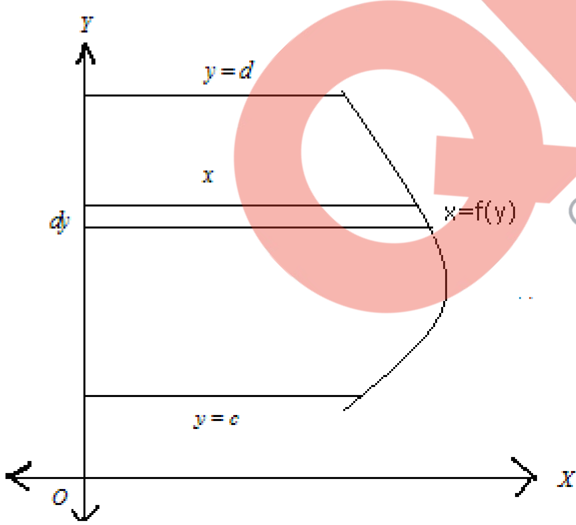


Area of elementary strip = y.dx

Total area = $\int_a^b dA = \int_a^b y dx = \int_a^b f(x) dx$

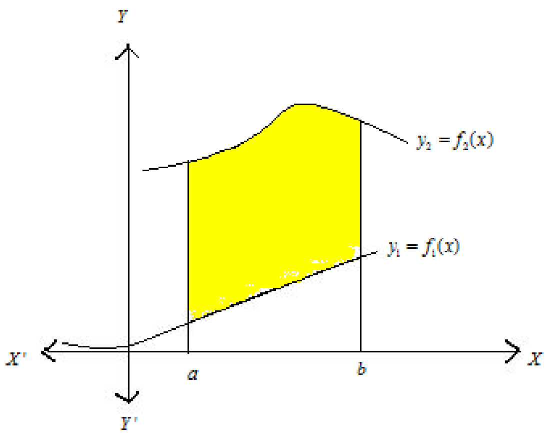
5. The area bounded by the curve x=f(y), the y-axis and the abscissa y=c and y=d is given by

$\int_c^d f(y) dy$ or, $\int_c^d x dy$



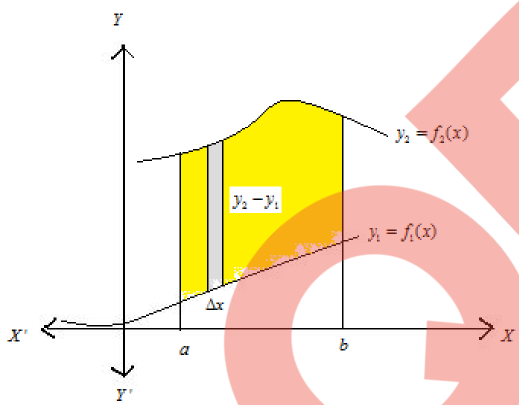
6. Area between $y_1 = f_1(x)$ and $y_2 = f_2(x)$, x = a and x = b is given by

$$\int_a^b y_2 dx - \int_a^b y_1 dx = \int_a^b (y_2 - y_1) dx$$



Area between two curves is the difference of the areas of the two graphs.

7. Area using strip



Each "typical" rectangle indicated has width Δx and height $y_2 - y_1$

Hence, Its area = $(y_2 - y_1)\ \Delta x$

Total Area = $\sum_{x=a}^{b} (y_2 - y_1)\Delta x$

Area = $\int_a^b (y_2 - y_1)dx$

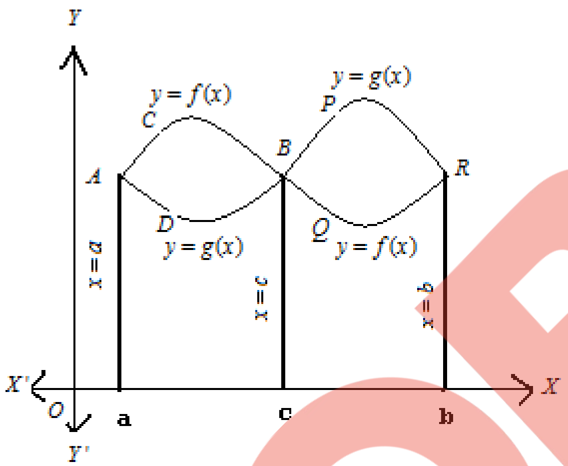
Area between two curves is also equal to integration of the area of an elementary rectangular strip within the region between the limits.

8. The area of the region bounded by the curve y = f (x), x-axis and the lines x = a and x = b (b > a) is Area= $\int_a^b y\,dx = \int_a^b f(x)\,dx$

9. The area of the region enclosed between two curves *y* = *f* (*x*), *y* = *g* (*x*) and the lines *x* = *a*, *x* = *b* is

   Area = $\int_a^b [f(x) - g(x)]dx$ where, f(x) > g(x) in [a,b]

10. If f(x) ≥g(x) in [a, c] and f(x)≤g(x) in [c, b], where a< c< b



then the area of the regions bounded by curves is
Total Area= Area of the region ACBDA + Area of the region BPRQB

$= \int_a^c [f(x) - g(x)]dx + \int_c^b [g(x) - f(x)]dx$

Key Formulae

1. Some standard Integrals

- $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
- $\int dx = x + C$
- $\int \cos x\,dx = \sin x + C$

- $\int \sin x \, dx = -\cos x + C$
- $\int \sec^2 x \, dx = \tan x + C$
- $\int \operatorname{co}\sec^2 x \, dx = -\cot x + C$
- $\int \sec x \tan x \, dx = \sec x + C$
- $\int \operatorname{co}\sec x \cot x \, dx = -\operatorname{cosec} x + C$
- $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$
- $\int \frac{dx}{\sqrt{1-x^2}} = -\cos^{-1} x + C$
- $\int \frac{dx}{1+x^2} = \tan^{-1} x + C$
- $\int \frac{dx}{1+x^2} = \cot^{-1} x + C$
- $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + C$
- $\int \frac{dx}{x\sqrt{x^2-1}} = -\operatorname{cos}\operatorname{ec}^{-1} x + C$
- $\int e^x dx = e^x + C$
- $\int a^x dx = \frac{a^x}{\log a} + C$
- $\int \frac{1}{x} dx = \log|x| + C$
- $\int \tan x \, dx = \log|\sec x| + C$
- $\int \cot x \, dx = \log|\sin x| + C$
- $\int \sec x \, dx = \log|\sec x + \tan x| + C$
- $\int \operatorname{co}\sec x \, dx = \log|\operatorname{co}\sec x - \cot x| + C$

2.Integral of some special functions

- $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + C$
- $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + C$
- $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$
- $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left|x + \sqrt{x^2 - a^2}\right| + C$

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- $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + C$
  - $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log\left|x + \sqrt{x^2 + a^2}\right| + C$

Class XII: Mathematics
Chapter 9: Differential Equations

Chapter Notes

Key Concepts

1. An equation involving derivatives of dependent variable with respect to independent variable is called a **differential equation**.

   For example: $\frac{dy}{dx} = \cos x \quad \frac{dy}{dx} = \frac{x^2 + y^2}{2x}$
2. Order of a differential equation is the order of the highest order derivative occurring in the differential equation.

   For example: order of $\frac{d^3y}{dx^3} + 3x(\frac{dy}{dx}) - 8y = 0$ is 3.

3. Degree of a differential equation is the highest power (exponent) of the highest order derivative in it when it is written as a polynomial in differential coefficients.

   Degree of equation $\left(\frac{d^2y}{dx^2}\right)^3 + (c+b)\left(\frac{dy}{dx}\right)^4 = y$ is 3
4. Both order as well as the degree of differential equation are positive intgers.

5. A function which satisfies the given differential equation is called its solution.

6. The solution which contains as many arbitrary constants as the order of the differential equation is called a **general solution**.

7. The solution which is free from arbitrary constants is called **particular solution**.
8. Order of differential equation is equal to the number of arbitrary constants present in the general solution.

9. An n^{th} order differential equation represents an n-parameter family of curves.
10. There are 3 Methods of Solving First Order, First Degree Differential Equations namely
    (i) Separating the variables if the variables can be separated.
    (ii) Substitution if the equation is homogeneous.
    (iii) Using integrating factor if the equation is linear different
11. **Variable separable** method is used to solve equations in which variables can be separated i.e terms containing y should remain with dy & terms containing x should remain with dx.

12.A differential equation which can be expressed in the form $\frac{dy}{dx} = f(x,y)$ or $\frac{dx}{dy} = g(x,y)$ where, f (x, y) & g(x, y) are homogenous functions is called a homogeneous differential equation.

13.Degree of each term is same in a homogeneous differential equation

14.A differential equation of the form $\frac{dy}{dx} + Py = Q$, where P and Q are constants or functions of x only is called a first order linear differential equation. If equation is of the form $\frac{dx}{dy} + Px = Q$ then P and Q are constants or functions of y

15.Steps to solve a homogeneous differential equation

$$\frac{dy}{dx} = F(x,y) = g\left(\frac{y}{x}\right) \qquad \text{.........(1)}$$

- Substitute y=v.x ....(2)
- Differentiate (2) wrt to x

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \qquad \text{......(3)}$$

- Substitute & separate the variables

$$\frac{dv}{g(v) - v} = \frac{dx}{x}$$

- Integrate, $\int \frac{dv}{g(v) - v} = \int \frac{dx}{x} + C$

16. $\frac{dy}{dx} + Py = Q$ where, P and Q are constants or functions of x only

Integrating factor (I.F)= $e^{\int Pdx}$

Solution: y (I.F) = ∫ (Q × I.F)dx + C

17. $\frac{dx}{dy} + P_1 y = Q_1$ where, P_1 & Q_1 are constants or functions of y only

Integrating factor (I.F)= $e^{\int P1dy}$

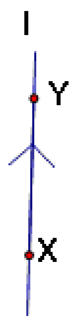
Solution: x (I.F) = ∫ (Q × I.F)dy + C

Class XII: Mathematics
Chapter 9: Vector Algebra
Chapter Notes

Key Concepts

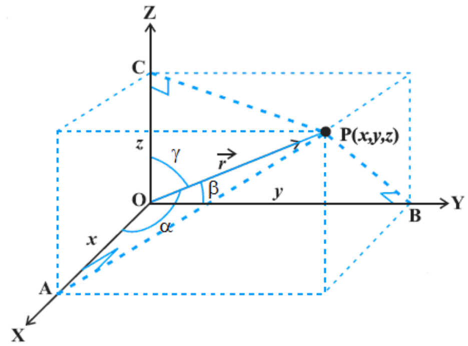
1. A quantity that has magnitude as well as direction is called a vector.

2. A directed line segment is called a vector.



The point X from where the vector starts is called the initial point and the point Y where it ends is called the terminal point.

3. For vector $\overrightarrow{XY}$, magnitude =distance between X and Y and is denoted by $|\overrightarrow{XY}|$, which is greater then or equal to zero.
4. The distance between the initial point and the terminal point is called the magnitude of the vector.
5. The position vector of point P ≡ (x_1, y_1, z_1) with respect to the origin is given by: $\overrightarrow{OP} = \vec{r} = \sqrt{x^2 + y^2 + z^2}$
6. If the position vector $\overrightarrow{OP}$ of a point P makes angles α, β and γ with x, y and z axis respectively, then α, β and γ are called the **direction angles** and $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are called the **Direction cosines** of the position vector $\overrightarrow{OP}$.
7. Then $\lambda = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$ are called the direction cosines of $\vec{r}$.



8. The numbers lr,mr, nr, proportional to l,m,n are called direction ratios of vector $\vec{r}$, and are denoted by a,b,c.
In general,$\lambda^2+m^2+n^2=1$ but $a^2+b^2+c^2 \neq 1$

9.Vectors can be classified on the basis of position and magnitude. On the basis of magnitude vectors are: zero vector and unit vector. On the basis of position, vectors are: coinitial vectors, parallel vectors,free vectors, and collinear vectors .

10. Zero vector is a vector whose initial and terminal points coincide and is denoted by $\vec{0}$. $\vec{0}$ is called the additive identity.

11. The Unit vector has a magnitude equal to 1.A unit vector in the direction of the given vector $\vec{a}$ is denoted by $\hat{a}$.

12. Co initial vectors are vectors having the same initial point.

13. Collinear vectors are parallel to the same line irrespective of their magnitudes and directions.

14. Two vectors are said to be parallel if they are non zero scalar multiples of one another.

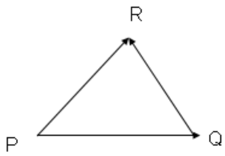
15. Equal vectors as the name suggests, are vectors which have same magnitude and direction irrespective of their initial points.

16. The negative vector of a given vector $\vec{a}$ is a vector which has the same magnitude as $\vec{a}$ but the direction is opposite of $\vec{a}$

17. A vector whose initial position is not fixed is called free vector.

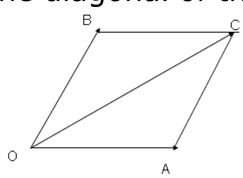
18. Two vectors can be added using the triangle law and parallelogram law of vector addition Vector addition is both commutative as well as associative

19. **Triangle Law of Vector Addition**: Suppose two vectors are represented by two sides of a triangle in sequence, then the third closing side of the triangle represents the sum of the two vectors



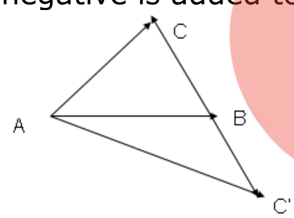
$\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$

20. **Parallelogram Law of Vector Addition**: If two vectors $\vec{a}$ and $\vec{b}$ are represented by two adjacent sides of a parallelogram in magnitude and direction, then their sum $\vec{a}+\vec{b}$ is represented in magnitude and direction by the diagonal of the parallelogram.



$\overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OC}$

21. Difference of vectors: To subtract a vector $\overrightarrow{BC}$ from vector $\overrightarrow{AB}$ its negative is added to $\overrightarrow{AB}$



$\overrightarrow{BC'} = -\overrightarrow{BC}$

$\overrightarrow{AB} + \overrightarrow{BC'} = \overrightarrow{AC'}$

$\Rightarrow \overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{AC'}$

22. If $\vec{a}$ is any vector and k is any scalar then scalar product of $\vec{a}$ and k is $k\vec{a}$.

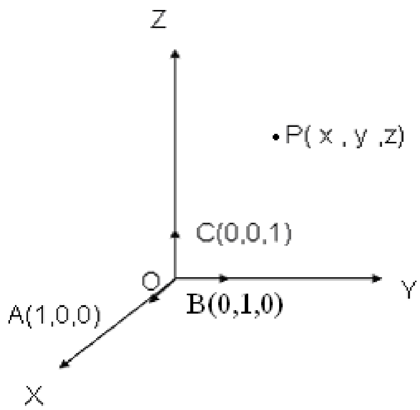
$k\vec{a}$ is also a vector , collinear to the vector $\vec{a}$.

$k>0 \Rightarrow k\vec{a}$ has the same direction as $\vec{a}$.

$k<0 \Rightarrow k\vec{a}$ has opposite direction as $\vec{a}$.

Magnitude of $k\vec{a}$ is |k| times the magnitude of vector $k\vec{a}$.

23. Unit vectors along OX, OY and OZ are denoted by of $\hat{i}, \hat{j}$ and $\hat{k}$ respectively.



Vector $\overrightarrow{OP} = \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is called the component form of vector r

Here, x, y and z are called the scalar components of $\vec{r}$ in the directions of $\hat{i}, \hat{j}$ and $\hat{k}$, and $x\hat{i}, y\hat{j}$ are $z\hat{k}$ are called the vector components of vector r along the respective axes.

24Two vectors are $\vec{a}$ and $\vec{b}$ are collinear $\Leftrightarrow$ $\vec{b} = k\vec{a}$, where k is a non zero scalar. Vectors $\vec{a}$ and $k\vec{a}$ are always collinear.

25. If $\vec{a}$ and $\vec{b}$ are equal then $|\vec{a}| = |\vec{b}|$.

26. If P(x_1, y_1, z_1) and Q(x_2, y_2, z_2) are any two points then vector joining P and Q is, $\overrightarrow{PQ}$=position vector of Q – position vector of P i.e $\overrightarrow{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

27.

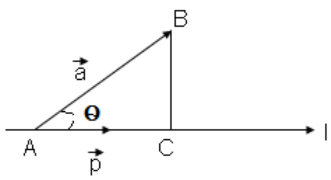
The position vector of a point R dividing a line segment joining the points P and Q whose position vectors are $\vec{a}$ and $\vec{b}$ respectively, in the ratio m : n

·(i) internally, is given by $\dfrac{n\vec{a}+m\vec{b}}{m+n}$

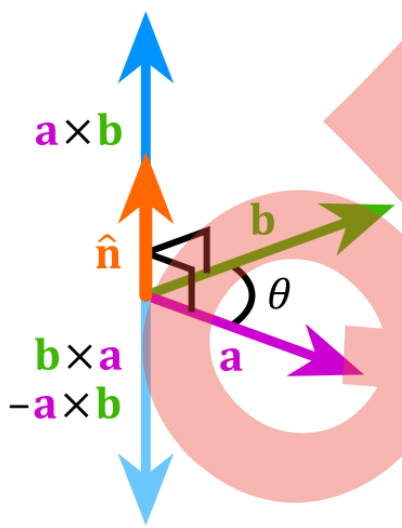
(ii) externally, is given by $\dfrac{m\vec{b}-n\vec{a}}{m-n}$

28. Scalar or dot product of two non zero vectors $\vec{a}$ and $\vec{b}$ is denoted by $\vec{a}\cdot\vec{b}$ given by $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$, where θ is the angle between vectors $\vec{a}$ and $\vec{b}$ and $0 \le \theta \le \pi$

29.Projection of vector **AB,** making an angle of θ with the line L, on line L is vector $\vec{P} = |\overrightarrow{AB}|$ **cos** θ



30. The vector product of two non zero vectors $\vec{a}$ and $\vec{b}$ denoted by $\vec{a}\times\vec{b}$ is defined as $\vec{a}\times\vec{b} = |a||b|\sin\theta\hat{n}$ where θ is the angle between $\vec{a}$ and $\vec{b}$, $0 \le \theta \le \pi$ and $\hat{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$,
Here $\vec{a},\vec{b}$ and $\hat{n}$ form a right handed system.



31. Area of a parallelogram is equal to modulus of the cross product of the vectors representing its adjacent sides.
32. Vector sum of the sides of a triangle taken in order is zero.

Key Formulae

1. Properties of addition of vectors

1) vector addition is commutative

$\vec{a}+\vec{b}=\vec{b}+\vec{a}$

2) vector addition is associative.

$\vec{a}+(\overrightarrow{b+c})=(\overrightarrow{a+b})+\vec{c}$

3) $\vec{0}$ is additive identity for vector addition

$\vec{a}+\vec{0}=\vec{0}+\vec{a}=\vec{a}$

2. Magnitude or Length of vector $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ is $|\vec{r}|=\sqrt{x^2+y^2+z^2}$

3. Vector addition in Component Form: Given $\vec{r}_1=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ and $\vec{r}_2=x_2\hat{i}+y_2\hat{j}+z_2\hat{k}$ then $\vec{r}_1+\vec{r}_2=(x_1+x_2)\hat{i}+(y_1+y_2)\hat{j}+(z_1+z_2)\hat{k}$

4. Difference of vectors: Given $\vec{r}_1=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ and $\vec{r}_2=x_2\hat{i}+y_2\hat{j}+z_2\hat{k}$ then $\vec{r}_1-\vec{r}_2=(x_1-x_2)\hat{i}+(y_1-y_2)\hat{j}+(z_1-z_2)\hat{k}$

5. Equal Vectors: Given $\vec{r}_1=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ and $\vec{r}_2=x_2\hat{i}+y_2\hat{j}+z_2\hat{k}$ then $\vec{r}_1=\vec{r}_2$ if and only if $x_1=x_2; y_1=y_2; z_1=z_2$

6. Multiplication of $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ with scalar k is given by $k\vec{r}=(kx)\hat{i}+(ky)\hat{j}+(kz)\hat{k}$

7. For any vector $\vec{r}$ in component form $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ then x, y, z are the direction ratios of $\vec{r}$ and $\frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{x^2+y^2+z^2}}, \frac{z}{\sqrt{x^2+y^2+z^2}}$ are its direction cosines.

8. Let $\vec{a}$ and $\vec{b}$ be any two vectors and k and m being two scalars then

(i) $k\vec{a}+m\vec{a}=(k+m)\vec{a}$

(ii) $k(m\vec{a})=(km)\vec{a}$

(iii) $k(\vec{a}+\vec{b})=k\vec{a}+k\vec{b}$

9. Vectors $\vec{r}_1=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ and $\vec{r}_2=x_2\hat{i}+y_2\hat{j}+z_2\hat{k}$ are collinear if

$$\left(x_1\hat{i}+y_1\hat{j}+z_1\hat{k}\right)=k\left(x_2\hat{i}+y_2\hat{j}+z_2\hat{k}\right)$$

i.e $x_1=kx_2$; $y_1=ky_2$; $z_1=k z_2$

or $\frac{x_1}{x_2}=\frac{y_1}{y_2}=\frac{z_1}{z_2}=k$

10. Scalar product of vectors a and b is $\vec{a}.\vec{b}=|\vec{a}||\vec{b}|\cos\theta$, where θ is the angle between vectors

11.Properties of Scalar product

(i)$\vec{a}\cdot\vec{b}$ is a real number.

(ii)If $\vec{a}$ and $\vec{b}$ are non zero vectors then $\vec{a}\cdot\vec{b}=0 \Leftrightarrow \vec{a}\perp\vec{b}$.

(iii) Scalar product is commutative : $\vec{a}\cdot\vec{b}=\vec{b}.\vec{a}$

(iv)If $\theta=0$ then $\vec{a}\cdot\vec{b}=|\vec{a}|.|\vec{b}|$

(v) If $\theta=\pi$ then $\vec{a}\cdot\vec{b}=-|\vec{a}|.|\vec{b}|$

(vi) scalar product distribute over addition

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors, then

$\vec{a}\cdot(\vec{b}+\vec{c})=\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$

(vii)Let $\vec{a}$ and $\vec{b}$ be two vectors, and λ be any scalar.

Then $(\lambda\vec{a}).\vec{b}=(\lambda\vec{a}).\vec{b}=\lambda(\vec{a}.\vec{b})=\vec{a}.(\lambda\vec{b})$

12. Angle between two non zero vectors $\vec{a}$ and $\vec{b}$ is given by $\cos\theta=\frac{\vec{a}.\vec{b}}{|\vec{a}|.|\vec{b}|}$

or $\theta=\cos^{-1}\left(\frac{\vec{a}.\vec{b}}{|\vec{a}|.|\vec{b}|}\right)$.

13. For unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$

$\hat{i}.\hat{i}=\hat{j}.\hat{j}=\hat{k}.\hat{k}=1$ and $\hat{i}.\hat{j}=\hat{j}.\hat{k}=\hat{k}.\hat{i}=0$

14. Unit vector in the direction of vector $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ is $\frac{\vec{r}}{|\vec{r}|}=\frac{x\hat{i}+y\hat{j}+z\hat{k}}{\sqrt{x^2+y^2+z^2}}$

15

Projection of a vector $\vec{a}$ on other vector $\vec{b}$ is given by:

$\vec{a}.\hat{b}=\vec{a}.\left(\frac{\vec{b}}{|\vec{b}|}\right)=\frac{1}{|\vec{b}|}(\vec{a}.\vec{b})$

16. Cauchy-Schwartz Inequality:

$|\vec{a}.\vec{b}|\le|\vec{a}|.|\vec{b}|$

o

17.Triangle Inequality: $|\vec{a}+\vec{b}| \le |\vec{a}| + |\vec{b}|$

18. Vector r product of vectors a and b is $\vec{a}\times\vec{b}=|a||b|\sin\theta\hat{n}$.

19. Properties of Vector Product:

(i) $\vec{a}\times\vec{b}$ is a vector

(ii) If $\vec{a}$ and $\vec{b}$ are non zero vectors then $\vec{a}\times\vec{b}$ =0 iff $\vec{a}$ and $\vec{b}$ are collinear i.e

$\vec{a}\times\vec{b}=0 \Leftrightarrow \vec{a}||\vec{b}$ Either θ =0 or $\theta = \pi$

(iii) If $\theta = \frac{\pi}{2}$, then $|\vec{a}\times\vec{b}| = |\vec{a}|.|\vec{b}|$

(iv) vector product distribute over addition

If $\vec{a},\vec{b}$ and $\vec{c}$ are three vectors and λ is a scalar, then

(i) $\vec{a}\times(\vec{b}+\vec{c}) = (\vec{a}\times\vec{b})+(\vec{a}\times\vec{c})$

(ii) $\lambda(\vec{a}\times\vec{b})=(\lambda\vec{a})\times\vec{b}=\vec{a}\times(\lambda\vec{b})$

(v) If we have two vectors $\vec{a}$ and $\vec{b}$ given in component form as

$\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$

then $\vec{a}\times\vec{b}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

(vi) For unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$

$\hat{i}\times\hat{i} = \hat{j}\times.\hat{j} = \hat{k}\times.\hat{k} = 0$ and $\hat{i}\times\hat{j} = \hat{k}$, $\hat{j}.\hat{k} = \hat{i}$, $\hat{k}.\hat{i} = \hat{j}$

(vii) $\vec{a}\times\vec{a}=\vec{0}$ as $\theta=0 \Rightarrow \sin\theta = 0$

$\vec{a}\times(-\vec{a})=\vec{0}$ as $\theta = \pi \Rightarrow \sin\theta=0$

$\vec{a} \perp \vec{b} \Rightarrow \theta = \frac{\pi}{2} \Rightarrow \sin\theta=1 \Rightarrow \vec{a}\times\vec{b} = |\vec{a}|\,|\vec{b}|\ \hat{n}$

(vii) . Angle between two non zero vectors $\vec{a}$ and $\vec{b}$ is given by

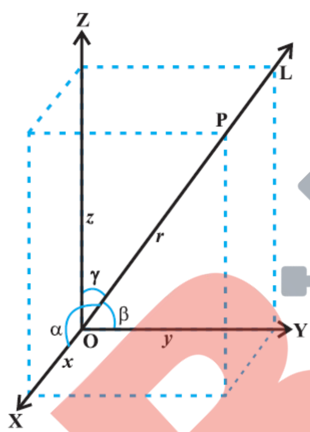
$\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$ or $\theta = \sin^{-1}\left(\frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}\right)$

Class XII: Mathematics
Chapter 10: Three Dimensional Geometry

Chapter Notes

Key Concepts

1. The angles α, β and γ which a directed line L through the origin makes with the x , y and z axes respectively are called direction angles.

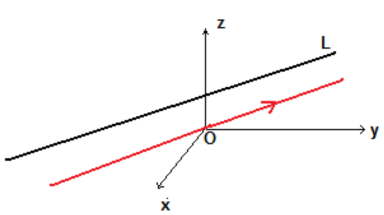


If the direction of line L is reversed then direction angles will be $\pi-\alpha$, $\pi-\beta$, $\pi-\gamma$.

2. If a directed line L passes through the origin and makes angles α , β and γ respectively with the x , y and z axes respectively , then $\lambda = \cos\alpha$, m= $\cos\beta$ and n= $\cos\gamma$ are called direction cosines of line L.

3. For a given line to have unique set of direction cosines take a directed line.

4. The direction cosines of the directed line which does not pass through the origin can be obtained by drawing a line parallel to it and passing through the origin



5.Any three numbers which are proportional to the direction cosines of the line are called direction ratios. If λ , m, n are the direction cosines and a, b,c are the direction ratios then λ=ka , m=kb, n=kc where k is any non zero real number.
6. For any line there are an infinite number of direction ratios.

7. Direction ratios of the line joining P(x_1, y_1, z_1) and Q(x_2, y_2, z_2) may be taken as ,

x_2-x_1 , y_2-y_1 , z_2-z_1 or x_1-x_2 , y_1-y_2 , z_1-z_2

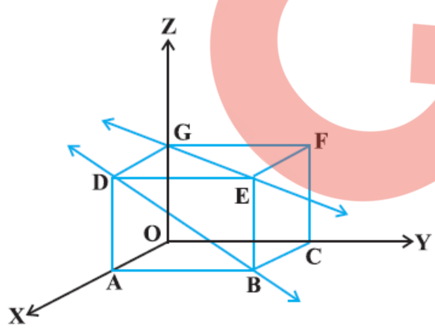
8. Direction cosines of x-axis are cos0, cos 90, cos90 i.e. 1,0,0
Similarly the direction cosines of y axis are 0,1, 0 and z axis are 0,0,1 respectively.

9. A line is uniquely determined if
1) It passes through a given point and has given direction
OR
2) It passes through two given points.

10. Two lines with direction ratios a_1, a_2, a_3 and b_1, b_2, b_3 respectively are perpendicular if:
$a_1b_1 + a_2b_2 + c_1c_2 = 0$

11. Two lines with direction ratios a_1, a_2, a_3 and b_1, b_2, b_3 respectively are parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

12. The lines which are neither intersecting nor parallel are called as skew lines. Skew lines are non coplanar i.e. they don't belong to the same 2D plane.

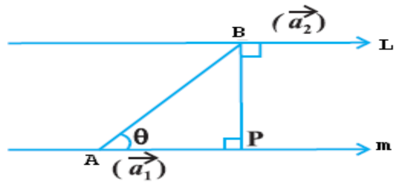


GE and DB are skew lines.

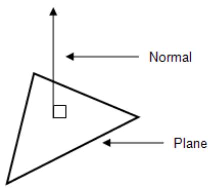
13. **Angle between skew lines** is the angle between two intersecting lines drawn from any point (preferably through the origin) parallel to each of the skew lines.

14. If two lines in space are intersecting then the shortest distance between them is zero.

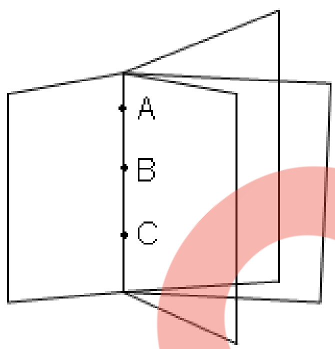
15. If two lines in space are parallel, then the shortest distance between them is the perpendicular distance.



16. The normal vector often simply called the "normal," to a surface; is a vector perpendicular to that surface.



17. If the three points are collinear, then the line containing those three points can be a part of many planes



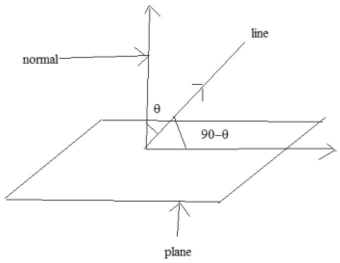
18. The angle between two planes is defined as the angle between their normals.

19. If the planes $A_1x+B_1y+C_1z+D_1=0$ and $A_2x+B_2y+C_2z+D_2=0$ are perpendicular to each other, then $A_1A_2+B_1B_2+C_1C_2=0$

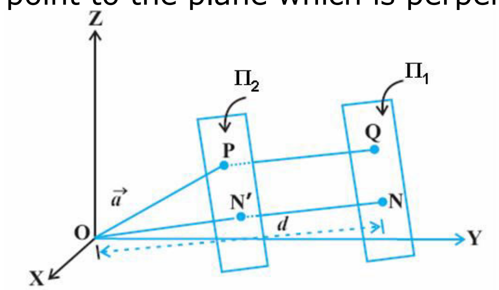
20. If the planes $A_1x+B_1y+C_1z+D_1=0$ and $A_2x+B_2y+C_2z+D_2=0$ are parallel, then

$$\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}$$

21. The angle between a line and a plane is the complement of the angle between the line and the normal to the plane.



22. Distance of a point from a plane is the length of the unique line from the point to the plane which is perpendicular to the plane.



Key Formulae

1. Direction cosines of the line L are connected by the relation $\ell^2+m^2+n^2=1$
2. If a, b, c are the direction ratios of a line and λ,m,n the direction cosines then,
$$\ell=\pm\frac{a}{\sqrt{a^2+b^2+c^2}},\quad m=\pm\frac{b}{\sqrt{a^2+b^2+c^2}},\quad n=\pm\frac{c}{\sqrt{a^2+b^2+c^2}}$$
The direction cosines of the line joining P(x_1,y_1,z_1) and Q(x_2,y_2,z_2) are
3. $\frac{x_2-x_1}{PQ},\frac{y_2-y_1}{PQ},\frac{z_2-z_1}{PQ}$
where $PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Vector equation of a line that passes through the given point whose position vector
4. is $\vec{a}$ and parallel to a given vector $\vec{b}$ is
$\vec{r}=\vec{a}+\lambda\vec{b}$

If coordinates of point A be (x_1,y_1,z_1) and Direction ratios of the line be a, b, c
5. Then, cartesian form of equation of line is :
$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

6.
If coordinates of point A be (x_1,y_1,z_1) and direction cosines of the line be ℓ,m,n
Then, cartesian equation of line is :

$$\frac{x-x_1}{\ell}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$$

7.
The vector equation of a line which passes throughtwo points whose position vectors are $\vec{a}$ and $\vec{b}$ is $\vec{r}=\vec{a}+\lambda(\vec{b}-\vec{a})$

8.
Cartesian equation of a line that passes through two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

9. Angle θ between two lines L_1 and L_2 passing through origin and having direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 is

$$\cos\theta=\left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$

Or $\sin\theta=\dfrac{\sqrt{(a_1b_2-a_2b_1)^2+(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2}}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

10. Shortest distance between two skew lines L and m,
$\vec{r}=\vec{a_1}+\lambda\vec{b_1}$ and $\vec{r}=\vec{a_2}+\mu\vec{b_2}$ is

$$d=\left|\frac{\vec{b_1}\times\vec{b_2}.(\vec{a_2}-\vec{a_1})}{|\vec{b_1}\times\vec{b_2}|}\right|$$

11. The shortest distance between the lines in Cartesian form

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1} \text{ and } \frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$$

is given by

$$d=\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}}$$

12. Distance between parallel lines $\vec{r} = \vec{a_1} + \lambda\vec{b}$ and $\vec{r} = \vec{a_2} + \mu\vec{b}$ is

$$d = \left|\frac{\vec{b} \times (\vec{a_2} - \vec{a_1})}{|\vec{b}|}\right|$$

13. Equation of a plane which is at a distance d from the origin, and $\hat{n}$ is the unit vector normal to the plane through the origin in vector form is

$$\vec{r}.\hat{n} = d$$

14. Equation of a plane which is at a distance of d from the origin and the direction cosines of the normal to the plane as l, m, n is
$lx + my + nz = d$.

15. Equation of a plane perpendicular to a given vector $\vec{N}$ and passing through a given point $\vec{a}$ is $(\vec{r} - \vec{a}).\vec{N} = 0$

16. Equation of a plane perpendicular to a given line with direction ratios A, B, C and passing through a given point (x_1, y_1, z_1) is
$A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$

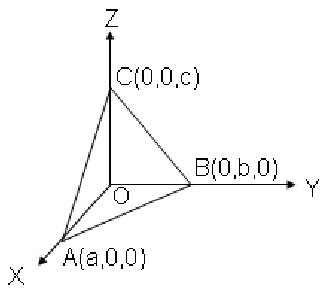
17. Equation of a plane passing through three non-collinear points in vector form is given as
$(\vec{r} - \vec{a}).[(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})] = 0$

18. Equation of a plane passing through three non collinear points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) in Cartesian form is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

19. Intercept form of equation of a plane
$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ where a, b and c are the intercepts on x, y and z-axes

respectively.



20. Any plane passing thru the intersection of two planes $\vec{r}.\vec{n_1} = d_1$ and $\vec{r}.\vec{n_2} = d_2$ is given by,

$\vec{r}.(\vec{n_1} + \lambda\vec{n_2}) = d_1 + \lambda d_2$

21. Cartesian Equation of plane passing through intersection of two planes

$(A_1x + B_1y + C_1z - d_1 + \lambda(A_2x + B_2y + C_2z - d_2) = 0$

22. The given lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ are coplanar if and only if $(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2}) = 0$

23. Let (x_1, y_1, z_1) and (x_2, y_2, z_2) be the coordinates of the points M and N respectively.
Let a_1, b_1, c_1 and a_2, b_2, c_2 be the direction ratios of $\vec{b_1}$ and $\vec{b_2}$ respectively.
The given lines are coplanar if and only if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

24. If $\vec{n_1}$ and $\vec{n_2}$ are normals to the planes $\vec{r}.\vec{n_1} = d_1$ and $\vec{r}.\vec{n_2} = d_2$ and θ is the angle between the normals drawn from some common point then

$$\cos\theta = \left|\frac{\vec{n_1}.\vec{n_2}}{|\vec{n_1}||\vec{n_2}|}\right|$$

25. Let θ is the angle between two planes $A_1x + B_1y + C_1z + D_1 = 0$, $A_2x + B_2y + C_2z + D_2 = 0$
The direction ratios of the normal to the planes are A_1, B_1, C_1 and A_2, B_2,, C_2.

$$\cos\theta = \left|\frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2}\sqrt{A_2^2 + B_2^2 + C_2^2}}\right|$$

o

26. The angle θ between the line and the normal to the plane is given by

$$\cos\theta = \left|\frac{|\vec{b}.\vec{n}|}{|\vec{b}||\vec{n}|}\right|$$

or $\sin\phi = \left|\frac{|\vec{b}.\vec{n}|}{|\vec{b}||\vec{n}|}\right|$ where $\phi = 90^o - \phi$

27. Distance of point P with position vector $\vec{a}$ from a plane $\vec{r}.\vec{N} = d$ is $\frac{|\vec{a}.\vec{N} - d|}{|\vec{N}|}$

where $\vec{N}$ is the normal to the plane

28. The length of perpendicular from origin O to the plane $\vec{r}.\vec{N} = d$ is $\frac{|d|}{|\vec{N}|}$

where $\vec{N}$ is the normal to the plane.

Class XII: Math
Chapter 12: Linear Programming

Chapter Notes

Key Concepts

1. Linear programming is the process of taking various linear inequalities relating to some situation, and finding the "best" value obtainable under those conditions.
2. Linear programming is part of a very important area of mathematics called "optimisation techniques.
3. Type of problems which seek to maximise (or minimise) profit (or cost) form a general class of problems called optimisation problems.
4. A problem which seeks to maximise or minimise a linear function subject to certain constraints as determined by a set of linear inequalities is called an optimisation problem.
5. A linear programming problem may be defined as the problem of maximising or minimising a linear function subject to linear constraints. The constraints may be equalities or inequalities.
6. **Objective Function:** Linear function Z = ax + by, where a, b are constants, x and y are decision variables, which has to be maximised or minimised is called a linear objective function. Objective function represents cost, profit, or some other quantity to be maximised of minimised subject to the constraints.
7. The linear inequalities or equations that are derived from the application problem are problem constraints.
8. The linear inequalities or equations or restrictions on the variables of a linear programming problem are called constraints.
9. The conditions $x \geq 0$, $y \geq 0$ are called non – negative restrictions. Non - negative constraints included because x and y are usually the number of items produced and one cannot produce a negative number of items. The smallest number of items one could produce is zero. These are not (usually) stated, they are implied.
10. A linear programming problem is one that is concerned with finding the optimal value (maximum or minimum value) of a linear function (called objective function) of several variables (say x and y), subject to the conditions that the variables are non – negative and satisfy a set of linear inequalities (called linear constraints).

11. In **Linear Programming** the term **linear** implies that all the mathematical relations used in the problem are linear and **Programming** refers to the method of determining a particular programme or plan of action.

12. Forming a set of linear inequalities (constrains) for a given situation is called formulation of the linear programming problem or LPP.

13. **Mathematical Formulation of Linear Programming Problems**

Step I: In every LPP certain decisions are to be made. These decisions are represented by decision variables. These decision variables are those quantities whose values are to be determined. Identify the variables and denote them by x_1, x_2, x_3, ... Or x,y and z etc

Step II: Identify the objective function and express it as a linear function of the variables introduced in step I.

Step III: In a LPP, the objective function may be in the form of maximising profits or minimising costs. So identify the type of optimisation i.e., maximisation or minimisation.

Step IV: Identify the set of constraints, stated in terms of decision variables and express them as linear inequations or equations as the case may be.

Step V: Add the non-negativity restrictions on the decision variables, as in the physical problems, negative values of decision variables have no valid interpretation.

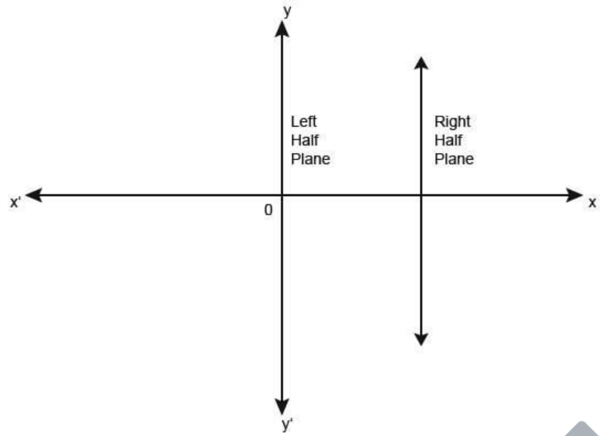
14. General LPP is of the form

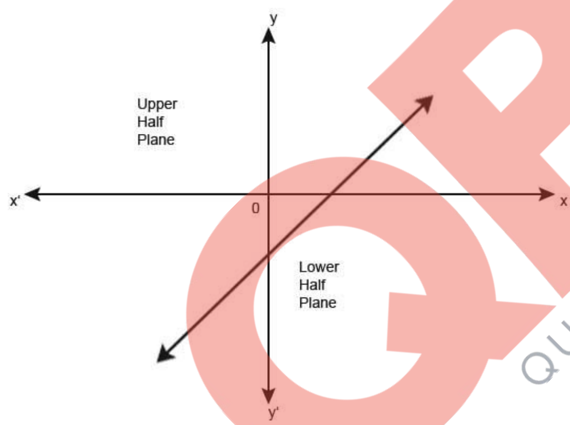
Max (or min) $Z = c_1x_1 + c_2x_2 + \dots + c_nx_n$ $c_1, c_2, \dots. c_n$ are constants

$x_1, x_2, \dots. x_n$ are called decision variable.

s.t $Ax \leq (\geq) B$ and $x_i \geq 0$

15. A linear inequality in two variables represents a half plane geometrically. Types of half planes





16. The common region determined by all the constraints including non – negative constraints x, y ≥ 0 of a linear programming problem is called the **feasible region** (or solution region) for the problem. The region other than feasible region is called an infeasible region.
17. Points within and on the boundary of the feasible region represent **feasible solution** of the constraints.
18. Any point in the feasible region that gives the optimal value (maximum or minimum) of the objective function is called an optimal solution.
19. Any point outside the feasible region is called an infeasible solution.

20.A corner point of a feasible region is the intersection of two boundary lines.

21.A feasible region of a system of linear inequalities is said to be bounded if it can be enclosed within a circle.

22.**Corner Point Theorem 1:** Let R be the feasible region (convex polygon) for a linear programming problem and let Z = ax + by be the objective function. When Z has an optimal value (maximum or minimum), where the variables x and y are subject to constraints described by linear inequalities, this optimal value must occur at a corner point (vertex) of the feasible region.

23. **Corner Point Theorem 2:** Let R be the feasible region for a linear programming problem, and let Z = ax + by be the objective function. If R is bounded, then the objectives function Z has both a maximum and a minimum value on R and each of these occurs at a corner point (vertex) of R.

24. If R is **unbounded**, then a maximum or a minimum value of the objectives functions may not exist.

25.The **graphical method** for solving linear programming problems in two unknowns is as follows.

A. Graph the feasible region.

B. Compute the coordinates of the corner points.

C. Substitute the coordinates of the corner points into the objective function to see which gives the optimal value.

D. When the feasible region is bounded, M and m are the maximum and minimum values of Z.

E. If the feasible region is not bounded, this method can be misleading: optimal solutions always exist when the feasible region is bounded, but may or may not exist when the feasible region is unbounded.

(i) M is the maximum value of Z, if the open half plane determined by ax + by > M has no point in common with the feasible region. Otherwise, Z has no maximum value.

(ii) Similarly, m is the minimum value of Z,if the open half plane determined by ax+by<m has no point in common with the feasible region. Otherwise, Z has no minimum value.

26. Points within and on the boundary of the feasible region represent feasible solutions of the constraints.

27. If two corner points of the feasible region are both optimal solutions of the same type, i.e., both produce the same maximum or minimum, then any point on the line segment joining these two points is also an optimal solution of the same type.

28. **Types of Linear Programming Problems**

1 **Manufacturing problems** : Problems dealing in finding the number of units of different products to be produced and sold by a firm when each product requires a fixed manpower, machine hours, labour hour per unit of product in order to make maximum profit.

2 **Diet problem:** Problems, dealing in finding the amount of different kinds of nutrients which should be included in a diet so as to minimise the cost of the desired diet such that it contains a certain minimum amount of each constituent/nutrients.

3 **Transportation problems:** Problems dealing in finding the transportation schedule of the cheapest way to transpor a product from plants/factories situated at different locations to different markets.

29. Advantages of LPP

(i) Linear programming technique helps to make the best possible use of available productive resources (such as time, labour, machines etc.)

(ii) A significant advantage of linear programming is highlighting of such bottle necks.

30. Disadvantages of LPP

(i) Linear programming is applicable only to problems where the constraints and objective functions are linear i.e., where they can be expressed as equations which represent straight lines.

(ii) Factors such as uncertainty, weather conditions etc. are not taken into consideration.

Class XII: Mathematics
Chapter : Probability

Chapter Notes

Key Concepts

1. The probability that event B will occur if given the knowledge that event A has already occurred is called conditional probability. It is denoted as P(B|A).

2. Conditional probability of B given A has occurred P(B|A) is given by the ratio of number of events favourable to both A and B to number of events favourable to A.

3. E and F be events of a sample space S of an experiment, then
(i). P(S|F) = P(F|F)=1
(ii) For any two events A and B of sample space S if F is another event such that $P(F) \neq 0$
$P((A \cup B)|F) = P(A|F)+P(B|F)-P((A \cap B)|F)$
(iii) $P(E'|F) = 1-P(E|F)$

4. Two events A and B are independent if and only if the occurrence of A does not depend on the occurrence of B and vice versa.

5. If events A and B are independent then P(B|A) = P(A) and P(A|B)=P(A)

6. Three events A,B, C are independent if they are pair wise independent i.e
$P(A \cap B) = P(A).P(B)$, $P(A \cap C) = P(A).P(C)$, $P(B \cap C) = P(B).P(C)$

7. Three events A,B, C are independent if
$P(A \cap B \cap C) = P(A).P(B).P(C)$
Independence implies pair wise independence, but not conversely.

8. In the case of independent events, event A has no effect on the probability of event B so the conditional probability of event B given event A has already occurred is simply the probability of event B, P(B|A)=P(B).

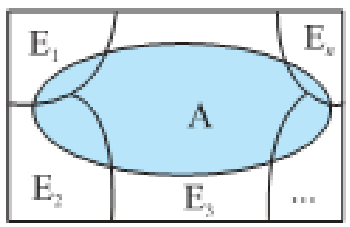
9. If E and F are independent events then so are the events
(i)E′ and F
(ii)E and F′
(iii)E′ and F′

10. If A and B are events such that $B \neq \varphi$ then B is said to affect A
i) favourably if P(A|B) > P(A)

ii) unfavourably if P(A|B) < P(A)
iii) not at all if P(A|B) = P(A).

11. Two events E and F are said to be dependent if they are not independent, i.e. if $P(E \cap F) \neq P(E).P(F)$

12. The events E_1, E_2, ..., En represent a partition of the sample space S if



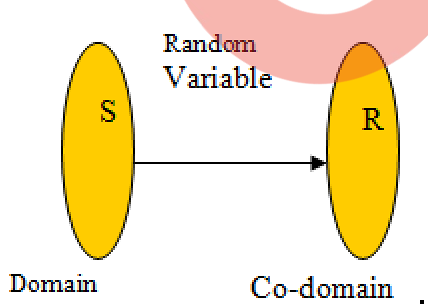
(1) They are pair wise disjoint,
(2) They are exhaustive and
(3) They have nonzero probabilities.

13.The events E_1, E_2,...,E_n are called hypotheses. The probability $P(E_i)$ is called the priori probability of the hypothesis E_i. The conditional probability $P(E_i| A)$ is called a posteriori probability of the hypothesis E_i

14. Bayes' Theorem is also known as the formula for the probability of **"causes".**

15. When the value of a variable is the outcome of a random experiment, that variable is a **random variable.**

16. A random variable is a function that associates a real number to each element in the sample space of random experiment.



17. A random variable which can assume only a finite number of values or countably infinite values is called a discrete random variable.
In experiment of tossing three coins a random variable X representing number of heads can take values 0, 1, 2, 3.

18. A random variable which can assume all possible values between certain limits is called a continuous random variable.
Examples include height, weight etc.

19. The probability distribution of a random variable X is the system of numbers

X	:	x_1	x	.....	x_n
P(X)	:	p_1	p	.....	p_n

where $p_i > 0, \sum_{i=1}^{n} p_i = 1, i = 1, 2, 3, ..., n$

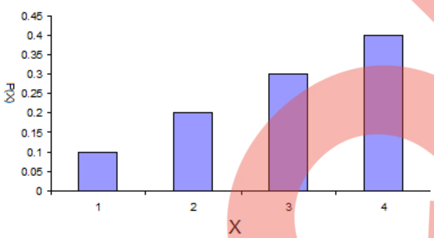
The real numbers $x_1, x_2, ..., x_n$ are the possible values of the random variable X and p_i ($i = 1,2,..., n$) is the probability of the random variable X taking the value x_i i.e. $P(X = x_i) = p_i$

20. In the probability distribution of x each probability pi is non negative, and sum of all probabilities is equal to 1.

21. Probability distribution of a random variable x can be represented using bar charts.

X	1	2	3	4
P(X)	.1	.2	.3	.4

Tabular Representation



Graphical Representation

22. The expected value of a random variable indicates its average or central value.

23. The expected value of a random variable indicates its average or central value. It is a useful summary value of the variable's distribution.

24. Let X be a discrete random variable which takes values $x_1, x_2, x_3, ...x_n$ with probability $p_i = P\{X = x_i\}$, respectively. The mean of X, denoted by μ, is summation pixi

25. Trials of a random experiment are called Bernoulli trials, if they satisfy the following conditions :

(i) There should be a finite number of trials.

(ii) The trials should be independent.

(iii) Each trial has exactly two outcomes: success or failure.

(iv) The probability of success remains the same in each trial.

26. **Binomial distribution** is the discrete probability distribution of the number of successes in a sequence of *n* independent binomial experiments, each of which yields success with probability p.

27. **Bernoulli experiment** is a random experiment whose trials have two outcomes that are mutually exclusive: they are, termed, success and failure.

28. Binomial distribution with n-Bernoulli trials, with the probability of success in each trial as p, is denoted by B (n, p). n and p are called the parameters of the distribution.

29. The random variable *X* follows the binomial distribution with parameters *n* and *p*, we write *XK* ~ B(*n*, *p*). The probability of getting exactly *k* successes in *n* trials is given by the probability mass function
$P(X = k) = {}^{n}C_{k}\, q^{n-k}\, p^{k}$

30. Equal means of two probability distributions does not imply same distributions.

Key Formulae

1. $0 \le P(B|A) \le 1$

2. If E and F are two events associated with the same sample space of a random experiment, the conditional probability of the event E given that F has occurred, i.e. P (E|F) is given by

$$P(E\,|\,F) = \frac{n(E \cap F)}{n(F)} \text{ provided } P(F) \neq 0 \text{ or}$$

$$P(F\,|\,E) = \frac{n(E \cap F)}{n(E)} \text{ provided } P(E) \neq 0$$

3. **Multiplication Theorem:**
   (a) For two events
   Let E and F be two events associated with a sample space S.
   $P(E \cap F) = P(E)\, P(F|E) = P(F)\, P(E|F)$ provided $P(E) \neq 0$ and $P(F) \neq 0$.

(b) For three events:
If E, F and G are three events of sample space S,

$P(E \cap F \cap G) = P(E)\,P(F|E)\,P(G|(E \cap F)) = P(E)\,P(F|E)\,P(G|EF)$

4. **Multiplication theorem for independent Events**
   (i) $P(E \cap F) = P(E)P(F)$
   (ii) $P(E \cap F \cap G) = P(E)P(F)P(G)$

5. Let E and F be two events associated with the same random experiment Two events E and F are said to be independent, if
   (i) $P(F|E) = P(F)$ provided $P(E) \neq 0$ and
   (ii) $P(E|F) = P(E)$ provided $P(F) \neq 0$
   (iii) $P(E \cap F) = P(E) \cdot P(F)$

6. Occurrence of atleast one of the two events A or B
   $P(A \cup B) = 1 - P(A')P(B')$

7. A set of events $E_1, E_2, \ldots, E_n$ is said to represent a partition of the sample space S if
(a) $E_i \cap E_j = \phi$, $i \neq j$, $i, j = 1, 2, 3, \ldots, n$
(b) $E_1 \cup E_2 \cup E_n = S$
(c) $P(E_i) > 0$ for all $i = 1, 2, \ldots, n$.

8. **Theorem of Total Probability**
Let $\{E_1, E_2, \ldots, E_n\}$ be a partition of the sample space S, and suppose that each of the events $E_1, E_2, \ldots, E_n$ has nonzero probability of occurrence. Let A be any event associated
with S, then
$P(A) = P(E_1)\,P(A|E_1) + P(E_2)\,P(A|E_2) + \ldots + P(E_n)\,P(A|E_n)$

$$= \sum_{j=1}^{n} P(E_j)\,P(A \mid E_j)$$

9. **Bayes' Theorem**
If $E_1, E_2, \ldots, E_n$ are n non-empty events which constitute a partition of sample space S and
A is any event of nonzero probability, then

$$P(E_i \mid A) = \frac{P(E_i)P(A \mid E_i)}{\sum_{j=1}^{n} P(E_j)P(A \mid E_j)} \text{ for any } I = 1,2,3,\ldots n$$

10. The mean or expected value of a random variable X, denoted by E(X) or μ is defined as

$$E(X) = \mu = \sum_{i=1}^{n} x_i p_i$$

11. The variance of the random variable X, denoted by Var (X) or σ_x^2 is defined as

$$\sigma_x^2 = Var(X) = \sum_{i=1}^{n}(x_i - \mu)^2 p(x_i) = E(X-\mu)^2$$

$$Var(X) = E(X^2) - [E(X)]^2$$

12. Standard Deviation of random variable X:

$$\sigma_x = \sqrt{Var(X)} = \sqrt{\sum_{i=1}^{n}(x_i - \mu)^2 p(x_i)}$$

13. For Binomial distribution B (n, p),
$P(X = x) = {}^nC_x\, q^{n-x} p^x,\ x = 0, 1, \ldots, n\ (q = 1 - p)$

14. Mean and Variance of a variable X following Binomial distribution
$E(X) = \mu = np$
$Var(X) = npq$
Where n is number of trials, p = probability of success
q = probability of failures
15. Standard Deviation of a variable X following Binomial distribution

$$\sigma_x = \sqrt{Var(X)} = \sqrt{npq}$$