

11th Standard –Mathematics

Introduction to Three Dimensional Geometry

Coordinate Axes

In three dimensions, the coordinate axes of a rectangular cartesian coordinate system are three mutually perpendicular lines. These axes are called the X, Y and Z axes.

Coordinate Planes

The three planes determined by the pair of axes are the coordinate planes. These planes are called XY, YZ and ZX plane and they divide the space into eight regions known as octants.

Coordinates of a Point in Space

The coordinates of a point in the space are the perpendicular distances from P on three mutually perpendicular coordinate planes YZ, ZX, and XY respectively. The coordinates of a point P are written in the form of triplet like (x, y, z) .

The coordinates of any point on

- X-axis is of the form $(x, 0, 0)$
- Y-axis is of the form $(0, y, 0)$
- Z-axis is of the form $(0, 0, z)$
- XY-plane are of the form $(x, y, 0)$
- YZ-plane is of the form $(0, y, z)$
- ZX-plane are of the form $(x, 0, z)$

Distance Formula

The distance between two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

The distance of a point P(x, y, z) from the origin O(0, 0, 0) is given by

$$OP = \sqrt{x^2 + y^2 + z^2}$$

Section Formula

The coordinates of the point R which divides the line segment joining two points P(x₁, y₁, z₁) and Q(x₂, y₂, z₂) internally or externally in the ratio m : n are given by

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n} \right) \text{ and}$$
$$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n} \right) \text{ respectively.}$$

The coordinates of the mid-point of the line segment joining two points P(x₁, y₁, z₁) and Q(x₂, y₂, z₂) are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right).$$

The coordinates of the centroid of the triangle, whose vertices are (x₁, y₁, z₁), (x₂, y₂, z₂) and (x₃, y₃, z₃) are

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3} \right).$$