11th Standard – Mathematics

Limits and Derivatives

Limit

Let y = f(x) be a function of x. If at x = a, f(x) takes indeterminate form, then we consider the values of the function which is very near to a. If these value tend to a definite unique number as x tends to a, then the unique number so obtained is called the limit of f(x) at x = a and we write it as limx→af(x).

Left Hand and Right-Hand Limits

If values of the function at the point which are very near to a on the left tends to a definite unique number as x tends to a, then the unique number so obtained is called the left-hand limit of f(x) at x = a, we write it as

$$f(a-0)=\lim_{x\to a^-} f(x)=\lim_{h\to 0} f(a-h)$$

Similarly, right hand limit is

$$f(a+0)=\lim_{x\to a^+} f(x)=\lim_{h\to 0} f(a+h)$$

Existence of Limit

$\lim_{x\to a} f(x)$ exists, if

(i) $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ both exists

(ii) $\lim_{x\to a^-} f(x)=\lim_{x\to a^+} f(x)$

Some Properties of Limits

Let f and g be two functions such that both limx→af(x) and lim limx→ag(x) exists, then

(i) $\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$

(ii) $\lim_{x \to a} kf(x) = k \lim_{x \to a} f(x)$

(iii) $\lim_{x \to a} f(x) \cdot g(x) = \lim_{x \to a} f(x) \times \lim_{x \to a} g(x)$

(iv) $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$, where $g(x) \neq 0$

Some Standard Limits

(i) $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$

(ii) $\lim_{x \to 0} \frac{\sin x}{x} = 1$

(iii) $\lim_{x \to 0} \frac{\tan x}{x} = 1$

(iv) $\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a$

(v) $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

(vi) $\lim_{x \to 0} \frac{\log(1 + x)}{x} = 1$

Derivatives

Suppose f is a real-valued function, then

$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$ is called the derivative of f at x

iff $\lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$ exists finitely.

Fundamental Derivative Rules of Function

Let f and g be two functions such that their derivatives are defined in a common domain, then

(i) $\frac{d}{dx}[f(x)+g(x)]=\frac{d}{dx}[f(x)]+\frac{d}{dx}[g(x)]$

(ii) $\frac{d}{dx}[(f(x)-g(x)]=\frac{d}{dx}[f(x)]-\frac{d}{dx}[g(x)]$

(iii) $\frac{d}{dx}[f(x).g(x)]=\left[\frac{d}{dx}f(x)\right].g(x)+f(x).\left[\frac{d}{dx}g(x)\right]$

(iv) $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{\left[\frac{d}{dx}f(x)\right]\cdot g(x)-f(x).\left[\frac{d}{dx}g(x)\right]}{[g(x)]^2}$

Some Standard Derivatives

(i) $\frac{d}{dx}(x^n)=nx^{n-1}$

(ii) $\frac{d}{dx}(\sin x)=\cos x$

(iii) $\frac{d}{dx}(\cos x)=-\sin x$

(iv) $\frac{d}{dx}(\tan x)=\sec^2 x$

(v) $\frac{d}{dx}(\cot x)=-\operatorname{cosec}^2 x$

(vi) $\frac{d}{dx}(\sec x)=\sec x \tan x$

(vii) $\frac{d}{dx}(\operatorname{cosec} x)=-\operatorname{cosec} x \cot x$

(viii) $\frac{d}{dx}(a^x)=a^x \log_e a$

(ix) $\frac{d}{dx}(e^x)=e^x$

(x) $\frac{d}{dx}(\log_e x)=\frac{1}{x}$