11th Standard - Mathematics

Mathematical Reasoning

Statements

A statement is a sentence which is either true or false, but not both simultaneously.

Note:

No sentence can be called a statement if

- It is an exclamation.
- It is an order or request.
- It is a question.

Simple Statements

A statement is called simple if it cannot be broken down into two or more statements.

Compound Statements

A compound statement is the one which is made up of two or more simple statement.

Connectives

The words which combine or change simple statements to form new statements or compound statements are called connectives.

Conjunction

If two simple statements p and q are connected by the word 'and', then the resulting compound statement "p and q" is called a conjunction of p and q is written in symbolic form as "$p \wedge q$".

Note:

- The statement $p \wedge q$ has the truth value T (true) whenever both p and q have the truth value T.
- The statement $p \wedge q$ has the truth value F (false) whenever either p or q or both have the truth value F.

Disjunction

If two simple statements p and q are connected by the word 'or', then the resulting compound statement "p or q" is called disjunction of p and q and is written in symbolic form as "$p \vee q$".

Note:

- The statement $p \vee q$ has the truth value F whenever both p and q have the truth value F.
- The statement $p \vee q$ has the truth value T whenever either p or q or both have the truth value T.

Negation

An assertion that a statement fails or denial of a statement is called the negation of the statement. The negation of a statement p in symbolic form is written as "~p".

Note:

- ~p has truth value T whenever p has truth value F.
- ~p has truth value F whenever p has truth value T.

Negation of Conjunction

The negation of a conjunction $p \wedge q$ is the disjunction of the negation of p and the negation of q.

Equivalently we write $\sim (p \wedge q) = \sim p \vee \sim q$.

Negation of Disjunction

The negation of a disjunction p v q is the conjunction of negation of p and the negation of q.

Equivalently, we write $\sim(p \vee q) = \sim p \wedge \sim q$.

Negation of Negation

Negation of negation of a statement is the statement itself.

Equivalently, we write $\sim(\sim p) = p$

The Conditional Statement

If p and q are any two statements, then the compound statement "if p then g" formed by joining p and q by a connective 'if-then' is called a conditional statement or an implication and is written in symbolically $p \rightarrow q$ or $p \Rightarrow q$, here p is called hypothesis (or antecedent) and q is called conclusion (or consequent) of the conditional statement $(p \Rightarrow q)$.

Contrapositive of Conditional Statement

The statement "$(\sim q) \rightarrow (\sim p)$ " is called the contrapositive of the statement $p \rightarrow q$.

Converse of a Conditional Statement

The conditional statement "$q \rightarrow p$" is called the converse of the conditional statement "$p \rightarrow q$".

Inverse of Conditional Statement

The Conditional statement "$q \rightarrow p$" is called inverse of $p \rightarrow q$.

The Biconditional Statement

If two statements p and q are connected by the connective 'if and only if', then the resulting compound statement "p if and only if q" is called biconditional of p and q and is written in symbolic form as $p \Leftrightarrow q$.

Quantifier

(i) For all or for every is called universal quantifier.

(ii) There exists is called existential quantifier.

Validity of Statements

A statement is said to valid or invalid according to as it is true or false.

If p and q are two mathematical statements, then the statement

(i) “p and q” is true if both p and q are true.

(ii) “p or g” is true if p is false

$\Rightarrow$ q is true orq is false $\Rightarrow$ p is true.

(iii) “If p, then q” is true p is true $\Rightarrow$ q is true

or

q is false

$\Rightarrow$ p is false

or

p is true and q is false less us to a contradiction,

(iv) “p if and only if q” is true, if

(a) p is true $\Rightarrow$ q is true and

(b) q is true $\Rightarrow$ p is true.