11th Standard – Physics

Motion in a Plane

• **Motion** in a plane is called as motion in two dimensions e.g., projectile motion, circular motion etc. For the analysis of such motion our reference will be made of an origin and two co-ordinate axes X and Y.

• Scalar and Vector Quantities

Scalar Quantities. The physical quantities which are completely specified by their magnitude or size alone are called scalar quantities.

Examples. Length, mass, density, speed, work, etc.

Vector Quantities. Vector quantities are those physical quantities which are characterised by both magnitude and direction.

Examples. Velocity, displacement, acceleration, force, momentum, torque etc.

• Characteristics of Vectors

Following are the characteristics of vectors:

(i) These possess both magnitude and direction.

(ii) These do not obey the ordinary laws of Algebra.

(iii) These change if either magnitude or direction or both change.

(iv) These are represented by bold-faced letters or letters having arrow over them.

• Unit Vector

A unit vector is a vector of unit magnitude and points in a particular direction. It is used to specify the direction only. Unit vector is represented by putting a cap (^) over the quantity.

The unit vector in the direction of $\vec{A}$ is denoted by $\hat{A}$ and defined by

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{\vec{A}}{A} \text{ or } \vec{A} = A\,\hat{A}$$

- **Equal Vectors**

Vectors $\vec{A}$ and $\vec{B}$ are said to be equal if $|\vec{A}| = |\vec{B}|$ as well as their directions

- **Zero Vector**

A vector with zero magnitude and an arbitrary direction is called a zero vector. It by $\vec{O}$ and also known as null vector.

- **Negative of a Vector**

The vector whose magnitude is same as that of $\vec{a}$ but the direction is opposite called the negative of $\vec{a}$ and is written as $-\vec{a}$.

- **Parallel Vectors**

$\vec{A}$ and $\vec{B}$ are said to be parallel vectors if they have same direction, and may or may magnitude $(\vec{A} \parallel \vec{B})$. If the directions are opposite, then $\vec{A}$ is anti-parallel to $\vec{B}$.

- **Coplanar Vectors**

Vectors are said to be coplanar if they lie in the same plane or they are parallel to the same plane, otherwise they are said to be non-coplanar vectors.

• Displacement Vector

The displacement vector is a vector which gives the position of a point with reference to a point other than the origin of the co-ordinate system.

Displacement vector $\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$.

• Parallelogram Law of Vector Addition

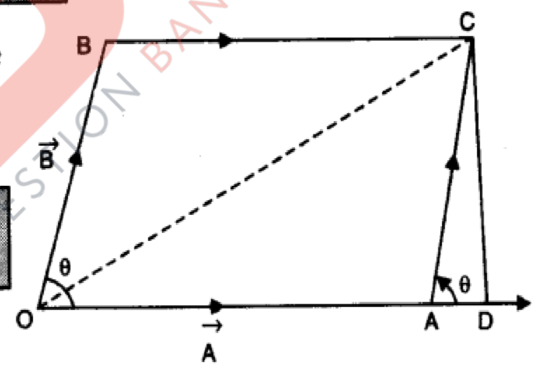
If two vectors, acting simultaneously at a point, can be represented both in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, then the resultant is represented completely both in magnitude and direction by the diagonal of the parallelogram passing through that point.

If $\vec{A}$ and $\vec{B}$ be two adjacent sides of a parallelogram, inclined at angle θ, then the magnitude of resultant vector is given as

$$\boxed{R = \sqrt{A^2 + B^2 + 2AB\cos\theta}}$$

Direction of resultant $\vec{R}$. Let α be the angle made by resultant $\vec{R}$ with vector $\vec{A}$. Then

$$\boxed{\alpha = \tan^{-1}\frac{B\sin\theta}{A + B\cos\theta}}$$

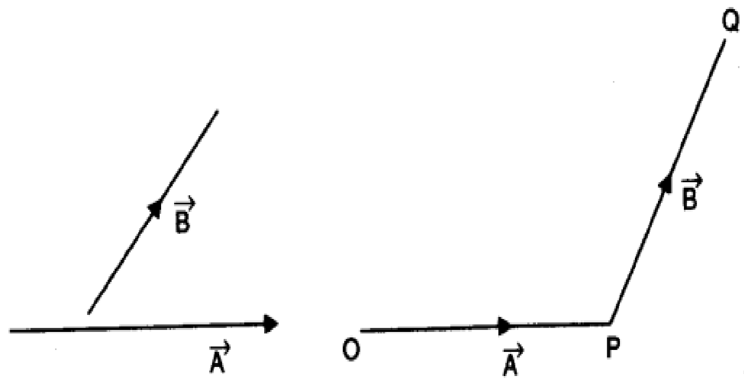


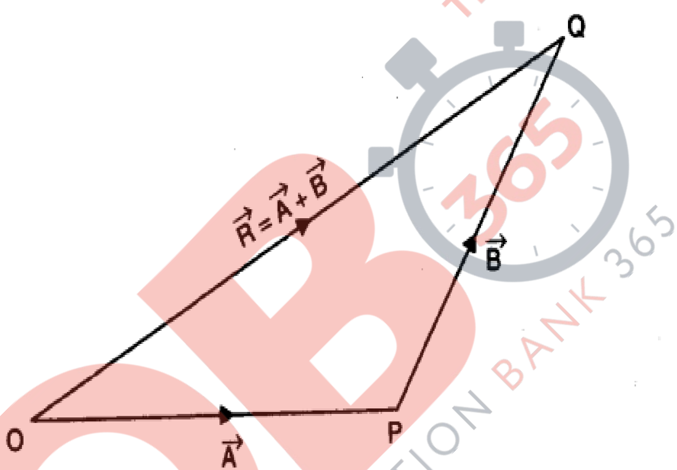
• Triangle Law of Vector Addition

If two vectors are represented both in magnitude and direction by the two sides of a triangle taken in the same order, then the resultant of these vectors is represented both in magnitude and direction by the third side of the triangle taken in the opposite order.

If two vectors $\vec{A}$ and $\vec{B}$ are to be added according to the triangle law of vector assume

$\overrightarrow{OP} = \vec{A}$ and $\overrightarrow{PQ} = \vec{B}$.





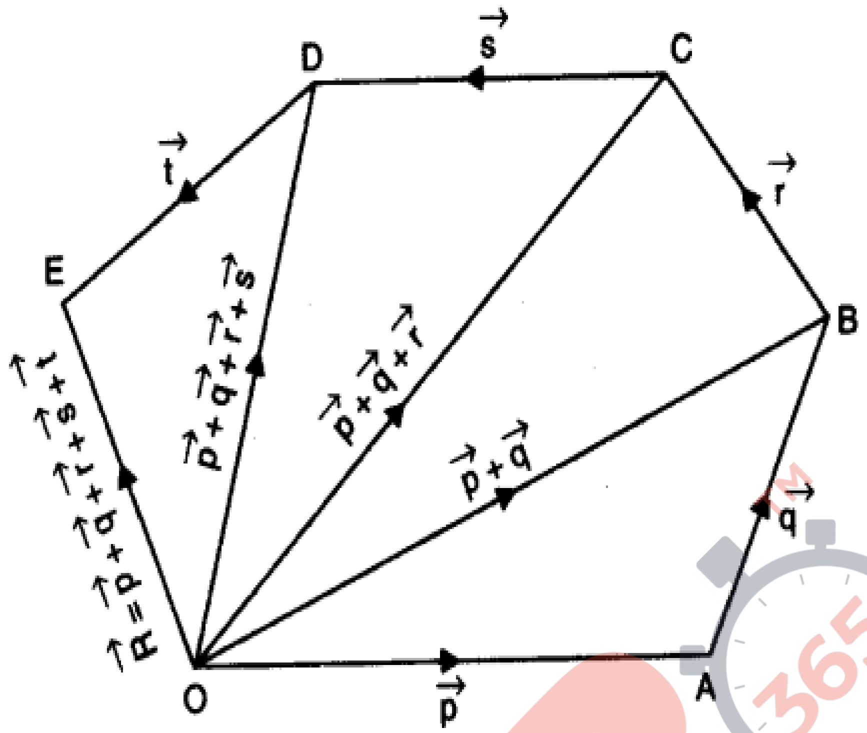
The sum of the resultant of $\vec{A}$ and $\vec{B}$ is represented by the vector $\overrightarrow{OQ}$ (joining tail of $\overrightarrow{OP}$ to the head of $\overrightarrow{PQ}$.

Hence, $\overrightarrow{OP} + \overrightarrow{PQ} = \overrightarrow{OQ}$

$$\overrightarrow{OQ} = \vec{A} + \vec{B} \quad \text{(Resultant vector)}$$

• Polygon Law of Vector Addition

If a number of vectors are represented both in magnitude and direction by the sides of a polygon taken in the same order, then the resultant vector is represented both in magnitude and direction by the closing side of the polygon taken in the opposite order.



Resultant $\vec{R} = \vec{p} + \vec{q} + \vec{r} + \vec{s} + \vec{t}$

• **Properties of Vector Addition**

Vector addition has following properties:

(i) It obeys commutative law

If $\vec{a}$ and $\vec{b}$ are any two vectors,

then $\vec{a} + \vec{b} = \vec{b} + \vec{a}$

(ii) It obeys associative law

If $\vec{a}, \vec{b}$ and $\vec{c}$ are any three vectors then

$$\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$$

(i) **It obeys commutative law**

If $\vec{a}$ and $\vec{b}$ are any two vectors,

then $$\vec{a}+\vec{b} = \vec{b}+\vec{a}$$

(ii) **It obeys associative law**

If $\vec{a},\vec{b}$ and $\vec{c}$ are any three vectors then

$$\vec{a}+\left(\vec{b}+\vec{c}\right) = \left(\vec{a}+\vec{b}\right)+\vec{c}$$

- **Resolution of Vectors**

It is a process of splitting a single vector into two or more vectors in different directions which together produce the same effect as is produced by the single vector alone.

The vectors into which the given single vector is splitted are called component of vectors. In fact, the resolution of a vector is just opposite to composition of vectors.

(iii) **It obeys distributive property**

If $\vec{a}$ and $\vec{b}$ are two vectors and λ is a real number then, $\lambda\left(\vec{a}+\vec{b}\right) = \lambda\vec{a}+\lambda\vec{b}$

- Equilibriant vector is a vector which balances two or more than two vectors acting simultaneously at a point. It is equal in magnitude and opposite in direction to the resultant vector of given vectors.

$\vec{R'}$ (equilibriant vector) $= -\vec{R} = -\left(\vec{A}+\vec{B}+.....\right)$

- If vector $\vec{A}$ is multiplied by a real number λ then it gives a vector $\vec{B}$ whose magnitude is λ times the magnitude of the vector $\vec{A}$ and whose direction is the same or opposite depending upon whether λ is positive or negative.
- Subtraction of vector can be defined in terms of addition of two vectors.

If $\vec{P}$ and $\vec{Q}$ two vectors are to be subtracted then we take them as follows:

$$\vec{P}-\vec{Q} = \vec{P}+\left(-\vec{Q}\right)$$

- Vector subtraction is non-commutative and non-associative.

$\Rightarrow \quad \vec{A}-\vec{B} \neq \vec{B}-\vec{A}$

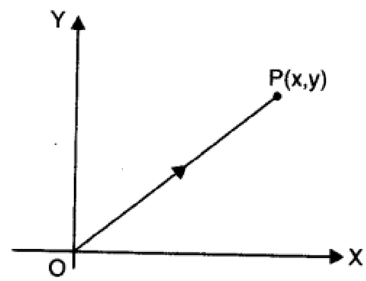
$\Rightarrow \quad \vec{A}-\left(\vec{B}-\vec{C}\right) \neq \left(\vec{A}-\vec{B}\right)-\vec{C}$

If the components of a given vector are perpendicular to each other, then they are called rectangular components.

• Position Vector

Position vector is a vector to represent any position of a body. The straight line joining the origin and the point represents the position vector. It is represented by both magnitude and direction.

It is represented by $\vec{r} = \overline{OP} = x\hat{i} + y\hat{i}$ where $\hat{i}$ and $\hat{j}$ are the unit vectors along x and y axis respectively.



If position vector $\vec{r}$ is in three dimensions, then it is given by $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ where, $\hat{i}, \hat{j}$ and $\hat{k}$ are the unit vectors along x, y and z co-ordinates respectively.

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$$

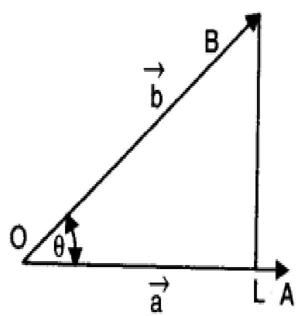
• Multiplication of Vectors

(i) Scalar product (Dot product). Scalar product of two vectors is defined as the product of the magnitude of two vectors with cosine of smaller angle between them.

It is always a scalar, so it is called as scalar product.

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$$

Geometrically, $\vec{a} \cdot \vec{b}$ = (Mod of $\vec{a}$) (Projection of b on $\vec{a}$)



(ii) **Vector product (Cross product).** The cross or vector product of two vectors $\vec{A}$ and $\vec{B}$ is defined as,

$$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}| \sin\theta\, \hat{n}, \quad \text{where}$$

θ – angle between $\vec{A}$ and $\vec{B}$ taken in anti-clockwise direction.

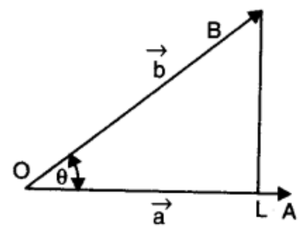
$\hat{n}$ – unit vector in the direction perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

Geometrically, $\vec{a} \times \vec{b}$ is a vector whose modulus is the area of the parallelogram formed by the two vectors as the adjacent sides and direction is perpendicular to both $\vec{a}$ and $\vec{b}$.

- **Properties of Scalar Product**

It is always a scalar, so it is called as scalar product.

$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$$



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Geometrically, $\vec{a}\times\vec{b}$ is a vector whose modulus is the area of the parallelogram formed by the two vectors as the adjacent sides and direction is perpendicular to both $\vec{a}$ and $\vec{b}$.

(vi) Dot product of unit vectors $\hat{i}, \hat{j}, \hat{k}$

$$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$$

$$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$$

(vii) Square of a vector $\vec{a}\cdot\vec{a} = |a|\,|a|\cos 0 = a^2$

(viii) If the two vectors $\vec{A}$ and $\vec{B}$, in terms of their rectangular components, are

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \quad \text{and} \quad \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k},$$ th

$$\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$$

• **Properties of Cross Product**

(i) Cross product of two vectors is not commutative

$$\vec{a}\times\vec{b} \neq \vec{b}\times\vec{a}$$

$$\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$$

(ii) Cross product is not associative

$$\vec{a}\times\left(\vec{b}\times\vec{c}\right) \neq \left(\vec{a}\times\vec{b}\right)\times\vec{c}$$

(iii) Cross product obeys distributive law

$$\vec{a}\times\left(\vec{b}+\vec{c}\right) = \vec{a}\times\vec{b}+\vec{a}\times\vec{c}$$

(iv) If $\theta = 0$ or π it means the two vectors are collinear.

$$\vec{a}\times\vec{b} = \vec{0}$$

and conversely, if $\vec{a}\times\vec{b} = \vec{0}$ then the vector $\vec{a}$ and $\vec{b}$ are parallel provided $\vec{a}$ and $\vec{b}$ are non-zero vectors.

(v) If $\theta = 90°$, and $\hat{n}$ is the unit vector perpendicular to both $\vec{a}$ and $\vec{b}$

$$\vec{a}\times\vec{b} = |a|\,|b| \sin 90° \;\hat{n} = |a|\,|b|\;\hat{n}$$

(vi) The vector product of any vector with itself is $\vec{0}$

$$\vec{a}\times\vec{a} = \vec{0}$$

(vii) If $\vec{a}\times\vec{b} = \vec{0}$, then

$$\vec{a} = 0 \quad\text{or}\quad \vec{b} = 0 \quad\text{or}\quad \vec{a} \parallel \vec{b}$$

(viii) If $\vec{a}$ and $\vec{b}$ are unit vectors, then $\vec{a}\times\vec{b} = 1\cdot 1 \sin\theta\;\hat{n} = \sin\theta\;\hat{n}$

(ix) Cross product of unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$

$$\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$$

$$\hat{i}\times\hat{j} = k = -\hat{j}\times\hat{j}$$

$$\hat{j}\times\hat{k} = \hat{i} = -\hat{k}\times\hat{j}$$

$$\hat{k}\times\hat{i} = \hat{j} = -\hat{i}\times\hat{k}$$

(x) If the two vectors $\vec{A}$ and $\vec{B}$ in terms of their rectangular components are

$$\vec{A} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$$

$$\vec{B} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$$

$$\vec{A}\times\vec{B} = \left(a_1\hat{i} + b_1\hat{j} + c_1\hat{k}\right)\times\left(a_2\hat{i} + b_2\hat{j} + c_2\hat{k}\right)$$

It can be found by the determinant method

i.e.,
$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \hat{i}\ (b_1 c_2 - b_2 c_1) - \hat{j}\ (a_1 c_2 - a_2 c_1) + \hat{k}\ (a_1 b_2 - a_2 b_1)$$

- For motion in a plane, velocity is defined as:

$$\vec{v} = \frac{\vec{r_2} - \vec{r_1}}{t_2 - t_1} = \frac{\left(x_2\,\hat{i} + y_2\,\hat{j}\right) - \left(x_1\,\hat{i} + y_1\,\hat{j}\right)}{(t_2 - t_1)} = \frac{x_2 - x_1}{t_2 - t_1}\,\hat{i} + \frac{y_2 - y_1}{t_2 - t_1}\,\hat{j} = v_x\,\hat{i}\cdot$$

and $v = \sqrt{a_x^2 + a_y^2}$.

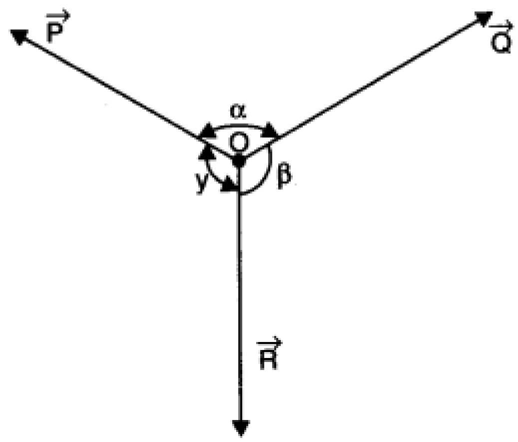
- For motion in a plane, acceleration is defined as

$$\vec{a} = \frac{\vec{v_2} - \vec{v_1}}{t_2 - t_1} = \frac{\left(v_{x_2}\hat{i} + v_{y_2}\hat{j}\right) - \left(v_{x_1}\hat{i} + v_{y_1}\hat{j}\right)}{(t_2 - t_1)} = \left(\frac{v_{x_2} - v_{x_1}}{t_2 - t_1}\right)\hat{i} + \left(\frac{v_{y_2} - v_{y_1}}{t_2 - t_1}\right)\hat{j}$$

and $a = \sqrt{a_x^2 + a_y^2}$.

• Lami's Theorem

Lami's theorem states, "If a particle under the simultaneous action of three forces is in equilibrium, then each force has a constant ratio with the sine of the angle between the other two forces."



If three forces $\vec{P}, \vec{Q}$ and $\vec{R}$ are acting on a particle O in directions given by angles α, β and γ, then, the particle O is in equilibrium, when

$$\boxed{\frac{P}{\sin\beta} = \frac{Q}{\sin\gamma} = \frac{R}{\sin\alpha}}$$

• **Projectile Motion**

The projectile is a general name given to an object that is given an initial inclined velocity and which subsequently follows a path determined by the gravitational force acting on it and by the frictional resistance of the air. The path followed by a projectile is called its trajectory.

Equation of projectile motion. The general case of projectile motion corresponds to that of an object that has been given an initial velocity u at some angle 8 above (or below) the horizontal.

The horizontal and vertical displacements x and y are given by

$$u_x = u\cos\theta$$

$$u_y = u\sin\theta$$

Then the equation of trajectory of a projectile is given as

$$\boxed{y = (\tan\theta)x - \frac{g}{2(u\cos\theta)^2}x^2}$$

The above equation is in the form of $y = ax + bx^2$ where a and b are constants. This is a equation of a parabola. Thus the trajectory of a projectile is parabolic.

Time of Flight. The time taken by a projectile to return to its initial elevation after projection is known as its time of flight (T). It is given by

$$\boxed{T = \frac{2u\sin\theta}{g}}$$

Horizontal Range. The maximum horizontal distance between the point of projection and the point on the horizontal plane where the projectile hits is called horizontal range.

$$\boxed{R = \frac{u^2\sin 2\theta}{g}}$$

The range of the projectile will be maximum if sin 2θ is maximum (*i.e.*, 1).

$$\sin 2\theta = 1$$

or

$$2\theta = 90° \Rightarrow \theta = 45°$$

Thus the projectile has maximum range if it is projected at an angle of 45° with the horizontal.

$$R_{max} = \frac{u^2}{g}$$

Maximum Height. The maximum vertical distance travelled by the projectile during its journey is called the maximum height attained by the projectile.

It is given by $$\boxed{H = \frac{u^2 \sin^2\theta}{2g}}$$

Projectile Given Horizontal Projection:

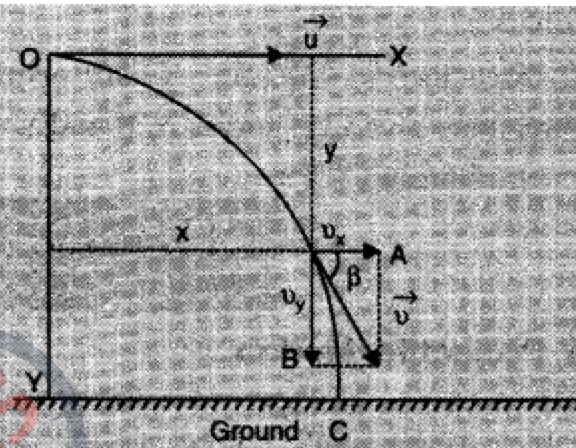
(i) Equation of path $y = kx^2$, which is a parabola

(ii) Time of flight $T = \sqrt{\frac{2h}{g}}$

(iii) Horizontal Range $R = u\sqrt{\frac{2h}{g}}$

(iv) Velocity at any time t is $v = \sqrt{u^2 + g^2t^2}$ and angle made by resultant velocity with horizontal $\beta = \tan^{-1}\left(\frac{gt}{u}\right)$.

(v) Velocity of projectile when it hits the ground $v = \sqrt{u^2 + 2gh}$.



• Angular Displacement

Angular displacement of the object moving around a circular path is defined as the angle traced out by the radius vector at the centre of the circular path in a given time.

θ (angle) = arc/radius

θ —> the magnitude of angular displacement. It is expressed in radians (rad).

• Angular Velocity

Angular velocity of an object in circular motion is defined as the time rate of change of its angular displacement.

It is denoted by ω and is measured in radians per second ($rad.s^{-1}$).

$$\omega = \frac{\text{angular displacement}}{\text{Time}} = \frac{\theta}{t} = \frac{d\theta}{dt}$$

• Angular Acceleration

Angular acceleration of an object in circular motion is defined as the time rate of change of its angular velocity.

It is denoted by 'α' and measured in rad s^{-2}.

$$\alpha = \frac{\text{angular velocity change}}{\text{time taken}} = \frac{d\omega}{dt}$$

- For uniform angular acceleration α, the equations of motion can be modified as,

$$\omega_f = \omega_i + \alpha t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\theta$$

$$\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

• Uniform Circular Motion

When a body moves in a circular path with a constant speed, then the motion of the body is known as uniform circular motion.

The time taken by the object to complete one revolution on its circular path is called time period. For circular motion, the number of revolutions completed per unit time is known as the frequency (v). Unit of frequency is 1 Hertz (1 Hz). It is found that

$$\nu \cdot T = 1 \quad \text{or} \quad \nu = \frac{1}{T}$$

- The relation between angular velocity, frequency and time period is given by

$$\omega = \frac{\theta}{t} = \frac{2\pi}{T} = 2\pi\nu$$

• Centripetal Acceleration

To maintain a particle in its uniform circular motion a radially inward acceleration should be continuously maintained. It is known as the centripetal acceleration.

$$a_c = \frac{v^2}{r} = r\omega^2 = \frac{r \cdot 4\pi^2}{T^2} = r \cdot 4\pi^2 \cdot \omega^2$$

• **IMPORTANT TABLES**

TABLE 4.1 Comparison of Equations of Motion of Linear and Circular Motions

S.No.	Linear Motion	Circular Motion
1.	$S = ut$	$\theta = \omega_0 t$
2.	$S = ut + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
3.	$v = u + at$	$\omega = \omega_0 + \alpha t$
4.	$v^2 - u^2 = 2aS$	$\omega^2 - \omega_0^2 = 2\,\alpha\theta$

TABLE 4.2

Physical Quantity	Symbol	Dimensions	Unit
Position vector	$\mathbf{r}$	[L]	m
Displacement	$\Delta\mathbf{r}$	[L]	m
Velocity		[LT^{-1}]	m s^{-1}
(a) Average	$\overline{\mathbf{V}}$		
(b) Instantaneous	$\mathbf{V}$		
Acceleration		[LT^{-2}]	m s^{-2}
(a) Average	$\bar{a}$		
(b) Instantaneous	a		
Projectile motion			
(a) Time of max. height	t_m	[T]	s
(b) Max. height	h_m	[L]	m
(c) Horizontal range	R	[L]	m
Circular motion			
(a) Angular speed	ω	[T^{-1}]	rad/s
(b) Centripetal acceleration	a_c	[LT^{-2}]	m s^{-2}