9th Standard-Maths

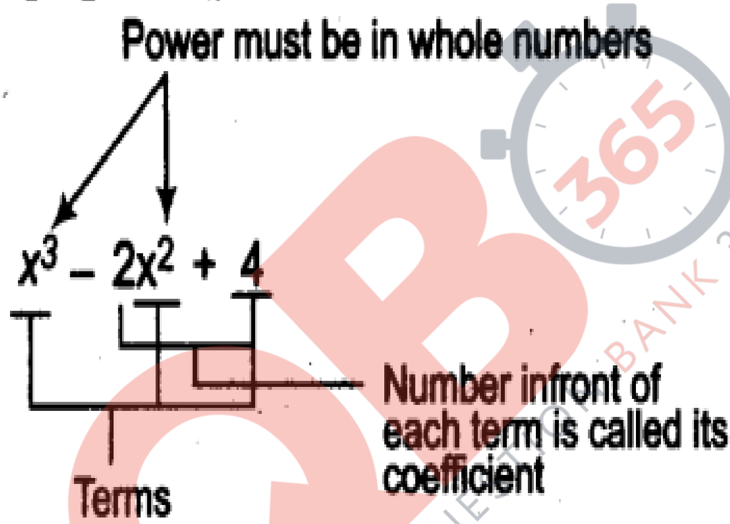
Polynomials

1. Polynomial: A polynomial in one variable x is an algebraic expression of the form

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_2 x^2 + a_1 x + a_0$$

where $a_0, a_1, a_2, \dots, a_n$ are constants and $a_n \neq 0$.

e.g.,



2. Terms: The several parts of a polynomial separated by '+' or '-' operations are called the terms of the expression.

3. Types of Polynomial

(i) Constant polynomial: A polynomial containing one term only, consisting of a constant is called a constant polynomial.

e.g., -6, 4, 23, −34 etc., are still constant polynomial.

Generally, each real number is a constant polynomial.

(ii) Zero polynomial: A polynomial consisting of one term, namely zero only, is called a zero polynomial.

(iii) Monomial: Polynomials having only one term are called monomials ('mono' means 'one').

e.g., μ^{43}, 73 xz and – 2 are all monomials.

(iv) Binomial: Polynomials having only two terms are called binomials ('bi' means 'two').

e.g., $(x^2 + x)$, $(y^{30} + \sqrt{2})$ and $(5x^2y + 6xz)$ are all binomials.

(v) Trinomial: Polynomials having only three terms are called trinomials ('tri' means 'three').

e.g., $(x^4 + x^3 + \sqrt{2})$, $(\mu^{43} + \mu^7 + \mu)$ and $(8y - 5xy + 9xy^2)$ are all trinomials.

4. The degree of a Polynomial: Highest power of the variable in a polynomial is the degree of the polynomial.

(a) In one variable: In case of a polynomial in one variable, the highest power of the variable is called the degree of the polynomial.

e.g.,

$\sqrt{2} - y^3 + y^5 + 2y^6$ is a polynomial in y of degree 6.

$6x + \sqrt{3}$ is a polynomial in x of degree 1.

(b) In two or more variables: In case of a polynomial in more than one variable, the sum of the powers of the variables in each term is taken up and the highest sum so obtained is called the degree of the polynomial.

e.g.,

$3x^3 - 7x^2y^2 + 8$ is a polynomial in x and y of degree 4.

$8a^8b - 4ab + \sqrt{2}$ is a polynomial in a and b of degree 9.

Note:

The degree of a non-zero constant polynomial is 'zero'.

The degree of a zero polynomial is not defined.

5. Linear Polynomial: A polynomial of degree one is called a linear polynomial.

e.g.,

$x + \sqrt{7}$ is a linear polynomial in x, y and z.

$\sqrt{2}\,\mu + 3$ is a linear polynomial in μ.

6. Quadratic Polynomial: A polynomial of degree two is called a quadratic polynomial.

e.g;

$xy + yz + zx$ is a quadratic polynomial in x, y and z.

$x^2 + 9x - 32$ is a quadratic polynomial in x.

7. Cubic Polynomial: A polynomial of degree three is called a cubic polynomial.

e.g.,

$ax^3 + bx^2 + cx + d$ is a cubic polynomial in x and a, b, c, d are constants.

$2y^3 + 3$ is a cubic polynomial in y.

$9x^2y + xy - 4$ is a cubic polynomial in x and y.

8. Value of a Polynomial: Value of a polynomial p(x) at x = a is p(a).
e.g., If $p(x) = x^2 + 2x + 6$ then, at x = 2, $p(2) = 2^2 + 2 \times 2 + 6 = 14$

9. Zeroes of a Polynomial: Zeroes of a polynomial p(x) is a number a such that p(a) = 0.

- Zero may be a zero of a polynomial.
- Every linear polynomial has one and only one zero.
- Zero of a polynomial is also called the root of the polynomial.
- A non-zero constant polynomial has no zero.
- Every real number is a zero of the zero polynomial.
- A polynomial can have more than one zero.

The maximum number of zeroes of a polynomial is equal to its degree.

10. Remainder Theorem: Let p (x) be any polynomial of degree n greater than or equal to one ($n \geq 1$) and let a be any real number. If p (x) is divided by the linear polynomial x – a, then the remainder is p (a).
Dividend = (Divisor × Quotient) + Remainder

11. Factor Theorem: Let q(x) be a polynomial of degree $n \geq 1$ and a be any real number, then
(i) (x – a) is a factor of q (x), if q(a) = 0 and
(ii) q(a) = 0, if x – a is a factor of q (x).

12. The factorisation of Quadratic Polynomials
(i) By splitting the middle term: Suppose a quadratic polynomial is $x^2 + lx + m$ where l and m are constants.

Now, we split the middle term lx as ax + bx, so that ab = m. Then,

$x^2 + lx + m = x^2 + ax + bx + ab = x(x + a) + b(x + a) = (x + a)(x + b)$

(ii) By using factor theorem: Suppose, $ax^2 + bx + c$ be a quadratic polynomial. Let two factors be $(x - \alpha)$ and $(x - \beta)$

$\therefore a(x - \alpha)(x - \beta) = ax^2 + bx + c$

$\Rightarrow ax^2 - a(\alpha + \beta)x + a\alpha\beta = ax^2 + bx + c$

On equating the coefficient of x and constant term.

We get, $\alpha + \beta = -ba$ and $\alpha\beta = ca$

On simplifying, we get the values of α and β.

(iii) The factorisation of cubic polynomials: The splitting middle term method is not applicable for cubic polynomials. Its need to find at least one factor first, and then adjust the given polynomial such that it becomes the product of term in which one of them is that factor and the other is a quadratic polynomial. Now, solve this quadratic polynomial by the previous method.

13. Algebraic Identities: An algebraic identity is an algebraic equation that is true for all values of the variable occurring in it.

Some algebraic identities are given below

- $(x + y)^2 = x^2 + 2xy + y^2$
- $(x - y)^2 = x^2 - 2xy + y^2$
- $x^2 - y^2 = (x + y)(x - y)$
- $(x + a)(x + b) = x^2 + (a + b)x + ab$
- $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

- $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$
- $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$
- $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

  If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$