8th Standard- Maths

Algebraic Expressions and Identities

A symbol which takes various numerical values is called a variable.

A combination of constants and variables connected by the signs of fundamental operations of addition, subtraction, multiplication and division is called an algebraic expression.

Various parts of an algebraic expression which are separated by the signs of '+' or '-' are called the terms of the expression.

An algebraic expression is called a monomial, a binomial, a trinomial, a quadrinomial accordingly as it contains one term, two terms, three terms and four terms, respectively.

While adding or subtracting polynomials, first look for like terms and then add or subtract these terms, then handle the, unlike terms.

Monomial multiplied by a monomial always gives a monomial.

While multiplying a polynomial by a monomial, we multiply every term in the polynomial by the monomial.

While multiplying a polynomial by a binomial (or trinomial), we multiply term by term, i.e., every term of a polynomial is multiplied by every term of in the binomial (or trinomial) and after that, we combined the like terms.

An identity is equality, which is true for all values of the variable in the equality.

The following are the useful identities and these identities are known as standard identities.

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- $(a + b)(a - b) = a^2 - b^2$
- $(x + a)(x + b) = x^2 + (a + b)x + ab$
- $(x + a)(x - b) = x^2 + (a - b)x - ab$
- $(x - a)(x + b) = x^2 - (a - b)x - ab$
- If x is a variable and m, n are positive integers, then $(x^m \times x^n) = x^{(m+n)}$ Thus, $x^2 \times x^4 = x^{(2+4)} = x^6$
- If x is a variable and m, n are positive integers such that m > n, then $(x^m \div x^n) = x^{m-n}$. Thus, $x^9 \div x^4 = x^{9-4} = x^5$
- Product of two monomials = (Product of their coefficients) × (Product of their variables)
- Division of two monomials = (Division of their coefficients) × (Division of their variables)

What are Expressions?

We know that a constant is a symbol having fixed numerical value whereas a variable is a symbol assuming various numerical values.

An algebraic expression is formed from variables and constants. A combination of variables and constants connected by the signs +, -, × and ÷ is called an algebraic expression.

The variable/variables in an algebraic expression can assume countless different values. The value of algebraic expression changes with the value (s) assumed by the variable (s) it contains.

Terms, Factors and Coefficients

Terms are added to form expressions. Terms themselves can be formed as the product of factors. The numerical factor (with sign) of a term is called it's coefficient.

Monomials, Binomials and Polynomials

An expression that contains exactly one, two or three terms is called a monomial, binomial or trinomial, respectively. In general, an expression containing, one or more terms with non-zero coefficients and with variable having non-negative exponents is called a polynomial.

Like and Unlike Terms

Like (or similar) terms are formed from the same variables and the powers of these variables are also the same. Coefficients of like terms need not be the same. In case otherwise, they are called, unlike (or dissimilar) terms.

Addition and Subtraction of Algebraic Expressions

While adding (or subtracting) polynomials, first look for like terms and add (or subtract) them; then handle the unlike terms. Note that the sum of a number of like terms is another like term whose coefficient is the sum of the coefficients of the like terms being added.

Multiplication of Algebraic Expressions Introduction

There exist a number of situations when we need to multiply algebraic expressions. For example, in finding area of a rectangle whose sides are given as expressions.

Multiplying a Monomial by a Monomial

A monomial multiplied by a monomial always gives a monomial.

Multiplying Two Monomials

In the product of two monomials

Coefficient = coefficient of the first monomial × coefficient of the second monomial

Algebraic factor = algebraic factor of a first monomial × algebraic factor of the second monomial

Multiplying Three or More Monomials

We first multiply the first two monomials and then multiply the resulting monomial by the third monomial. This method can be extended to the product of any number of monomials.

Rules of Signs

The product of two factors is positive or negative accordingly as the two factors have like signs or unlike signs. Note that

(i) (+) × (+) = +

(ii) (+) × (-) = –

(iii) (-) × (+) = –

(iv) (-) × (-) = +

If x is a variable and p, q are positive integers, then $x^p \times x^q = x^{p+q}$

Multiplying a Monomial by a Polynomial

While multiplying a polynomial by a monomial, we multiply every term in the polynomial by the monomial.

Multiplying a Monomial by a Binomial

By using the distributive law, we carry out the multiplication term by term.

It states that if P, Q and R are three monomials, then

- $P \times (Q + R) = (P \times Q) + (P \times R)$
- $(Q + R) \times P = (Q \times P) + (R \times P)$

Multiplying a Monomial by a Trinomial

By using the distributive law, we carry out the multiplication term by term.

Multiplying A Polynomial by a Polynomial

We multiply each term of one polynomial by each term of the other polynomial. Also, we combine the like terms in the product.

Multiplying a Binomial by a Binomial

We use distributive law and multiply each of the two terms of one binomial by each of the two terms of the other binomial and combine like terms in the product.

Thus, if P, Q, R and S are four monomials, then

(P + Q) × (R + S) = P × (R + S) + Q × (R + S)

= (P × R + P × S) + (Q × R + Q × S)

= PR + PS + QR + QS.

Multiplying a Binomial by a Trinomial

We use distributive law and multiply each of the three terms in the trinomial by each of the two terms in the binomial and combine like terms in the product.

What is Identity?

An identity is equality, which is true for all values of the variables, in the equality. On the other hand, an equation is true only for certain values of its variables. An equation is not always an identity. However, obviously, an identity is always an equation.

Standard Identities

(i) $(a + b)^2 = a^2 + 2ab + b^2$

i.e., square of the sum of two terms = (square of the first term) + 2 × (first term) × (second term) + (square of the second term)

(ii) $(a - b)^2 = a^2 - 2ab + b^2$

i.e., square of the difference of two terms = (square of the first term) – 2 × (first term) × (second term) + (square of the second term)

(iii) $(a + b)(a - b) = a^2 - b^2$

i.e., (first term + second term) (first term – second term) = $(\text{first term})^2$ – $(\text{second term})^2$

(iv) $(x + a)(x + b) = x^2 + (a + b)x + ab$

Applying Identities

The above identities are useful in carrying out squares and products of algebraic expressions. They provide us with easy alternative methods to calculate products of numbers and so on.