8th Standard- Maths

Cubes and Cube Roots

Cubes

The cube of a number is that number raised to the power 3. If x is a number, then $x^3 = x \times x \times x$.

A natural number n is a perfect cube if $n = m^3$ for some natural number m.

The cube of an even natural number is even.

The cube of an odd natural number is odd.

The cube of a negative number is always negative.

The sum of the cubes of first n natural numbers is equal to the square of their sum.
i.e., $1^3 + 2^3 + 3^3 + \ldots\ldots + n^3 = (1 + 2 + 3 + \ldots\ldots + n)^2$

Cubes of the numbers ending with the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 end with digits 0, 1, 8, 1, 7, 5, 6, 3, 2, 9 respectively. Here, cubes of numbers ending with digits 0, 1, 4, 5, 6 and 9 end with same digits.

Cubes of the number ending with digit 2 ends in 8 or cube of the number ending with digit 8 ends in 2.

Cube of the number ending with digit 3 ends in 7 and cube of the number ending with digit 7 ends in 3.

Cubes root

The cube root of a number x is the number whose cube is x. It is denoted by x−−√3.

For finding the cube root of a perfect cube, resolve it into prime factors; make triplets of similar factors and take the product of prime factors, choosing one out of every triplet.

For any positive integer x, we have −x−−−√3=−x−−√3

For any integers a and b, we have:

(i) $\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b}$

(ii) $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$

Hardy-Ramanujan Number

Look at the following relations:

$1729 = 1728 + 1 = 12^3 + 1^3$

$1729 = 1000 + 729 = 10^3 + 9^3$

1729 is the smallest Hardy-Ramanujan Number (a number which can be expressed as a sum of two cubes in two different ways is known as Hardy-Ramanujan Number). There are infinitely many such numbers.

Few are 4104 (2, 16; 9, 15), 13832 (18, 20; 2, 24).

Cubes

Numbers obtained when a number is multiplied by itself three times are known as cube numbers or perfect cubes.

For example: 1, 8, 27, ..., etc.

The cube of a natural number m is denoted by m3 and is expressed as $m^3 = m \times m \times m$.

Thus, $1^3 = 1 \times 1 \times 1 = 1$

$2^3 = 2 \times 2 \times 2 = 8$

$3^3 = 3 \times 3 \times 3 = 27$, and so on.

Some Interesting Patterns

If in the prime factorisation of any number, each prime factor appears three times, then the number is a perfect cube.

For example, $216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^3 \times 3^3 = (2 \times 3)^3 = 6^3$ which is a perfect cube.

Smallest Multiple that is a Perfect Cube

Sometimes we have to find the smallest natural number by which a number is multiplied or divided to make it a perfect cube.

Cube Roots

The cube root is the inverse operation of finding the cube.

$2^3 = 8 \Rightarrow 2$ is the cube root of 8.

The symbol √ denotes the cube root. Thus, 8–√3=2

Cube Root Through Prime Factorisation Method

We express the given number into a product of its prime factors and make triplets (groups of three) of similar factors. Then, we take one factor from each triplet and multiply. The product so obtained gives the cube root of the given number.

Cube Root of a Cube Number

Steps

1. Obtain the given number. Start making groups of three digits starting from the rightmost digit of the number.
2. The first group will give one's (unit's) digit of the required cube root.
3. Then, take another group. Find two closest cube numbers between which this group lies. Take the one's place of the smaller number as the ten's place of the required cube root.