Question Paper-Delhi (2013)

General Instructions:

(i) *All questions are compulsory.*

(ii) *The question paper consists of 29 questions divided into three Sections A, B and C, Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.*

(iii) *All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.*

(iv) *There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.*

(v) *Use of calculators is not permitted.*

SECTION-A

Question numbers 1 to 10 carry 1 mark each.

Q1. Write the principal value of $\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)$

Q2. Write the value of tan $\left(2\tan^{-1}\frac{1}{5}\right)$.

Q3. Find the value of a if $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix}=\begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

Q4. If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+3 \end{vmatrix}=\begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$, then write the value of x.

Q5. If $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix}=\text{A}+\begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$, then find the matrix A.

Q6. Write the degree of the differential equation $x^3\left(\frac{d^2y}{dx^2}\right)^2+x\left(\frac{dy}{dx}\right)^4=0$.

Q7. If $\vec{a} = x\hat{i}+2\hat{j}-z\hat{k}$ and $\vec{b} = 3\hat{i}-y\hat{j}+\hat{k}$ are two equal vectors, then write the value of $x + y + z$.

Q8. If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}, \frac{\pi}{4}$, with $\hat{j}$ and an acute angle θ with $\hat{k}$, then find the value of θ.

Q9. Find the Cartesian equation of the line which passes through the point (–2, 4, –5) and is parallel to the line $\frac{x+3}{3}=\frac{4-y}{5}=\frac{z+8}{6}$.

Q10. The amount of pollution content added in air in a city due to *x*-diesel vehicles is given by $P(x) = 0.005x^3 + 0.02x^2 + 30x$. Find the marginal increase in pollution content when 3 diesel vehicles are added and write which value is indicated in the above question.

SECTION-B

Question numbers 11 to 22 carry 4 marks each.

Q11. Show that the function f in A = IR $-\left\{\frac{2}{3}\right\}$ defined as $f(x)\ \frac{4x+3}{6x-4}$ is one-one and onto. Hence find f^{-1}.

Q12. Find the value of the following :

$$\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right], |x|<1, y>0 \text{ and } xy<1.$$

OR

Prove that: $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}$

Q13. Using properties of determinants, prove the following

$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2.$$

Q14. Differentiate the following function with respect to x :
$(\log x)^x + x^{\log x}$

Q15. If y = $\log[x+\sqrt{x^2+a^2}]$, show that $(x^2+a^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=0$.

Q16. Show that the function $f(x) = |x - 3|$, $x \in \mathbb{R}$, is continuous but not differentiable at $x = 3$.

OR

If $x = a \sin t$ and $y = a(\cos t + \log \tan t/2)$, find $\frac{d^2y}{dx^2}$.

Q17. Evaluate: $\int \frac{\sin(x-a)}{\sin(x+a)}dx$

OR

Evaluate: $\int \frac{5x-2}{1+2x+3x^2}dx$

Q18. Evaluate: $\int \frac{x^2}{(x^2+4)(x^2+9)}dx$

Q19. Evaluate: $\int_0^4 (|x|+|x-2|+|x-4|)dx$

Q20. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}+\vec{b}| = |\vec{a}|$, then prove that vector $2\vec{a}+\vec{b}$ is perpendicular to vector $\vec{b}$.

Q21. Find the coordinates of the point, where the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2}$ intersects the plane $x - y + z - 5 = 0$. Also find the angle between the line and the plane.

OR

Find the vector equation of the plane which contains the line of intersection of the planes $\vec{r}.(\hat{i}+2\hat{j}+3\hat{k})-4=0$ and $\vec{r}.(2\hat{i}+\hat{j}-\hat{k})+5=0$ and which is perpendicular to the plane $\vec{r}.(5\hat{i}+3\hat{j}-6\hat{k})+8=0$.

Q22. A speaks truth in 60% of the cases, while B in 90% of the cases. In what percent of cases are they likely to contradict each other in stating the same fact? In the cases of contradiction do you think, the statement of B will carry more weight as he speaks truth in more number of cases than A?

SECTION -C

Question numbers 23 to 29 carry 6 marks each.

Q23. A school wants to award its students for the values of Honesty, Regularity and Hard work with a total cash award of ₹ 6,000. Three times the award money for Hardwork added to that given for honesty amounts to ₹ 11,000. The award money given for Honesty and Hard work together is double the one given for Regularity. Represent the above situation algebraically and find the award money for each value, using matrix method. Apart from these values, namely, Honesty, Regularity and Hard work, suggest one more value which the school must include for awards.

Q24. Show that the height of the cylinder of maximum volume, that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

OR

Find the equation of the normal at a point on the curve $x^2 = 4y$ which passes through the point (1,2). Also find the equation of the corresponding tangent.

Q25. Using integration, find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

OR

Using integration, find the area of the region enclosed between the two circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

Q26. Show that the differential equation $2ye^{x/y}dx + (y - 2xe^{x/y})dy = 0$ is homogeneous. Find the particular solution of this differential equation, given that $x = 0$ when $y = 1$.

Q27. Find the vector equation of the plane passing through three points with position vectors $\hat{i} + \hat{j} - 2\hat{k}, 2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$. Also find the coordinates of the point of intersection of this plane and the line $\vec{r} = 3\hat{i} - \hat{j} - \hat{k} + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$.

Q28. A cooperative society of farmers has 50 hectares of land to grow two crops A and B. The profits from crops A and B per hectare are estimated as ₹ 10,500 and ₹ 9,000 respectively. To control weeds, a liquid herbicide has to be used for crops A and B at the rate of 20 litres and 10 litres per hectare, respectively. Further not more than 800 litres of herbicide should be used in order to protect fish and wildlife using a pond which collects drainage from this land. Keeping in mind that the protection of fish and other wildlife is more important than earning profit, how much land should be allocated to each crop so as to maximize the total profit? Form an LPP from the above and solve it graphically. Do you agree with the message that the protection of wildlife is utmost necessary to preserve the balance in environment ?

Q29. Assume that the chances of a patient having a heart attack is 40%. Assuming that a meditation and yoga course reduces the risk of heart attack by 30% and prescription of certain drug reduces its chance by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options, the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga. Interpret the result and state which of the above stated methods is more beneficial for the patient.

Marking Scheme

Q.No.	Value Points/Solution	Marks.

SECTION-A

1-10

1. $\frac{11\pi}{12}$ **2.** $\frac{5}{12}$ **3.** $a = 1$ **4.** $x = 2$

5. $\begin{bmatrix} 8 & -3 & 5 \\ -2 & -3 & -6 \end{bmatrix}$ **6.** 2 **7.** zero

8. $\frac{\pi}{3}$ **9.** $\frac{x+2}{3} = \frac{y-4}{-5} = \frac{z+5}{6}$

10. 30. 255, (i) Pollution control in the environment (ii) or, any other vlaue suggested with just fication)

SECTION-B

11. Let x_1, x_2, be the elements of A, then $f(x_1) = f(x_2)$

$$\Rightarrow \quad \frac{4x_1+3}{6x_1-4} = \frac{4x_2+3}{6x_2-4}$$

$\Rightarrow x_1 = x_2 \Rightarrow f$ is one-one 1½

Let $y = \frac{4x+3}{6x-4} \Rightarrow 6xy - 4y = 4x + 3$

$$\Rightarrow x = \frac{4y+3}{6y-4}$$ 1

$\Rightarrow$ For each $y \in$ R – {2/3},

there exists $x = \frac{4y+3}{6y-4} \in R - \left\{\frac{2}{3}\right\}$ such that $f(x) = y$ ½

$\therefore$ f is an onto function 1

Also $f^{-1}(x) = \frac{4x+3}{6x-4}$

12. $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right] = \tan\frac{1}{2}\left[2\tan^{-1}x+2\tan^{-1}y\right]$ 1½

$= \tan[\tan^{-1}x+\tan^{-1}y]=\tan\left[\tan^{-1}\frac{x+y}{1-xy}\right]$ 1½

$= \frac{x+y}{1-xy}$ 1

OR

LHS $= \tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{8}\right)$ 1+1

$= \tan^{-1}\left[\frac{\frac{1}{2}+\frac{1}{5}}{1-\frac{1}{2}\times\frac{1}{5}}\right]+\tan^{-1}\frac{1}{8}=\tan^{-1}\left[\frac{7}{9}\right]+\tan^{-1}\left(\frac{1}{8}\right)$ 1

$= \tan^{-1}\left[\frac{\frac{7}{9}+\frac{1}{8}}{1-\frac{7}{72}}\right]=\tan^{-1}\left(\frac{65/72}{65/72}\right)=\tan^{-1}1$ 1

$= \frac{\pi}{4}$ = RHS 1

13. Using $R_1 \to R_1 + R_2 + R_3$

$\Delta = \begin{vmatrix} 1+x+x^2 & 1+x+x^2 & 1+x+x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$ 1

$= (1+x+x^2)\begin{vmatrix} 1 & 1 & 1 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$: using $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$ 1

$\Delta = (1+x+x^2)\begin{vmatrix} 1 & 0 & 0 \\ x^2 & x^2-x & x-x^2 \\ x & x^2-x & 1-x \end{vmatrix}$ 1

$= (1+x+x^2)(1-x)(1-x)\begin{vmatrix} 1+x & x \\ -x & 1 \end{vmatrix}$ 1

$= (1-x)^2(1+x+x^2)^2$

$= (1-x^3)^2$ 1

14. $y = (\log x)^x + x^{\log x} = u + v(\text{say}) \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ ½

$\log u = x \log(\log x) \Rightarrow \frac{du}{dx} = (\log x)^x \left[\frac{1}{\log x} + \log(\log x)\right]$ 1½

$\log v = \log x . \log x = (\log x)^2 \Rightarrow \frac{dv}{dx} = x^{\log x}\left[\frac{2}{x}\log x\right]$ 1

$\therefore \frac{dy}{dx} = (\log x)^x \left[\frac{1}{\log x} + \log(\log x)\right] + x^{\log x}\left[\frac{x}{2}\log x\right]$ 1

15. $y = \log[x + \sqrt{x^2 + a^2}] \Rightarrow \frac{dy}{dx} = \frac{1}{x + \sqrt{x^2+a^2}}\left[1 + \frac{\not{2}x}{\not{2}\sqrt{x^2+a^2}}\right]$ 1

$= \frac{1}{x\sqrt{x^2+a^2}}\left[\frac{x + \sqrt{x^2+a^2}}{\sqrt{x^2+a^2}}\right]$ 1

$\therefore \sqrt{x^2+a^2}\frac{dy}{dx} = 1$ ½

Diff. again, $\sqrt{x^2+a^2}\frac{d^2y}{dx^2} + \frac{\not{2}x}{\not{2}\sqrt{x^2+a^2}}\frac{dy}{dx} = 0$ 1

or $(x^2+a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$ ½

16. $f(x) = |x - 3| = (x - 3)$ if $x \geq 3$

$(3 - x)$ if $x < 3$ 1

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-}(3 - x) = 0$ 1

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+}(x - 3) = 0$ and $f(3) = 0$ ½

LHD $= \lim_{h \to 0}\frac{f(3-h) - f(3)}{(3-h) - 3} = ...\lim_{h \to 0}\frac{[3-(3-h)] - 0}{(3-h) - 3} = -1$ 1

RHD $= \lim_{h \to 0}\frac{f(3+h) - f(3)}{(3+h) - 3} = \frac{(h+3-3) - 0}{3+h-3} = 1$ ½

As LHD $\neq f$ is not differentiable at $x = 3$. ½

OR

$x = a \sin t \Rightarrow \frac{dx}{dt} = a\cos t$ 1

$y = a(\cos t + \log\tan t/2) \Rightarrow \dfrac{dy}{dt} = a\left(-\sin t + \dfrac{1}{2}.\dfrac{\sec^2 t/2}{\tan t/2}\right)$ 1

$= a\left(-\sin t + \dfrac{1}{\sin t}\right) = a\dfrac{\cos^2 t}{\sin t}$ 1½

$\therefore \dfrac{dy}{dx} = \dfrac{a\cos^2 t}{\sin t}.\dfrac{1}{a\cos t} = \cot t$ ½

$\therefore \dfrac{d^2y}{dx^2} = -\text{cosec}^2 t.\dfrac{dt}{dx} = -\dfrac{\text{cosec}^2 t}{a\cos t}$ 1

17. $I = \displaystyle\int \frac{\sin(x-a)}{\sin(x+a)}dx = \int \frac{\sin\{(x+a)-2a\}}{\sin(x+a)}dx$ 1

$= \cos 2a\displaystyle\int dx - \int \cot(x+a).\sin 2a\, dx$ 1

$= x\cos 2a - \sin 2a \log|\sin(x+a)| + c$ 1

OR

$I = \displaystyle\int \frac{5x-2}{1+2x+3x^2}dx$

$5x - 2 = A(6x+2) + B \Rightarrow A = \dfrac{5}{6}, B = \dfrac{-11}{3}$ ½+½

$\therefore$ $I = \dfrac{5}{6}\displaystyle\int \frac{6x+2}{1+2x+3x^2}dx - \frac{11}{3x3}\int \frac{1}{\left(x+\frac{1}{3}\right)^2 + \left(\frac{\sqrt{2}}{3}\right)^2}$ 1

$= \dfrac{5}{6}\log\left|1+2x+3x^2\right| - \dfrac{11}{3\sqrt{2}}\tan^{-1}\left(\dfrac{3x+1}{\sqrt{2}}\right) + C$ 1+1

18. $\dfrac{x^2}{(x^2+4)(x^2+9)}$: Let $x^2 = t$

$\therefore$ Given expression can be written as

$\dfrac{t}{(t+4)(t+9)} = \dfrac{A}{t+4} + \dfrac{B}{t+9}$ ½

On solving to get A = $-\dfrac{4}{25}$ and B = $\dfrac{9}{5}$ 1

Replacing t by x^2, we get

$\Rightarrow I = \displaystyle\int \frac{x^2}{(x^2+4)(x^2+9)}dx = \int \frac{-4}{5}\frac{dx}{x^2+4} + \int \frac{9}{5}\frac{dx}{x^2+9}$ 1

$= \frac{-2}{5}\tan^{-1}\frac{x}{2}+\frac{3}{5}\tan^{-1}\frac{x}{3}+C$ ½

19. I = $\int_0^4 (|x|+|x-2|+|x-4|)dx$ ½

$= \int_0^4 xdx+\int_0^2 (2-x)dx+\int_2^4 (x-2)dx+\int_0^4 (4-x)dx$ 1½

$= \left|\frac{x^2}{2}\right|_0^4+\left|2x-\frac{x^2}{2}\right|_0^2+\left|\frac{x^2}{2}-2x\right|_2^4+\left|4x-\frac{x^2}{2}\right|_0^4$ 1½

$= 8+(4-2)+[(8-8)-(2-4)]+[(16-8)]$ 1

= 8 + 2 + 2 + 8

= 20

20. $|\vec{a}+\vec{b}|=|\vec{a}| \Rightarrow |\vec{a}+\vec{b}|^2=|\vec{a}|^2$ 1

$\Rightarrow \ |\vec{a}|^2+|\vec{b}|^2+2\vec{a}.\vec{b}=|\vec{a}|^2$ 1

$\Rightarrow \ 2\vec{a}.\vec{b}+|\vec{b}|=0$ 1

or $(2\vec{a}+\vec{b}).\vec{b}=0$ 1½

which gives $2\vec{a}+\vec{b}$ is $\perp$ $\vec{b}$ ½

21. A general point on the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}$ is ...(i) 1

$3\lambda + 2,\ 4\lambda - 1,\ 2\lambda + 2$

If this point lies on the plane $x - y + z - 5 = 0$, ...(ii)

it should satisfy it

$\Rightarrow (3\lambda+2)-(4\lambda-1)+(2\lambda+2)-5=0$ 1

$\Rightarrow \lambda = 0$

$\therefore$ The point of intersection of line (i) and plane (ii) is (2, –1, 2) ½

Angle between line (i) and plane (ii) is given by

$$\sin\theta = \left|\frac{3.1+4(-1)+1(1)}{\sqrt{3^2+4^2+2^2}\sqrt{1^2+1^2+1^2}}\right| = \frac{1}{\sqrt{87}}$$ 1

or $\theta = \sin^{-1}\left(\frac{1}{\sqrt{87}}\right)$ ½

OR

Vector equation of a plane containing line of intersection of planes $\vec{r}.(\hat{i}+2\hat{j}+3\hat{k})-4=0$ and

$\vec{r}.(2\hat{i}+\hat{j}-\hat{k}+5)]=0$ is $[\vec{r}.(\hat{i}+2\hat{j}+3\hat{k}-4)+\lambda[\vec{r}.(2\hat{i}+\hat{j}-\hat{k}+5]=0$

$\Rightarrow \vec{r}.[(1+2\lambda)\hat{i}+(2+\lambda)\hat{j}+(3-\lambda)\hat{k}]=4-5\lambda$ ...(i) 1½

(i) is given $\perp$ to the plane $\vec{r}.(5\hat{i}+3\hat{j}-6\hat{k})+8=0$

$\therefore$ $(1+2\lambda)5+(2+\lambda)3+(3-\lambda)(-6)=0$

$19\lambda = 7$ or $\lambda = \frac{7}{19}$ 1

$\therefore$ Regd equation of plane is

$\vec{r}.\left[\left(1+\frac{14}{19}\right)\hat{i}+\left(2+\frac{7}{19}\right)\hat{j}+\left(3-\frac{7}{19}\right)\hat{j}\right]=4-\frac{35}{19}$ 1

or $r.[33\hat{i}+45\hat{j}+50\hat{k}]=41$ ½

22. Let the events be defined as below :

E : A speak the truth

F : B speaks the truth

$P(E) = \frac{60}{100} = 0.6$, $P(F) = \frac{9}{100} = 0.9$

$P(\bar{E}) = 0.4$, $P P(\bar{F}) = 0.1$

$\therefore$ Required probability

$= P(E\cap\bar{F})+P(\bar{E}\cap F)$

$= 0.6 \times 0.1 + 0.4 \times 0.9$

$= 0.42$ 1

$\therefore$ In 42% of the cases, A and B are likely to contradict each other.

Any value suggested with justificaion may be accepted. 1

SECTION-C

23. Let the values honesty, regularity and bard work be denotes by x, y and z respectively ½

From the question

(i) $x+y+z=6000$ (ii) $3z+x=11000$ (iii) $x+z=2y$ or $x+2-2y=0$ 1

In matrix from, the system of = x can be written as

$$\begin{pmatrix}1 & 1 & 1\\ 1 & 0 & 3\\ 1 & -2 & 1\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix} = \begin{pmatrix}6000\\ 11000\\ 0\end{pmatrix}$$

In the for AX = B, where A^{-1} exists of $|A| \neq 0$ 1

$|A| = 1\times6-1(-2)+1(-2)=6\neq0 \Rightarrow A^{-1}$ exists

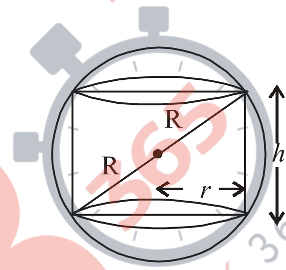
$$\text{Adj } A = \begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}, \therefore A^{-1} = \frac{1}{6}\begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}$$ 1½

$$x = A^{-1}B = \frac{1}{6}\begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}\begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix} = \begin{bmatrix} 500 \\ 2000 \\ 3500 \end{bmatrix}$$ 1

$\therefore$ Award money fo (i) Honest = Rs 500 (ii) Regularity = Rs. 2000 (iii) Hard work = Rs. 3500

Any other value suggested with full justification may be accepted. 1

24. From the figure, $h^2 + 4r^2 = 4R^2$ ....(i)



$v = \pi r^2 h = \pi \frac{[4R^2 - h^2]h}{4}$ [using (i)] 1

$\frac{dv}{dh} = \frac{\pi}{4}[4R^2 - 3h^2]$ 1½

$\frac{dv}{dh} = 0 \Rightarrow h = \frac{2R}{\sqrt{3}}$ 1½

Showing $\frac{d^2v}{dh^2}$ is negative when $h = \frac{2R}{\sqrt{3}}$ 1

$\therefore$ V is maximum at $h = \frac{2R}{\sqrt{3}}$ 1½

and maximum volume $= \frac{\pi}{4}\left[\frac{8R^3}{\sqrt{3}} - \frac{8R^3}{3\sqrt{3}}\right] = \frac{4\pi R^3}{3\sqrt{3}}$ 1

OR

The equation of curve is $y = \frac{x^2}{4} \Rightarrow \frac{dy}{dx} = \frac{x}{2}$ 1½

$\therefore$ Slope of normal is $\frac{-2}{x}$, at (x_1, y_1)

$\therefore$ Equation of normal to the curve at (x_1, y_1) is

$$y - y_1 = -\frac{2}{x_1}(x - x_1) \quad ....(i)$$ 1

(i) passes through (1, 2) and using $y_1 = \frac{x_1^2}{4}$, we get

$$x_1 = 2, \; y_1 = 1$$ 1½

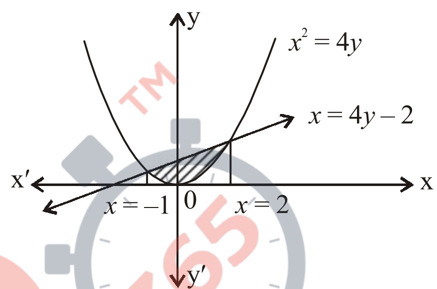
$\therefore$ Equation of normal is $y - 1 = -1(x - 2) \Rightarrow x + y = 3$ 1

Equation of corresponding tangent is

$$y - 1 = 1(x - 2) \Rightarrow x - y = 1$$ 1

25. Curves are intersecting figure 1

at $x = 2, -1$ 1½



Regq. area $= \int_{-1}^{2} \frac{x+2}{4} dx - \int_{-1}^{2} \frac{x^2}{4} dx$ 2

$$= \frac{1}{4}\left[\left(\frac{x^2}{2} + 2x\right) - \left(\frac{x^3}{3}\right)\right]_{-1}^{2}$$ 1

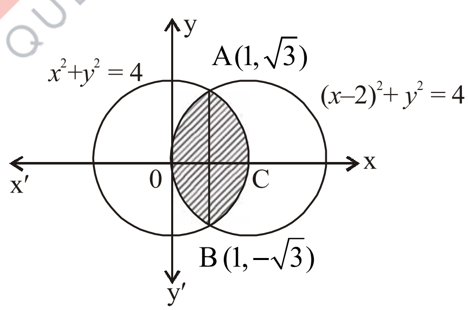
$= \frac{9}{8}$ sq. units. ½

OR

Points of intersection of two curres are $(1, \pm\sqrt{3})$ 1

The area is symmetrical about AB

$\therefore$ Reqd. area



$$= 4\left[\int_{1}^{2} y dx\right] = 4\left[\int_{1}^{2} \sqrt{4x^2} dx\right]$$ 2

$$= 4\left[\frac{1}{2} x\sqrt{4 - x^2} + 2\sin^{-1}\frac{x}{2}\right]_1^2$$ 2

$$= \frac{8\pi}{3} - 2\sqrt{3}$$

26. The given diff. equation can be written as

$$\frac{dx}{dy} = \frac{2xe^{\frac{x}{y}}}{2ye^{\frac{x}{y}}} = \frac{2\frac{x}{y}e^{\frac{x}{y}}}{2e^{x/y}} \qquad ...(i)$$

(i) is a homogenous function of $\frac{x}{y}$ 2

$\therefore$ we put $x = vy \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$ ½

(i) can be written as $v + y\frac{dv}{dy} = \frac{2ve^v - 1}{2e^v}$ 1

$\therefore \quad y\frac{dv}{dy} = \frac{2ve^v - 1}{2e^v} - v = \frac{-1}{2e^v}$ ½

$\Rightarrow \; 2e^v dv - \frac{dv}{y}$

$\Rightarrow \; 2e^v = -\log y + c$

or $2e^{\frac{x}{y}} = C - \log y$

when $x = 0$, $y = 1 \Rightarrow c = z$ ½

$\therefore$ The particular solution of the given diff equation is

$2e^{\frac{x}{y}} + \log y = 2$ ½

27. Let A = $\hat{i} + \hat{j} - 2\hat{k}$, B $= 2\hat{i} - \hat{j} + \hat{k}$ and C $= \hat{i} + 2\hat{j} + \hat{k}$ 1

$\therefore \; \overrightarrow{AB} = \hat{i} - 2\hat{j} + 3\hat{k}, \overrightarrow{AC} = \hat{j} + 3\hat{k}$ 1

$$\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 0 & 1 & 3 \end{vmatrix} = -9\hat{i} - 3\hat{j} + \hat{k}$$ 1

Equation of plane is $\vec{r}.\vec{n} = \vec{a}.\vec{n}$

or $\vec{r}.(-9\hat{i} - 3\hat{j} + \hat{k}) = -9 - 3 - 2 = -14$ 1

or $\vec{r}.(9\hat{i} + 3\hat{j} + \hat{k}) = 14$ 1

The equation of line is $\frac{x-3}{2} = \frac{y+1}{-2} = \frac{z+1}{1}$ ....(i)

A general point on (i) is $2\lambda + 3, -2\lambda - 1, \lambda - 1$

The equation of plane is $9x + 3y - z - 14 = 0$

The point should satify plane if it lies on it

$\Rightarrow \lambda = -1$ 1

$\therefore$ The point is (1, 1, – 2) 1

28. Let crop A be grown on x hectares of land and crop B be grown on y hectares of land

Profit function = $10500\,x + 9000\,y = p$ ½

Subject to constraints

(i) $x + y \le 50$

(ii) $20x + 10y \le 800$ or $2x + y \le 80$

$x \ge 0,\ y \ge 0$

Corneas of feaible region are

A(0, 50), C(40, 0), O(0, 0), B(30, 20) 1½ + 1½

P(A) = 450000

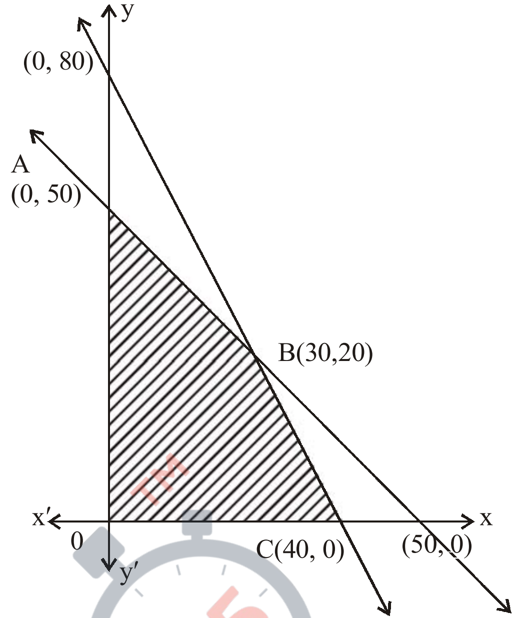
P(B) = 315000 + 180000 = 495000

P(C) = 420000

∴ For maximum profit 1

Crop A : 30 hectares

Crop B : 20 hectares



Value : (i) yes or no (any response written) or (ii) Any other value suggested withful justification may also accepted.

29. E_1 : The patient follows a course of meditation and yoga

E_2 : The patient takes certain drugs; A : The patient suffers a heart attack 2

$P(E_1) = \frac{1}{2}$, $P(E_2) = \frac{1}{2}$, $P(A/E_1) = \frac{70}{100}\times\frac{40}{100}$ of $P(A/E_2) = \frac{75}{100}\times\frac{40}{100}$

Reqd. probability = $P(E_1/A) = \dfrac{P(E_1)\,.\,P(A/E_1)}{\sum_{i=1}^{2} P(E_i).P(A/E_i)}$ 1

$$= \frac{\frac{1}{2}\times\frac{70}{100}\times\frac{40}{100}}{\frac{1}{2}\times\frac{70}{100}\times\frac{40}{100}+\frac{1}{2}\times\frac{75}{100}\times\frac{40}{100}} = \frac{14}{29}$$ 2

∴ Meditation and yoga is very important and beneficial for human values.