Maths (Standard) Delhi (Set 2)

General Instructions :
(i) This question paper comprises four sections – A, B, C and D. This question paper carries 40 questions. All questions are compulsory:
(ii) Section A : Q. No. 1 to 20 comprises of 20 questions of one mark each.
(iii) Section B : Q. No. 21 to 26 comprises of 6 questions of two marks each.
(iv) Section C: Q. No. 27 to 34 comprises of 8 questions of three marks each.
(v) Section D : Q. No. 35 to 40 comprises of 6 questions of four marks each.
(vi) There is no overall choice in the question paper. However, an internal choice has been provided in 2 questions of one mark each, 2 questions of two marks each, 3 questions of three marks each and 3 questions of four marks each. You have to **attempt only one of the choices** in such questions.
(vii) In addition to this, separate instructions are given with each section and question, wherever necessary.
(viii) Use of calculators is not permitted.

Question: 1

The HCF and the LCM of 12, 21, 15 respectively are
(a) 3, 140
(b) 12, 420
(c) 3, 420
(d) 420, 3

Solution:

Here,
$12 = 2^2 \times 3$
$21 = 3 \times 7$
$15 = 3 \times 5$
Therefore, HCF(12, 21, 15) = 3 and
LCM(12, 21, 15) = $2^2 \times 3 \times 5 \times 7 = 420$

Hence, the correct answer is option C.

Question: 2

The value of x for which $2x$, $(x + 10)$ and $(3x + 2)$ are the three consecutive terms of an AP, is
(a) 6
(b) −6

(c) 18
(d) −18

Solution:

Given $2x,\ x+10,\ 3x+2$ are the consecutive terms of an AP.
Therefore, the common difference will be same.
$\Rightarrow (x+10)-2x=(3x+2)-(x+10)$
$\Rightarrow x+10-2x=3x+2-x-10$
$\Rightarrow 10-x=2x-8$
$\Rightarrow 3x=18$
$\Rightarrow x=6$
Hence, the correct answer is option (a).

Question: 3

The value of k for which the system of equations $x + y - 4 = 0$ and $2x + ky = 3$, has no solution, is
(a) − 2
(b) ≠ 2
(c) 3
(d) 2

Solution:

For a system of a quadratic equation to have no solution, the condition is $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$.
Given equations are $x+y-4=0 \text{ and } 2x+ky-3=0$, where
$a_1=1,\ b_1=1,\ c_1=-4,\ a_2=2,\ b_2=k,\ c_2=-3.$
We have,
$\frac{1}{2}=\frac{1}{k}\neq\frac{-4}{-3}$
Now,
$\frac{1}{2}=\frac{1}{k}$
$\Rightarrow k=2$
Hence, the correct answer is option (d).

Question: 4

The first term of an AP is p and the common difference is q, then its 10th term is
(a) $q + 9p$
(b) $p - 9p$
(c) $p + 9q$
(d) $2p + 9q$

Solution:

The nth term of an AP = a + (n − 1)d, where a and d are the first term and common difference respectively.

Therefore, 10^{th} term = p + (10 − 1)q = p + 9q.

Hence, the correct answer is option (c).

Question: 5

The quadratic polynomial, the sum of whose zeroes is −5 and their product is 6, is
(a) $x^2 + 5x + 6$
(b) $x^2 - 5x + 6$
(c) $x^2 - 5x - 6$
(d) $-x^2 + 5x + 6$

Solution:

Let the zeroes be α and β respectively.
Therefore, $\alpha + \beta = -5$ and $\alpha\beta = 6$.
Hence, the required polynomial is
$$x^2 - (\alpha + \beta)x + \alpha\beta$$
$$= x^2 - (-5)x + 6$$
$$= x^2 + 5x + 6$$

Hence, the correct answer is option A.

Question: 6

(a) $a^2 + b^2$
(b) $a^2 - b^2$
(c) $\sqrt{a^2 + b^2}$
(d) $\sqrt{a^2 - b^2}$

Solution:

The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Thus, the distance between the two given points is given by

$$= \sqrt{[0 - (a\cos\theta + b\sin\theta)]^2 + [(a\sin\theta - b\cos\theta) - 0]^2}$$

$$= \sqrt{(a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2}$$

$$= \sqrt{a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta + a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta}$$

$$= \sqrt{a^2\left(\cos^2\theta + \sin^2\theta\right) + b^2\left(\sin^2\theta + \cos^2\theta\right)}$$

$$= \sqrt{a^2 \times 1 + b^2 \times 1}$$

$$= \sqrt{a^2 + b^2}$$

Hence, the correct answer is option (c).

Question: 7

The total number of factors of a prime number is

(a) 1

(b) 0

(c) 2

(d) 3

Solution:

The factors of a prime number are 1 and the number itself.

Therefore, the total number of factors of a prime number is 2.

Hence, the correct answer is option (c).

Question: 8

If the point P(*k*, 0) divides the line segment joining the points A(2, −2) and B(−7, 4) in the ratio 1 : 2, then the value of *k* is

(a) 1

(b) 2

(c) −2

(d) −1

Solution:

Using the Section Formula, we have

$k=\frac{1\times(-7)+2\times2}{1+2}$

$\Rightarrow \quad k=\frac{-7+4}{3}$

$\Rightarrow \quad k=\frac{-3}{3}$

$\Rightarrow \quad k=-1$

Hence, the correct answer is option D.

Question: 9

The value of p, for which the points A(3, 1), B(5, p) and C(7, −5) are collinear, is
(a) −2
(b) 2
(c) −1
(d) 1

Solution:

Given $A(3,\ 1),\ \ B(5,\ p)$ and $C(7,\ \ -5)$ are collinear.

$\Rightarrow \text{Area of } \Delta\,\text{ABC},\ A=0$

$\Rightarrow \frac{1}{2}\left[x_1\left(y_2-y_3\right)+x_2\left(y_3-y_1\right)+x_3\left(y_1-y_2\right)\right]=0$

$\Rightarrow \left[x_1\left(y_2-y_3\right)+x_2\left(y_3-y_1\right)+x_3\left(y_1-y_2\right)\right]=0$

$\Rightarrow [3(p+5)+5(-5-1)+7(1-p)]=0$

$\Rightarrow [3p+15-30+7-7p]=0$

$\Rightarrow -4p-8=0$

$\Rightarrow 4p=-8$

$\Rightarrow p=-2$

Hence, the correct answer is option A.

Question: 10

If one of the zeroes of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is
(a) 10
(b) −10
(c) −7
(d) −2

Solution:

Let the given polynomial be $p(x) = x^2 + 3x + k$
Since, one of the zeroes is 2.
Therefore, the value of $p(x)$ at $x = 2$ will be zero.
Therefore,

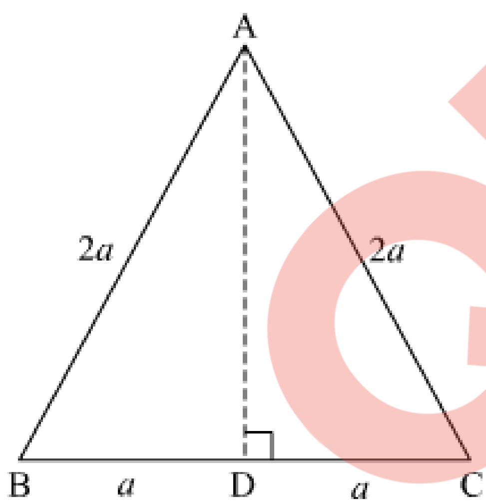
$$2^2 + 3 \times 2 + k = 0$$
$$\Rightarrow 4 + 6 + k = 0$$
$$\Rightarrow 10 + k = 0$$
$$\Rightarrow k = -10$$

Hence, the correct answer is option (b).

Question: 11

Fill in the blanks.
ABC is an equilateral triangle of side $2a$, then length of one of its altitude is __________.

Solution:



We have the above equilateral triangle in which the length of each side is 2a units.
Drop a perpendicular from A on BC, intersecting it at D.

In $\triangle$ABD and $\triangle$ACD, we have
AB = AC (Sides of an equilateral triangle)
$\angle$ABD = $\angle$ACD (Angles of an equilateral triangle)
$\angle$ADB = $\angle$ADC = 90° (By construction)
Therefore, $\triangle ABD \cong \Delta ACD$ (By AAS rule)
$\Rightarrow$BD = CD = a (By CPCT)

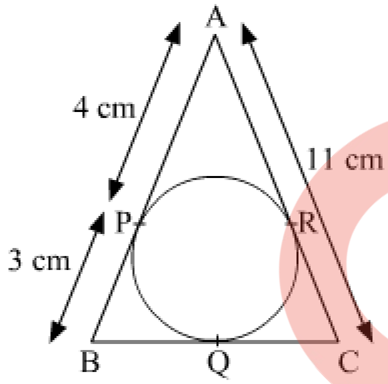
Now, using Pythagoras Theorem in $\triangle$ABD, we have

$$AB^2 = AD^2 + BD^2$$
$$\Rightarrow (2a)^2 = AD^2 + a^2$$
$$\Rightarrow AD^2 = 4a^2 - a^2 = 3a^2$$
$$\Rightarrow AD = \sqrt{3}a$$

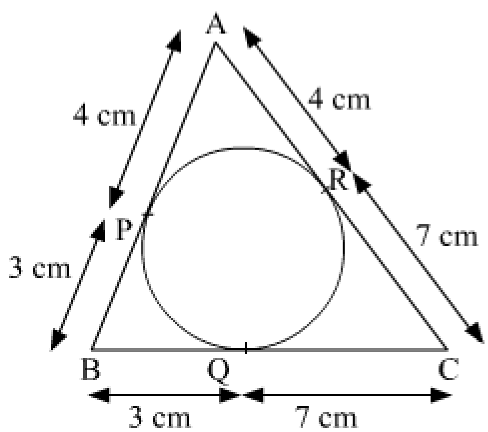
This is the required length of the altitude.

Question: 12

Fill in the blank.
In the given figure ΔABC is circumscribing a circle, the length of BC is _____ cm.



Solution:



Since we know that the lengths of tangents drawn from an exterior point to a circle are equal.

Therefore, AP = AR = 4 cm, BP = BQ = 3 cm.
Therefore, CR = AC − AR = $11 - 4 = 7$ cm.
Hence, BC = BQ + CQ = BQ + CR = $3 + 7$ cm = 10 cm.

Question: 13

Fill in the blank.
The value of $\left(\sin^2\theta + \frac{1}{1+\tan^2\theta}\right)$ = ___________.

OR

Fill in the blank.
The value of $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$ = ___________.

Solution:

$$\sin^2\theta + \frac{1}{1+\tan^2\theta}$$
$$= \sin^2\theta + \frac{1}{\sec^2\theta} = \sin^2\theta + \cos^2\theta = 1$$

OR

The given expression is
$$(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$$
$$= (1 + \tan^2\theta)(1 - \sin^2\theta)$$
$$= \sec^2\theta . \cos^2\theta$$
$$= \frac{1}{\cos^2\theta} . \cos^2\theta$$
$$= 1$$
Thus, the value of $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$ is 1.

Question: 14

$$\left(\frac{\sin\ 35^\circ}{\cos\ 55^\circ}\right)^2 + \left(\frac{\cos\ 43^\circ}{\sin\ 47^\circ}\right)^2 - 2\ \cos\ 60^\circ = \_\_\_\_\_\_\_.$$

Solution:

Consider the given expression,

$$\left(\frac{\sin\ 35^\circ}{\cos\ 55^\circ}\right)^2+\left(\frac{\cos\ 43^\circ}{\sin\ 47^\circ}\right)^2-2\cos 60^\circ$$
$$=\left[\frac{\sin(90^\circ-55^\circ)}{\cos 55^\circ}\right]^2+\left[\frac{\cos(90^\circ-47^\circ)}{\sin 47^\circ}\right]^2-2\cos 60^\circ$$
$$=\left(\frac{\cos 55^\circ}{\cos 55^\circ}\right)^2+\left(\frac{\sin 47^\circ}{\sin 47^\circ}\right)^2-2\cos 60^\circ$$
$$=1^2+1^2-2\times\frac{1}{2}$$
$$=2-1=1$$

Hence, the answer is 1.

Question: 15

ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the areas of triangles ABC and BDE is ________.

Solution:

Given: Two equilateral triangles ABC and BDE
Since two equilateral triangles are always similar, thus ratio of sides will be equal.

Since, it is given that D is the mid-point of the side BC of triangle ABC

Therefore, BD = CD or we can say BD = $\frac{1}{2}$BC.

Let BC = x, then we can say BD = $\frac{1}{2}x$

$$\Rightarrow\frac{ar(\triangle\mathrm{ABC})}{ar(\triangle\mathrm{BDE})}=\left(\frac{\mathrm{BC}}{\mathrm{BD}}\right)^2$$
$$\Rightarrow\frac{ar(\triangle\mathrm{ABC})}{ar(\triangle\mathrm{BDE})}=\left(\frac{x}{\frac{x}{2}}\right)^2$$
$$\Rightarrow\frac{ar(\triangle\mathrm{ABC})}{ar(\triangle\mathrm{BDE})}=4$$

Hence, the answer is 4.

Question: 16

A die is thrown once. What is the probability of getting a number less than 3?

OR

If the probability of winning a game is 0.07, what is the probability of losing it?

Solution:

When a die is thrown, all the outcomes are = {1, 2, 3, 4, 5, 6}
Total number of outcomes = 6
Favourable outcomes = {1, 2}
Favourable number of outcomes = 2

P(a number less than 3) = $\frac{2}{6} = \frac{1}{3}$

OR

P(winning) = 0.07
P(losing) = 1 — P(winning)
P(losing) = 1 — 0.07 = 0.93

Question: 17

If the mean of the first *n* natural number is 15, then find *n*.

Solution:

Given: mean of the first *n* natural numbers is 15.

$$\therefore \frac{1+2+3+\ldots+n}{n} = 15$$
$$\Rightarrow 1+2+3+\ldots+n = 15n$$
$$\Rightarrow \frac{n(n+1)}{2} = 15n$$
$$\Rightarrow n^2 + n = 30n$$
$$\Rightarrow n^2 - 29n = 0$$
$$\Rightarrow n\left(n-29\right) = 0$$
$$\Rightarrow n = 0,\ 29$$

So, *n* = 29 (Since *n* cannot be zero)

Question: 18

Two cones have their heights in the ratio 1 : 3 and radii in the ratio 3 : 1. What is the ratio of their volumes?

Solution:

Let the heights, radii and volumes of the two cones be $(h_1,\ r_1,\ V_1)$ and $(h_2,\ r_2,\ V_2)$.
Given: $\frac{h_1}{h_2} = \frac{1}{3}$ and $\frac{r_1}{r_2} = \frac{3}{1}$

The required ratio of their volumes = $\frac{V_1}{V_2}$

$= \frac{\frac{1}{3}\pi {r_1}^2 h_1}{\frac{1}{3}\pi {r_2}^2 h_2}$

$= \frac{{r_1}^2}{{r_2}^2} \times \frac{h_1}{h_2}$

$= \frac{3^2}{1} \times \frac{1}{3}$

$= \frac{3}{1}$

$= 3 : 1$

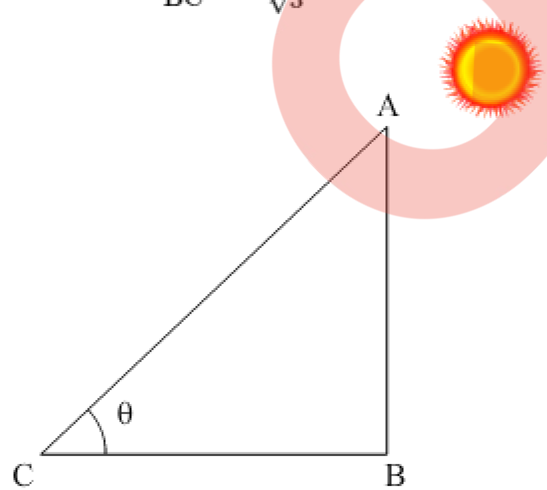
Hence, the required ratio of the volumes is 3 : 1.

Question: 19

The ratio of the length of a vertical rod and the length of its shadow is 1: √3. Find the angle of elevation of the sun at that moment?

Solution:

Given that $\frac{\mathrm{AB}}{\mathrm{BC}} = \frac{1}{\sqrt{3}}$



From the figure, it is clear that △△ABC is a right-angled triangle in which AB is the vertical rod and BC is its shadow.

We have,

$\tan\theta = \frac{AB}{BC} = \frac{1}{\sqrt{3}}$
$\Rightarrow \tan\theta = \tan 30^{\circ}$
$\Rightarrow \theta = 30^{\circ}$
Hence, the required angle of elevation of the sun is 30°.

Question: 20

A die is thrown once. What is the probability of getting an even prime number?

Solution:

Total number of possible outcomes = 1, 2, 3, 4, 5, 6
Even prime number on a die = 2

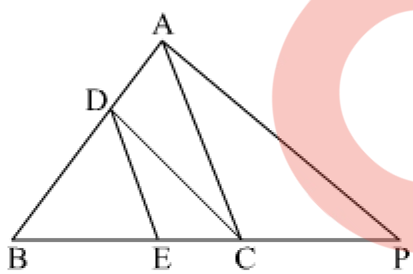
Thus, we conclude following

Probability of getting a even prime number = $\frac{\text{Number of even prime numbers}}{\text{Total possible outcomes}} = \frac{1}{6}$

Hence, the answer is 1/6.

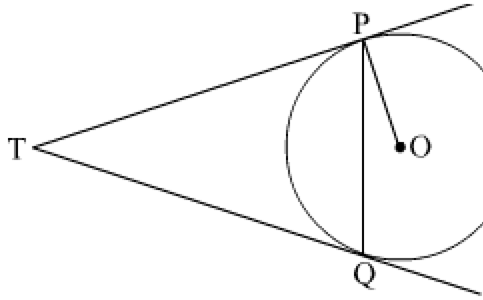
Question: 21

In the given Figure, DE || AC and DC || AP. Prove that $\frac{BE}{EC} = \frac{BC}{CP}$



OR

In the given Figure, two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that ∠PTQ = 2 ∠OPQ.



Solution:

In ΔABP, DC || AP
By Basic Proportionality theorem,
$\frac{BD}{DA} = \frac{BC}{CP}$ .....(i)
In ΔBAC, DE || AC
By Basic Proportionality theorem,
$\frac{BD}{DA} = \frac{BE}{EC}$ .....(ii)
Thus, from (i) and (ii) we have
$\frac{BE}{EC} = \frac{BC}{CP}$

Hence proved.

Hence proved.

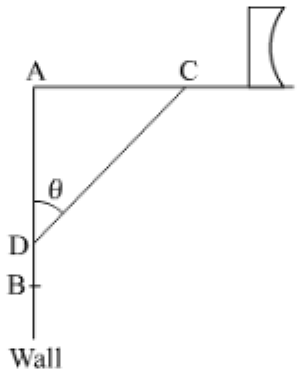
OR

Given: PT and TQ are the tangents to the circle with centre O.
To prove: $\angle PTQ = 2\angle OPQ$
Proof:
In $\triangle$PTQ,
PT = PQ (Tangents from an external point to the circle are equal)
$\Rightarrow \angle TPQ = \angle TQP$ (Angles opposite to equal sides are equal)
Let $\angle PTQ = \theta$
So, in $\triangle$PTQ
$\angle TPQ + \angle TQP + \angle PTQ = 180^\circ$
$\Rightarrow \angle TPQ = \angle TQP = \frac{1}{2}(180^\circ - \theta) = 90^\circ - \frac{1}{2}\theta$
We know, angle made by the tangent with the radius is 90°.
So, $\angle OPT = 90^\circ$
Now,
$\angle OPT = \angle OPQ + \angle TPQ$
$\Rightarrow 90^\circ = \angle OPQ + \left(90^\circ - \frac{1}{2}\theta\right)$
$\Rightarrow \angle OPQ = \frac{1}{2}\theta = \frac{1}{2}\angle PTQ$
$\Rightarrow \angle PTQ = 2\angle OPQ$

Hence Proved.

Question: 22

The rod AC of a TV disc antenna is fixed at right angles to the wall AB and a rod CD is supporting the disc as shown in the given figure. If AC = 1.5 m long and CD = 3 m, find (i) tanθ (ii) secθ + cosecθ



Solution:

In $\triangle$ACD, we have
AC = 1.5 cm, CD = 3 cm.

Since $\triangle$ACD is a right-angled triangle, so using Pythagoras Theorem, we have

$$\begin{aligned} \mathrm{AD}^2 &= \mathrm{CD}^2 - \mathrm{AC}^2 \\ &= 3^2 - 1.5^2 \\ &= 6.75 \end{aligned}$$
$$\therefore \mathrm{AD} = \sqrt{6.75} = 2.5 \text{ cm}$$

Consider
(i) $\tan\theta = \frac{\mathrm{AC}}{\mathrm{AD}} = \frac{1.5}{2.5} = \frac{3}{5}$
(ii) $\sec\theta + \operatorname{cosec}\theta = \frac{\mathrm{CD}}{\mathrm{AD}} + \frac{\mathrm{CD}}{\mathrm{AC}} = \frac{3}{2.5} + \frac{3}{1.5} = \frac{6}{5} + 2 = \frac{16}{5}$

Question: 23

If a number x is chosen at random from the numbers −3, −2, −1, 0, 1, 2, 3. What is probability that $x^2 \leq 4$?

Solution:

The given numbers are $-3, -2, -1, 0, 1, 2, 3$.
Total number of possible outcomes = 7
Now, the favorable outcomes are given by $x^2 \leq 4$
i.e. $-2 \leq x \leq 2$
i.e. $-2, -1, 0, 1, 2$
Total number of favorable outcomes = 5
Hence, the required probability = $\frac{5}{7}$.

Question: 24

Find the mean of the following distribution:

Class:	3 – 5	5 – 7	7 – 9	9 – 11	11 – 13
Frequency:	5	10	10	7	8

OR

Find the mode of the following data :

Class:	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120	120 – 140
Frequency:	6	8	10	12	6	5	3

Solution:

Class	Frequency(f_i)	Class Mark(x_i)	$f_i x_i$
3 - 5	5	4	20
5 - 7	10	6	60
7 - 9	10	8	80
9 - 11	7	10	70
11 - 13	8	12	96
	$\sum f_i = 40$		$\sum f_i x_i$ = 326

Mean, $\overline{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{326}{40} = 8.15$

Thus, mean = 8.15

OR

In the given data, the maximum class frequency is 12.
The class corresponding to the given class is 60 - 80, which is the modal class.

We have

Lower limit of modal class, l = 60
Frequency of modal class, f_1 = 12

Frequency of a class preceding to modal class, f_0 = 10
Frequency of a class succeeding to modal class, f_2 = 6
Class size h = 20

Mode = $l + \frac{(f_1 - f_0)}{(2f_1 - f_o - f_2)} \times h$

$= 60 + \frac{(12-10)}{(24-10-6)} \times 20$

$= 60 + \frac{2}{8} \times 20$

$= 60 + 5$

$= 65$

Hence, the mode of the given data is 65.

Question: 25

Find the sum of first 20 terms of the following AP:
1, 4, 7, 10, ________

Solution:

Given: The arithmetic progression is 1, 4, 7, 10, ________
As we know, Sum of an AP is given by $S_n = \frac{n}{2}[2a + (n-1)d]$
From the given AP, we conclude that $a = 1$, $d = 3$ and $n = 20$

$S_{20} = \frac{20}{2}[2 \times 1 + (20-1)3]$

$\Rightarrow S_{20} = 10(2 + 19 \times 3)$

$\Rightarrow S_{20} = 10 \times 59$

$\Rightarrow S_{20} = 590$

Hence, the sum of the first 20 terms is 590.

Question: 26

The perimeter of a sector a circle of radius 5.2 cm is 16.4 cm. Find the area of the sector.

Solution:

Given:
Perimeter of a sector of a circle = 16.4cm.
Radius = 5.2cm

Perimeter of a sector of the circle = 2 × Radius + length of an arc

$$\Rightarrow 16.5 = 2 \times 5.2 + \text{Length of the arc}$$
$$\Rightarrow \text{Length of the arc} = 6.1\,\text{cm}$$

Also, length of an arc$= \frac{\theta}{360} \times 2\pi r$

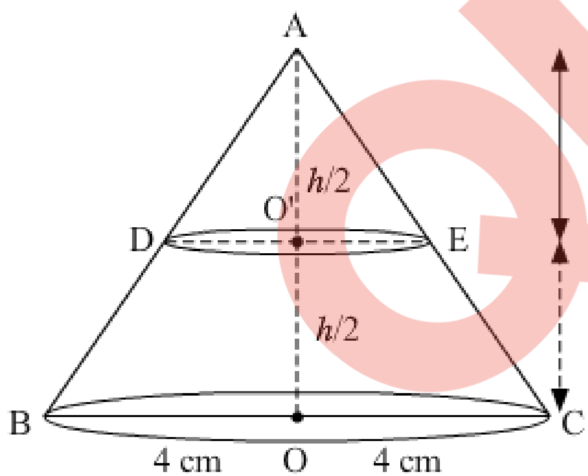
$$\Rightarrow \frac{\theta}{360} \times 2\pi r = 6.1$$
$$\Rightarrow \frac{\theta}{360} \times \pi r = \frac{6.1}{2}$$
$$\Rightarrow \frac{\theta}{360} \times \pi r^2 = \frac{6.1}{2} \times r$$

Thus, Area of the sector$= \frac{6.1}{2} \times 5.2 = 15.86$

Question: 27

A cone of base radius 4 cm is divided into two parts by drawing a plane through the mid-points of its height and parallel to its base. Compare the volume of the two parts.

Solution:



Given: $\mathrm{OC}=4\,\mathrm{cm}, \quad \mathrm{AO'}=\mathrm{OO'}$
Let $\mathrm{AO}=h$
$\Rightarrow \mathrm{AO'}=\mathrm{OO'}=\frac{h}{2}$

In $\triangle\ \mathrm{AO'C}$ and $\triangle\ \mathrm{AOC}$
$\angle\mathrm{E}=\angle\mathrm{C}$ [Corresponding angles]
$\angle\mathrm{A}=\angle\mathrm{A}$ [Common angle]
$\Rightarrow\triangle\ \mathrm{AO'E}\cong\triangle\ \mathrm{AOC}$ [By SS similarity criterion]

Therefore, $\frac{\mathrm{O'E}}{\mathrm{OC}}=\frac{\mathrm{AO'}}{\mathrm{AO}}=\frac{1}{2}$
$\Rightarrow \frac{\mathrm{O'E}}{\mathrm{OC}}=\frac{1}{2}$
Let $V_1,\ V_2$ are the volumes of the cone ADE and cone ABC respectively.

$$\frac{V_1}{V_2}=\frac{\left[\frac{1}{3}\pi(\mathrm{O'E})^2\ \mathrm{AO'}\right]}{\left[\frac{1}{3}\pi(\mathrm{OC})^2\ \mathrm{AO}\right]}$$
$$=\left(\frac{\mathrm{O'E}}{\mathrm{OC}}\right)^2\left(\frac{\mathrm{AO'}}{\mathrm{AO}}\right)$$
$$=\frac{1}{4}\times\frac{1}{2}$$
$$=\frac{1}{8}$$
$$\frac{\text{Volume of the upper part of the cone}}{\text{Volume of the lower part of the cone}}=\frac{V_1}{V_2-V_1}$$
$$=\frac{\left(\frac{V_1}{V_2}\right)}{1-\left(\frac{V_1}{V_2}\right)}$$
$$=\frac{1}{7} \qquad \left(\because \frac{V_1}{V_2}=\frac{1}{8}\right)$$

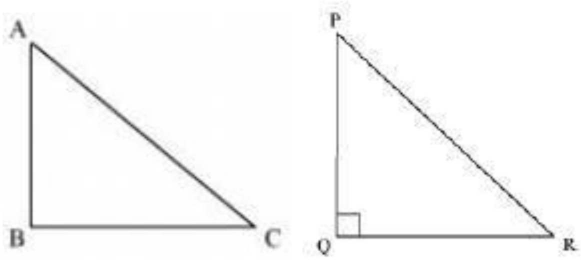
Question: 28

In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then prove that the angle opposite to the first side is a right angle.

Solution:

Given: In ΔABC, $AC^2 = AB^2 + BC^2$
To prove: ∠B = 90°

Construction: ΔPQR right-angled at Q such that PQ = AB and QR = BC



In ΔPQR,
$PR^2 = PQ^2 + QR^2$ (By Pythagoras Theorem, as $\angle Q = 90°$)

$\Rightarrow PR^2 = AB^2 + BC^2$ ..... (1) (By construction)
However, $AC^2 = AB^2 + BC^2$ ..... (2) (Given)

From (1) and (2), we obtain
$AC = PR$ ..... (3)

Now, In ΔABC and ΔPQR, we obtain
$AB = PQ$ (By construction)
$BC = QR$ (By construction)
$AC = PR$ [From (3)]

Therefore, $\Delta ABC \cong \Delta PQR$ (by SSS congruency criterion)
$\Rightarrow \angle B = \angle Q$ (By CPCT)
However, $\angle Q = 90°$ (By construction)

$\therefore \angle B = 90°$
Hence proved.

Question: 29

Find the area of triangle PQR formed by the points P(−5, 7), Q(−4, −5) and R (4, 5).

OR

If the point C(–1, 2) divides internally the line segment joining A(2, 5) and B(x, y) in the ratio 3 : 4, find the coordinates of B.

Solution:

Given: Vertices of the triangle are $\mathrm{P}(-5,7),\ \mathrm{Q}(-4,-5)$ and $\mathrm{R}(4,5)$.
Let A be the area of the triangle.
Using the formula to calculate the area of the triangle, we have

$$A=\frac{1}{2}\left[x_1\left(y_2-y_3\right)+x_2\left(y_3-y_1\right)+x_3\left(y_1-y_2\right)\right]$$
$$=\frac{1}{2}\left[(-5)(-5-5)+(-4)(5-7)+(4)(7+5)\right]$$
$$=\frac{1}{2}[50+8+48]$$
$$=53$$

Hence, the area of the triangle is 53 square units.

OR

Since the point $\mathrm{C}(-1,\ 2)$ divides the line segment joining $\mathrm{A}(2,\ 5)$ and $\mathrm{B}(x,\ y)$ in the ratio $3:4$.
Therefore using the section-formula of internal division, we get
For x- coordinate,

$$-1=\frac{3\times x+4\times 2}{3+4}$$

$$\Rightarrow 3x+8=-7$$
$$\Rightarrow x=-5$$

For y- coordinate,

$$2=\frac{3\times y+4\times 5}{3+4}$$

$$\Rightarrow 3y+20=14$$
$$\Rightarrow y=-2$$

Hence, the coordinates of B are $(-5,\ -2)$.

Question: 30

Find a quadratic polynomial whose zeroes are reciprocals of the zeroes of the polynomial $f(x) = ax^2 + bx + c,\ a \neq 0,\ c \neq 0$.

OR

Divide the polynomial $f(x) = 3x^2 - x^3 - 3x + 5$ by the polynomial $g(x) = x - 1 - x^2$ and verify the division algorithm.

Solution:

The given quadratic polynomial is
$f(x) = ax^2 + bx + c,\ a \neq 0,\ c \neq 0$
Let α and β be the two zeroes of the given quadratic polynomial.
Then,
Sum of zeroes = $\alpha + \beta = \frac{-b}{a}$
Product of zeroes = $\alpha\beta = \frac{c}{a}$

$\Rightarrow \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{\frac{-b}{a}}{\frac{c}{a}} = \frac{-b}{c}$ and $\frac{1}{\alpha}.\frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{\frac{c}{a}} = \frac{a}{c}$

So, the required new quadratic polynomial is
$k\left[x^2 - (\text{sum of zeroes})x + \text{product of zeroes}\right]$
$= k\left[x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \frac{1}{\alpha}.\frac{1}{\beta}\right]$
$= k\left[x^2 - \left(\frac{-b}{c}\right)x + \frac{a}{c}\right]$
where k is a real number.

OR

Given,
$f(x) = 3x^2 - x^3 - 3x + 5$
$g(x) = x - 1 - x^2$

$$
\begin{array}{r}
x-2 \\
-x^2+x-1 \overline{\big)\, -x^3+3x^2-3x+5} \\
-x^3+x^2-x \\
+\quad-\quad+ \\
\hline
2x^2-2x+5 \\
2x^2-2x+2 \\
-\quad+\quad- \\
\hline
3
\end{array}
$$

So,
$q(x) = (x-2)$ and $r(x) = 3$

To verify: $f(x) = g(x).q(x) + r(x)$
Verification:
$g(x).q(x) + r(x) = \left(-x^2 + x - 1\right)(x-2) + 3$
$= -x^2(x-2) + x(x-2) - 1(x-2) + 3$
$= -x^3 + 2x^2 + x^2 - 2x - x + 2 + 3$
$= -x^3 + 3x^2 - 3x + 5$
$= f(x)$
Hence verified.

Question: 31

Determine graphically the coordinates of the vertices of a triangle, the equations of whose sides are given by $2y - x = 8$, $5y - x = 14$ and $y - 2x = 1$.

OR

If 4 is a zero of the cubic polynomial $x^3 - 3x^2 - 10x + 24$, find its other two zeroes.

Solution:

The first given equation is $2y - x = 8$

x	0	−8
y	4	0

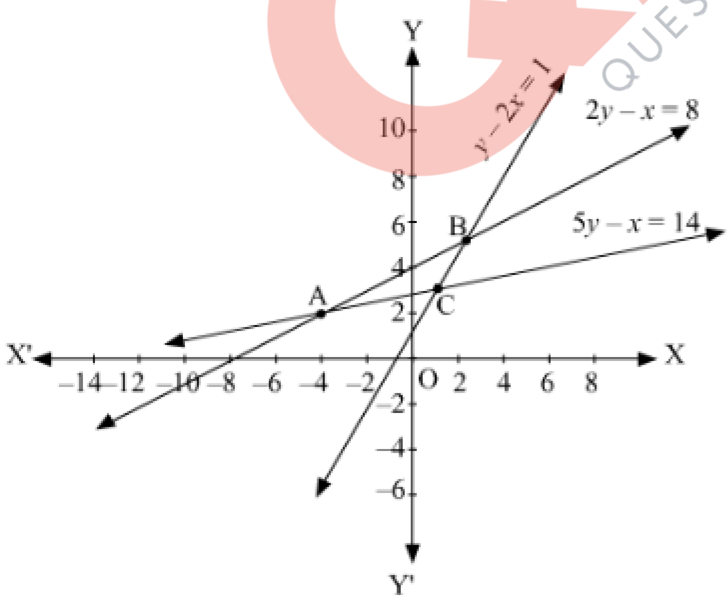
The second given equation is $5y - x = 14$

x	0	−14
y	2.8	0

The third given equation is $y - 2x = 1$

x	0	−0.5
y	1	0

Plotting the three given lines on the graph paper we get



The coordinates of the vertices of the triangle ABC are A(−-4, 2), B(2, 5) and C(1, 3).

OR

Given 4 is a zero of a cubic polynomial $x^3 - 3x^2 - 10x + 24$

$\Rightarrow (x-4)$ is the factor of polynomial $x^3 - 3x^2 - 10x + 24$

Therefore, we have

$$
\begin{array}{r|l}
 & x^2 + x - 6 \\
\hline
x - 4 & x^3 - 3x^2 - 10x + 24 \\
 & x^3 - 4x^2 \\
 & \underline{- \quad +} \\
 & \qquad x^2 - 10x + 24 \\
 & \qquad x^2 - 4x \\
 & \qquad \underline{- \quad +} \\
 & \qquad\qquad -6x + 24 \\
 & \qquad\qquad -6x + 24 \\
 & \qquad\qquad \underline{+ \quad -} \\
 & \qquad\qquad\qquad 0
\end{array}
$$

To find the other two zeroes of the given polynomial, we need to find the zeroes of the quotient $x^2 + x - 6$.

$$
\begin{aligned}
&\text{i.e. } x^2 + x - 6 = 0 \\
&\Rightarrow x^2 + 3x - 2x - 6 = 0 \\
&\Rightarrow x(x+3) - 2(x+3) = 0 \\
&\Rightarrow (x+3)(x-2) = 0 \\
&\Rightarrow x + 3 = 0 \text{ or } x - 2 = 0 \\
&\Rightarrow x = -3 \text{ or } x = 2
\end{aligned}
$$

Hence, the other two zeroes of the given polynomial are 2 and -3.

Question: 32

A train covers a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. Find the original speed of the train.

Solution:

Given: Distance = 480km.

Let the original speed be *x*km/h.

Time taken (t_1) = $\frac{480}{x}$ h

Now, reduced speed = (x – 8) km/h.

Time taken (t_2) = $\frac{480}{x-8}$ h

$\Delta\,\mathrm{ACD},\ \angle\mathrm{A} = 45^{\circ}$ According to the given condition,

$\frac{480}{x-8} - \frac{480}{x} = 3$

$\Rightarrow 480\left[\frac{1}{x-8} - \frac{1}{x}\right] = 3$

$\Rightarrow \frac{x-(x-8)}{x(x-8)} = \frac{1}{160}$

$\Rightarrow \frac{8}{x(x-8)} = \frac{1}{160}$

$\Rightarrow 8\,(160) = x\,(x-8)$

$\Rightarrow x^2 - 8x - 1280 = 0$

$\Rightarrow x^2 - 40x + 32x - 1280 = 0$

$\Rightarrow x(x-40) + 32(x-40) = 0$

$\Rightarrow (x-40)(x+32) = 0$

$\Rightarrow x = 40,\ -32$

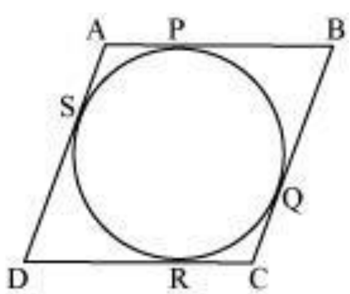
But speed can never be negative.
Thus, we conclude speed of train is 40 km/h.

Question: 33

Prove that the parallelogram circumscribing a circle is a rhombus.

Solution:

Since ABCD is a parallelogram,
AB = CD .....(1)
BC = AD .....(2)



It can be observed that

DR = DS (Tangents to the circle from point D)
CR = CQ (Tangents to the circle from point C)

BP = BQ (Tangents to the circle from point B)
AP = AS (Tangents to the circle from point A)

Adding all these equations, we obtain
DR + CR + BP + AP = DS + CQ + BQ + AS
(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)

CD + AB = AD + BC
2AB = 2BC [From (1) and (2)]

AB = BC .....(3)
From (1), (2), and (3), we obtain

AB = BC = CD = DA

Hence, ABCD is a rhombus.

Question: 34

Prove that: $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$.

Solution:

$$\begin{aligned}
\text{LHS} &= 2\left(\sin^6\theta + \cos^6\theta\right) - 3\left(\sin^4\theta + \cos^4\theta\right) + 1 \\
&= 2\left[\left(\sin^2\theta\right)^3 + \left(\cos^2\theta\right)^3\right] - 3\left(\sin^4\theta + \cos^4\theta\right) + 1 \\
&= 2\left[\left(\sin^2\theta + \cos^2\theta\right)\left(\sin^4\theta - \sin^2\theta \times \cos^2\theta + \cos^4\theta\right)\right] - 3\left(\sin^4\theta + \cos^4\theta\right) + 1 \\
&= 2(1)\left(\sin^4\theta - \sin^2\theta \times \cos^2\theta + \cos^4\theta\right) - 3\left(\sin^4\theta + \cos^4\theta\right) + 1 \\
&= 2\sin^4\theta - 2\sin^2\theta \times \cos^2\theta + 2\cos^4\theta - 3\sin^4\theta - 3\cos^4\theta + 1 \\
&= -\sin^4\theta - \cos^4\theta - 2\sin^2\theta \times \cos^2\theta + 1 \\
&= -\left(\sin^4\theta + 2\sin^2\theta \times \cos^2\theta + \cos^4\theta\right) + 1 \\
&= -\left(\sin^2\theta + \cos^2\theta\right)^2 + 1 \\
&= -(1)^2 + 1 \\
&= 0 \\
&= \text{RHS}
\end{aligned}$$

Question: 35

The following table gives production yield per hectare (in quintals) of wheat of 100 farms of a village :

Production yield/hect.	40 – 45	45 – 50	50 – 55	55 – 60	60 – 65	65 – 70
No. of farms	4	6	16	20	30	24

Change the distribution to 'a more than' type distribution and draw its ogive.

OR

The median of the following data is 525. Find the values of *x* and *y*, if total frequency is 100:

Class :	**Frequency:**
0 – 100	2
100 – 200	5
200 – 300	x
300 – 400	12
400 – 500	17
500 – 600	20
600 – 700	y
700 – 800	9
800 – 900	7
900 – 1000	4

Solution:

Given:

Production yield/hect.	40 – 45	45 – 50	50 – 55	55 – 60	60 – 65	65 – 70
No. of farms	4	6	16	20	30	24

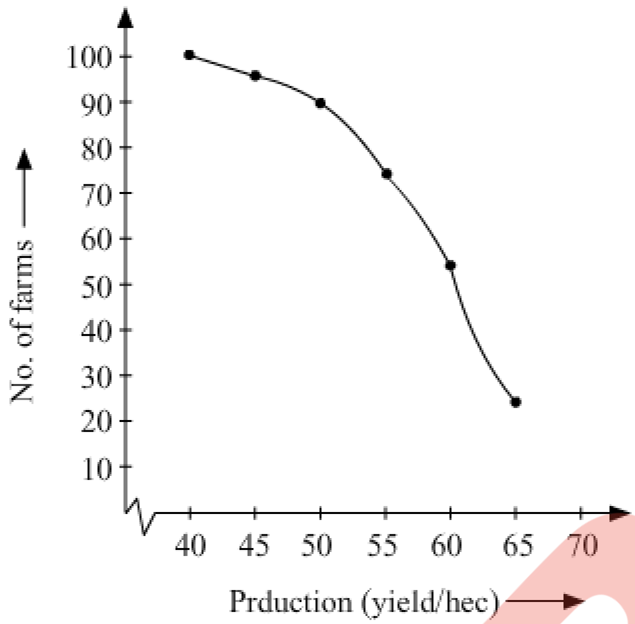
"more than type" distribution table is as follows-

Production (yield/hec)	No. of farms
More than 40	100
More than 45	96
More than 50	90
More than 55	74
More than 60	54
More than 65	24

To draw the ogive, we have the following points-

(40, 100), (45, 96), (50, 90), (55, 74), (60, 54), (65, 24)

Plotting these points, we get the following ogive-



OR

Given, median = 525

We prepare the cumulative frequency table, as given below.

Class interval:	Frequency: (f_i)	Cumulative frequency $(c.f.)$
0-100	2	2
100-200	5	7
200-300	f_1	$7+f_1$
300-400	12	$19+f_1$
400-500	17	$36+f_1$
500-600	20	$56+f_1$
600-700	f_2	$56+f_1+f_2$
700-800	9	$65+f_1+f_2$
800-900	7	$72+f_1+f_2$
900−1000	4	$76+f_1+f_2$
	$N=100=76+f_1+f_2$	

$$N = 100$$
$$76 + f_1 + f_2 = 100$$
$$f_2 = 24 - f_1 \quad .....(1)$$
$$\frac{N}{2} = 50$$

Since median = 525,
So, the median class is $500 - 600$.

Here, $l = 500, f = 20, F = 36 + f_1$ and $h = 100$

We know that

$$\text{Median} = l + \left\{ \frac{\frac{N}{2} - F}{f} \right\} \times h$$
$$525 = 500 + \left\{ \frac{50 - (36 + f_1)}{20} \right\} \times 100$$
$$25 = \frac{(14 - f_1) \times 100}{20}$$
$$25 \times 20 = 1400 - 100 f_1$$
$$100 f_1 = 1400 - 500$$
$$f_1 = \frac{900}{100}$$
$$= 9$$

Putting the value of f_1 in (1), we get
$$f_2 = 24 - 9$$
$$= 15$$

Hence, the missing frequencies are 9 and 15.

Question: 36

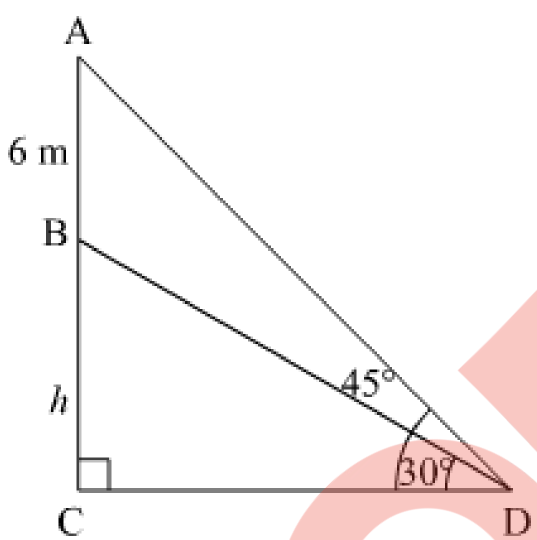
A vertical tower stands on a horizontal plane and is surmounted by a vertical flag-staff of height 6 m. At a point on the plane, the angle of elevation of the bottom and top of the flag-staff are 30° and 45° respectively. Find the height of the tower.
(Take √3 = 1.73)

Solution:

Let BC be the tower of height h m, AB be the flag staff of height 7 m on tower and D be the point on the plane making an angle of elevation of the top of the flag staff as 45° and angle of elevation of the bottom of the flag staff as 30°.

Let CD = x, AB = 6m and $\angle BDC = 30°$ and $\angle ADC = 45°$.

We need to find the height of the tower i.e. h.
We have the corresponding figure as follows:



So we use trigonometric ratios.

In a triangle BCD :

$$\Rightarrow \quad \tan D = \frac{BC}{CD}$$

$$\Rightarrow \quad \tan 30° = \frac{h}{x}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\Rightarrow \quad x = \sqrt{3}h$$

Again in a triangle ADC:

$$\tan\ D = \frac{AB+BC}{CD}$$

$\Rightarrow \tan 45^\circ = \frac{h+6}{x}$
$\Rightarrow 1 = \frac{h+6}{x}$
$\Rightarrow x = h + 6$
$\Rightarrow \sqrt{3}h = h + 6$
$\Rightarrow h\left(\sqrt{3} - 1\right) = 6$
$\Rightarrow h = \frac{6}{\sqrt{3}-1} = \frac{6(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{6(\sqrt{3}+1)}{(\sqrt{3})^2-1^2} = \frac{6(\sqrt{3}+1)}{3-1} = 3\left(\sqrt{3}+1\right)$
$\Rightarrow h = 3\left(\sqrt{3}+1\right) = 3(1.732+1) = 3 \times 2.732 = 8.196\text{m}$
Hence the height of the tower is 8.196 m.

Question: 37

Show that the square of any positive integer cannot be of the form ($5q$ + 2) or ($5q$ + 3) for any integer q.

OR

Prove that one of every three consecutive positive integers is divisible by 3.

Solution:

Let b be an arbitrary positive integer.
By Euclid's division lemma,
$b = aq + r$, where $0 \leq r < a$

Now, if we divide b by 5, then b can be written in the form of $5m$, $5m+1$, $5m+2$, $5m+3$ or $5m+4$.
This implies that we have five possible cases.

Case I:
If $b = 5m$

Squaring both sides, we get

$b^2 = (5m)^2 = 25m^2 = 5\left(5m^2\right)$
$\Rightarrow b^2 = 5q$
where $q = 5m^2$ is an integer.

Case II:
If $b = 5m + 1$,
Squaring both sides, we get
$b^2 = (5m+1)^2 = 25m^2 + 1 + 10m$
$\Rightarrow b^2 = 5\left(5m^2 + 2m\right) + 1$
$\Rightarrow b^2 = 5q + 1$
where $q = 5m^2 + 2m$ is an integer.

Case III:
If $b = 5m + 2$
Squaring both sides, we get
$b^2 = (5m+2)^2 = 25m^2 + 4 + 20m$
$\Rightarrow b^2 = 5\left(5m^2 + 4m\right) + 4$
$\Rightarrow b^2 = 5q + 4$

where $q = 5m^2 + 4m$ is an integer.

Case IV:
If $b = 5m + 3$
Squaring both sides, we get
$b^2 = (5m+3)^2 = 25m^2 + 9 + 30m$
$\Rightarrow b^2 = 25m^2 + 5 + 4 + 30m$
$\Rightarrow b^2 = 5\left(5m^2 + 1 + 6m\right) + 4$
$\Rightarrow b^2 = 5q + 4$
where $q = 5m^2 + 1 + 6m$ is an integer.

Case V:
If $b = 5m + 4$
Squaring both sides, we get
$b^2 = (5m+4)^2 = 25m^2 + 16 + 40m$
$\Rightarrow b^2 = 25m^2 + 15 + 1 + 40m$
$\Rightarrow b^2 = 5\left(5m^2 + 3 + 8m\right) + 1$
$\Rightarrow b^2 = 5q + 1$
where $q = 5m^2 + 3 + 8m$ is an integer.

Hence, we can conclude that the square of any positive integer cannot be of the form $5q + 2$ or $5q + 3$ for any integer.

OR

Let n, $n + 1$, $n + 2$ be three consecutive positive integers, where n is any natural number.
By Euclid's division lemma,
$n = aq + r$, where $0 \leq r < a$.

Now, if we divide n by 3, then n can be written in the form of $3q$, $3q+1$ or $3q+2$.
This implies that we have three possible cases.

Case I:
If $n = 3q$, then n is divisible by 3.
However, $n + 1$ and $n + 2$ are not divisible by 3.

Case II:
If $n = 3q + 1$, then $n + 2 = 3q + 3 = 3(q + 1)$, which is divisible by 3.
However, n and $n + 1$ are not divisible by 3.

Case III:
If $n = 3q + 2$, then $n + 1 = 3q + 3 = 3(q + 1)$, which is divisible by 3.
However, n and $n + 2$ are not divisible by 3.

Hence, we conclude that one of any three consecutive positive integers must be divisible by 3.

Question: 38

The sum of four consecutive numbers in AP is 32 and the ratio of the product of the first and last terms to the product of two middle terms is 7 : 15. Find the numbers.

OR

Solve : $1 + 4 + 7 + 10 + ... + x = 287$

Solution:

Let the four terms of the AP be $a - 3d$, $a - d$, $a + d$ and $a + 3d$.
Given:
$(a - 3d) + (a - d) + (a + d) + (a + 3d) = 32$
$\Rightarrow 4a = 32$
$\Rightarrow a = 8$

Also,

$\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$

$\Rightarrow \frac{a^2-9d^2}{a^2-d^2} = \frac{7}{15}$

$\Rightarrow \frac{(8)^2-9d^2}{(8)^2-d^2} = \frac{7}{15}$

$\Rightarrow \frac{64-9d^2}{64-d^2} = \frac{7}{15}$

$\Rightarrow 960 - 135d^2 = 448 - 7d^2$
$\Rightarrow 512 = 128d^2$
$\Rightarrow d^2 = 4$
$\Rightarrow d = \pm 2$

When a = 8 and d = 2, then the terms are 2, 6, 10, 14.
When a = 8 and d = −2, then the terms are 14, 10, 6, 2.

OR

In the given AP, we have
$a = 1,\ d = 3,\ S_n = 287$
The formula for sum of n terms of an AP is given by $S_n = \frac{n}{2}[2a + (n-1)d]$.
This implies
$\frac{n}{2}[2(1) + (n-1)(3)] = 287$
$\Rightarrow n(2 + 3n - 3) = 574$
$\Rightarrow 3n^2 - n - 574 = 0$
$\Rightarrow 3n^2 - 42n + 41n - 574 = 0$
$\Rightarrow 3n(n-14) + 41(n-14) = 0$
$\Rightarrow (n-14) = 0 \text{ or } 3n + 41 = 0$
$\Rightarrow n = 14 \text{ or } n = -\frac{41}{3}$
$\therefore\ n = 14$
$x = a_{14} = a + (14-1)d = 1 + 13(3) = 1 + 39 = 40$
Thus, the value of x is 40.

Question: 39

A bucket is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper circular ends as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the bucket, at the rate of ₹ 40 per litre. (Use π = 3.14)

Solution:

The bucket is in the shape of a frustum of a cone.

It is given that radius of upper end of the bucket, r_1 = 20 cm

Radius of lower end of the bucket, r_2 = 8 cm
Height of the bucket, h = 16 cm

Therefore, the volume of the bucket

$$
\begin{aligned}
&= \tfrac{\pi}{3} h \left(r^2{}_1 + r^2{}_2 + r_1 r_2\right) \\
&= 3.14 \times \tfrac{16}{3}\left(20^2 + 8^2 + 20 \times 8\right) \\
&= 10450 \text{ cm}^3
\end{aligned}
$$

$$\text{Amount of milk the bucket can hold} = \frac{10450}{1000} = 10.45 \text{ L}$$

$$\text{Total cost of milk} = 10.45 \times 40 = \text{Rs } 418$$

Question: 40

Construct a triangle with sides 4 cm, 5 cm and 6 cm. Then construct another triangle whose sides are 2/3 times the corresponding sides of the first triangle.

Solution:

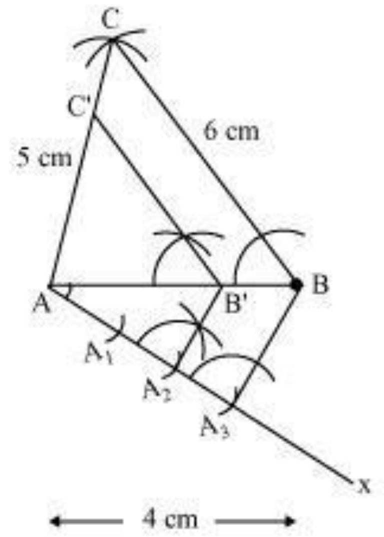
Step 1
Draw a line segment AB= 4 cm. Taking point A as the centre, draw an arc of 5 cm radius. Similarly, taking point B as its center, draw an arc of 6 cm radius. These arcs will intersect each other at point C. Now, AC = 5 cm and BC = 6 cm and ΔABC is the required triangle.

Step 2
Draw a ray AX making an acute angle with line AB on the opposite side of vertex C.

Step 3
Locate 3 points A_1,A_2, A_3 (as 3 is greater between 2 and 3) online AX such that $AA_1 = A_1A_2 = A_2A_3$.

Step 4
Join BA_3 and draw a line through A2 parallel to BA_3 to intersect AB at point B'.

Step 5
Draw a line through B' parallel to the line BC to intersect AC at C'.



$\Delta AB'C'$ is the required triangle.