CBSE Board Paper Solution-2020

Class	:	**X**
Subject	:	**Mathematics (Basic)**
Set	:	**1**
Code No	:	**430/5/1**
Time allowed	:	**3 Hours**
Maximum Marks	:	**80 Marks**

General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) This question paper comprises **four** sections – A, B, C and D. This question. Paper carries **40** questions. All questions are compulsory.

(ii) **Section A:** Question Number **1 to 20** comprises of **20** questions of one mark each.

(iii) **Section B:** Question Number **21 to 26** comprises of 6 questions of two marks each.

(iv) **Section C:** Question Number **27 to 34** comprises of 8 questions of three marks each.

(v) **Section D:** Question Number **35 to 40** comprises of 6 questions of four marks each.

(vi) There is no overall choice in the question Paper. However, an internal choice has been provided in **2** questions of one mark, **2** questions of two marks, **3** questions of three marks and **3** questions of four marks. You have to attempt only one of the choices in such questions.

(vii) In addition to this, separate instructions are given with each section and question, wherever necessary.

(viii) Use of calculators is not permitted.

Section A

Question numbers 1 to 20 carry 1 mark each.

Choose the correct option in question numbers 1 to 10.

1. If a pair of linear equations is consistent, then the lines represented by them are

(A) parallel

(B) intersecting or coincident

(C) always coincident

(D) always intersecting

Answer:

Correct Answer: (B) intersecting or coincident

If a pair of linear equations is consistent, then the lines represented by them are intersecting or coincident.

2. The distance between the points (3, −2) and (−3, 2) is

(A) $\sqrt{52}$ **units**

(B) $4\sqrt{10}$ **units**

(C) $2\sqrt{10}$ **units**

(D) 40 units

Answer:

Correct Answer: (A) √(52) units

Explanation:

$$\text{Distance} = \sqrt{(3-(-3))^2 + (-2-2)^2}$$
$$= \sqrt{(3+3)^2 + (-4)^2}$$
$$= \sqrt{36+16}$$
$$= \sqrt{52}$$

3. 8 cot² A – 8 cosec² A equal to

(A) 8

(B) $\frac{1}{8}$

(C) -8

(D) -$\frac{1}{8}$

Answer:

Correct Answer: (C) -8

Explanation:

$$8\cot^2 A - 8\operatorname{cosec}^2 A$$
$$= 8(\cot^2 A - \operatorname{cosec}^2 A)$$
$$= 8 \times -1$$
$$= -8$$

4. The total surface area of a frustum-shaped glass tumbler is ($r_1 > r_2$)

(A) $\pi r_1 l + \pi r_2 l$

(B) $\pi l(r_1 + r_2) + \pi r_2^2$

(C) $\frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

(D) $\sqrt{h^2 + (r_1 - r_2)^2}$

Answer:

Correct Answer: (C) $\frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

Explanation:

The total surface area of a frustum-shaped glass tumbler is $\frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$ where radii $r_1 > r_2$.

5. 120 can be expressed as a product of its prime factors as

(A) $5 \times 8 \times 3$

(B) 15×2^3

(C) $10 \times 2^2 \times 3$

(D) $5 \times 2^3 \times 3$

Answer:

Correct Answer: (D) $5 \times 2^3 \times 3$

Explanation:

$120 = 20 \times 6$

$= 5 \times 4 \times 2 \times 3$

$= 5 \times 2^3 \times 3$

6. The discriminant of the quadratic equation $4x^2-6x+3=0$ is

(A) 12

(B) 84

(C) $2\sqrt{3}$

(D) −12

Answer:

Correct Answer: (D)−12

Explanation:

The given equation is:

$4x^2 - 6x + 3 = 0$

Discriminant = $b^2 - 4ac$

Here, b =−6, a =4, and c =3

So, Discriminant = $(-6)^2 - 4\times4\times3$

$= 36 - 48 = -12$

7. If (3, – 6) is the mid-point of the line segment joining (0, 0) and (x, y), then the point (x, y) is

(A) (−3, 6)

(B) (6, – 6)

(C) (6, −12)

(D) $\left(\frac{3}{2}, -3\right)$

Answer:

Correct Answer: (C) (6, −12)

Explanation:

(3, −6) is the mid-point of the line segment joining

(0, 0) and (x, y).
So, $(0+x)/2 = 3$ or, $x = 6$
And $(0+y)/2 = -6$ or, $y = -12$

8) In the circle given in Figure-1, the number of tangents parallel to tangent PQ is

Figure-1

(A) 0

(B) many

(C) 2

(D) 1

Answer:
Correct Answer: (D) 1
In the given figure, number of tangents parallel to tangent PQ is 1.

9) For the following frequency distribution:

Class:	0–5	5–10	10–15	15–20	20–25
	8	10	19	25	8

The upper limit of median class is

(A) 15

(B) 10

(C) 20

(D) 25

Answer:
Correct Answer: (A) 15

Explanation:

Class	Frequency	Cumulative frequency
0-5	8	8
5-10	10	18
10-15	19	37
15-20	25	62
20-25	8	70
Sum:	70	

Sum of frequencies (n)= 70

Middle observation = ((n/2)+1) th observation

= (70/2 +1) th observation

=36^{th} observation

36^{th} observation lies in class interval 10-15. So, median class is 10-15 and its upper limit is 15.

10) The probability of an impossible event is

(A) 1

(B) $\frac{1}{2}$

(C) not defined

(D) 0

Answer:
Correct Answer: (D) 0
Explanation:
The probability of an impossible event is 0.

Fill in the blanks in question numbers 11 to 15.

11) A line intersecting a circle in two points is called a_______.
Answer:
secant

12) If 2 is a zero of the polynomial $ax^2 - 2x$, then the value of 'a' is ___________.
Answer:
1

$$a(2)^2 - 2 \times 2 = 0$$
$$\Rightarrow \quad 4a - 4 = 0$$
$$\Rightarrow \quad a = 1$$

13) All squares are__________. (congruent/similar)
Answer:
similar

14) If the radii of two spheres are in the ratio 2:3, then the ratio of their respective volumes is________.

Answer:

8:27

$r_1 : r_2 = 2 : 3$

$$\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{r_1^3}{r_2^3}$$

$$\frac{V_1}{V_2} = \frac{8}{27}$$

15) If ar (ΔPQR) is zero, then the points P, Q and R are________.

Answer:

collinear

Answer the following question numbers 16 to 20:

16) In Figure-2, the angle of elevation of the top of a tower AC from a point B on the ground is 60°. If the height of the tower is 20 m, find the distance of the point from the foot of the tower.

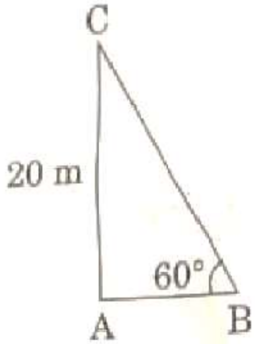


Figure-2

Answer:

$$\tan 60^\circ = \frac{20}{AB}$$

$$\sqrt{3} = \frac{20}{AB}$$

$$AB = \frac{20}{\sqrt{3}}$$

So, the required distance is $\frac{20}{\sqrt{3}}$ m.

17) Evaluate:

tan 40° × tan 50°

OR

If cos A = sin 42°, then find the value of A.

Answer:

$$\tan 40° \times \tan 50°$$
$$= \tan(90° - 50°) \times \tan 50°$$
$$= \cot 50° \times \tan 50°$$
$$= 1 \quad (\because \tan\theta \cot\theta = 1)$$

OR

$$\cos A = \sin 42^\circ$$
$$\Rightarrow \cos A = \sin(90^\circ - 48^\circ)$$
$$\Rightarrow \cos A = \cos 48^\circ$$
$$\Rightarrow \quad A = 48^\circ$$

18) A coin is tossed twice. Find the probability of getting head both the times.

Answer:

All possible outcomes are HH, HT, TT, TH.

Probability of an event $= \dfrac{\text{Number of favourable outcomes}}{\text{Number of all possible outcomes}}$

Probability of getting head both the times= $\dfrac{1}{4}$

19) Find the height of a cone of radius 5 cm and slant height 13 cm.

Answer:

Height of the cone $= \sqrt{13^2 - 5^2}$

$= \sqrt{169 - 25}$

$= \sqrt{144}$

$= 12$

Therefore, the height of the cone is $12\,m$.

20) Find the value of x so that –6, x, 8 are in A.P.

OR

Find the 11th term of the A.P. –27, –22, –17, –12, ...

Answer:

$-6, x, 8$ are in A.P.

$\Rightarrow 2x = -6 + 8$

$\Rightarrow 2x = 2$

$\Rightarrow x = 1$

OR

$-27, -22, -17, -12, \ldots$

$a_n = a + (n-1)d$

$a_{11} = -27 + (11-1) \times 5$

$= -27 + 50$

$= 23$

Section - B

Question numbers 21 to 26 carry 2 marks each.

21) Find the roots of the quadratic equation.

$3x^2 - 4\sqrt{3}x + 4 = 0$

Answer:

$$\begin{aligned} 3x^2 - 4\sqrt{3}x + 4 &= 3x^2 - 2\sqrt{3}x - 2\sqrt{3}x + 4 \\ &= \sqrt{3}x\left(\sqrt{3}x - 2\right) - 2\left(\sqrt{3}x - 2\right) \\ &= \left(\sqrt{3}x - 2\right)\left(\sqrt{3}x - 2\right) \end{aligned}$$

So, the roots of the equation are the values of x for which

$$\left(\sqrt{3}x - 2\right)\left(\sqrt{3}x - 2\right) = 0$$

Now, $\sqrt{3}x - 2 = 0$ for x = $\frac{2}{\sqrt{3}}$

So, this root is repeated twice, one for each repeated factor $\sqrt{3}x - 2$.

Therefore, the roots of $3x^2 - 4\sqrt{3}x + 4$ are $\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}$.

22) Check whether 6^n can end with the digit '0' (zero) for any natural number n.

OR

Find the LCM of 150 and 200.

Answer:

If the number 6^n, for any n, were to end with the digit zero, then it would be divisible by 5.

That is, the prime factorisation of 6^n would contain the prime 5. This is not possible

$\because\ 6^n = (2 \times 3)^n$

So, the prime numbers in the factorisation of 6^n are 2 and 3.

So, the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorisation of 6^n.

So, there is no natural number n for which 6^n ends with the digit zero.

OR

We have,

$$150 = 5^2 \times 3 \times 2$$

and, $200 = 5^2 \times 2^3$

Here, $2^3, 3^1$ and 5^2 are the greatest powers of the prime factors 2, 3 and 5 respectively involved in the two numbers.

So, $\text{LCM}(150, 200) = 2^3 \times 3^1 \times 5^2 = 600$

23) **If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = \frac{1}{\sqrt{3}}, 0 < A+B \leq 90°, A > B$, then find the values of A and B.**

Answer:

We have

$$\tan(A+B) = \sqrt{3}$$

or $\tan(A+B) = \tan 60°$

or $A+B = 60°$ ...(1)

Again, we have

$$\tan(A-B) = \frac{1}{\sqrt{3}}$$

or $\tan(A-B) = \tan 30°$

or $A-B = 30°$ ...(2)

On adding equations (1) and (2), we get

$$2A = 90°$$

or $A = 45°$

On putting this value of A in equation (1), we get

$$B = 15°$$

24. In Figure-3, ΔABC and ΔXYZ are shown. If AB = 3 cm, BC = 6 cm, AC = $2\sqrt{3}$ cm, $\angle$ A = 80^0, $\angle$B 60^0, XY = $4\sqrt{3}$ cm, YZ = 12 cm and XZ = 6 cm, then find the value of $\angle$Y.

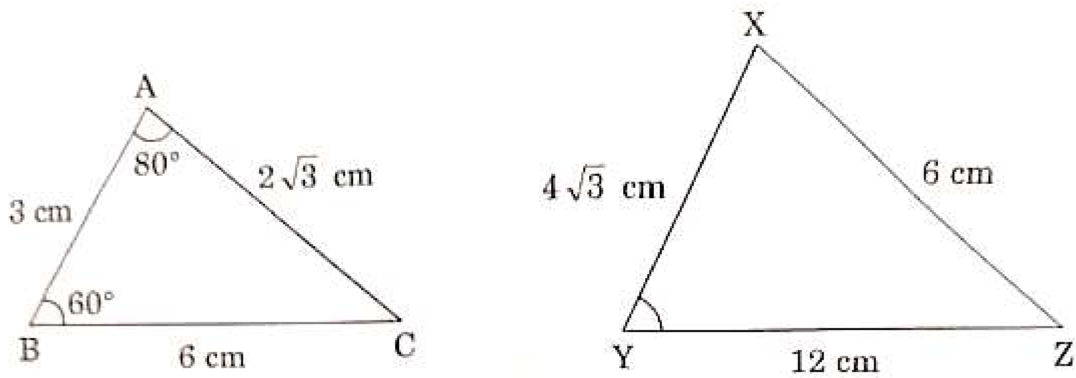


Figure-3

Answer:

In ΔABC and ΔXZY,

$$\frac{BC}{ZY} = \frac{6}{12}$$

$$= \frac{1}{2}$$

$$\frac{AC}{XY} = \frac{2\sqrt{3}}{4\sqrt{3}}$$

$$= \frac{1}{2}$$

$$\frac{AB}{XZ} = \frac{3}{6}$$

$$= \frac{1}{2}$$

Ratios of the corresponding sides of the given pair of triangles are equal.

i.e., $\frac{BC}{YZ} = \frac{AC}{XY} = \frac{AB}{XZ} = \frac{1}{2}$

Therefore, by SSS similarity critarion, $\Delta ABC \sim \Delta XZY$.

The corresponding angles are equal in ΔABC and ΔXZY. i.e.,

$\angle A = \angle X = 80^\circ$,

$\angle B = \angle Z = 60^\circ$

and

$\angle C = \angle Y$

In ΔABC,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow 80^\circ + 60^\circ + \angle C = 180^\circ$$

$$\Rightarrow \angle C = 180^\circ - 140^\circ$$

$$\Rightarrow \angle C = 40^\circ$$

$$\Rightarrow \angle Y = 40^\circ$$

25. 14 defective bulbs are accidentally mixed with 98 good ones. It is not possible to just look at the bulb and tell whether it is defective or not. One bulb is taken out at random from this lot. Determine the probability that the bulb taken out is a good one.

Answer:

Number of defective bulbs $= 14$

Number of good bulbs $= 98$

Total number of outcomes $= 98 + 14 = 112$

Probability of getting a good bulbs

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{98}{112}$$

$$= \frac{7}{8}$$

26. Find the mean for the following distribution:

Classes	**5 – 15**	**15 – 25**	**25 – 35**	**35 – 45**
Frequency	**2**	**4**	**3**	**1**

OR

The following distribution shows the transport expenditure of 100 employees:

Expenditure (in ₹) :	**200-400**	**400-600**	**600-800**	**800-1000**	**1000-1200**
Number of employees :	**21**	**25**	**19**	**23**	**12**

Find the mode of the distribution.

Answer:

Classes	5-15	15-25	25-35	35-45
Frequency	2	4	3	1

Here, we observe that class marks and frequencies are small quantities.

So, we use direct method to compute the mean and proceed as below.

Classes	Frequency (f_i)	x_i	$f_i x_i$
$5-15$	2	10	20
$15-25$	4	20	80
$25-35$	3	30	90
$35-45$	1	40	40
Total	10		230

Mean, $\overline{x} = \dfrac{\sum f_i x_i}{\sum f_i}$

$= \dfrac{230}{10}$

$= 23$

Therefore, mean for the following distribution is 23.

OR

Expenditure	200-400	400-600	600-800	800-1000	1000-1200
Number of employees	21	25	19	23	12

From the given data, we have

$l = 400,\ f_1 = 25,\ f_0 = 21,\ f_2 = 19,\ h = 200$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 400 + \left(\frac{25 - 21}{2 \times 25 - 21 - 19}\right) \times 200$$

$$= 480$$

$\therefore$ Mode of the given data is 480.

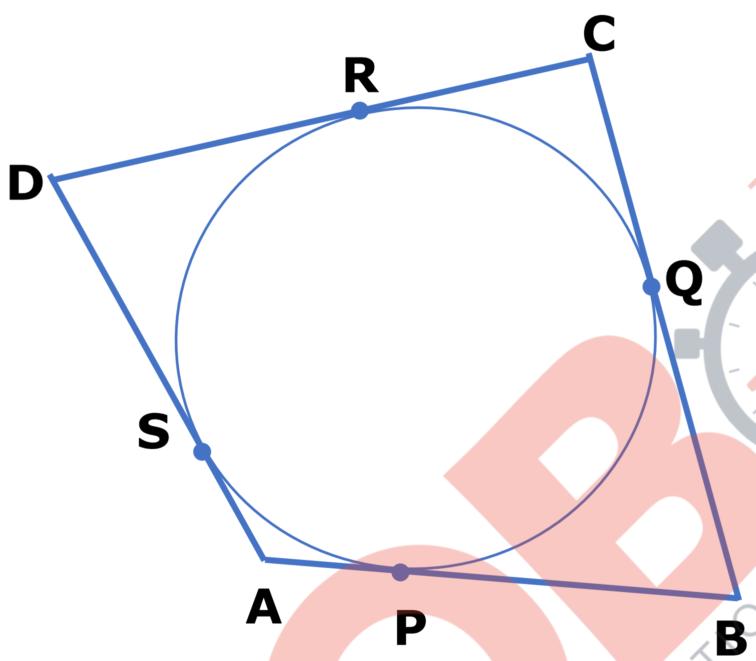
SECTION C

Question number 27 to 34 carry 3 marks each.

27. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that

AB + CD = AD + BC.

Answer:



In the given figure, quadrilatetral ABCD is circumscribing the given circle and its sides are touching the circle at P, Q, R and S.

We have to prove that

$AB + CD = AD + BC$

We know that lengths of tangents drawn from a point to a circle are equal.

Therefore, from figure, we have

$DR = DS,\ CR = CQ,\ AS = AP,\ BP = BQ$

Now,

$$\begin{aligned} L.H.S. = AB + CD &= (AP + BP) + (CR + DR) \\ &= (AS + BQ) + (CQ + DS) \\ &= AS + DS + BQ + CQ \\ &= AD + BC \\ &= R.H.S. \end{aligned}$$

28. The difference between two numbers is 26 and the larger number exceeds thrice of the smaller number by 4. Find the numbers.

OR

Solve for x and y:

$$\frac{2}{x} + \frac{3}{y} = 13 \quad \text{and} \quad \frac{5}{x} - \frac{4}{y} = -2$$

Answer:

Let the larger number be y and the smaller number be x.

According to question,

$y - x = 26 \quad \ldots(1)$

and $y = 3x + 4 \quad \ldots(2)$

Substituting the value of y from equation (2) in equation (1), we get

$3x + 4 - x = 26$

or $2x = 26 - 4$

or $2x = 22$

or $x = 11$

Putting this value of x in equation (1), we get

$y - 11 = 26$

or $y = 26 + 11 = 37$

Hence, the numbers are 11 and 37.

OR

$$\frac{2}{x}+\frac{3}{y}=13 \text{ and } \frac{5}{x}-\frac{4}{y}=-2$$

Let $\frac{1}{x}=p$ and $\frac{1}{y}=q$

then given equations can be written as:

$2p+3q=13$

$2p+3q-13=0 \quad ...(1)$

and

$5p-4q=-2$

$5p-4q+2=0 \quad ...(2)$

Using cross-multiplication method, we get

$$\frac{p}{6-52}=\frac{q}{-65-4}=\frac{1}{-8-15}$$

$$\Rightarrow \frac{p}{-46}=\frac{q}{-69}=\frac{1}{-23}$$

$$\Rightarrow \frac{p}{-46}=\frac{1}{-23} \text{ and } \frac{q}{-69}=\frac{1}{-23}$$

$$\Rightarrow p=\frac{-46}{-23} \text{ and } q=\frac{-69}{-23}$$

$$\Rightarrow p=2 \quad \text{and} \quad q=3$$

$$\Rightarrow \frac{1}{x}=2 \quad \text{and} \quad \frac{1}{y}=3$$

$$\Rightarrow x=\frac{1}{2} \quad \text{and} \quad y=\frac{1}{3}$$

29. Prove that $\sqrt{3}$ is an irrational number.

Answer:

Let us assume that $\sqrt{3}$ is rational.

So we can find integers r and $s\left(\neq 0\right)$ such that

$\sqrt{3}=\frac{r}{s}$.

Suppose r and s have a common factor other than 1.

Then we divide r and s by the common factor and get

$\sqrt{3}=\frac{a}{b}$

where a and b are coprime.

So, $\sqrt{3}$ b=a

Squaring on both sides, we get

$3b^2=a^2$

Therefore,

a^2 is divisible by 3, and so a is also divisible by 3.

So, we can write $a=3c$ for some integer c.

Now,

$$3b^2 = a^2$$
$$\Rightarrow 3b^2 = 9c^2$$
$$\Rightarrow b^2 = 3c^2.$$

This means that b^2 is divisible by 3, and so b is also divisible by 3.

Therefore,

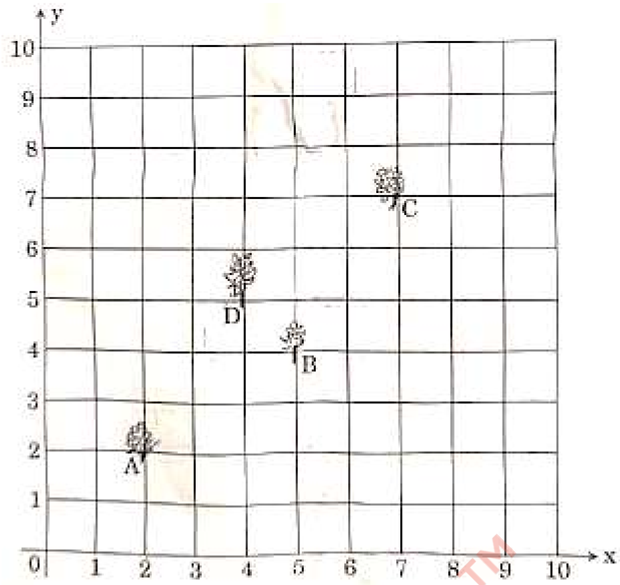
a and b have at least 3 as a common factor.

But this contradicts the fact that a and b are coprime.

So, our assumption that $\sqrt{3}$ is a rational is wrong.

Hence, $\sqrt{3}$ is an irrational number.

30. Krishna has an apple orchard which has a 10 m ×10 m sized kitchen garden attached to it. She divides it into a 10 ×10 grid and puts soil and manure into it. She grows a lemon plant at A, a coriander plant at B, an onion plant at C and a tomato plant at D. Her husband Ram praised her kitchen garden and points out that on joining A, B, C and D they may form a parallelogram. Look at the below figure carefully and answer the following questions:



(i) **Write the coordinates of the points A, B, C and D, using the 10 ×10 grid as coordinate axes.**

(ii) **Find whether ABCD is a parallelogram or not.**

Answer:

(i)

From the given figure, the coordinates of points A, B, C and D can be written as below:

A(2, 2), B(5, 4), C(7, 7) and D(4, 5).

(ii)

We know that a quadrilateral is a parallelogram if its opposite sides are equal.

Now, using distance formula, we will find the length of each side of the quadrilatral ABCD.

$$AB = \sqrt{(5-2)^2 + (4-2)^2} = \sqrt{9+4} = \sqrt{13},$$

$$BC = \sqrt{(7-5)^2 + (7-4)^2} = \sqrt{4+9} = \sqrt{13},$$

$$CD = \sqrt{(4-7)^2 + (5-7)^2} = \sqrt{9+4} = \sqrt{13},$$

$$DA = \sqrt{(2-4)^2 + (2-5)^2} = \sqrt{4+9} = \sqrt{13}$$

We see that sides AB, BC, CD and DA are equal in lengths. Therefore, quadrilateral ABCD is a parallelogram.

31. If the sum of the first 14 terms of an A.P. is 1050 and its first term is 10, then find the 21^{st} term of the A.P.

Answer:

Here, $S_{14} = 1050,\ a = 10$

We have to find a_{21}.

We know that sum of first n terms of an AP is given by

$$S_n = \frac{n}{2}\left[2a + \ n - 1\ d\right]$$

So, $S_{14} = \frac{14}{2}\ 2 \times 10 + 13 \times d$

$1050 = 7(20 + 13d)$

or $\quad d = 10$

We know that

$$a_n = a + (n-1)d$$

So, $a_{21} = 10 + (21-1)10$

$= 10 + 20 \times 10$

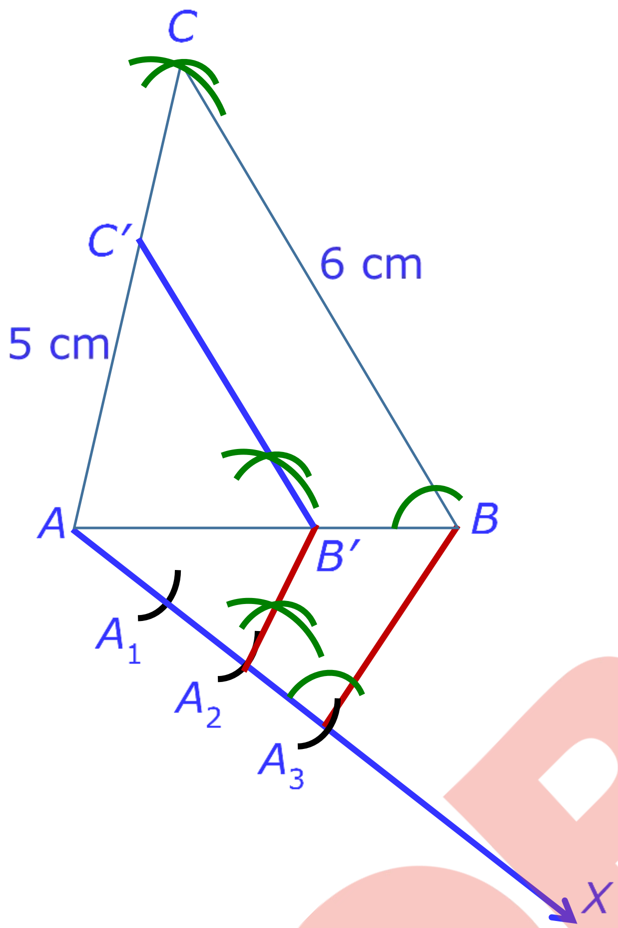
$= 210$

32. Construct a triangle with its sides 4 cm, 5 cm and 6 cm. Then construct a triangle similar to it whose sides are $\frac{2}{3}$ of the corresponding sides of the first triangle.

OR

Draw a circle of radius 2.5 cm. Take a point P at a distance of 8 cm from its centre. Construct a pair of tangents from the point P to the circle.

Answer:



Step 1: Draw a line segment $AB = 4$ cm. Taking point A as centre, draw an arc of 5 cm radius. Again, taking point B as centre, draw an arc of 6 cm. These arcs intersect each other at point C. So, we have $AC = 5$cm and $BC = 6$ cm. ΔABC is the required triangle.

Step 2: Draw a ray AX making an acute angle with line AB on the opposite side of vertex C.

Step 3: Locate 3 points A_1, A_2, A_3 on AX such that $AA_1 = A_1A_2 = A_2A_3$.

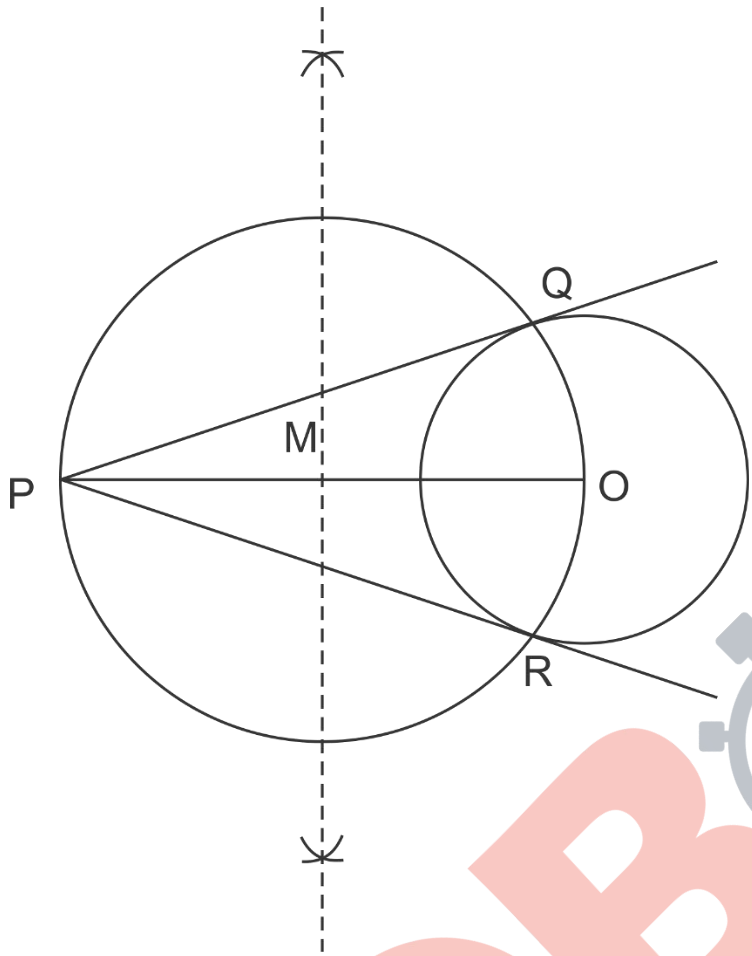
Step 4: Join the points B and A_3.

Step 5: Through the point A_2, draw a line parallel to BA_3 intersecting AB at point B'.

Step 6: Draw a line through B' parallel to the line BC to intersect AC at C'.

The required triangle is $\Delta AB'C'$.

OR



Steps of Construction :

Step 1: Draw a circle of radius 2.5 cm with centre at point O.

Locate a point P, at a distance of 8 cm from O, and join O and P.

Step 2: Bisect OP. Let M be the mid-point of OP.

Step 3: Draw a circle with centre at M and MO as radius.

Q and R are points of intersections of this circle with the circle having centre at O.

Step 4: Join PQ and PR.

PQ and PR are the required tangents.

33. Prove that:

$$\mathbf{cosec\ A - sin\ A \quad sec\ A - cos\ A = \frac{1}{tan\ A + cot\ A}}$$

Answer:

$$\text{LHS} = (\text{cosec A} - \sin\text{ A})\ (\sec\text{ A} - \cos\text{ A})$$

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$

$$= \left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right)$$

$$= \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A}$$

$$= \frac{\sin A \cos A}{1}$$

$$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A}$$

$$= \frac{1}{\dfrac{\sin^2 A + \cos^2 A}{\sin A \cos A}}$$

$$= \frac{1}{\dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}}$$

$$= \frac{1}{\tan A + \cot A}$$

$$= \text{RHS}$$

34. In Figure – 4, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller

circle. If OA = 7 cm, then find the area of the shaded region.

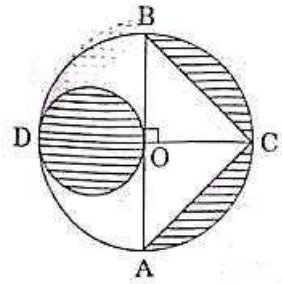


Figure-4

OR

In Figure – 5 ABCD is a square with side 7 cm. A circle is drawn circumscribing the square. Find the area of the shaded region.

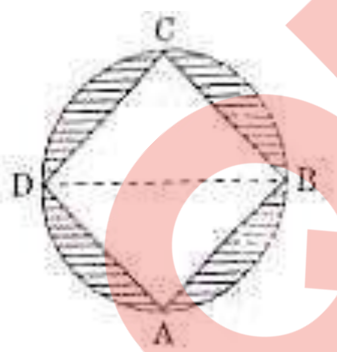


Figure-5

Answer:

For bigger circle, $OA = 7\,cm$

Diameter of the smaller circle $= 7\,cm$

Radius of the smaller circle $= \frac{7}{2}\,cm$

$$\text{Area of the smaller circle} = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$= \frac{77}{2}\ cm^2$$

Area of shaded region

$= \text{Area of the smaller circle} + 2 \times \text{Area of segment}\,OCB$

$= \frac{77}{2}\ cm^2 + 2 \times \left(\text{Area of quadrant} - \text{Area}\,\Delta ABC\right)$

$= \frac{77}{2}\ cm^2 + 2 \times \left(\frac{1}{4} \times \frac{22}{7} \times 7^2 - \frac{1}{2} \times 7 \times 7\right) cm^2$

$= \frac{77}{2}\ cm^2 + 49\left(\frac{11}{7} - 1\right) cm^2$

$= \frac{77}{2}\ cm^2 + 49\left(\frac{4}{7}\right) cm^2$

$= \frac{77}{2}\ cm^2 + 28\,cm^2$

$= \frac{77 + 56}{2}\ cm^2$

$= \frac{133}{2}\ cm^2$

$= 66.5\ cm^2$

OR

ABCD is a square with side 7 cm. Then,

Length of the diagonal of square $= 7\sqrt{2}$ cm

$$\text{Diameter of circle} = \text{Diagonal of square}$$

$$\Rightarrow \qquad BD = 7\sqrt{2} \text{ cm}$$

$$\text{Radius of circle} = \frac{BD}{2}$$

$$= \frac{7\sqrt{2}}{2} \text{ cm}$$

$$\text{Area of shaded region} = \text{Area of circle} - \text{Area of the sqaure}$$

$$= \frac{22}{7} \times \frac{7\sqrt{2}}{2} \times \frac{7\sqrt{2}}{2} - 7 \times 7$$

$$= 77 - 49$$

$$= 28 \text{ cm}^2$$

Therefore, the area of the saded region is 28 cm^2.

Section D

Question numbers 35 to 40 carry 4 marks each.

35. Find other zeroes of the polynomial

$$\mathbf{p\ x\ = 3x^4 - 4x^3 - 10x^2 + 8x + 8,}$$

if two of its zeroes are $\sqrt{2}$ and - $\sqrt{2}$.

OR

Divide the polynomial $g\ x\ = x^3 - 3x^2 + x + 2$ by the polynomial $x^2 - 2x + 1$ and verify the division algorithm.

Answer:

The given polynomial is $p(x) = 3x^4 - 4x^3 - 10x^2 + 8x + 8$

The two zeroes of p(x) are $\sqrt{2}$ and $-\sqrt{2}$.

Therefore, $\left(x - \sqrt{2}\right)$ and $\left(x + \sqrt{2}\right)$ are factors of p(x).

Also, $\left(x - \sqrt{2}\right)\left(x + \sqrt{2}\right) = x^2 - 2$

and so $x^2 - 2$ is a factor of p(x).

Now,

$$
\begin{array}{r|l}
 & 3x^2 - 4x - 4 \\
\hline
x^2 - 2 & 3x^4 - 4x^3 - 10x^2 + 8x + 8 \\
 & 3x^4 \qquad\quad - 6x^2 \\
 & - \qquad\quad\ \ + \\
\hline
 & \quad - 4x^3 - 4x^2 + 8x + 8 \\
 & \quad - 4x^3 \qquad\quad + 8x \\
 & \quad + \qquad\qquad - \\
\hline
 & \qquad\quad - 4x^2 \qquad + 8 \\
 & \qquad\quad - 4x^2 \qquad + 8 \\
 & \qquad\quad + \qquad\quad - \\
\hline
 & \qquad\qquad\qquad 0
\end{array}
$$

$$3x^4 - 4x^3 - 10x^2 + 8x + 8 = (x^2-2)(3x^2-4x-4)$$
$$= (x^2-2)(3x^2-6x+2x-4)$$
$$= (x^2-2)(3x+2)(x-2)$$

Equating $(x^2-2)(3x+2)(x-2)$ to zero, we get the zeroes of the given polynomial.

Hence, the zeroes of the given polynomial are :

$\sqrt{2}, -\sqrt{2}, -\dfrac{2}{3}$ and 2.

OR

The given polynomial is $g(x) = x^3 - 3x^2 + x + 2.$

Here, divisor is $x^2 - 2x + 1.$

Divide $g(x) = x^3 - 3x^2 + x + 2$ by $x^2 - 2x + 1$ and find the remainder.

$$
\begin{array}{r|l}
 & \quad\quad\quad\quad\quad\quad x-1 \\
x^2-2x+1 & x^3-3x^2+x+2 \\
 & x^3-2x^2+x \\
 & - \quad + \quad - \\
\hline
 & \quad\quad -x^2 \quad\quad +2 \\
 & \quad\quad -x^2+2x-1 \\
 & \quad\quad + \quad - \quad + \\
\hline
 & \quad\quad\quad -2x+3
\end{array}
$$

So, Quotient $= x-1$ and Remainder $= -2x+3$.

The division alogorithm states that

Dividend = Divisor $\times$ Qoutient + Remainder

RHS = Divisor $\times$ Qoutient + Remainder

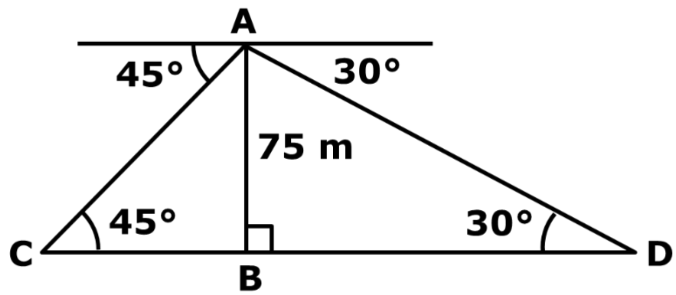
$$= (x^2-2x+1)(x-1)-2x+3$$
$$= x^3-2x^2+x-x^2+2x-1-2x+3$$
$$= x^3-3x^2+x+2$$
$$= \text{LHS}$$

Thus, the division alogorithm is verified.

36. From the top of a 75 m high lighthouse from the seal level, the angles of depression of two ships are 30°and 45° if the ships are on the opposite sides of the lighthouse, then find the distance between the two ships.

Answer:

Let AB be a lighthouse and ships be at points C and D. It is given that AB = 75 m. We have to find the distance CD.



In ΔABC, we have

$$\tan 45° = \frac{AB}{BC}$$

or $$1 = \frac{AB}{BC}$$

or $$BC = AB = 75 \qquad \ldots(1)$$

Now,

In ΔABD, we have

$$\tan 30° = \frac{AB}{BD}$$

or $$\frac{1}{\sqrt{3}} = \frac{75}{BD}$$

or $$BD = 75\sqrt{3} \qquad \ldots(2)$$

From (1) and (2), we get

$$CD = BC + BD = 75 + 75\sqrt{3} = 75\left(1+\sqrt{3}\right)$$

Therefore, the distance between the two ships is $75\left(1+\sqrt{3}\right)$ m.

37. **If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio.**

OR

In Figure-6, in an equilateral triangle ABC, AD⊥ BC, BE ⊥ AC and CF ⊥ AB. Prove that $4(AD^2+BE^2+CF^2) = 9\ AB^2$.

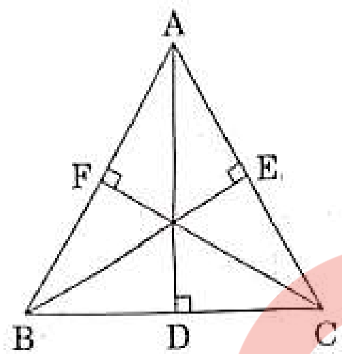
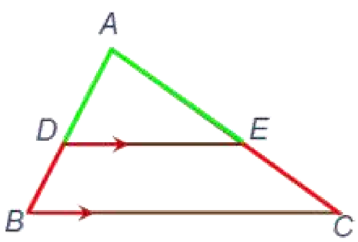


Figure-6

Answer:



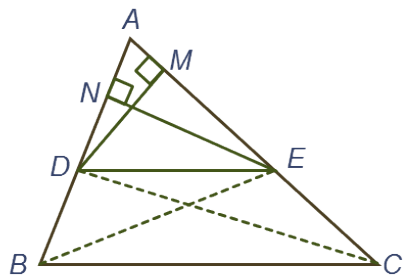
Given : In ΔABC, $DE \parallel BC$.

To Prove : $\dfrac{AD}{DB} = \dfrac{AE}{EC}$.

Construction :

i Join BE and CD.

ii Draw $DM \perp AC$ and $EN \perp AB$.



Proof :

$$\text{area}(\Delta ADE) = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times AD \times EN$$

and

$$\text{area}(\Delta BDE) = \frac{1}{2} \times BD \times EN$$

Therefore,

$$\frac{\text{area } \Delta ADE}{\text{area } \Delta BDE} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times BD \times EN} = \frac{AD}{BD} \quad \dots 1$$

Similarly,

$$\frac{\text{area } \Delta ADE}{\text{area } \Delta DEC} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \quad \dots 2$$

But area (ΔBDE) = area (ΔDEC) $\quad \dots (3)$

(Triangles on the same base and between the same parallels are equal in area.)

Therefore, from (1), (2) and (3), we get

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence, proved.

OR

ABC is an equilateral triangle.

Therefore, AB = BC = AC

Now,

$$BD = DC = \frac{BC}{2} = \frac{AB}{2} \quad \left(\begin{array}{l}\text{D is the midpoint of BC} \\ \text{and AB = BC}\end{array}\right)$$

Using Pythagoras theorem in $\triangle ADC$, we get

$$AC^2 = AD^2 + DC^2$$

$$AB^2 = AD^2 + \left(\frac{AB}{2}\right)^2 \qquad \left(AC = AB \text{ and } DC = \frac{AB}{2}\right)$$

$$AB^2 = AD^2 + \frac{AB^2}{4}$$

$$AB^2 - \frac{AB^2}{4} = AD^2$$

$$\frac{3AB^2}{4} = AD^2$$

$$3AB^2 = 4AD^2 \qquad \ldots (1)$$

Similarly, using Pythagoras theorem in $\triangle AEB$, we get

$$3AB^2 = 4BE^2 \qquad \ldots (2)$$

Again, using Pythagoras theorem in $\triangle AFC$, we get

$$3AB^2 = 4CF^2 \qquad \ldots (3)$$

On adding equations (1), (2) and (3), we get

$$3AB^2 + 3AB^2 + 3AB^2 = 4AD^2 + 4BE^2 + 4CF^2$$

or, $9AB^2 = 4\left(AD^2 + BE^2 + CF^2\right)$

or, $4\left(AD^2 + BE^2 + CF^2\right) = 9AB^2$

Hence, proved

38. A container open at the top and made up of a metal sheet, is in the form of a frustum of a cone of height 14 cm with radii of its lower and upper circular ends as 8 cm and 20 cm. respectively. Find the capacity of the container.

Answer:

Height of the frustum $= h = 14$ cm

Radius of upper end of the frustum $= r_1 = 20$ cm

Radius of lower end of the frustum $= r_2 = 8$ cm

Capacity of container = Volume of the frustum

$$= \frac{1}{3}\pi h\left(r^2{}_1 + r_2{}^2 + r_1 r_2\right)$$

$$= \frac{14}{3} \times \frac{22}{7}\left(20^2 + 8^2 + 20 \times 8\right)$$

$$= \frac{44}{3}(400 + 64 + 160)$$

$$= \frac{44}{3} \times 624$$

$$= 9152 \text{ cm}^3$$

39. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

OR

A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m. Find the length and breadth of the park.

Answer:

Let A and B be the time taken by the smaller and the larger taps respectively to fill the tank.

Since both the taps together can fill the tank in

$9\frac{3}{8}$ hours $= \frac{75}{8}$ hours.

So, $\frac{1}{A} + \frac{1}{B} = \frac{1}{\frac{75}{8}}$

Or, $\frac{1}{A} + \frac{1}{B} = \frac{8}{75}$ ... 1

Tap with larger diameter takes 10 hours less than smaller one to fill the tank.

So, $A - 10 = B$

Or, $\frac{1}{B} = \frac{1}{A-10}$ ... 2

By placing the value of $\frac{1}{B}$ from 2 in to 1, we get

$$\frac{1}{A}+\frac{1}{A-10}=\frac{8}{75} \qquad \dots 1$$

Or, $\frac{A-10+A}{A^2-10A}=\frac{8}{75}$

Or, $\frac{A-5}{A^2-10A}=\frac{4}{75}$

Or, $75A-375=4A^2-40A$

Or, $4A^2-40A-75A+375=0$

Or, $4A^2-115A+375=0$

Or, $4A^2-100A-15A+375=0$

Or, $4A(A-25)-15(A-25)=0$

Or, $(A-25)(4A-15)=0$

$A=25$ hours $\left(A\neq\frac{15}{4}\text{ hours, because B becomes negative.}\right)$

So, $B=25-10=15$ hours

OR

Let L be the length of the rectangle.

So, breadth of the rectangle $= L-3$

Area of the rectangle $= L(L-3)$ ... (1)

Base of the isosceles triangle $= L-3$

Altitude of the isosceles triangle $= 12$ m

Area of the isosceles triangle $= \frac{1}{2}(12)(L-3)$ ... (2)

Given that

$$L(L-3) = \frac{1}{2}(12)(L-3) + 4$$

Or, $L^2 - 3L = 6L - 18 + 4$

Or, $L^2 - 9L + 14 = 0$

Or, $L^2 - 7L - 2L + 14 = 0$

Or, $L(L-7) - 2(L-7) = 0$

Or, $(L-7)(L-2) = 0$

So, $L = 7\text{m}$ $\left(L \neq 2 \quad \because L-3 \text{ is negative.}\right)$

Breadth $= 7\text{m} - 3\text{m} = 4\text{m}$

Length and breadth of the rectangle are 7m and 4m respectively.

40. Draw a 'less than' ogive for the following frequency distribution:

Classes:	0-10	10–20	20–30	30–40	40–50	50–60	60–70	70–80
Frequency:	7	14	13	12	20	11	15	8

Answer:

Marks	Cumulative frequency
Less than 10	7
Less than 20	7 + 14 = 21
Less than 30	21 + 13 = 34
Less than 40	34 + 12 = 46
Less than 50	46 + 20 = 66
Less than 60	66 + 11 = 77
Less than 70	77 + 15 = 92
Less than 80	92 + 8 = 100

Marks	Frequency(f)	Cumulative frequency (cf)
0 – 10	7	7
10 – 20	14	21
20 – 30	13	34
30 – 40	12	46
40 – 50	20	66
50 – 60	11	77
60 – 70	15	92
70 – 80	8	100

Now, plot (10, 7), (20, 21), …, (80, 100) on the graph.