Class X
(CBSE 2019)
Mathematics
Delhi (Set-3)

General Instructions:

(i) **All** questions are compulsory.

(ii) The question paper consists of **30** questions divided into four sections **– A, B, C** and **D**.

(iii) Section **A** comprises **6** questions of **1** mark each. Section **B** contains **6** questions of **2** marks each. Section **C** contains **10** questions of **3** marks each. Section **D** contains **8** questions of 4 marks each.

(iv) There is no overall choice. However, an internal choice has been provided in **two** questions of **1** mark, **two** questions of **2** marks, **four** questions of **3** marks each and **three** questions of **4** marks each. You have to attempt only **one** of the alternative in all such questions.

(v) Use of calculators is **not** permitted.

Question 1

Two positive integers a and b can be written as $a = x^3y^2$ and $b = xy^3$. x, y are prime numbers. Find LCM (a, b).

SOLUTION:

Given: $a = x^3y^2 \text{ and } b = xy^3$ where a and b are positive integers and x and y are prime numbers.

$a = x^3y^2$
$\Rightarrow a = x \times x \times x \times y \times y$
And $b = xy^3$
$\Rightarrow b = x \times y \times y \times y$
So, the LCM of a and b will be x^3y^3.

Question 2

How many two digits numbers are divisible by 3?

SOLUTION:

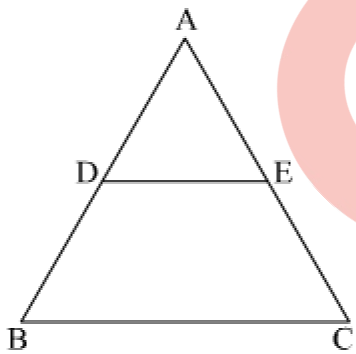
The first two digit number that is divisible by 3 is 12
So, the series starts from 12
And the highest two digit number that is divisible by 3 is 99
So, the sequence becomes:
12, 15, ..., 99.
We, need to find the numbers in the given sequence
Using $a_n = a + (n-1)d$
a is the first term, *d* is the common difference, *n* is the number of terms and a_n is the *n*th term *a* = 12, *d* = 3, a_n= 99 substituting the values we get

$$99 = 12 + (n-1)3$$
$$99 = 12 + 3n - 3$$
$$90 = 3n$$
$$n = \frac{90}{3}$$
$$n = 30.$$

Therefore, there are total 30 two digit numbers that are divisible by 3.

Question 3

In Fig. 1, DE || BC, AD = 1 cm and BD = 2 cm. What is the ratio of the ar (Δ ABC) to the ar (Δ ADE)?



SOLUTION:

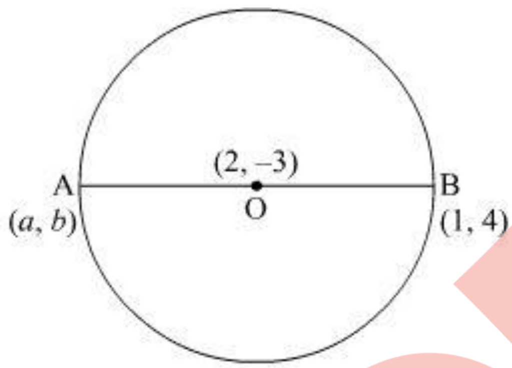
It is given that AD=1cm, BD=2 cm and DE||BC
In $\Delta \mathrm{ADE}$ and ΔABC
$\angle \mathrm{ADE} = \angle \mathrm{ABC}$ (corresponding angles)
$\angle A = \angle A$ (common angle)
By AA similarity
$\Delta \mathrm{ADE} \sim \Delta \mathrm{ABC}$
Ratio of area of similar triangles is equal to the square of the ratio of corresponding sides.

$$\therefore \frac{ar\ (\Delta ABC)}{ar(\Delta ADE)} = \frac{AB^2}{AD^2}$$
$$\Rightarrow \frac{ar\ (\Delta ABC)}{ar(\Delta ADE)} = \frac{3^2}{1^2}$$
$$\Rightarrow \frac{ar\ (\Delta ABC)}{ar(\Delta ADE)} = \frac{9}{1}$$
Therefore, the ratio of the $ar\ (\Delta ABC) : ar\ (\Delta ADE)$ is $9 : 1$.

Question 4

Find the coordinates of a point A, where AB is diameter of a circle whose centre is (2, –3) and B is the point (1, 4).

SOLUTION:



Let the centre of the circle be O.
Since AB is the diameter so, O is the midpoint of AB.
Thus, using the section formula,
$$\frac{a+1}{2} = 2$$
$$\Rightarrow a = 4 - 1 = 3$$
And
$$\frac{b+4}{2} = -3$$
$$\Rightarrow b = -10$$
So, the coordinate of point A is $(3, -10)$.

Question 5

For what value of k, the roots of the equation $x^2 + 4x + k = 0$ are real?

OR

Find the value of k for which the roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other.

SOLUTION:

The given equation is $x^2+4x+k=0$.
For real roots, $D \geq 0$

$\Rightarrow b^2-4ac \geq 0$

$\Rightarrow 4^2-4(1)(k) \geq 0$

$\Rightarrow 4-k \geq 0$

$\Rightarrow k \leq 4$

For $k \leq 4$, the given equation $x^2+4x+k=0$ has real roots.

OR

The given equation is $3x^2-10x+k=0$.
Roots of the given equation are reciprocal of each other.
Let α and $\frac{1}{\alpha}$ be the roots of the given equation.
Product of roots= $\frac{c}{a}$

$\Rightarrow \alpha \cdot \frac{1}{\alpha}=\frac{k}{3}$

$\Rightarrow \frac{k}{3}=1$

$\Rightarrow k=3$

Question 6

Find A if tan 2A = cot (A – 24°)

OR

Find the value of ($\sin^2 33° + \sin^2 57°$)

SOLUTION:

Given:

$\tan 2A=\cot(A-24^\circ)$

$\Rightarrow \tan 2A=\tan[90^\circ-(A-24^\circ)]$

$\Rightarrow \tan 2A=\tan[90^\circ-A+24^\circ]$

$\Rightarrow \tan 2A=\tan[114^\circ-A]$

$\Rightarrow 2A=114^\circ-A$

$\Rightarrow 3A=114^\circ$

$\Rightarrow A=\frac{114^\circ}{3}$

$\Rightarrow A=38^\circ$

OR

Given:

$$\sin^2 33^\circ + \sin^2 57^\circ$$
$$= \sin^2 33^\circ + [\cos(90^\circ - 57^\circ)]^2$$
$$= \sin^2 33^\circ + \cos^2 33^\circ$$
$$= 1$$

Question 7

Find, how many two digit natural numbers are divisible by 7.

Or

If the sum of first *n* terms of an AP is n^2, then find its 10th term.

SOLUTION:

The first two digit number that is divisible by 7 is 14
So, the sequence starts from 14
And the highest two digit number that is divisible by 7 is 98
So, the sequence becomes:
14, 21, ..., 98.
We, need to find the numbers in the given sequence
Using an = a + (n−1) d
a is the first term, *d* is the common difference, *n* is the number of terms and a_n is the *n*th term *a* = 14, *d* = 7, a_n= 98 substituting the values we get

$$98 = 14 + (n-1)7$$
$$98 = 14 + 7n - 7$$
$$91 = 7n$$
$$n = \frac{91}{7}$$
$$n = 13.$$

Therefore, there are total 13 two digit numbers that are divisible by 7.

OR

We know sum of *n* terms of an AP is

$$S_n = n^2$$

For $n=1, S_1=(1)^2=1$
So, $a_1=1$
For $n=2, S_2=(2)^2=4$
For $n=3, S_3=(3)^2=9$
$S_2-S_1=a_2$
$\Rightarrow a_2=4-1=3$
So, $d=a_2-a_1=3-1=2$
We know nth term of an AP is
$a_n=a+(n-1)d$
For $n=10$,
$a_{10}=1+(10-1)2$
$\Rightarrow a_{10}=1+9\times 2$
$\Rightarrow a_{10}=19$
Thus, the 10th term of the AP is 19.

Question 8

A game consists of tossing a coin 3 times and noting the outcome each time. If getting the same result in all the tosses is a success, find the probability of losing the game.

SOLUTION:

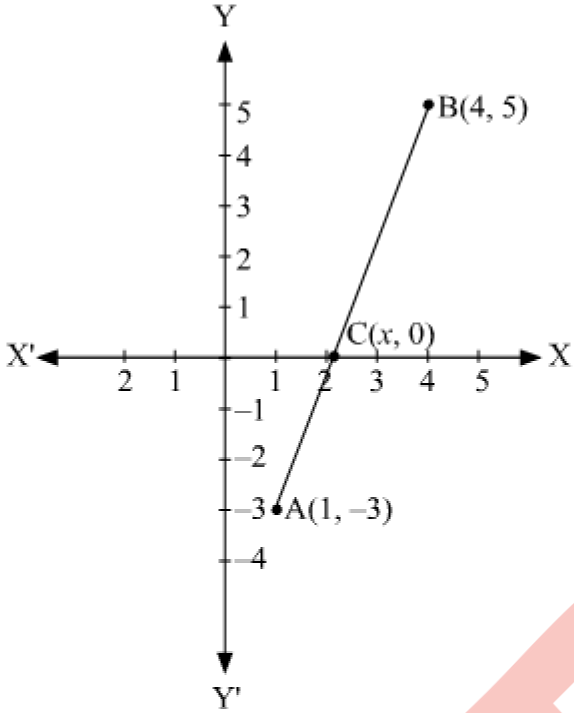
Possible outcomes of tossing a coin three times will be
{HHH, TTT, HHT, HTH, THH, TTH, THT, HTT} = 8
Getting the same result in all tosses is a success.
We need to find the probability of losing the game that means not the same result in all tosses
Favourable outcomes are {HHT, HTH, THH, TTH, THT, HTT} = 6
Probability of losing a game $=\frac{6}{8}=\frac{3}{4}$
Therefore, probability of losing a game is $\frac{3}{4}$.

Question 9

Find the ratio in which the segment joining the points (1, –3) and (4, 5) is divided by *x*-axis? Also find the coordinates of this point on *x*-axis.

SOLUTION:

Let C(x, 0) divides the line-segment A(1, –3) and B(4, 5) in k : 1 ratio.
By section formula,



$$(x,\ y) = \left(\frac{mx_2+nx_1}{m+n},\ \frac{my_2+ny_1}{m+n}\right)$$

$$\Rightarrow (x,\ 0) = \left(\frac{4k+1\times 1}{k+1},\ \frac{5k+1\times(-3)}{k+1}\right)$$

$$\Rightarrow (x,\ 0) = \left(\frac{4k+1}{k+1},\ \frac{5k-3}{k+1}\right)$$

$$\Rightarrow \frac{5k-3}{k+1} = 0$$

$$\Rightarrow 5k - 3 = 0$$

$$\Rightarrow 5k = 3$$

$$\Rightarrow k = \frac{3}{5}$$

$$\text{and } x = \frac{4k+1}{k+1} = \frac{4\times\frac{3}{5}+1}{\frac{3}{5}+1}$$

$$\Rightarrow x = \frac{\frac{12+5}{5}}{\frac{3+5}{5}}$$

$$\Rightarrow x = \frac{17}{8}$$

The ratio in which C divides A and B is k: 1 i.e., 3: 5 and the coordinate of C is $\left(\frac{17}{0}, 0\right)$.

Question 10

A die is thrown once. Find the probability of getting a number which (i) is a prime number (ii) lies between 2 and 6.

SOLUTION:

Total possible outcomes that occur after throwing a die once are 1,2,3,4,5,6
Number of possible outcomes = 6
We need to find the probability of getting a prime number
Prime number is a number not divisible by any number except itself
Prime numbers on a dice are 2,3 and 5.
Number of favourable outcomes = 3
Probability of getting a prime number

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

Probability of getting a prime number$= \frac{3}{6} = \frac{1}{2}$

Therefore, probability of getting a prime number is $\frac{1}{2}$

(b) Probability of getting a number lying between 2 and 6
Number lying between 2 and 6 on a dice are 3,4 and 5.

Probability of getting a number lying between 2 and 6 $= \frac{3}{6} = \frac{1}{2}$

Therefore, Probability of getting a number lying between 2 and 6 is $\frac{1}{2}$

Question 11

Find c if the system of equations $cx + 3y + (3 - c) = 0$; $12x + cy - c = 0$ has infinitely many solutions?

SOLUTION:

$cx + 3y + (3 - c) = 0$ .....(i)
$12x + cy - c = 0$ .....(ii)

For infinitely many solutions, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{c}{12} = \frac{3}{c} = \frac{3-c}{-c}$

$\Rightarrow \frac{c}{12} = \frac{3}{c}$ or $\frac{3}{c} = \frac{3-c}{-c}$

$\Rightarrow c^2 = 36$ $\qquad \Rightarrow c(c-6) = 0$

$\Rightarrow c = \pm 6$ $\qquad \Rightarrow c = 0, 6$

Hence, $c = 6$ is the solution which gives infinitely many solution.

Question 12

Find the HCF of 1260 and 7344 using Euclid's algorithm.

OR

Show that every positive odd integer is of the form (4*q* + 1) or (4*q* + 3), where *q* is some integer.

SOLUTION:

The given numbers are 1260 and 7344.

Now 7344 > 1260. So, on applying Euclid's algorithm we get

$$7344 = 1260 \times 5 + 1044$$

Now the remainder is not 0 so, we repeat the process again on 1260 and 1044

$$1260 = 1044 \times 1 + 216$$

The algorithm is applied again but this time on the numbers 1044 and 216

$$1044 = 216 \times 4 + 180$$

Now, the algorithm is applied again until the remainder is 0.

$$216 = 180 \times 1 + 36$$

$$180 = 36 \times 5 + 0$$

Thus, the HCF obtained is 36.

OR

According to Euclid's division lemma,

$a = bq + r$ where $0 \le r < b$

Now, let *a* be any odd positive integer and *b* = 4.

When $0 \le r < 4$ so, the possible values of *r* will be 0, 1, 2, 3.

Now, the possible values of *a* will be thus, 4q, 4q + 1, 4q + 2, 4q + 3

where *q* is an integer.

But, we already know that *a is* any odd positive integer.

So, *a* will be 4q + 1 and 4q + 3.

Question 13

Find all zeros of the polynomial $3x^3 + 10x^2 - 9x - 4$ if one of its zero is 1.

SOLUTION:

Given: $3x^3 + 10x^2 - 9x - 4$

Since 1 is a zero of the given polynomial. So, $(x - 1)$ will be a factor of the given polynomial.

$$
\begin{array}{r|l}
 & 3x^2 + 13x + 4 \\
\hline
(x-1) & 3x^3 + 10x^2 - 9x - 4 \\
 & \underline{3x^3 - 3x^2} \\
 & \quad 13x^2 - 9x \\
 & \quad \underline{13x^2 - 13x} \\
 & \qquad 4x - 4 \\
 & \qquad \underline{4x - 4} \\
 & \qquad\quad 0
\end{array}
$$

So, $3x^3 + 10x^2 - 9x - 4 = (x - 1)\left(3x^2 + 13x + 4\right)$

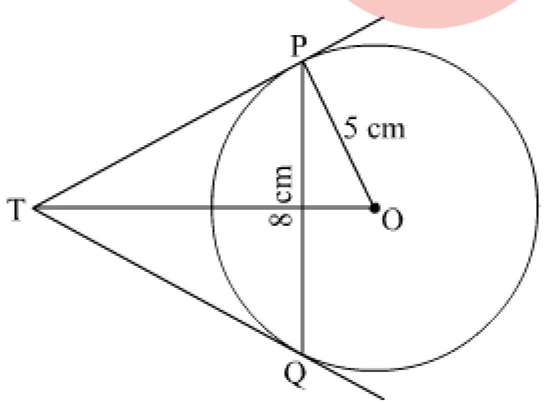
By splitting the middle term in $\left(3x^2 + 13x + 4\right)$ we factorise $\left(3x^2 + 13x + 4\right)$ as $(3x + 1)(x + 4)$.

So, the zeroes are given by $x = -4, \frac{-1}{3}$.

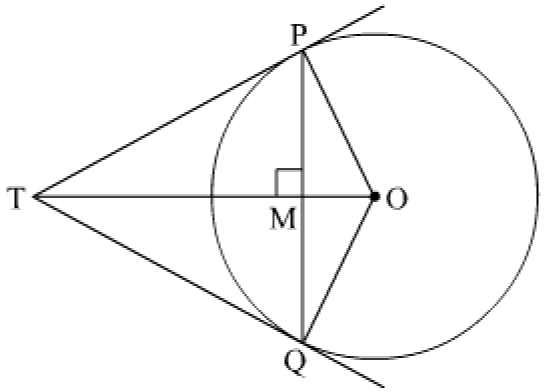
Thus, all the zeroes are $x = -4, \frac{-1}{3}, 1$

Question 14

PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at point T. Find the length of TP.



SOLUTION:



Given,
Radius, OP = 5 cm
PQ = 8 cm,
TP = TQ (Tangents drawn from a common point to a circle are equal)
ΔPTQ is isosceles.
Let PQ and OT intersect at point M.
TO is the angle bisector of ∠PTQ.
So, ∠PMT = $90°$ ⇒ PM = QM = 4 cm (Perpendicular drawn from the centre of the circle to the chord bisects the chord)

In ΔPMO,

$$OP^2 = OM^2 + PM^2$$
$$\Rightarrow 5^2 = OM^2 + 4^2$$
$$\Rightarrow OM = 3 \text{ cm}$$

Let PT = x and TM = y
In ΔPMT,

$$PT^2 = TM^2 + PM^2$$
$$x^2 = y^2 + 16 \quad \ldots\ldots (i)$$

In ΔTPO,

$$OT^2 = PT^2 + OP^2$$
$$(y+3)^2 = x^2 + 5^2$$
$$y^2 + 9 + 6y = x^2 + 25 \quad \ldots\ldots (ii)$$

From (i) and (ii), we get

$$y^2 + 9 + 6y = y^2 + 16 + 25$$

$6y = 32$

$y = \frac{16}{3}$ cm

substituting $y = \frac{16}{3}$ in equation (i),

$x^2 = \left(\frac{16}{3}\right)^2 + 16$

$x^2 = \frac{400}{9}$

$x = \frac{20}{3}$ cm

Therefore, PT = $\frac{20}{3}$ cm.

Question 15

Prove that $\frac{2+\sqrt{3}}{5}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

SOLUTION:

To prove $\frac{2+\sqrt{3}}{5}$ is irrational, let us assume that $\frac{2+\sqrt{3}}{5}$ is rational.

$\frac{2+\sqrt{3}}{5} = \frac{a}{b}$; $b \neq 0$ and *a* and *b* are integers.

$\Rightarrow 2b + \sqrt{3}b = 5a$

$\Rightarrow \sqrt{3}b = 5a - 2b$

$\Rightarrow \sqrt{3} = \frac{5a-2b}{b}$

Since *a* and *b* are integers so, $5a - 2b$ will also be an integer.

So, $\frac{5a-2b}{b}$ will be rational which means $\sqrt{3}$ is also rational.

But we know $\sqrt{3}$ is irrational(given).

Thus, a contradiction has risen because of incorrect assumption.

Thus, $\frac{2+\sqrt{3}}{5}$ is irrational.

Question 17

A father's age is three times the sum of the ages of his two children. After 5 years his age will be two times the sum of their ages. Find the present age of the father.

Or

A fraction becomes $\frac{1}{3}$ when 2 is subtracted from the numerator and it becomes $\frac{1}{2}$ when 1 is subtracted from the denominator. Find the fraction.

SOLUTION:

Let the present age of father be *x* years and the present ages of his two children's be *y* and *z* years.

The present age of father is three times the sum of the ages of the two children's. Thus, we have

$$x = 3(y+z)$$
$$\Rightarrow y+z = \frac{x}{3}$$

After 5 years, father's age will be $(x+5)$ years and the children's age will be $(y+5)$ and $(z+5)$ years. Thus using the given information, we have

$$x+5 = 2\{(y+5)+(z+5)\}$$
$$\Rightarrow x+5 = 2(y+5+z+5)$$
$$\Rightarrow x = 2(y+z)+20-5$$
$$\Rightarrow x = 2(y+z)+15$$

So, we have two equations

$$y+z = \frac{x}{3}$$

$$x = 2(y+z)+15$$

Here *x, y* and *z* are unknowns. We have to find the value of *x*.

Substituting the value of $(y+z)$ from the first equation in the second equation, we have

By using cross-multiplication, we have

$$x = \frac{2x}{3}+15$$
$$\Rightarrow x - \frac{2x}{3} = 15$$
$$\Rightarrow x(1-\frac{2}{3}) = 15$$
$$\Rightarrow \frac{x}{3} = 15$$

$$\Rightarrow x = 15\times 3$$
$$\Rightarrow x = 45$$

Hence, the present age of father is $\boxed{45}$ years.

Or

Let's assume the fraction be $\frac{x}{y}$

1st condition:

$\frac{x-2}{y}=\frac{1}{3}$

$\Rightarrow 3x-6=y$

$\Rightarrow 3x-y=6\ldots(1)$

2nd condition:

$\frac{x}{y-1}=\frac{1}{2}$

$\Rightarrow 2x=y-1$

$\Rightarrow 2x-y=-1$

Using elimination method:

Multiplying (2) by -1 and then adding (1) and (2)

$\Rightarrow 3x-y=6$

$\Rightarrow -2x+y=1$

$\Rightarrow x=7$

Now, from (1),

$\Rightarrow 3x-y=6$

$\Rightarrow 3(7)-y=6$

$\Rightarrow 21-y=6$

$\Rightarrow y=15$

Hence, the required fraction is $\frac{7}{15}$.

Question 18

Find the point on *y*-axis which is equidistant from the points (5, −2) and (−3, 2).

OR

The line segment joining the points A(2, 1) and B(5, −8) is trisected at the points P and Q such that P is nearer to A. If P also lies on the line given by $2x - y + k = 0$, find the value of *k*.

SOLUTION:

Since the point is on *y*-axis so, X-coordinate is zero.

Let the point be (0, *y*)

It's distance from (5, –2) and (–3, 2) are equal

$$\therefore \sqrt{(0-5)^2+(y+2)^2} = \sqrt{(0+3)^2+(y-2)^2}$$
$$\Rightarrow 25+y^2+4y+4 = 9+y^2-4y+4 \quad \text{[squaring both sides]}$$
$$\Rightarrow 4y+29 = -4y+13$$
$$\Rightarrow 4y+4y = 13-29$$
$$\Rightarrow 8y = -16$$
$$\therefore y = \frac{-16}{8} = -2$$

Thus, the point is (0, −2)

OR

We have two points A (2, 1) and B (5,−8). There are two points P and Q which trisect the line segment joining A and B.

Now according to the section formula if any point P divides a line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio m : n internally then,

$$P(x, y) = \left(\frac{nx_1+mx_2}{m+n}, \frac{ny_1+my_2}{m+n}\right)$$

The point P is the point of trisection of the line segment AB. So, P divides AB in the ratio 1: 2

Now we will use section formula to find the co-ordinates of unknown point A as,

$$P(x_1, y_1) = \left(\frac{1(5)+2(2)}{1+2}, \frac{2(1)+1(-8)}{1+2}\right)$$
$$= (3, -2)$$

Therefore, co-ordinates of point P is(3,−2)

It is given that point P lies on the line whose equation is

$$2x-y+k=0$$

Since, point P satisfies this equation.

2 (3) − (−2) + k = 0

So,

$\boxed{k=-8}$

Question 19

Find the mode of the following frequency distribution.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	8	10	10	16	12	6	7

SOLUTION:

The maximum class frequency is 16. And the corresponding class is 30 - 40.
∴ Modal class = 30 - 40
Lower limit of modal class(l) = 30
Class size(h) = 10
Frequency(f_1) of modal class = 16
Frequency(f_0) of class preceding the modal class = 10
Frequency(f_2) of class succeeding the modal class = 12

mode = $l+\left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right)\times h$

$$\text{Mode} = 30+\frac{16-10}{2\times16-10-12}\times 10$$
$$=30+\frac{6}{32-22}\times 10$$
$$=30+\frac{6}{10}\times 10$$
$$=36$$

Question 20

Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/hour. How much area will it irrigate in 30 minutes; if 8 cm standing water is needed?

SOLUTION:

The canal is 6 m wide and 1.5 m deep. The water is flowing in the canal at 10 km/hr. Hence, in 30 minutes, the length of the flowing standing water is

$$=10\times\frac{30}{60}\text{ km}$$

$$=5\text{ km}$$

$$=5000\text{ m}$$

Therefore, the volume of the flowing water in 30 min is

$$V_1 = 5000 \times 1.5 \times 6 \text{ m}^3$$

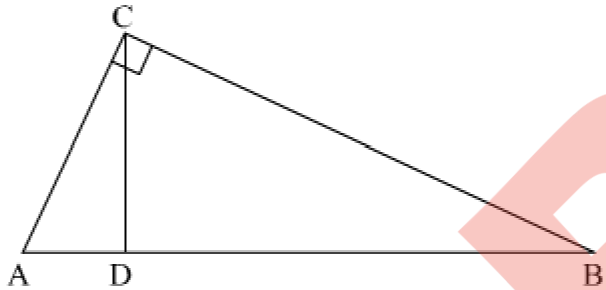
Thus, the irrigated area in 30 min of 8 cm=0.08 m standing water is

$$= \frac{5000 \times 1.5 \times 6}{0.08}$$

$$= \boxed{562500 \text{ m}^2}$$

Question 21

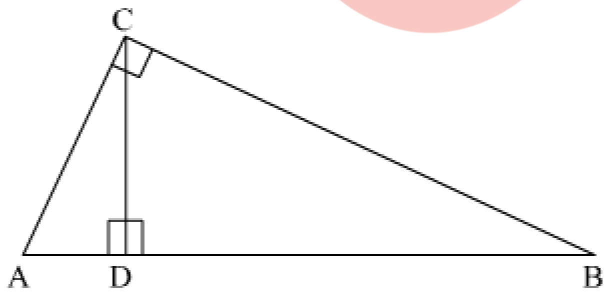
In Fig. 3, ∠ACB = 90° and CD ⊥ AB, prove that $CD^2 = BD \times AD$.



OR

If P and Q are the points on side CA and CB respectively of Δ ABC, right angled at C, prove that $(AQ^2 + BP^2) = (AB^2 + PQ^2)$

SOLUTION:



Given that : CD ⊥ AB

∠ACB = 90°

To Prove : $CD^2 = BD \times AD$

Using Pythagoras Theorem in Δ ACD

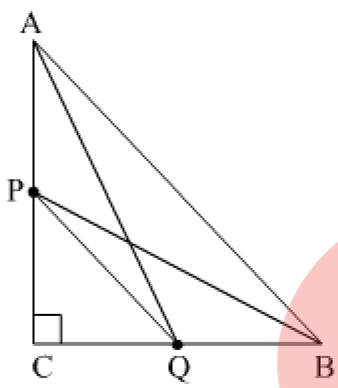
$AC^2 = AD^2 + CD^2$ .....(1)

Using Pythagoras Theorem in Δ CDB

$CB^2 = BD^2 + CD^2$ .....(2)

Similarly in Δ ABC,
$AB^2 = AC^2 + BC^2$ .....(3)
As AB = AD + DB
⇒ AB = AD + BD .....(4)
Squaring both sides of equation (4), we get
$(AB)^2 = (AD + BD)^2$
⇒ $AB^2 = AD^2 + BD^2 + 2 \times BD \times AD$
From equation (3) we get
$AC^2 + BC^2 = AD^2 + BD^2 + 2 \times BD \times AD$
Substituting the value of AC^2 from equation (1) and the value of BC^2 from equation (2), we get
$AD^2 + CD^2 + BD^2 + CD^2 = AD^2 + BD^2 + 2 \times BD \times AD$
⇒ $2\,CD^2 = 2 \times BD \times AD$
⇒ $CD^2 = BD \times AD$
Hence Proved.

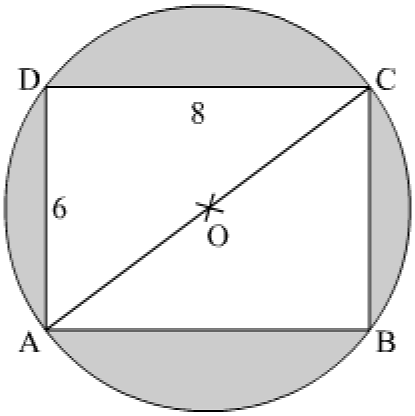
OR



Using the Pythagoras theorem in Δ ABC, Δ ACQ, Δ BPC, Δ PCQ, we get
$AB^2 = AC^2 + BC^2$ .....(1)
$AQ^2 = AC^2 + CQ^2$ .....(2)
$BP^2 = PC^2 + BC^2$ .....(3)
$PQ^2 = PC^2 + CQ^2$ .....(4)
Adding the equations (2) and (3) we get
$AQ^2 + BP^2 = AC^2 + CQ^2 + PC^2 + BC^2$
$= (AC^2 + BC^2) + (CQ^2 + PC^2)$
$= AB^2 + PQ^2$
As
L.H.S = $AQ^2 + BP^2$
$= AB^2 + PQ^2$
= R.H.S
Hence Proved

Question 22

Find the area of the shaded region in Fig. 4, if ABCD is a rectangle with sides 8 cm and 6 cm and O is the centre of circle. (Take π = 3.14)



SOLUTION:

Here, diagonal AC also represents the diameter of the circle.
Using Pythagoras theorem:

$\mathrm{AC}=\sqrt{\mathrm{AB}^2+\mathrm{BC}^2}$

$\mathrm{AC}=\sqrt{8^2+6^2}$

$\mathrm{AC}=\sqrt{64+36}$

$\mathrm{AC}=\sqrt{100}$

$\mathrm{AC}=10$

∴ Radius of the circle, $\mathrm{OC}=\frac{\mathrm{AC}}{2}=5$ cm

Now, area of shaded region = area of circle - area of rectangle

$=\pi r^2-\mathrm{AB}\times\mathrm{BC}$

$=\pi(\mathrm{OC})^2-\mathrm{AB}\times\mathrm{BC}$

$=3.14\times5^2-8\times6$

$=78.5-48$

$=30.5$

Therefore, the area of shaded region is 30.5 cm^2.

Question 23

If $\sec\theta = x + \frac{1}{4x}, x \neq 0$, find ($\sec\theta + \tan\theta$).

SOLUTION:

Given: $\sec\theta = x + \frac{1}{4x}, x \neq 0,$

Squaring both sides

$\sec^2\theta = \left(x + \frac{1}{4x}\right)^2$

We know

$\tan^2\theta = \sec^2\theta - 1$

$\Rightarrow \tan^2\theta = \left(x + \frac{1}{4x}\right)^2 - 1$

$\Rightarrow \tan^2\theta = \left(x + \frac{1}{4x} - 1\right)\left(x + \frac{1}{4x} + 1\right)$

$\Rightarrow \tan^2\theta = \left(x - \frac{1}{4x}\right)^2$

$\Rightarrow \tan\theta = \pm\left(x - \frac{1}{4x}\right)$

When $\tan\theta = x - \frac{1}{4x}$

$\sec\theta + \tan\theta = x + \frac{1}{4x} + x - \frac{1}{4x}$

$= 2x$

When $\tan\theta = -\left(x - \frac{1}{4x}\right) = \frac{1}{4x} - x$

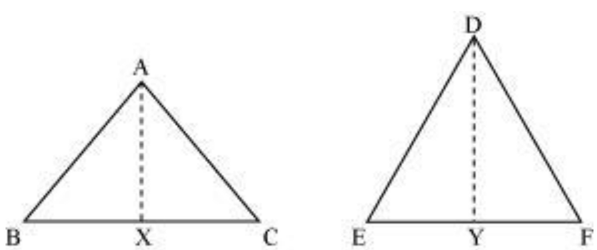
$\sec\theta + \tan\theta = x + \frac{1}{4x} + \frac{1}{4x} - x$

$= \frac{1}{2x}$

Question 24

Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

SOLUTION:



Let ΔABC and ΔDEF be such that ΔABC ~ ΔDEF.

To prove: $\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{CA}{FD}\right)^2$

Construction: Draw AX⊥BC and DY⊥EF

Proof: ar (ΔABC) = $\frac{1}{2}$ × BC × AX

ar (ΔDEF) = $\frac{1}{2}$ × EF × DY

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC \times AX}{EF \times DY} \quad \ldots (1)$$

In ΔABX and ΔDEY:

∠B = ∠E {∵ ΔABC ~ ΔDEF}

∠X = ∠Y = 90°

∴ ΔABX ~ ΔDEY {By AA similarity criterion}

$$\therefore \frac{AX}{DY} = \frac{AB}{DE} \quad \ldots (2)$$

It is given that: ΔABC ~ ΔDEF

$$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} \quad \ldots (3)$$

Using (1) and (2):

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC \times AB}{EF \times DE}$$

$$= \frac{BC}{EF} \times \frac{BC}{EF} \quad [\text{Using } (3)]$$

$$= \left(\frac{BC}{EF}\right)^2$$

Therefore, using (3):

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{CA}{FD}\right)^2$$

Thus, the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Question 25

The following distribution gives the daily income of 50 workers of a factory.

Daily income (in ₹)	200-220	220-240	240-260	260-280	280-300
Number of workers	12	14	8	6	10

Convert the distribution above to a 'less than type' cumulative frequency distribution and draw its ogive.

Or

The table below shows the daily expenditure on food of 25 households in a locality. Find the mean daily expenditure on food.

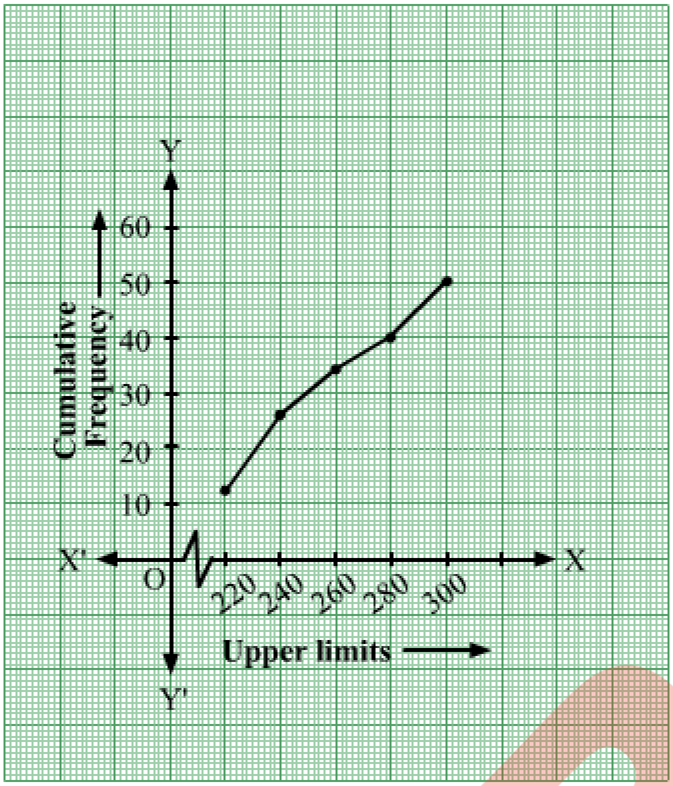
Daily expenditure (in ₹) :	100-150	150-200	200-250	250-300	300-350
Number of households :	4	5	12	2	2

SOLUTION:

The less than type cumulative frequency distribution table will be as follows:

Daily income(in ₹)	**Number of Workers**	**Daily Income Less than**	**Cumulative Frequency**
200 - 220	12	220	12
220 - 240	14	240	12 + 14 = 26
240 - 260	8	260	26 + 8 = 34
260 - 280	6	280	34 + 6 = 40
280 - 300	10	300	40 + 10 = 50

The ogive thus formed will be



OR

Daily Expenditure(in ₹)	Number of Households(f_i)	x_i	$d_i = x_i - 225$	$f_i d_i$
100 - 150	4	125	-100	-400
150 - 200	5	175	-50	-250
200 - 250	12	225	0	0
250 - 300	2	275	50	100
300 - 350	2	325	100	200
	$\sum f_i$ = 25			$\sum f_i d_i = -350$

$\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$

$\bar{x} = 225 + \frac{-350}{25}$

$\bar{x} = 225 - 14$

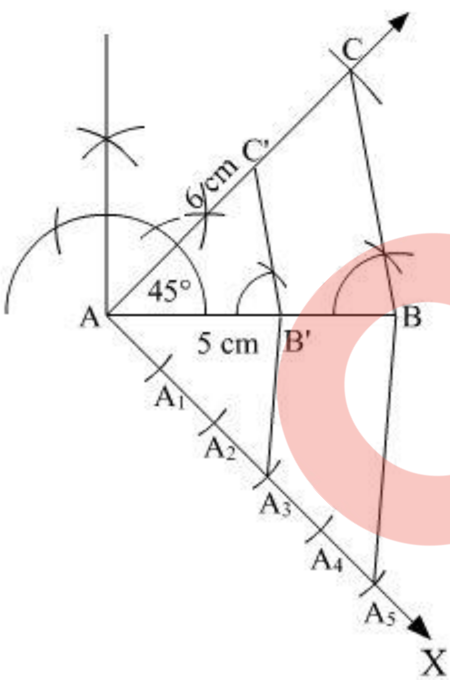
$\bar{x} = 211$

Thus, the mean daily expenditure on food is ₹211.

Question 26
Construct a ΔABC in which CA = 6 cm, AB = 5 cm and ∠BAC = 45°. Then construct a triangle whose sides are $\frac{3}{5}$ of the corresponding sides of ΔABC.

SOLUTION:

Steps of construction:
1. Draw AB = 5 cm. With A as centre, draw ∠BAC = 45°. Join BC. △ABC is thus formed.
2. Draw AX such that ∠BAX is an acute angle.
3. Cut 5 equal arcs $AA_1,\ A_1A_2,\ A_2A_3,\ A_3A_4$ and A_4A_5.
4. Join A_5 to B and draw a line through A_3 parallel to A_5B which meets AB at B'.
Here, AB' = $\frac{3}{5}$AB
5.Now draw a line through B' parallel to BC which joins AC at C'.
Here, B'C' = $\frac{3}{5}$BC and AC'=$\frac{3}{5}$AC
Thus, AB'C' is the required triangle.



Question 27
A bucket open at the top is in the form of a frustum of a cone with a capacity of 12308.8 cm^3. The radii of the top and bottom of circular ends of the bucket are 20 cm and 12 cm respectively. Find the height of the bucket and also the area of the metal sheet used in making it. (Use π = 3.14)

SOLUTION:

Let the depth of the bucket is *h* cm. The radii of the top and bottom circles of the frustum bucket are r_1 =20cm and r_2 =12cm respectively.

The volume/capacity of the bucket is

$$V = \frac{1}{3}\pi(r_1^2 + r_1 r_2 + r_2^2) \times h$$
$$= \frac{1}{3}\pi(20^2 + 20 \times 12 + 12^2) \times h$$
$$= \frac{1}{3} \times \frac{22}{7} \times 784 \times h$$
$$= \frac{1}{3} \times 22 \times 112 \times h \text{ cm}^3$$

Given that the capacity of the bucket is 12308.8 Cubic cm. Thus, we have

$$\frac{1}{3} \times 22 \times 112 \times h = 12308.8$$
$$\Rightarrow h = \frac{12308.8 \times 3}{22 \times 112}$$
$$\Rightarrow h = 15$$

Hence, the height of the bucket is $\boxed{15 \text{ cm}}$

The slant height of the bucket is

$$l = \sqrt{(r_1 - r_2)^2 + h^2}$$
$$= \sqrt{(20 - 12)^2 + 15^2}$$
$$= \sqrt{289}$$
$$= 17 \text{ cm}$$

The surface area of the used metal sheet to make the bucket is

$$S_1 = \pi(r_1 + r_2) \times l + \pi r_2^2$$
$$= \pi \times (20 + 12) \times 17 + \pi \times 12^2$$
$$= \pi \times 32 \times 17 + 144\pi$$
$$= 2160.32 \text{ cm}^2$$

Hence $\boxed{\text{Surface area of the metal} = 2160.32 \text{ cm}^2}$

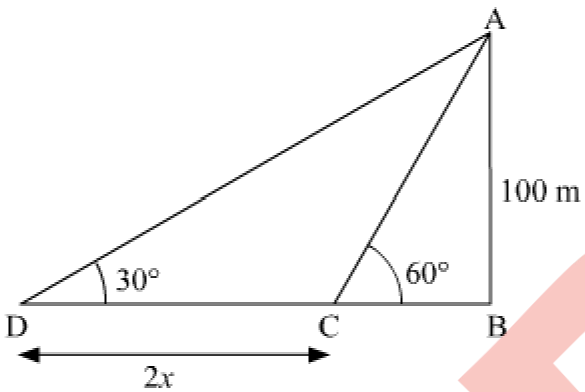
Question 28

A man in a boat rowing away from a light house 100 m high takes 2 minutes to change the angle of elevation of the top of the light house from 60° to 30°.
Find the speed of the boat in metres per minute. [Use $\sqrt{3}$ = 1.732]

OR

Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distances of the point from the poles.

SOLUTION:



AB is a lighthouse of height 100m.
Let the speed of boat be *x* metres per minute.
And CD is the distance which man travelled to change the angle of elevation.

So, $\mathrm{CD} = 2x \quad [\because \text{Distance} = \text{Speed} \times \text{Time}]$

$\tan(60^\circ) = \frac{AB}{BC}$

$\sqrt{3} = \frac{100}{BC}$

$\Rightarrow BC = \frac{100}{\sqrt{3}}$

$\tan(30^\circ) = \frac{AB}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{BD}$

$BD = 100\sqrt{3}$

$CD = BD - BC$

$2x = 100\sqrt{3} - \frac{100}{\sqrt{3}}$

$2x = \frac{300-100}{\sqrt{3}}$

$\Rightarrow x = \frac{200}{2\sqrt{3}}$

$\Rightarrow x = \frac{100}{\sqrt{3}}$

Using $\sqrt{3} = 1.73$
$x = \frac{100}{1.73} \approx 57.80$
Hence, the speed of the boat is 57.8057.80 metres per minute.

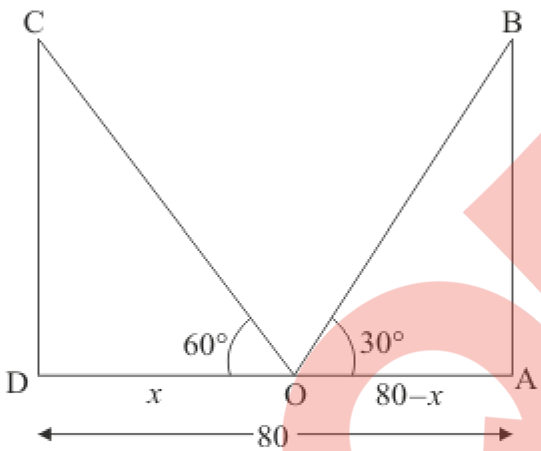
OR

Let AB and *CD* be the two poles of equal height *h* m. *O* be the point makes an angle of elevation from the top of poles are 60° and 30° respectively.

Let $OA = 80 - x$, $OD = x$. And $\angle BOA = 30°$, $\angle COD = 60°$.

Here we have to find the height of poles and distance of the points from poles.

We have the corresponding figure as follows.



So we use trigonometric ratios.

In a triangle *COD*,

$$\Rightarrow \quad \tan 60° = \frac{CD}{DO}$$
$$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$$
$$\Rightarrow \quad x = \frac{h}{\sqrt{3}}$$

Again in a triangle *AOB*,

$$\Rightarrow \quad \tan 30^\circ = \frac{AB}{OA}$$
$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{80-x}$$
$$\Rightarrow \quad \sqrt{3}h = 80 - x$$
$$\Rightarrow \quad \sqrt{3}h = 80 - \frac{h}{\sqrt{3}}$$

$$\Rightarrow \sqrt{3}h + \frac{h}{\sqrt{3}} = 80$$
$$\Rightarrow \quad 3h + h = 80\sqrt{3}$$
$$\Rightarrow \quad 4h = 80\sqrt{3}$$
$$\Rightarrow \quad h = 20\sqrt{3}$$

$$\Rightarrow \quad x = \frac{20\sqrt{3}}{\sqrt{3}}$$
$$\Rightarrow \quad = 20$$

And

$$\Rightarrow \quad OA = 80 - x$$
$$\Rightarrow \quad = 80 - 20$$
$$\Rightarrow \quad = 60$$

Hence, the height of pole is $\boxed{20\sqrt{3}}$ m. and distances are $\boxed{20}$ m, $\boxed{60}$ m respectively.

Question 29

Two water taps together can fill a tank in $1\frac{7}{8}$ hours. The tap with longer diameter takes 2 hours less than the tap with smaller one to fill the tank separately. Find the time in which each tap can fill the tank separately.

OR

A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream. Determine the speed of the stream and that of the boat in still water.

SOLUTION:

Let the first tap takes *x* hours to completely fill tank

⇒ Second tap will take 2 hours less
⇒ According to question

$\frac{1}{x}+\frac{1}{x-2}=\frac{8}{15}$

$\frac{x-2+x}{x(x-2)}=\frac{8}{15}$

$\frac{2x-2}{x(x-2)}=\frac{8}{15}$

$\frac{2(x-1)}{x(x-2)}=\frac{8}{15}$

$15(x-1)=4x(x-2)$

$15x-15=4x^2-8x$

$4x^2-23x+15=0$

$4x^2-20x-3x+15=0$

$4x(x-5)-3(x-5)=0$

$(x-5)(4x-3)=0$

$x=5 \text{ or } \frac{3}{4}$

Since $\frac{3}{4}-2=\text{Negative time } \frac{3}{4} \text{ is not possible.}$
Which is not possible
⇒ $x = 5$
Rate of 1st pipe = 5 hours
Rate of 2nd pipe = 5 – 2 = 3 hours

OR

Let the speed of the boat in still water be x km/hr and the speed of the stream be y km/hr

Speed upstream = $(x-y)$ km/hr

Speed down stream = $(x+y)$ km/hr

Now,

Time taken to cover 30 km upstream = $\frac{30}{x-y}$ hrs

Time taken to cover 44 km down stream = $\frac{44}{x+y}$ hrs

But total time of journey is 10 hours

$\frac{30}{x-y}+\frac{44}{x+y}=10 \cdots(i)$

Time taken to cover 40 km upstream= $\frac{40}{x-y} hrs$

Time taken to cover 55 km down stream = $\frac{55}{x+y} hrs$

In this case total time of journey is given to be 13 hours

Therefore, $\frac{40}{x-y}+\frac{55}{x+y}=13$ ...(ii)

Putting $\frac{1}{x-y}$ = u and $\frac{1}{x+y}=v$ in equation (i) and (ii) we get

$30u+44v-10=0 \cdots(iii)$

$40u+55v-13=0 \cdots(iv)$

Solving these equations by cross multiplication we get

$$\frac{u}{44\times-13-55\times-10}=\frac{-v}{30\times-13-40\times-10}=\frac{1}{30\times55-40\times44}$$
$$\frac{u}{-572+550}=\frac{-v}{-390+400}=\frac{1}{1650-1760}$$
$$\frac{u}{-22}=\frac{-v}{10}=\frac{1}{-110}$$
$$u=\frac{\not-22}{\not-110}$$

$$v=\frac{\not-10}{\not-110}$$

$u=\frac{2}{10}$ and $v=\frac{1}{11}$

Now,

$$u=\frac{2}{10}$$
$$\frac{1}{x-y}=\frac{2}{10}$$
$$1\times10=2(x-y)$$
$$10=2x-2y\div2$$
$$u=\frac{2}{10}$$
$$\frac{1}{x-y}=\frac{2}{10}$$
$$1\times10=2(x-y)$$
$$10=2x-2y$$

$$5=x-y\cdots(v)$$

$$v=\frac{1}{11}$$
$$\frac{1}{x+y}=\frac{1}{11}$$
$$1\times11=1(x+y)$$

$$11=x+y\cdots(vi)$$

By solving equation (v) and (vi) we get ,

$$x-y=5$$
$$x+y=11$$
$$2x=16$$
$$x=\frac{16}{2}$$
$$x=8$$

Substituting $x=8$ in equation (vi) we get ,

$x+y=11$

$8+y=11$

$y=11-8$

$y=3$

Hence, speed of the boat in still water is $\boxed{8\ km/hr}$

Speed of the stream is $\boxed{3\ km/hr}$

Question 30

If the sum of first four terms of an AP is 40 and that of first 14 terms is 280. Find the sum of its first *n* terms.

SOLUTION:

Given that, $S_4 = 40$ and $S_{14} = 280$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_4 = \frac{4}{2}[2a + (4-1)d] = 40$

$\Rightarrow 2a + 3d = 20 \qquad \ldots\ldots (i)$

$S_{14} = \frac{14}{2}[2a + (14-1)d] = 280$

$\Rightarrow 2a + 13d = 40 \qquad \ldots\ldots (ii)$

$(ii) - (i)$,

$10d = 20 \Rightarrow d = 2$

substituting the value of *d* in (i), we get

$2a + 6 = 20 \Rightarrow a = 7$

Sum of first n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[14 + (n-1)2]$

$= n(7 + n - 1)$

$= n(n+6)$

$= n^2 + 6n$

Therefore, $S_n = n^2 + 6n$