Class X

(CBSE 2019)

Mathematics

Delhi (Set-2)

General Instructions:

(i) **All** questions are compulsory.

(ii) The question paper consists of **30** questions divided into four sections – **A, B, C** and **D**.

(iii) Section **A** comprises **6** questions of **1** mark each. Section **B** contains **6** questions of **2** marks each. Section **C** contains **10** questions of **3** marks each. Section **D** contains **8** questions of 4 marks each.

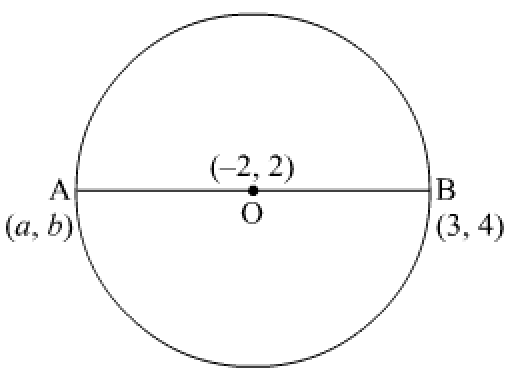
(iv) There is no overall choice. However, an internal choice has been provided in **two** questions of **1** mark, **two** questions of **2** marks, **four** questions of **3** marks each and **three** questions of **4** marks each. You have to attempt only **one** of the alternative in all such questions.

(v) Use of calculators is **not** permitted.

Question 1

Find the coordinates of a point A, where AB is a diameter of the circle with centre (–2, 2) and B is the point with coordinates (3, 4).

SOLUTION:



Let the centre of the circle be O.
Since AB is the diameter so, O is the midpoint of AB.
Thus, using the section formula,

$\frac{a+3}{2} = -2$
$\Rightarrow a = -4 - 3 = -7$
And
$\frac{b+4}{2} = 2$
$\Rightarrow b = 4 - 4 = 0$
So, the coordinate of point A is $(-7, 0)$

Question 2

Find a rational number between $\sqrt{2}$ and $\sqrt{3}$.

SOLUTION:

As we know, $\sqrt{2}$ and $\sqrt{3}$ are irrational numbers.
$\sqrt{2} = 1.414$ (approx.)
$\sqrt{3} = 1.732$ (approx.)
A rational number between $\sqrt{2}$ and $\sqrt{3}$ will be
$1.414 < 1.5 < 1.732$
$\Rightarrow \sqrt{2} < \frac{3}{2} < \sqrt{3}$
Therefore, $1.5 = \frac{3}{2}$ is a rational number between $\sqrt{2}$ and $\sqrt{3}$.

Question 3

How many two digits numbers are divisible by 3?

SOLUTION:

The first two digit number that is divisible by 3 is 12
So, the series starts from 12
And the highest two digit number that is divisible by 3 is 99
So, the sequence becomes:
12, 15, ..., 99.
We, need to find the numbers in the given sequence
Using $a_n = a + (n-1)d$
a is the first term, *d* is the common difference, *n* is the number of terms and a_n is the *n*th term
a = 12, *d* = 3, a_n = 99 substituting the values we get
$99 = 12 + (n-1)3$
$99 = 12 + 3n - 3$
$90 = 3n$
$n = \frac{90}{3}$
$n = 30.$
Therefore, there are total 30 two digit numbers that are divisible by 3.

Question 4

Find A if tan 2A = cot (A – 24°)

OR

Find the value of ($\sin^2 33° + \sin^2 57°$)

SOLUTION:

Given:

$\tan 2A = \cot (A - 24^\circ)$

$\Rightarrow \tan 2A = \tan [90^\circ - (A - 24^\circ)]$

$\Rightarrow \tan 2A = \tan [90^\circ - A + 24^\circ]$

$\Rightarrow \tan 2A = \tan [114^\circ - A]$

$\Rightarrow 2A = 114^\circ - A$

$\Rightarrow 3A = 114^\circ$

$\Rightarrow A = \frac{114^\circ}{3}$

$\Rightarrow A = 38^\circ$

OR

Given:

$\sin^2 33^\circ + \sin^2 57^\circ$

$= \sin^2 33^\circ + [\cos (90^\circ - 57^\circ)]^2$

$= \sin^2 33^\circ + \cos^2 33^\circ$

$= 1$

Question 5

For what value of *k*, the roots of the equation $x^2 + 4x + k = 0$ are real?

OR

Find the value of *k* for which the roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other.

SOLUTION:

The given equation is $x^2 + 4x + k = 0$.

For real roots, $D \geq 0$

$\Rightarrow b^2 - 4ac \geq 0$

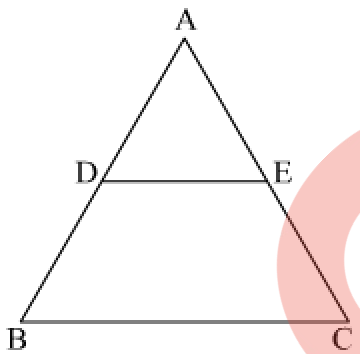
$\Rightarrow 4^2 - 4(1)(k) \geq 0$

$\Rightarrow 4-k \geq 0$
$\Rightarrow k \leq 4$
For $k \leq 4$, the given equation $x^2+4x+k=0$ has real roots.

OR

The given equation is $3x^2-10x+k=0$.
Roots of the given equation are reciprocal of each other.
Let α and $\frac{1}{\alpha}$ be the roots of the given equation.
Product of roots$=\frac{c}{a}$
$\Rightarrow \alpha \cdot \frac{1}{\alpha}=\frac{k}{3}$
$\Rightarrow \frac{k}{3}=1$
$\Rightarrow k=3$

Question 6
In Fig. 1, DE || BC, AD = 1 cm and BD = 2 cm. What is the ratio of the ar (Δ ABC) to the ar (Δ ADE)?



SOLUTION:
It is given that AD=1cm, BD=2 cm and DE||BC
In $\Delta\,\mathrm{ADE}$ and ΔABC
$\angle\mathrm{ADE}=\angle\mathrm{ABC}$ (corresponding angles)
$\angle A=\angle A$ (common angle)
By AA similarity
$\Delta\,\mathrm{ADE} \sim \Delta\,\mathrm{ABC}$

Ratio of area of similar triangles is equal to the square of the ratio of corresponding sides.

$$\therefore \frac{\text{ar }(\Delta\text{ ABC})}{\text{ar}(\Delta\text{ ADE})} = \frac{\text{AB}^2}{\text{AD}^2}$$
$$\Rightarrow \frac{\text{ar }(\Delta\text{ ABC})}{\text{ar}(\Delta\text{ ADE})} = \frac{3^2}{1^2}$$
$$\Rightarrow \frac{\text{ar }(\Delta\text{ ABC})}{\text{ar}(\Delta\text{ ADE})} = \frac{9}{1}$$

Therefore, the ratio of the $\text{ar}\,(\Delta\text{ ABC}) : \text{ar}\,(\Delta\text{ ADE})$ is $9:1$.

Question 7

Find the value of *k* for which the following pair of linear equations have infinitely many solutions. $2x + 3y = 7$, $(k + 1)x + (2k - 1)y = 4k + 1$

SOLUTION:

We have,

$$2x + 3y = 7 \Rightarrow 2x + 3y - 7 = 0$$
$$(k+1)x + (2k-1)y = 4k+1 \Rightarrow (k+1)x + (2k-1)y - (4k+1) = 0$$

For infinitely many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$
$$\Rightarrow \frac{2}{k+1} = \frac{3}{(2k-1)} = \frac{-7}{-(4k+1)}$$

$$\begin{array}{llll}
\Rightarrow & \frac{2}{k+1} = \frac{3}{2k-1} & \text{or} \quad \Rightarrow & \frac{2}{k+1} = \frac{-7}{-(4k+1)} \\
\Rightarrow & 2(2k-1) = 3(k+1) & \Rightarrow & 2(4k+1) = 7(k+1) \\
\Rightarrow & 4k - 2 = 3k + 3 & \Rightarrow & 8k + 2 = 7k + 7 \\
\Rightarrow & 4k - 3k = 3 + 2 & \Rightarrow & 8k - 7k = 7 - 2 \\
 & k = 5 & & k = 5
\end{array}$$

Hence, the value of *k* is 5 for which given equations have infinitely many solutions.

Question 8

A die is thrown once. Find the probability of getting a number which (i) is a prime number (ii) lies between 2 and 6.

SOLUTION:

Total possible outcomes that occur after throwing a die once are 1,2,3,4,5,6
Number of possible outcomes = 6
We need to find the probability of getting a prime number
Prime number is a number not divisible by any number except itself
Prime numbers on a dice are 2,3 and 5.
Number of favourable outcomes = 3

Probability of getting a prime number

$= \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

Probability of getting a prime number $= \frac{3}{6} = \frac{1}{2}$

Therefore, probability of getting a prime number is $\frac{1}{2}$

(b) Probability of getting a number lying between 2 and 6

Number lying between 2 and 6 on a dice are 3,4 and 5.

Probability of getting a number lying between 2 and 6 $= \frac{3}{6} = \frac{1}{2}$

Therefore, Probability of getting a number lying between 2 and 6 is $\frac{1}{2}$

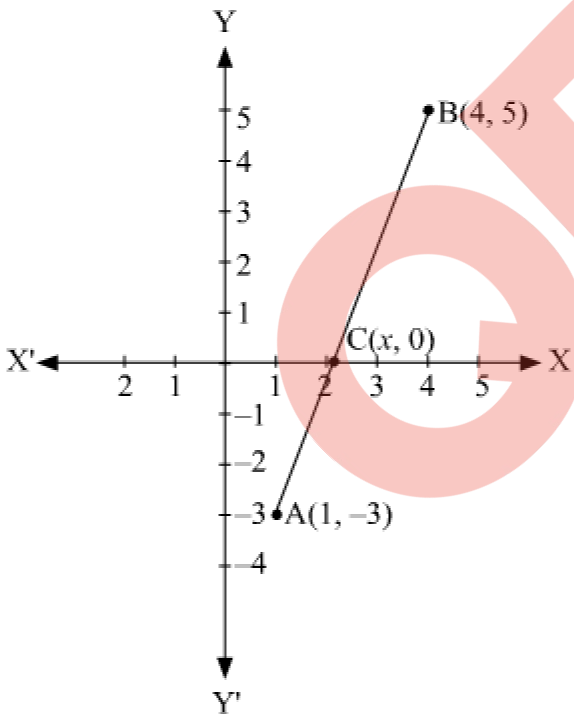
Question 9

Find the ratio in which the segment joining the points (1, –3) and (4, 5) is divided by *x*-axis? Also find the coordinates of this point on *x*-axis.

SOLUTION:

Let C(*x*, 0) divides the line-segment A(1, –3) and B(4, 5) in *k* : 1 ratio.

By section formula,



$$(x,\ y) = \left(\frac{mx_2+nx_1}{m+n},\ \frac{my_2+ny_1}{m+n}\right)$$
$$\Rightarrow (x,\ 0) = \left(\frac{4k+1\times1}{k+1},\ \frac{5k+1\times(-3)}{k+1}\right)$$
$$\Rightarrow (x,\ 0) = \left(\frac{4k+1}{k+1},\ \frac{5k-3}{k+1}\right)$$
$$\Rightarrow \frac{5k-3}{k+1} = 0$$
$$\Rightarrow 5k-3=0$$
$$\Rightarrow 5k=3$$
$$\Rightarrow k=\frac{3}{5}$$
$$\text{and } x = \frac{4k+1}{k+1} = \frac{4\times\frac{3}{5}+1}{\frac{3}{5}+1}$$
$$\Rightarrow x = \frac{\frac{12+5}{5}}{\frac{3+5}{5}}$$
$$\Rightarrow x = \frac{17}{8}$$
The ratio in which C divides A and B is $k:1$ i.e., 3 : 5 and the coordinate of C is $\left(\frac{17}{8},\ 0\right)$.

Question 10

A game consists of tossing a coin 3 times and noting the outcome each time. If getting the same result in all the tosses is a success, find the probability of losing the game.

SOLUTION:

Possible outcomes of tossing a coin three times will be
{HHH, TTT, HHT, HTH, THH, TTH, THT, HTT} = 8
Getting the same result in all tosses is a success.
We need to find the probability of losing the game that means not the same result in all tosses
Favourable outcomes are {HHT, HTH, THH, TTH, THT, HTT} = 6
Probability of losing a game $= \frac{6}{8} = \frac{3}{4}$
Therefore, probability of losing a game is $\frac{3}{4}$.

Question 11

Which term of the AP 3, 15, 27, 39, ... will be 120 more than its 21st term?

OR

If S_n, the sum of first n terms of an AP is given by $S_n = 3n^2 - 4n$, find the nth term.

SOLUTION:

In the given problem, let us first find the 21st term of the given A.P.
A.P. is 3, 15, 27, 39 …
Here,
First term (a) = 3

Common difference of the A.P. (d) $= 15 - 3 = 12$

Now, as we know, $a_n = a + (n-1)d$

So, for 21st term (n = 21),

$$a_{21} = 3 + (21-1)(12)$$
$$= 3 + 20(12)$$
$$= 3 + 240$$
$$= 243$$

Let us take the term which is 120 more than the 21st term as a_n.

$$a_n = 120 + a_{21}$$

So, $= 120 + 243$
$$= 363$$

Also, $a_n = a + (n-1)d$

$$363 = 3 + (n-1)12$$
$$363 = 3 + 12n - 12$$
$$363 = -9 + 12n$$
$$363 + 9 = 12n$$

Further simplifying, we get

$$372 = 12n$$
$$n = \frac{370}{12}$$
$$n = 31$$

Therefore, the $\boxed{31^{st}\text{ term}}$ of the given A.P. is 120 more than the 21st term.

OR

We have, $S_n = 3n^2 - 4n$

for $n = 1 \Rightarrow S_1 = 3(1)^2 - 4(1) = 3 - 4 = -1$

$\Rightarrow a_1 = S_1 = -1$

for $n = 2 \Rightarrow S_2 = 3(2)^2 - 4(2)$

$= 12 - 8 = 4$

$S_2 = 4$

$a_2 = S_2 - S_1$

$= 4 - (-1)$

$a_2 = 5$

Common difference, $d = a_2 - a_1$

$d = 5 - (-1)$

$d = 6$

$\Rightarrow a_n = a + (n - 1)d$
$= (-1) + (n - 1)6$
$= -1 + 6n - 6$
$a_n = 6n - 7$
$\Rightarrow n^{th}$ term is $(6n - 7)$

Question 12
Find the HCF of 1260 and 7344 using Euclid's algorithm.

OR

Show that every positive odd integer is of the form $(4q + 1)$ or $(4q + 3)$, where q is some integer.

SOLUTION:
The given numbers are 1260 and 7344.
Now 7344 > 1260. So, on applying Euclid's algorithm we get
$7344=1260\times5+1044$
Now the remainder is not 0 so, we repeat the process again on 1260 and 1044
$1260=1044\times1+216$
The algorithm is applied again but this time on the numbers 1044 and 216
$1044=216\times4+180$
Now, the algorithm is applied again until the remainder is 0.
$216=180\times1+36$
$180=36\times5+0$
Thus, the HCF obtained is 36.

OR

According to Euclid's division lemma,

$a = bq+r$ where $0 \le r < b$

Now, let a be any odd positive integer and $b = 4$.

When $0 \le r < 4$ so, the possible values of r will be 0, 1, 2, 3.

Now, the possible values of a will be thus, 4q,4q+1,4q+2,4q+3 where q is an integer.

But, we already know that *a is* any odd positive integer.

So, a will be $4q$+1 and $4q$+3.

Question 13

The arithmetic mean of the following frequency distribution is 53. Find the value of *k*.

Class	0-20	20-40	40-60	60-80	80-100
Frequency	12	15	32	*k*	13

SOLUTION:

Class	Frequency (f_i)	x_i	$d_i = x_i - 50$	$f_i d_i$
0 – 20	12	10	−40	−480
20 – 40	15	30	−20	−300
40 – 60	32	50	0	0
60 – 80	k	70	20	$20k$
80 – 100	13	90	40	520
	$\Sigma f_i = 72 + k$			$f_i d_i = -260 + 20k$

$$\overline{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

$$53 = 50 + \frac{(-260+20k)}{72+k}$$

$$\Rightarrow 3 = \frac{-260+20k}{72+k}$$

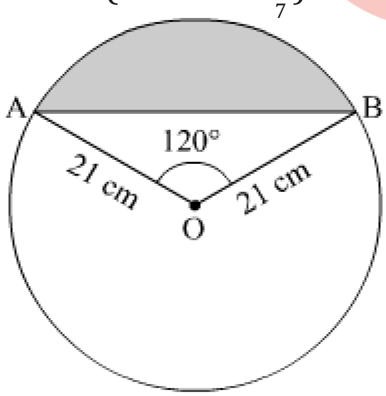
$$\Rightarrow 216 + 3k = -260 + 20k$$

$$\Rightarrow 476 = 17k$$

$$\Rightarrow k = 28$$

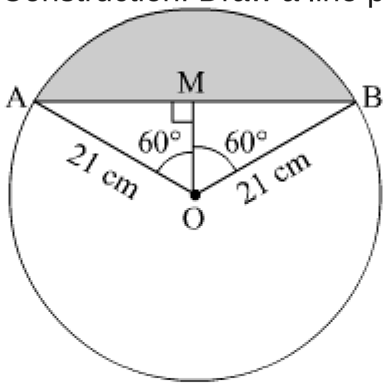
Question 14

Find the area of the segment shown in Fig. 2, if radius of the circle is 21 cm and ∠AOB = 120° (Use $\pi = \frac{22}{7}$)



SOLUTION:

Construction: Draw a line passing through O and perpendicular to AB.



In Δ AOM and Δ BOM,
$\angle AMO = \angle BMO$ (by construction)
$AO = BO$ (radius of same circle)
$OM = OM$ (common side)
$\therefore \ \Delta AOM \cong \Delta BOM$ (By RHS congruence rule)
We have, $\angle AOM = \angle BOM = 60^\circ$ (By CPCT) ..... (i)
$AM = BM$ (By CPCT) ..... (ii)
In Δ AOM,

$$\sin 60^\circ = \frac{AM}{OA} = \frac{AM}{21}$$

$$\Rightarrow \frac{AM}{21} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow AM = \frac{21\sqrt{3}}{2} \text{ cm}$$

$$\text{Also, } \cos 60^\circ = \frac{OM}{OA}$$

$$\Rightarrow \frac{OM}{OA} = \frac{1}{2}$$

$$\Rightarrow OM = \frac{21}{2} \text{cm}$$

$$AB = AM + MB = 2AM = 21\sqrt{3} \text{ cm} \qquad \text{[from (ii)]}$$

Area of sector AOB$= \frac{120}{360} \cdot \pi r^2 = \frac{1}{3} \cdot \frac{22}{7} \cdot 21^2 = 462 \text{ cm}^2$

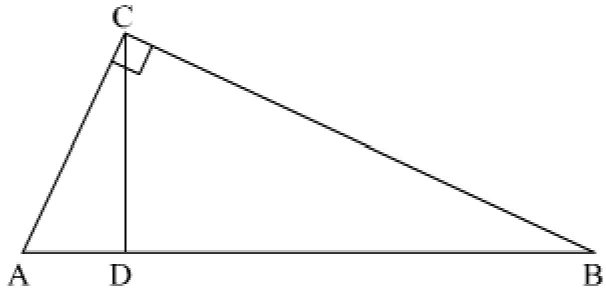
Area of ΔAOB = $\frac{1}{2} \times OM \times AB = \frac{1}{2} \times \frac{21}{2} \times 21\sqrt{3} = \frac{441\sqrt{3}}{4} \text{ cm}^2 \approx 191 \text{ cm}^2$

Required area of segment = Area of sector AOB – Area of ΔAOB

$= 462 - 191 = 271 \text{ cm}^2 \ (\text{approx.})$

Question 15

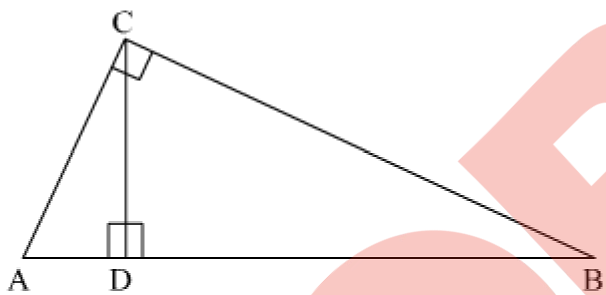
In Fig. 3, $\angle ACB = 90°$ and $CD \perp AB$, prove that $CD^2 = BD \times AD$.



OR

If P and Q are the points on side CA and CB respectively of Δ ABC, right angled at C, prove that $(AQ^2 + BP^2) = (AB^2 + PQ^2)$

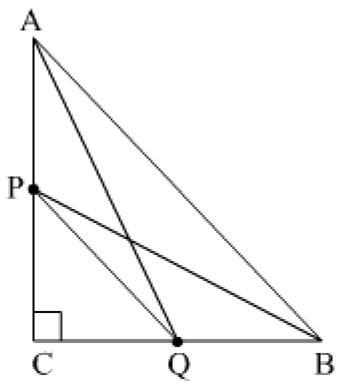
SOLUTION:



Given that : $CD \perp AB$
$\angle ACB = 90°$
To Prove : $CD^2 = BD \times AD$
Using Pythagoras Theorem in Δ ACD
$AC^2 = AD^2 + CD^2$ .....(1)
Using Pythagoras Theorem in Δ CDB
$CB^2 = BD^2 + CD^2$ .....(2)
Similarly in Δ ABC,
$AB^2 = AC^2 + BC^2$ .....(3)
As AB = AD + DB
$\Rightarrow$ AB = AD + BD .....(4)
Squaring both sides of equation (4), we get
$(AB)^2 = (AD + BD)^2$
$\Rightarrow AB^2 = AD^2 + BD^2 + 2 \times BD \times AD$
From equation (3) we get
$AC^2 + BC^2 = AD^2 + BD^2 + 2 \times BD \times AD$
Substituting the value of AC^2 from equation (1) and the value of BC^2 from equation (2), we get
$AD^2 + CD^2 + BD^2 + CD^2 = AD^2 + BD^2 + 2 \times BD \times AD$

$\Rightarrow 2\ CD^2 = 2 \times BD \times AD$
$\Rightarrow CD^2 = BD \times AD$
Hence Proved.

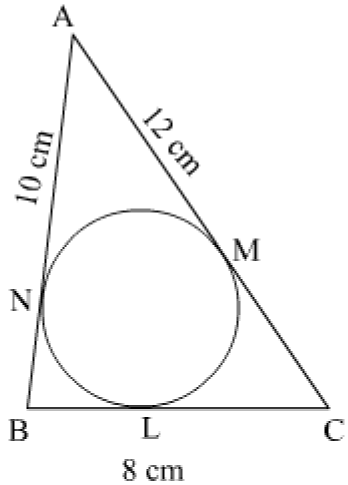
OR



Using the Pythagoras theorem in Δ ABC, Δ ACQ, Δ BPC, Δ PCQ, we get
$AB^2 = AC^2 + BC^2$ .....(1)
$AQ^2 = AC^2 + CQ^2$ .....(2)
$BP^2 = PC^2 + BC^2$ .....(3)
$PQ^2 = PC^2 + CQ^2$ .....(4)
Adding the equations (2) and (3) we get
$AQ^2 + BP^2 = AC^2 + CQ^2 + PC^2 + BC^2$
$= (AC^2 + BC^2) + (CQ^2 + PC^2)$
$= AB^2 + PQ^2$
As
L.H.S = $AQ^2 + BP^2$
$= AB^2 + PQ^2$
= R.H.S
Hence Proved

Question 16

In Fig. 4, a circle is inscribed in a Δ ABC having sides BC = 8 cm, AB = 10 cm and AC = 12 cm. Find the lengths BL, CM and AN.



SOLUTION:

BN and BL are tangents from the same point to the circle

∴ BN = BL

Similarly

AM = AN and CL = CM

Given that

AB = 10 cm, BC = 8 cm and AC = 12 cm

Let

AN = AM = x

CM = CL = y

BN = BL = z

$$\text{AB} = \text{AN} + \text{NB} = 10$$
$$x + z = 10 \qquad \ldots \quad (1)$$
$$\text{BC} = \text{BL} + \text{LC} = 8$$
$$z + y = 8 \qquad \ldots \quad (2)$$
$$\text{AC} = \text{AM} + \text{MC}$$
$$= x + y = 12 \qquad \ldots \quad (3)$$

Adding equations (1), (2) and (3), we get

$2\,(x + y + z) = 30$

$\Rightarrow x + y + z = 15 \quad \ldots \quad (4)$

Subtracting (1) from (4) we get

$y = 5$

Subtracting (2) from (4) we get

$x = 7$

Subtracting (3) from (4) we get

$z = 3$

BL = z = 3 cm

CM = y = 5 cm

AN = x = 7 cm

Question 17
Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/hour. How much area will it irrigate in 30 minutes; if 8 cm standing water is needed?

SOLUTION:

The canal is 6 m wide and 1.5 m deep. The water is flowing in the canal at 10 km/hr. Hence, in 30 minutes, the length of the flowing standing water is

$$= 10 \times \frac{30}{60} \text{ km}$$

$$= 5 \text{ km}$$

$$= 5000 \text{ m}$$

Therefore, the volume of the flowing water in 30 min is

$$V_1 = 5000 \times 1.5 \times 6 \text{ m}^3$$

Thus, the irrigated area in 30 min of 8 cm=0.08 m standing water is

$= \frac{5000 \times 1.5 \times 6}{0.08}$

$= \boxed{562500 \text{ m}^2}$

Question 18

Prove that √2 is an irrational number.

SOLUTION:

Let $\sqrt{2}$ is rational.

$\therefore \sqrt{2} = \frac{p}{q}$ where p and q are co-prime integers and $q \neq 0$.

$\Rightarrow \sqrt{2}q = p$

$\Rightarrow 2q^2 = p^2$ .....(1)

$\Rightarrow$ 2 divides p^2

$\Rightarrow$ 2 divides p .....(A)

Let $p = 2c$ where c is an integer

$\Rightarrow p^2 = 4c^2$

$\Rightarrow 2q^2 = 4c^2$ [from (1)]

$\Rightarrow q^2 = 2c^2$

$\Rightarrow$ 2 divides q^2

$\Rightarrow$ 2 divides q .....(B)

From statements (A) and (B), 2 divides p and q both that means p and q are not co-prime which contradicts our assumption.
So, our assumption is wrong.
Hence √2 is irrational.
Proved.

Question 19

Find the value of k such that the polynomial $x^2 - (k + 6)x + 2(2k - 1)$ has sum of its zeros equal to half of their product.

SOLUTION:

Given polynomial is $x^2 - (k + 6)x + 2(2k - 1)$

Here

$a = 1$, $b = -(k + 6)$, $c = 2(2k - 1)$

Given that,

Sum of zeroes $= \frac{1}{2}$ product of zeroes

$$\Rightarrow \frac{-[-(k+6)]}{1} = \frac{1}{2} \times \frac{2(2k-1)}{1}$$
$$\Rightarrow k+6 = 2k-1$$
$$\Rightarrow 6+1 = 2k-k$$
$$\Rightarrow k = 7$$

Question 20

Find the point on *y*-axis which is equidistant from the points (5, −2) and (−3, 2).

OR

The line segment joining the points A(2, 1) and B(5, −8) is trisected at the points P and Q such that P is nearer to A. If P also lies on the line given by $2x - y + k = 0$, find the value of *k*.

SOLUTION:

Since the point is on *y*-axis so, X-coordinate is zero.

Let the point be (0, *y*)

It's distance from (5, –2) and (–3, 2) are equal

$$\therefore \sqrt{(0-5)^2 + (y+2)^2} = \sqrt{(0+3)^2 + (y-2)^2}$$
$$\Rightarrow 25 + y^2 + 4y + 4 = 9 + y^2 - 4y + 4 \quad \text{[squaring both sides]}$$
$$\Rightarrow 4y + 29 = -4y + 13$$
$$\Rightarrow 4y + 4y = 13 - 29$$
$$\Rightarrow 8y = -16$$
$$\therefore y = \frac{-16}{8} = -2$$

Thus, the point is (0, −2)

OR

We have two points A (2, 1) and B (5,−8). There are two points P and Q which trisect the line segment joining A and B.

Now according to the section formula if any point P divides a line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio m : n internally then,

$$P(x,y)=\left(\frac{nx_1+mx_2}{m+n},\frac{ny_1+my_2}{m+n}\right)$$

The point P is the point of trisection of the line segment AB. So, P divides AB in the ratio 1: 2

Now we will use section formula to find the co-ordinates of unknown point A as,

$$P(x_1,y_1)=\left(\frac{1(5)+2(2)}{1+2},\frac{2(1)+1(-8)}{1+2}\right)$$
$$=(3,-2)$$

Therefore, co-ordinates of point P is(3,−2)

It is given that point P lies on the line whose equation is

$$2x-y+k=0$$

Since, point P satisfies this equation.

2(3) − (−2) + k=0

So,

$$\boxed{k=-8}$$

Question 21
A father's age is three times the sum of the ages of his two children. After 5 years his age will be two times the sum of their ages. Find the present age of the father.

Or

A fraction becomes 1313 when 2 is subtracted from the numerator and it becomes 1212 when 1 is subtracted from the denominator. Find the fraction.

SOLUTION:

Let the present age of father be *x* years and the present ages of his two children's be *y* and *z* years.

The present age of father is three times the sum of the ages of the two children's. Thus, we have

$$x = 3(y+z)$$
$$\Rightarrow y+z = \frac{x}{3}$$

After 5 years, father's age will be $(x+5)$ years and the children's age will be $(y+5)$ and $(z+5)$ years. Thus using the given information, we have

$$x+5 = 2\{(y+5)+(z+5)\}$$
$$\Rightarrow x+5 = 2(y+5+z+5)$$
$$\Rightarrow x = 2(y+z)+20-5$$
$$\Rightarrow x = 2(y+z)+15$$

So, we have two equations

$$y+z = \frac{x}{3}$$

$$x = 2(y+z)+15$$

Here *x, y* and *z* are unknowns. We have to find the value of *x*.

Substituting the value of $(y+z)$ from the first equation in the second equation, we have

By using cross-multiplication, we have

$$x = \frac{2x}{3}+15$$
$$\Rightarrow x - \frac{2x}{3} = 15$$
$$\Rightarrow x(1-\frac{2}{3}) = 15$$
$$\Rightarrow \frac{x}{3} = 15$$

$$\Rightarrow x = 15 \times 3$$
$$\Rightarrow x = 45$$

Hence, the present age of father is $\boxed{45}$ years.

Or

Let's assume the fraction be $\frac{x}{y}$

1st condition:

$\frac{x-2}{y} = \frac{1}{3}$

$\Rightarrow 3x - 6 = y$

$\Rightarrow 3x - y = 6 \ldots (1)$

2nd condition:

$\frac{x}{y-1} = \frac{1}{2}$

$\Rightarrow 2x = y - 1$

$\Rightarrow 2x - y = -1$

Using elimination method:

Multiplying (2) by -1 and then adding (1) and (2)

$\Rightarrow 3x - y = 6$

$\Rightarrow -2x + y = 1$

$\Rightarrow x = 7$

Now, from (1),

$\Rightarrow 3x - y = 6$

$\Rightarrow 3(7) - y = 6$

$\Rightarrow 21 - y = 6$

$\Rightarrow y = 15$

Hence, the required fraction is $\frac{7}{15}$.

Question 22

Prove that (sin θ + cosec θ)^2 + (cos θ + sec θ)^2 = 7 + tan^2 θ + cot^2 θ.

Or

Prove that (1 + cot A - cosec A) (1 + tan A + sec A) = 2

SOLUTION:

$$\begin{aligned} \text{L.H.S} &= (\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 \\ &= \left(\sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta\operatorname{cosec}\theta + \cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta\right) \\ &= \left(\sin^2\theta + \cos^2\theta\right) + \left(\operatorname{cosec}^2\theta + \sec^2\theta\right) + 2\sin\theta\left(\frac{1}{\sin\theta}\right) + 2\cos\theta\left(\frac{1}{\cos\theta}\right) \\ &= (1) + \left(1 + \cot^2\theta + 1 + \tan^2\theta\right) + (2) + (2) \\ &= 7 + \tan^2\theta + \cot^2\theta \\ &= \text{R.H.S} \end{aligned}$$

Or

We have to prove $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$

We know that, $\sin^2 A + \cos^2 A = 1$.

So,

$$(1+\cot A-\text{cosec}A)(1+\tan A+\sec A) = \left(1+\frac{\cos A}{\sin A}-\frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right)$$

$$= \left(\frac{\sin A+\cos A-1}{\sin A}\right)\left(\frac{\cos A+\sin A+1}{\cos A}\right)$$

$$= \frac{(\sin A+\cos A-1)(\sin A+\cos A+1)}{\sin A\cos A}$$

$$= \frac{\{(\sin A+\cos A)-1\}\{(\sin A+\cos A)+1\}}{\sin A\cos A}$$

$$= \frac{(\sin A+\cos A)^2-1}{\sin A\cos A}$$

$$= \frac{\sin^2 A+2\sin A\cos A+\cos^2 A-1}{\sin A\cos A}$$

$$= \frac{(\sin^2 A+\cos^2 A)+2\sin A\cos A-1}{\sin A\cos A}$$

$$= \frac{1+2\sin A\cos A-1}{\sin A\cos A}$$

$$= \frac{2\sin A\cos A}{\sin A\cos A}$$

$$= 2$$

Hence proved.

Question 23

Prove that $\frac{\tan^2 A}{\tan^2 A-1}+\frac{\operatorname{cosec}^2 A}{\sec^2 A-\operatorname{cosec}^2 A}=\frac{1}{1-2\cos^2 A}$

SOLUTION:

Taking L.H.S.

$$\frac{\tan^2 A}{\tan^2 A-1}+\frac{\operatorname{cosec}^2 A}{\sec^2 A-\operatorname{cosec}^2 A}$$

$$\frac{\frac{\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A}{\cos^2 A}-1}+\frac{\frac{1}{\sin^2 A}}{\frac{1}{\cos^2 A}-\frac{1}{\sin^2 A}} \qquad \left(\because \tan A=\frac{\sin A}{\cos A}\right)$$

$$=\frac{\sin^2 A}{\sin^2 A-\cos^2 A}+\frac{1}{\sin^2 A}\cdot\frac{\sin^2 A\cos^2 A}{\sin^2 A-\cos^2 A}$$

$$=\frac{\sin^2 A}{\sin^2 A-\cos^2 A}+\frac{\cos^2 A}{\sin^2 A-\cos^2 A}$$

$$=\frac{\sin^2 A+\cos^2 A}{\sin^2 A-\cos^2 A}$$

$$=\frac{1}{1-\cos^2 A-\cos^2 A} \qquad \left(\because \sin^2 A=1-\cos^2 A\right)$$

$$=\frac{1}{1-2\cos^2 A}$$

$$=\text{R.H.S}$$

Question 24

The first term of an AP is 3, the last term is 83 and the sum of all its terms is 903. Find the number of terms and the common difference of the AP.

SOLUTION:

Given:

a = 3

l = 83

Sum all the n terms = 903

S_n = 903

$\frac{n}{2}[a+l]=903$

$\frac{n}{2}\cdot(3+83)=903$

$n\cdot 43=903$

$n=\frac{903}{43}$

$n=21$

Number of terms = 21

$\because l=83$

$a+(n-1)d=83$

$3+(21-1)\cdot d=83$

$20d=80$

$d=4$

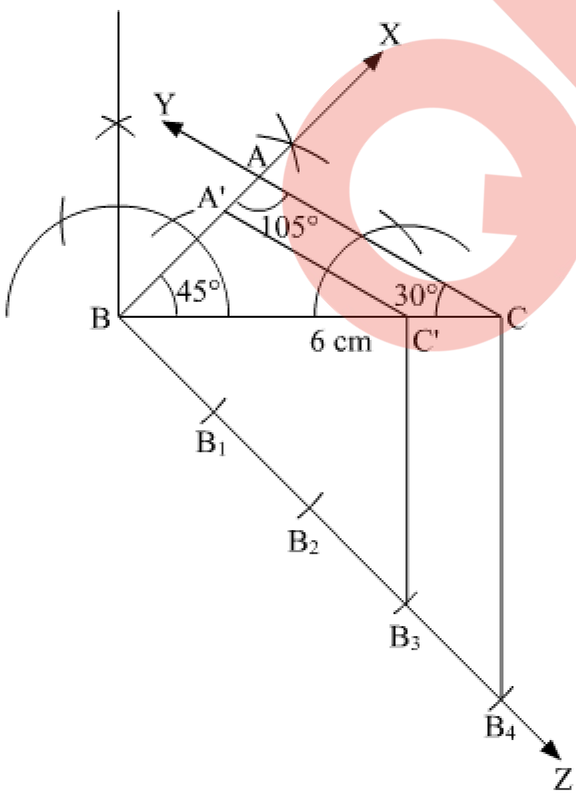
$\therefore$ Common difference = 4

$\therefore n=21$ and $d=4$

Question 25

Construct a triangle ABC with side BC = 6 cm, ∠B = 45°, ∠A = 105°. Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the Δ ABC.

SOLUTION:



Steps of construction

(1) Draw BC = 6 cm.

(2) At point B, draw ∠XBC = 45°

(3) Using Angle Sum Property in Δ ABC,

∠A + ∠B + ∠C = 180°

105° + 45° + ∠C = 180°

∠C = 30°

And, now draw ∠YCB = 30°

(4) Now, line YC and XB intersect at point A. Thus this is our required triangle ABC.

(5) Draw an acute angle ∠CBZ at B.

(6) Now cut 4 equal arcs BB_1, B_1B_2, B_2B_3 and B_3B_4 on BZ.

(7) Now, join B_4 to C.

(8) Draw a line parallel to B_4C from B_3 which intersects BC at C'.

(9) Now, draw a line parallel to AC from C' which intersects BX at A'.

(10) Thus, Δ A'BC' is our required triangle whose sides are $\frac{3}{4}$ times the corresponding sides of Δ ABC.

Question 26

If the median of the following frequency distribution is 32.5. Find the values of f_1 and f_2.

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	Total
Frequency	f_1	5	9	12	f_2	3	2	40

OR

The marks obtained by 100 students of a class in an examination are given below.

Mark	**No. of Students**
0 – 5	2
5 – 10	5
10 – 15	6
15 – 20	8
20 – 25	10
25 – 30	25
30 – 35	20

35 – 40	18
40 – 45	4
45 – 50	2

Draw 'a less than' type cumulative frequency curves (ogive). Hence find median.

SOLUTION:

Given: Median = 32.5

We prepare the cumulative frequency table, as given below

Class interval:	Frequency: (f_i)	Cumulative frequency $(c.f.)$
0-10	f_1	f_1
10-20	5	$5+f_1$
20-30	9	$14+f_1$
30-40	12	$26+f_1$
40-50	f_2	$26+f_1+f_2$
50-60	3	$29+f_1+f_2$
60-70	2	$31+f_1+f_2$
	$N=40=31+f_1+f_2$	

Now, we have

$$N=40$$
$$31+f_1+f_2=40$$

$f_2=9-f_1$ .....(1)

Also, $\frac{N}{2}=20$

Since median = 32.5 so the median class is $30-40$.

Here, $l=30, f=12, F=14+f_1$ and $h=10$

We know that

$$\text{Median} = l + \left\{\frac{\frac{N}{2} - F}{f}\right\} \times h$$

$$32.5 = 30 + \left\{\frac{20 - (14 + f_1)}{12}\right\} \times 10$$

$$2.5 = \frac{(6 - f_1) \times 10}{12}$$

$$2.5 \times 12 = 60 - 10 f_1$$

$$f_1 = \frac{30}{10}$$
$$= 3$$

Putting the value of f_1 in (1), we get

$$f_2 = 9 - 3$$
$$= 6$$

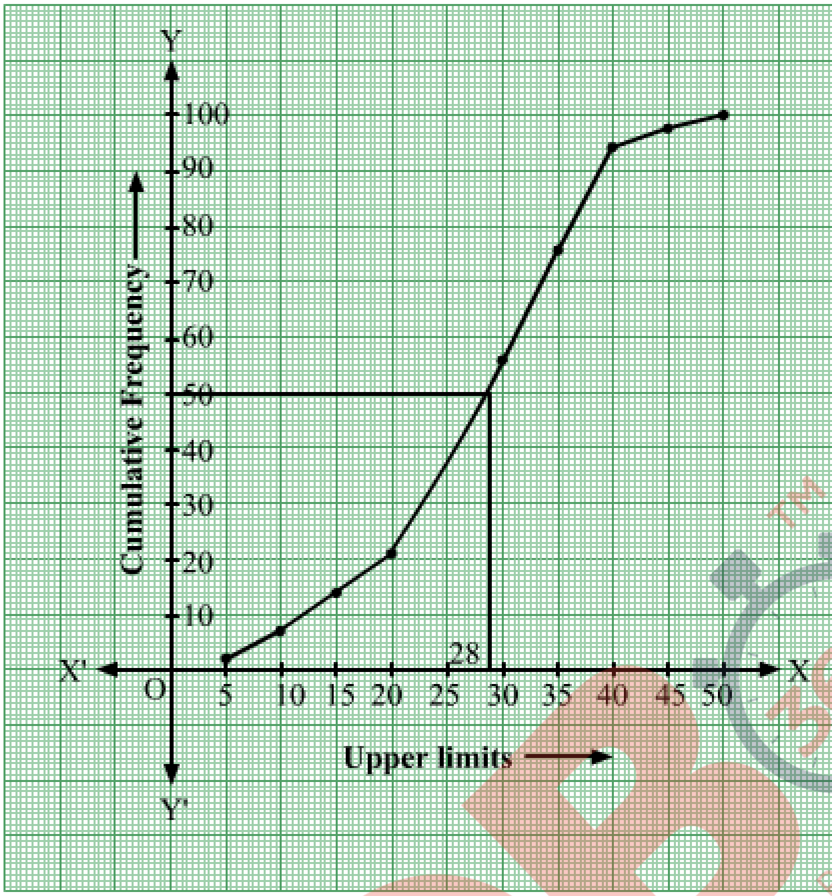
Hence, the missing frequencies are 3 and 6.

OR

We first prepare the cumulative frequency table by less than method as given below–

Marks	No. of Students	Marks less than	Cumulative Frequency
0 – 5	2	5	2
5 – 10	5	10	7
10 – 15	6	15	13
15 – 20	8	20	21
20 – 25	10	25	31
25 – 30	25	30	56
30 – 35	20	35	76
35 – 40	18	40	94
40 – 45	4	45	98
45 – 50	2	50	100

Thus we will plot the points (5, 2), (10, 7), (15, 13), (20, 21), (25, 31), (30, 56), (35, 76), (40, 94), (45, 98) and (50, 100).



∴ From the above ogive, the horizontal line drawn from $\frac{N}{2} = 50$ intersects the ogive at a point whose *x*-coordinate is approximately 28.
∴ Hence, Median ≈ 28.

Question 27
Prove that in a right angle triangle, the square of the hypotenuse is equal the sum of squares of the other two sides.

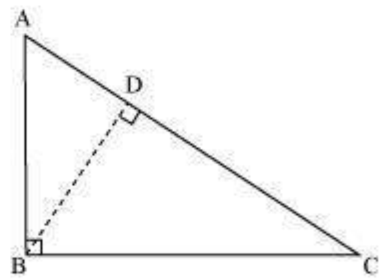
SOLUTION:

"In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides."

Proof: Let ABC be a right triangle where ∠B = 90°.

It has to be proved that $AC^2 = AB^2 + BC^2$

Construction: Draw BD ⊥ AC



In ΔADB and ΔABC,

∠ADB = ∠ABC [Each is right angle]

∠BAD = ∠BAC [Common angle]

Therefore, by AA similarity criterion, ΔADB ~ ΔABC

$\therefore \frac{AD}{AB} = \frac{AB}{AC}$ [Sides are proportional in similar triangles]

$\Rightarrow AD \times AC = AB^2$ … (1)

Similarly, it can be proved that ΔBDC ~ ΔABC

$\therefore \frac{CD}{BC} = \frac{BC}{AC}$

$\Rightarrow AC \times CD = BC^2$ … (2)

Adding equations (1) and (2), we obtain

$AB^2 + BC^2 = AD \times AC + AC \times CD$

$\Rightarrow AB^2 + BC^2 = AC (AD + CD)$

$\Rightarrow AB^2 + BC^2 = AC \times AC$

$\Rightarrow AB^2 + BC^2 = AC^2$

This proves the Pythagoras Theorem.

Question 28

A bucket open at the top is in the form of a frustum of a cone with a capacity of 12308.8 cm^3. The radii of the top and bottom of circular ends of the bucket are 20 cm and 12 cm respectively. Find the height of the bucket and also the area of the metal sheet used in making it. (Use π = 3.14)

SOLUTION:

Let the depth of the bucket is h cm. The radii of the top and bottom circles of the frustum bucket are r_1 =20cm and r_2 =12cm respectively.

The volume/capacity of the bucket is

$$\begin{aligned} V &= \frac{1}{3}\pi(r_1^2 + r_1r_2 + r_2^2) \times h \\ &= \frac{1}{3}\pi(20^2 + 20\times 12 + 12^2)\times h \\ &= \frac{1}{3}\times\frac{22}{7}\times 784\times h \\ &= \frac{1}{3}\times 22\times 112\times h \text{ cm}^3 \end{aligned}$$

Given that the capacity of the bucket is 12308.8 Cubic cm. Thus, we have

$$\begin{aligned} &\frac{1}{3}\times 22\times 112\times h = 12308.8 \\ &\Rightarrow h = \frac{12308.8\times 3}{22\times 112} \\ &\Rightarrow h = 15 \end{aligned}$$

Hence, the height of the bucket is $\boxed{15 \text{ cm}}$

The slant height of the bucket is

$$\begin{aligned} l &= \sqrt{(r_1 - r_2)^2 + h^2} \\ &= \sqrt{(20-12)^2 + 15^2} \\ &= \sqrt{289} \\ &= 17 \text{ cm} \end{aligned}$$

The surface area of the used metal sheet to make the bucket is

$$S_1 = \pi(r_1 + r_2) \times l + \pi r_2^2$$
$$= \pi \times (20 + 12) \times 17 + \pi \times 12^2$$
$$= \pi \times 32 \times 17 + 144\pi$$
$$= 2160.32 \text{ cm}^2$$

Hence $\boxed{\text{Surface area of the metal} = 2160.32 \text{ cm}^2}$

Question 29

Two water taps together can fill a tank in $1\frac{7}{8}$ hours. The tap with longer diameter takes 2 hours less than the tap with smaller one to fill the tank separately. Find the time in which each tap can fill the tank separately.

OR

A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream. Determine the speed of the stream and that of the boat in still water.

SOLUTION:

Let the first tap takes *x* hours to completely fill tank
⇒ Second tap will take 2 hours less
⇒ According to question

$$\frac{1}{x} + \frac{1}{x-2} = \frac{8}{15}$$
$$\frac{x-2+x}{x(x-2)} = \frac{8}{15}$$
$$\frac{2x-2}{x(x-2)} = \frac{8}{15}$$
$$\frac{2(x-1)}{x(x-2)} = \frac{8}{15}$$
$$15(x-1) = 4x(x-2)$$
$$15x - 15 = 4x^2 - 8x$$
$$4x^2 - 23x + 15 = 0$$
$$4x^2 - 20x - 3x + 15 = 0$$
$$4x(x-5) - 3(x-5) = 0$$
$$(x-5)(4x-3) = 0$$
$$x = 5 \text{ or } \frac{3}{4}$$

Since $\frac{3}{4} - 2 =$ Negative time $\frac{3}{4}$ is not possible.
Which is not possible
$\Rightarrow x = 5$
Rate of 1st pipe = 5 hours
Rate of 2nd pipe = 5 − 2 = 3 hours

OR

Let the speed of the boat in still water be *x* km/hr and the speed of the stream be *y* km/hr

Speed upstream = $(x-y)$ km/hr

Speed down stream = $(x+y)$ km/hr

Now,

Time taken to cover 30 km upstream = $\frac{30}{x-y}$ hrs

Time taken to cover 44 km down stream = $\frac{44}{x+y}$ hrs

But total time of journey is 10 hours

$$\frac{30}{x-y} + \frac{44}{x+y} = 10 \quad \dots(i)$$

Time taken to cover 40 km upstream= $\frac{40}{x-y} hrs$

Time taken to cover 55 km down stream = $\frac{55}{x+y} hrs$

In this case total time of journey is given to be 13 hours

Therefore, $\frac{40}{x-y} + \frac{55}{x+y} = 13$ ...(ii)

Putting $\frac{1}{x-y} = u$ and $\frac{1}{x+y} = v$ in equation (i) and (ii) we get

$30u+44v-10=0 \cdots(iii)$

$40u+55v-13=0 \cdots(iv)$

Solving these equations by cross multiplication we get

$$\frac{u}{44\times-13-55\times-10}=\frac{-v}{30\times-13-40\times-10}=\frac{1}{30\times55-40\times44}$$
$$\frac{u}{-572+550}=\frac{-v}{-390+400}=\frac{1}{1650-1760}$$
$$\frac{u}{-22}=\frac{-v}{10}=\frac{1}{-110}$$
$$u=\frac{\cancel{-}22}{\cancel{-}110}$$

$$v=\frac{\cancel{-}10}{\cancel{-}110}$$

$u=\frac{2}{10}$ and $v=\frac{1}{11}$

Now,

$$u=\frac{2}{10}$$
$$\frac{1}{x-y}=\frac{2}{10}$$
$$1\times10=2(x-y)$$
$$10=2x-2y\div2$$
$$u=\frac{2}{10}$$
$$\frac{1}{x-y}=\frac{2}{10}$$
$$1\times10=2(x-y)$$
$$10=2x-2y$$

$5=x-y \cdots(v)$

$$v=\frac{1}{11}$$
$$\frac{1}{x+y}=\frac{1}{11}$$
$$1\times 11=1(x+y)$$

$$11=x+y \cdots(vi)$$

By solving equation (v) and (vi) we get ,

$$\begin{aligned} x-\cancel{y}&=5\\ x+\cancel{y}&=11\\ \hline 2x&=16\\ x&=\frac{16}{2}\\ x&=8 \end{aligned}$$

Substituting $x=8$ in equation (vi) we get ,

$$\begin{aligned} x+y&=11\\ 8+y&=11\\ y&=11-8\\ y&=3 \end{aligned}$$

Hence, speed of the boat in still water is $\boxed{8\ km/hr}$

Speed of the stream is $\boxed{3\ km/hr}$

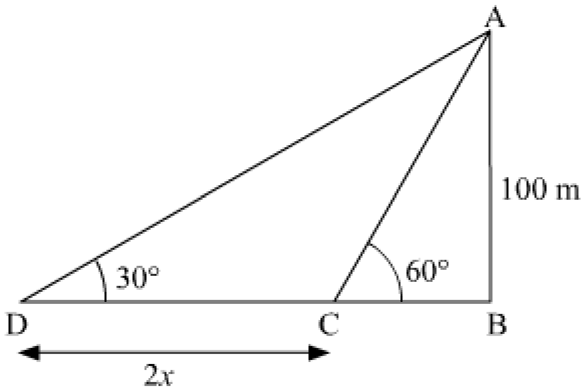
Question 30

A man in a boat rowing away from a light house 100 m high takes 2 minutes to change the angle of elevation of the top of the light house from 60° to 30°.
Find the speed of the boat in metres per minute. [Use $\sqrt{3}$=1.732]

OR

Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distances of the point from the poles.

SOLUTION:



AB is a lighthouse of height 100m.
Let the speed of boat be *x* metres per minute.
And CD is the distance which man travelled to change the angle of elevation.

So, $\mathrm{CD} = 2x \quad [\because \text{Distance} = \text{Speed} \times \text{Time}]$

$\tan(60^\circ) = \frac{AB}{BC}$

$\sqrt{3} = \frac{100}{BC}$

$\Rightarrow BC = \frac{100}{\sqrt{3}}$

$\tan(30^\circ) = \frac{AB}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{BD}$

$BD = 100\sqrt{3}$

$CD = BD - BC$

$2x = 100\sqrt{3} - \frac{100}{\sqrt{3}}$

$2x = \frac{300-100}{\sqrt{3}}$

$\Rightarrow x = \frac{200}{2\sqrt{3}}$

$\Rightarrow x = \frac{100}{\sqrt{3}}$

Using $\sqrt{3} = 1.73$

$x = \frac{100}{1.73} \approx 57.80$

Hence, the speed of the boat is 57.80 metres per minute.

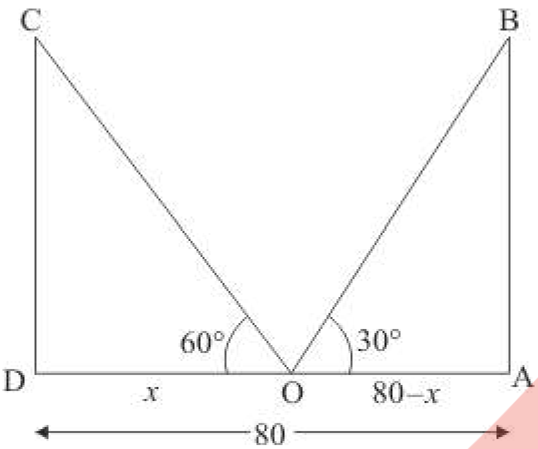
OR

Let AB and CD be the two poles of equal height h m. O be the point makes an angle of elevation from the top of poles are 60° and 30° respectively.

Let $OA = 80 - x$, $OD = x$. And $\angle BOA = 30°$, $\angle COD = 60°$.

Here we have to find the height of poles and distance of the points from poles.

We have the corresponding figure as follows.



So we use trigonometric ratios.

In a triangle COD,

$$\Rightarrow \quad \tan 60° = \frac{CD}{DO}$$
$$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$$
$$\Rightarrow \quad x = \frac{h}{\sqrt{3}}$$

Again in a triangle AOB,

$$\Rightarrow \quad \tan 30° = \frac{AB}{OA}$$
$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{80 - x}$$
$$\Rightarrow \quad \sqrt{3}h = 80 - x$$
$$\Rightarrow \quad \sqrt{3}h = 80 - \frac{h}{\sqrt{3}}$$

$$\Rightarrow \sqrt{3}h + \frac{h}{\sqrt{3}} = 80$$
$$\Rightarrow \quad 3h + h = 80\sqrt{3}$$
$$\Rightarrow \quad 4h = 80\sqrt{3}$$
$$\Rightarrow \quad h = 20\sqrt{3}$$

$$\Rightarrow \quad x = \frac{20\sqrt{3}}{\sqrt{3}}$$
$$\Rightarrow \quad = 20$$

And

$$\Rightarrow \quad OA = 80 - x$$
$$\Rightarrow \quad = 80 - 20$$
$$\Rightarrow \quad = 60$$

Hence, the height of pole is $\boxed{20\sqrt{3}}$ m. and distances are $\boxed{20}$ m, $\boxed{60}$ m respectively.