# Class X
# (CBSE 2019)
# Mathematics
# All India (Set-2)

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**General Instructions:**

*(i)* **All** *questions are compulsory.*

*(ii) The question paper consists of* **30** *questions divided into four sections –* **A, B, C** *and* **D**.

*(iii) Section* **A** *comprises* **6** *questions of* **1** *mark each. Section* **B** *contains* **6** *questions of* **2** *marks each. Section* **C** *contains* **10** *questions of* **3** *marks each. Section* **D** *contains* **8** *questions of 4 marks each.*

*(iv) There is no overall choice. However, an internal choice has been provided in* **two** *questions of* **1** *mark,* **two** *questions of* **2** *marks,* **four** *questions of* **3** *marks each and* **three** *questions of* **4** *marks each. You have to attempt only* **one** *of the alternative in all such questions.*

*(v) Use of calculators is* **not** *permitted.*

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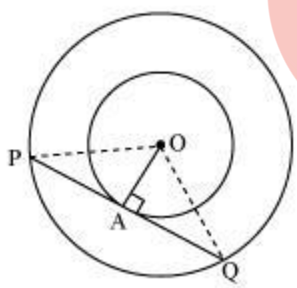
**Question 1**

Two concentric circles of radii *a* and *b* (*a* > *b*) are given. Find the length of the chord of the larger circle which touches the smaller circle.

**SOLUTION:**




Let the two concentric circles be centred at point O.
Also, *a* > *b* where *a* is radius of larger circle and *b* is radius of smaller circle.
And let PQ be the chord of the larger circle which touches the smaller circle at point A.
Therefore, PQ is tangent to the smaller circle.

OA ⊥ PQ (As OA is the radius of the circle)

Applying Pythagoras theorem in ΔOAP, we obtain

$OA^2 + AP^2 = OP^2$

$$b^2 + PA^2 = a^2$$
$$\Rightarrow PA^2 = a^2 - b^2$$
$$\Rightarrow PA = \sqrt{a^2 - b^2}$$

In ΔOPQ,

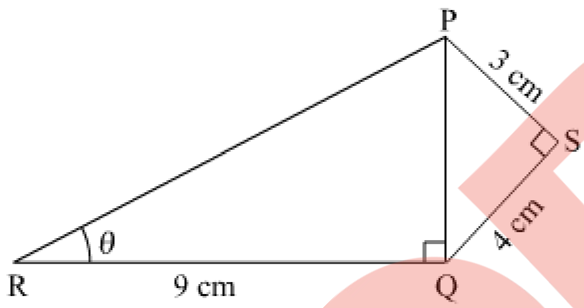
Since OA ⊥ PQ,

PA = AQ (Perpendicular from the centre of the circle to the chord bisects the chord)

$\therefore$ PQ = 2PA = $2\sqrt{a^2 - b^2}$

Therefore, the length of the chord of the larger circle is $2\sqrt{a^2 - b^2}$ cm.

**Question 2**




In Figure 1, PS = 3 cm, QS = 4 cm, ∠PRQ = θ, ∠PSQ = 90°, PQ ⊥ RQ and RQ = 9 cm. Evaluate tan θ.

**OR**

If $\tan \alpha = \frac{5}{12}$, find the value of sec α.

**SOLUTION:**

Given,
PS = 3 cm, QS = 4 cm, RQ = 9cm
In ΔPSQ,

$$PQ^2 = PS^2 + QS^2$$
$$PQ^2 = 3^2 + 4^2 = 25$$
$$\Rightarrow PQ = 5 \text{ cm}$$

In ΔPQR,

$$\tan \theta = \frac{PQ}{RQ}$$

$\Rightarrow \tan\theta = \frac{5}{9}$

**OR**

Given,
$\tan\alpha = \frac{5}{12}$

We know, $\sec^2\alpha = 1 + \tan^2\alpha$
$\Rightarrow \sec^2\alpha = 1 + \left(\frac{5}{12}\right)^2 = \frac{169}{144}$
$\Rightarrow \sec\alpha = \pm\sqrt{\frac{169}{144}} = \pm\frac{13}{12}$

## Question 3

Write the discriminant of the quadratic equation $(x + 5)^2 = 2(5x - 3)$.

## SOLUTION:

Given quadratic equation is $(x+5)^2 = 2(5x-3)$
$\Rightarrow x^2 + 25 + 10x = 10x - 6$
$\Rightarrow x^2 + 31 = 0$
$\Rightarrow x^2 + 0x + 31 = 0$
So, the discriminant will be
$D = b^2 - 4ac$
$= 0^2 - 4 \times 1 \times 31$
$= -124$

## Question 4

Find after how many places of decimal the decimal form of the number $\frac{27}{2^3 \cdot 5^4 \cdot 3^2}$ will terminate.

**OR**

Express 429 as a product of its prime factors.

## SOLUTION:

$$\frac{27}{2^3 \cdot 5^4 . 3^2} = \frac{3}{2^3 \cdot 5^4} = \frac{3\times 2}{2^4 \times 5^4} = \frac{3\times 2}{(2\times 5)^4} = \frac{6}{10^4} = 0.0006$$

Therefore, after 4 place of decimal the decimal form of given number will terminate.

**OR**

429 = 3 × 11 × 13

where 3, 11 and 13 are prime numbers.

## Question 5

Find the sum of first 10 multiples of 6.

## SOLUTION:

First 10 multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, 54, 60.
This forms an AP with first term $a = 6$ and common difference, $d = 6$.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow S_{10} = \frac{10}{2}[2 \times 6 + (10-1)6]$$

$$\Rightarrow S_{10} = 5[12 + 54]$$

$$\Rightarrow S_{10} = 5 \times 66 = 330$$

Thus, sum of first 10 multiples of 6 is 330.

## Question 6

Find the positive value of *m* for which the distance between the points A(5, −3) and B(13, m) is 10 units.

## SOLUTION:

It is given that distance between $\mathrm{A}(5, -3)$ and $\mathrm{B}(13, m)$ is 10 units.

$$\Rightarrow AB = \sqrt{(5-13)^2 + (-3-m)^2}$$

$$\Rightarrow 10 = \sqrt{64 + 9 + 6m + m^2}$$

$$\Rightarrow 100 = 73 + 6m + m^2$$

$$\Rightarrow m^2 + 6m - 27 = 0$$

$$\Rightarrow m^2 + 9m - 3m - 27 = 0$$

$$\Rightarrow m(m+9) - 3(m+9) = 0$$

$$\Rightarrow (m-3)(m+9) = 0$$

$$\Rightarrow m = 3, -9$$

Hence, the positive value of $m$ is 3.

## Question 7

A die is thrown once. Find the probability of getting (i) a composite number, (ii) a prime number.

**SOLUTION:**

Total outcomes of throwing a dice once are 1,2 3,4,5 and 6

(1) Probability of getting a composite number

Composite numbers are those that are formed by multiplying two smaller numbers

So, there are two composite numbers in throwing a dice once which is 4 and 6.

probability of getting a composite number $= \frac{2}{6} = \frac{1}{3}$

(2) Probability of getting a prime number

Prime numbers are 2, 3 and 5 in throwing a die once

Probability of getting a prime number $= \frac{3}{6} = \frac{1}{2}$

**Question 8**

Cards numbered 7 to 40 were put in a box. Poonam selects a card at random. What is the probability that Poonam selects a card which is a multiple of 7 ?

**SOLUTION:**

There are total 34 cards numbered from 7 to 40.

Numbers multiple of 7 are 7, 14, 21, 28 and 35.

total number of favourable outcomes = 5

Let E be the event that Poonam selects a card which is multiple of 7.

$$P(E) = \frac{\text{Number of cards which are multiple of 7}}{\text{Total number of cards}}$$

$$\Rightarrow P(E) = \frac{5}{34}$$

**Question 9**

Points A(3, 1), B(5, 1), C(*a*, *b*) and D(4, 3) are vertices of a parallelogram ABCD. Find the values of *a* and *b*.

**OR**

Points P and Q trisect the line segment joining the points A(−2, 0) and B(0, 8) such that P is near to A. Find the coordinates of points P and Q.

**SOLUTION:**

The given points of the parallelogram are A(3, 1), B(5, 1), C(*a, b*) and D(4, 3).

We know that the diagonals of a parallelogram bisect each other. So, O is the mid point of AC and DB.

So,

$$\left(\frac{3+a}{2}, \frac{1+b}{2}\right) = \left(\frac{5+4}{2}, \frac{1+3}{2}\right)$$
$$\Rightarrow \left(\frac{3+a}{2}, \frac{1+b}{2}\right) = \left(\frac{9}{2}, \frac{4}{2}\right)$$
$$\Rightarrow \left(\frac{3+a}{2}, \frac{1+b}{2}\right) = \left(\frac{9}{2}, 2\right)$$

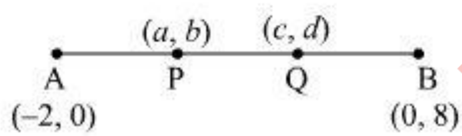
On comparing we get

$$\frac{3+a}{2} = \frac{9}{2}$$
$$\Rightarrow 3 + a = 9$$
$$\Rightarrow a = 6$$

Also,

$$\frac{1+b}{2} = 2$$
$$\Rightarrow 1 + b = 4$$
$$\Rightarrow b = 3$$

Thus, $a = 6, b = 3$.




P and Q trisect line joining the points A and B.
Let the coordinates of P and Q be (*a, b*) and (*c, d*) respectively.
P is the mid point of AQ.

$$\frac{-2+c}{2} = a \text{ and } \frac{0+d}{2} = b$$
$$\Rightarrow c = 2a + 2 \text{ and } d = 2b \qquad \ldots\ldots(1)$$

Also, Q is the mid point of PB.

$$\frac{a+0}{2} = c \text{ and } \frac{b+8}{2} = d$$
$$\Rightarrow \frac{a}{2} = c \text{ and } \frac{b+8}{2} = d \qquad \ldots\ldots(2)$$

From (1) and (2) we have

$$2a + 2 = \frac{a}{2}$$
$$\Rightarrow 4a + 4 = a$$
$$\Rightarrow 3a = -4$$
$$\Rightarrow a = \frac{-4}{3}$$

Also,

$$2b = \frac{b+8}{2}$$

$\Rightarrow 4b = b + 8$

$\Rightarrow b = \frac{8}{3}$

Putting these values of *a* and *b* in (2)

$\frac{\frac{-4}{3}}{2} = c$

$\Rightarrow \frac{-2}{3} = c$

And

$\frac{\frac{8}{3}+8}{2} = d$

$\Rightarrow \frac{\frac{8+24}{3}}{2} = d$

$\Rightarrow \frac{\frac{32}{3}}{2} = d$

$\Rightarrow \frac{16}{3} = d$

Thus, the points are P$\left(\frac{-4}{3}, \frac{8}{3}\right)$ and Q$\left(\frac{-2}{3}, \frac{16}{3}\right)$.

## Question 10

Solve the following pair of linear equations:

3*x* - 5y = 4
2*y* + 7 = 9*x*

## SOLUTION:

Given,

$3x - 5y = 4$ ..... (i)

$2y + 7 = 9x$ ..... (ii)

From (ii), $x = \frac{2y+7}{9}$ ..... (iii)

substitute $x = \frac{2y+7}{9}$ in (i), then

$3\left(\frac{2y+7}{9}\right) - 5y = 4$

$\Rightarrow \frac{2y+7}{3} - 5y = 4$

$\Rightarrow \frac{2y+7-15y}{3} = 4$

$\Rightarrow 7 - 13y = 12$

$\Rightarrow y=-\frac{5}{13}$

put $y=-\frac{5}{13}$ in $(\mathrm{iii})$, then

$x=\frac{2\left(-\frac{5}{13}\right)+7}{9}=\frac{-10+91}{9\times 13}$

$\Rightarrow x=\frac{9}{13}$

Hence, $x=\frac{9}{13}$ and $y=-\frac{5}{13}$ is the solution for the given pair of linear equations.

**Question 11**

If HCF of 65 and 117 is expressible in the form 65*n* − 117, then find the value of *n*.

**OR**

On a morning walk, three persons step out together and their steps measure 30 cm, 36 cm and 40 cm respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps ?

**SOLUTION:**

The given numbers are 65 and 117 where 117 > 65.
Applying Euclid's division lemma,

$117=65\times 1+52$ .....(1)

The remainder is not 0 so we apply the process again on the numbers 65 and 52.

$65=52\times 1+13$ .....(2)

Again the remainder is not 0 so the process is repeated with the numbers 52 and 13.

$52=13\times 4+0$

The last non-zero remainder obtained was 13 which is the HCF of 65 and 117.

From (2) we get

$65=52\times 1+13$

$\Rightarrow 13=65-52\times 1$

$\Rightarrow 13=65-(117-65\times 1)$ $\quad\ldots\ldots(\text{From }(1))$

$\Rightarrow 13=65-117+65\times 1$

$\Rightarrow 13=65\times 2+117\times(-1)$

$\Rightarrow 13=65\times 2-117$

On comparing it with $65n-117$ we get the value of *n* as 2.

**OR**

GIVEN: In a morning walk, three persons step off together. Their steps measure 30 cm, 36 cm and 40 cm.

The distance covered by each of them is required to be same as well as minimum. The required distance each should walk would be the L.C.M of the measures of their steps i.e. 30 cm, 36 cm, and 40 cm,

So, we have to find the L.C.M of 30 cm, 36 cm and 40 cm.

30 = 3 × 5 × 2
36 = 3 × 2 × 3 × 2
40 = 2 × 2 × 5 × 2

LCM of 30, 36 and 40 will be 360.

Hence minimum 360 cm distance each should walk so that all can cover the same distance in complete steps.

**Question 12**
In the quadratic equation $kx^2 - 6x - 1 = 0$, determine the values of $k$ for which the equation does not have any real root.

**SOLUTION:**
Given,
$kx^2 - 6x - 1 = 0,$
For no real roots, $\mathrm{D} < 0$
$\Rightarrow b^2 - 4ac < 0$
$\Rightarrow (-6)^2 - 4(k)(-1) < 0$
$\Rightarrow 36 + 4k < 0$
$\Rightarrow k < -9$
For all $k < -9$, the given quadratic equation does not have any real roots.

**Question 13**
A, B and C are interior angles of a triangle ABC. Show that
(i) $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$
(ii) If ∠A = 90°, then find the value of $\tan\left(\frac{B+C}{2}\right)$.

**OR**

If tan (A + B) = 1 and $\tan(A - B) = \frac{1}{\sqrt{3}}$, 0° < A + B < 90°, A > B, then find the values of A and B.

**SOLUTION:**

(i) We know that for a triangle ABC,

∠A + ∠B + ∠C = 180°

∠B + ∠C = 180° − ∠A

$$\frac{\angle B+\angle C}{2}=90^\circ-\frac{\angle A}{2}$$

$$\sin\left(\frac{B+C}{2}\right)=\sin\left(90^\circ-\frac{A}{2}\right)$$

$$=\cos\left(\frac{A}{2}\right)$$

(ii) Given: ∠ A = 90°,
∠A + ∠B + ∠C = 180°

∠B + ∠C = 180° - ∠A

$$\frac{\angle B+\angle C}{2}=90^\circ-\frac{\angle A}{2}$$

$$\Rightarrow \tan\left(\frac{\angle B+\angle C}{2}\right)=\tan\left(90^\circ-\frac{\angle A}{2}\right)$$

$$\Rightarrow \tan\left(\frac{\angle B+\angle C}{2}\right)=\cot\left(\frac{\angle A}{2}\right)$$

$$\Rightarrow \tan\left(\frac{\angle B+\angle C}{2}\right)=\cot\left(\frac{90^\circ}{2}\right)=\cot 45^\circ$$

$$\Rightarrow \tan\left(\frac{\angle B+\angle C}{2}\right)=1$$

OR

$$\tan(A+B)=1$$

$$\Rightarrow \tan(A+B)=\tan 45^\circ$$

$$\Rightarrow A+B=45^\circ \quad .....(1)$$

$$\tan(A-B)=\frac{1}{\sqrt{3}}$$

⇒ tan(A - B) = tan 30°

⇒ A - B = 30 .....(2)

On adding (1) and (2), we obtain

$$2A=75^\circ$$

$$\Rightarrow A=37.5^\circ$$

Putting the value of A in (1) we get

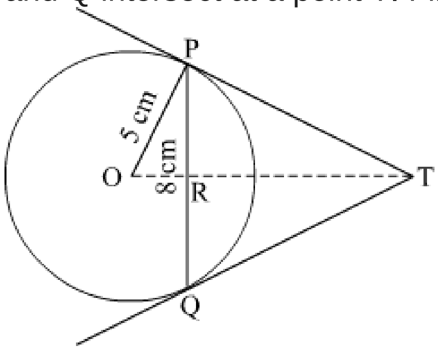
$$37.5^\circ+B=45^\circ$$

$$\Rightarrow B=7.5^\circ$$

Therefore, ∠A = 37.5° and ∠B = 7.5°

**Question 14**

In Figure 2, PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length TP.




**OR**

Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

**SOLUTION:**

PT = TQ (Tangents drawn from a common point to the circle are equal)
So, ΔTPQ is isosceles with PT = TQ and TO is thus the angle bisector of ∠PTQ.
So, OT ⊥ PQ and thus, OT bisects PQ. (Perpendicular drawn from the centre of the circle to the chord bisects the chord)
Thus, PR = RQ = 4 cm
Also, in ΔOPR,

$$OR = \sqrt{OP^2 - PR^2}$$
$$= \sqrt{5^2 - 4^2}$$
$$= 3 \text{ cm}$$

In ΔTPR,

$$TR^2 + PR^2 = TP^2$$
$$\Rightarrow TR^2 + 4^2 = TP^2$$
$$\Rightarrow TR^2 + 16 = TP^2$$

Let TP = $x$ and TR = $y$

$$\Rightarrow y^2 + 16 = x^2 \qquad .....(1)$$

In ΔOPT,

$$TP^2 + OP^2 = OT^2$$
$$\Rightarrow TP^2 + 5^2 = (TR + 3)^2$$
$$\Rightarrow x^2 + 25 = (y + 3)^2 \qquad .....(2)$$

Using (1) and (2) we get

$$y = \frac{16}{3}$$

Putting this value of $y$ in (1) we get

$$\left(\frac{16}{3}\right)^2 + 16 = x^2$$

$\Rightarrow \frac{256}{9} + 16 = x^2$

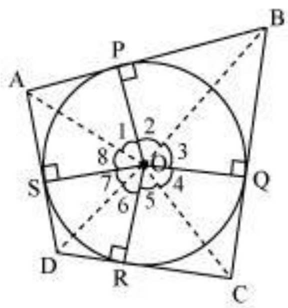
$\Rightarrow \frac{256+144}{9} = x^2$

$\Rightarrow \frac{400}{9} = x^2$

$\Rightarrow x = \frac{20}{3}$

Thus, the length of TP = $\frac{20}{3}$

OR




Let ABCD be a quadrilateral circumscribing a circle centred at O such that it touches the circle at point P, Q, R, S. Let us join the vertices of the quadrilateral ABCD to the center of the circle.

Consider ΔOAP and ΔOAS,

AP = AS (Tangents from the same point)

OP = OS (Radii of the same circle)

OA = OA (Common side)

ΔOAP ≅ ΔOAS (SSS congruence criterion)

Therefore, A ↔ A, P ↔ S, O ↔ O

And thus, ∠POA = ∠AOS

∠1 = ∠8

Similarly,

∠2 = ∠3

∠4 = ∠5

∠6 = ∠7
∠1 + ∠2 + ∠3 + ∠4 + ∠5 + ∠6 + ∠7 + ∠8 = 360º
(∠1 + ∠8) + (∠2 + ∠3) + (∠4 + ∠5) + (∠6 + ∠7) = 360º
2∠1 + 2∠2 + 2∠5 + 2∠6 = 360º
2(∠1 + ∠2) + 2(∠5 + ∠6) = 360º
(∠1 + ∠2) + (∠5 + ∠6) = 180º
∠AOB + ∠COD = 180º

Similarly, we can prove that ∠BOC + ∠DOA = 180º

Hence, opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

**Question 15**
A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.

| Number of days: | 0-6 | 6-12 | 12-18 | 18-24 | 24-30 | 30-36 | 36-42 |
|---|---|---|---|---|---|---|---|
| Number of students: | 10 | 11 | 7 | 4 | 4 | 3 | 1 |

**SOLUTION:**
We will do it by direct method
Using: mean $\frac{\sum f_i x_i}{\sum f_i}$

| Number of days | Number of students $(f_i)$ | Class mark $(x_i)$ | $f_i x_i$ |
|---|---|---|---|
| 0-6 | 10 | $\frac{0+6}{2}=3$ | 30 |
| 6-12 | 11 | $\frac{6+12}{2}=9$ | 99 |
| 12-18 | 7 | $\frac{12+18}{2}=15$ | 105 |
| 18-24 | 4 | $\frac{18+24}{2}=21$ | 84 |
| 24-30 | 4 | $\frac{24+30}{2}=27$ | 108 |
| 30-36 | 3 | $\frac{30+36}{2}=33$ | 99 |
| 36-42 | 1 | $\frac{36+42}{2}=39$ | 39 |
| | $\sum f_i = 40$ | | $\sum f_i x_i = 564$ |

substituting the values in the formula
mean$= \frac{564}{40} = 14.1$

**Question 16**

A car has two wipers which do not overlap. Each wiper has a blade of length 21 cm sweeping through an angle 120°. Find the total area cleaned at each sweep of the blades. (Take $\pi = \frac{22}{7}$)

**SOLUTION:**

Radius$= r = 21\ \text{cm}$
sweeping angle$= 120^\circ$
Total area cleaned by two wipers$= 2 \times \text{area cleaned by one wiper}$

Total area cleaned by two wipers $= 2 \times \text{area of sector with } 120^\circ$

$$= 2 \times \frac{\theta}{360} \times \pi r^2$$

On substituting the values

$$= 2 \times \frac{120}{360} \times \frac{22}{7} \times (21)^2$$

$$= 2 \times \frac{120}{360} \times \frac{22}{7} \times 441 = \frac{2 \times 22 \times 147}{7}$$

$$= \frac{6468}{7} = 924$$

Therefore, area cleaned by both wipers is $924\text{cm}^2$.

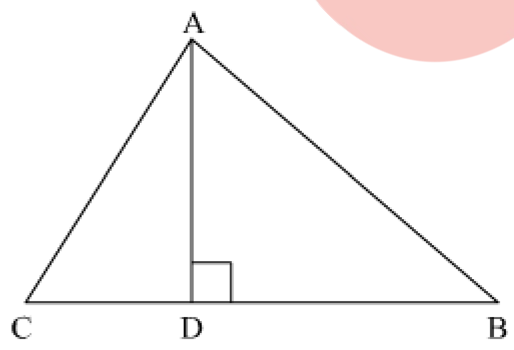
**Question 17**

The perpendicular from A on side BC of a Δ ABC meets BC at D such that DB = 3CD. Prove that $2AB^2 = 2AC^2 + BC^2$.

**OR**

AD and PM are medians of triangles ABC and PQR respectively where Δ ABC ~ Δ PQR. Prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.

SOLUTION:




Applying Pythagoras theorem for ΔACD, we obtain

$$AC^2 = AD^2 + DC^2$$

$$AD^2 = AC^2 - DC^2 \qquad \text{... (1)}$$

Applying Pythagoras theorem in ΔABD, we obtain

$$AB^2 = AD^2 + DB^2$$
$$AD^2 = AB^2 - DB^2 \quad \text{... (2)}$$

From equation (1) and equation (2), we obtain

$$AC^2 - DC^2 = AB^2 - DB^2 \quad \text{... (3)}$$

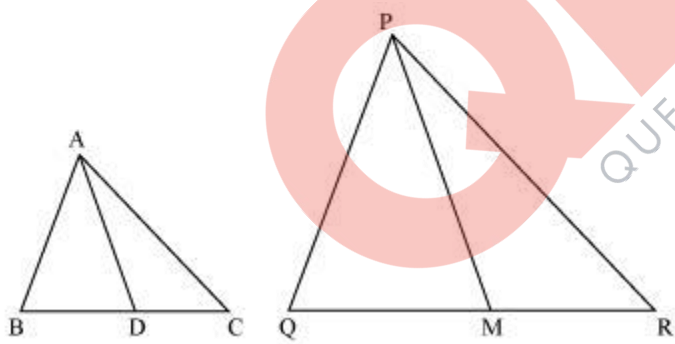
It is given that 3DC = DB

$$\therefore DC = \frac{BC}{4} \text{ and } DB = \frac{3BC}{4}$$

Putting these values in equation (3), we obtain

$$AC^2 - \left(\frac{BC}{4}\right)^2 = AB^2 - \left(\frac{3BC}{4}\right)^2$$
$$AC^2 - \frac{BC^2}{16} = AB^2 - \frac{9BC^2}{16}$$
$$16AC^2 - BC^2 = 16AB^2 - 9BC^2$$
$$16AB^2 - 16AC^2 = 8BC^2$$
$$2AB^2 = 2AC^2 + BC^2$$

**OR**




It is given that ΔABC ~ ΔPQR

We know that the corresponding sides of similar triangles are in proportion.

$$\therefore \frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR} \quad \text{..... (1)}$$

Also, ∠A = ∠P, ∠B = ∠Q, ∠C = ∠R .....(2)
Since AD and PM are medians, they will divide their opposite sides.

∴ $BD=\frac{BC}{2}$ and $QM=\frac{QR}{2}$ … (3)

From equations (1) and (3), we obtain

$\frac{AB}{PQ}=\frac{BD}{QM}$ … (4)

In ΔABD and ΔPQM,

∠B = ∠Q [Using equation (2)]

$\frac{AB}{PQ}=\frac{BD}{QM}$ [Using equation (4)]

∴ ΔABD ~ ΔPQM (By SAS similarity criterion)

⇒ $\frac{AB}{PQ}=\frac{BD}{QM}=\frac{AD}{PM}$

## Question 18

Check whether $g(x)$ is a factor of $p(x)$ by dividing polynomial $p(x)$ by polynomial $g(x)$, where $p(x) = x^5 - 4x^3 + x^2 + 3x + 1$, $g(x) = x^3 - 3x + 1$

## SOLUTION:

We have been given two polynomials

$p(x) = x^5 - 4x^3 + x^2 + 3x + 1$ and $g(x) = x^3 - 3x + 1$

We will say $g(x)$ is factor of $p(x)$ if remainder is zero when we divide $p(x)$ by $g(x)$.

$$\begin{array}{r|l} & x^2 - 1 \\ x^3 - 3x + 1 & x^5 - 4x^3 + x^2 + 3x + 1 \\ & x^5 - 3x^3 + x^2 \\ & - \quad + \quad - \\ \hline & -x^3 + 3x + 1 \\ & -x^3 + 3x - 1 \\ & + \quad - \quad + \\ \hline & 2 \\ \hline \end{array}$$

Here, remainder is $2 \neq 0$

$g(x)$ is not a factor of $p(x)$

## Question 19

Prove that √3 is an irrational number.

**OR**

Find the largest number which on dividing 1251, 9377 and 15628 leaves remainders 1, 2 and 3 respectively.

**SOLUTION:**

Let √3 is rational.

∴ $\sqrt{3} = \frac{p}{q}$ where *p* and *q* are co-prime integers and $q \neq 0$.

⇒ $\sqrt{3}q = p$

⇒ $3q^2 = p^2$ .....(1)

⇒ 3 divides $p^2$

⇒ 3 divides *p* .....(A)

Let $p = 3c$ where *c* is an integer

⇒ $p^2 = 9c^2$

⇒ $3q^2 = 9c^2$ [from (1)]

⇒ $q^2 = 3c^2$

⇒ 3 divides $q^2$

⇒ 3 divides *q* .....(B)

From statements (A) and (B), 3 divides *p* and *q* both that means *p* and *q* are not co-prime which contradicts our assumption.
So, our assumption is wrong.
Hence √3 is irrational.

**OR**

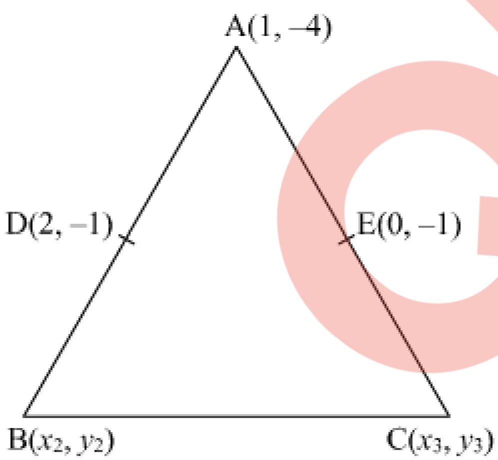
It is given that 1, 2 and 3 are the remainders of 1251, 9377 and 15628, respectively.
Subtracting these remainders from the respective numbers, we get
1251 − 1 = 1250
9377 − 2 = 9375
15628 − 3 = 15625
Now, 1250, 9375 and 15625 are divisible by the required number.
Required number = HCF of 1250, 9375 and 15625

By Euclid's division algorithm $a = bq + r,\ 0 \leq r < b$
For largest number, put *a* = 15625 and *b* = 9375
15625 = 9375 × 1 + 6250
$\Rightarrow 9375 = 6250 \times 1 + 3125$
$\Rightarrow 6250 = 3125 \times 2 + 0$
Since remainder is zero, therefore, HCF(15625 and 9375) = 3125
Further, take *c* = 1250 and *d* = 3125. Again using Euclid's division algorithm
$d = cq + r,\ \ 0 \leq r < c$
$\Rightarrow 3125 = 1250 \times 2 + 625 \quad [\because r \neq 0]$
$\Rightarrow 1250 = 625 \times 2 + 0$

Since remainder is zero, therefore, HCF(1250, 9375 and 15625) = 625
Hence, 625 is the largest number which divides 1251, 9377 and 15628 leaving remainder 1, 2 and 3, respectively.

**Question 20**
Find the area of the triangle ABC with the coordinates of A as (1, −4) and the coordinates of the mid-points of sides AB and AC respectively are (2, −1) and (0, −1).

**SOLUTION:**




Let D is mid-point of AB and E is midpoint of AC.
Coordinates of A $(1, -4)$

Using mid-point $x = \frac{x_1+x_2}{2}$, $y = \frac{y_1+y_2}{2}$
we will find coordinates of B using mid-point in AB
$2 = \frac{1+x_2}{2}, \; -1 = \frac{-4+y_2}{2}$
$x_2 = 3, \; y_2 = 2$
Coordinates of B $(3, 2)$
Now, we will find coordinates of C from mid-point formula in AC
$0 = \frac{1+x_3}{2}, \; -1 = \frac{-4+y_3}{2}$
$x_3 = -1, \; y_3 = 2$
Coordinates of C $(-1, 2)$

Now, using coordinates A (1,−4), B(3,2 ) and C(−1,2) in the formula of area of triangle which is:
$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$
$\frac{1}{2}[1(2-2) + 3(2-(-4)) + (-1)(-4-2)]$
$= \frac{1}{2}[0 + 18 + 6]$
$= \frac{1}{2}(24) = 12 \text{ sq. units}$

**Question 21**
Two numbers are in the ratio of 5 : 6. If 7 is subtracted from each of the numbers, the ratio becomes 4 : 5. Find the numbers.

**SOLUTION:**
Let the two numbers be *x* and *y*.
We have been given the ratio of two numbers 5:6

$\frac{x}{y} = \frac{5}{6} \qquad \ldots(1)$
when 7 is subtracted from the each number the ratio becomes 4:5
$\frac{x-7}{y-7} = \frac{4}{5} \qquad \ldots(2)$
$6x = 5y$
$\Rightarrow 6x - 5y = 0 \qquad \ldots(3)$
$5(x-7) = 4(y-7)$
$\Rightarrow 5x - 35 = 4y - 28$
$\Rightarrow 5x - 4y = 7 \qquad \ldots(4)$
Solving $(3)$ and $(4)$ by elimination method
Multiply $(3)$ by 5 and $(4)$ by 6
$30x - 25y = 0 \qquad \ldots(5)$

$30x-24y=42 \qquad \dots(6)$

Subtracting $(5)$ from $(6)$

$y=42$

Substitute $y=42$ in $(3)$ we get

$6x-5(42)=0$

$\Rightarrow 6x=210$

$\Rightarrow x=35$

Therefore, the two numbers are 35 and 42.

**Question 22**

Water is flowing at the rate of 2.52 km/h through a cylindrical pipe into a cylindrical tank, the radius of whose base is 40 cm. If the increase in the level of water in the tank, in half an hour is 3.15 m, find the internal diameter of the pipe.

**SOLUTION:**

Let the internal diameter of pipe be *x* m

water flows in 1hour = 2.52 km

water flows in $\frac{1}{2}$ hour = $\frac{2.52}{2}=1.26\text{ km}=1260\text{ m}$

Volume of water flows in $\frac{1}{2}$ hour=$\pi r^2 h$

$$\pi r^2\times 1260$$

$$=\pi\times\left(\frac{40}{100}\right)^2\times 3.15$$

Volume of water flow = volume of increased water

$\pi r^2\times 1260=\pi\left(\frac{2}{5}\right)^2\times 3.15$

$1260r^2=\frac{2}{5}\times\frac{2}{5}\times 3.15$

$\Rightarrow r^2=\frac{4}{25}\times\frac{315}{100}\times\frac{1}{1260}$

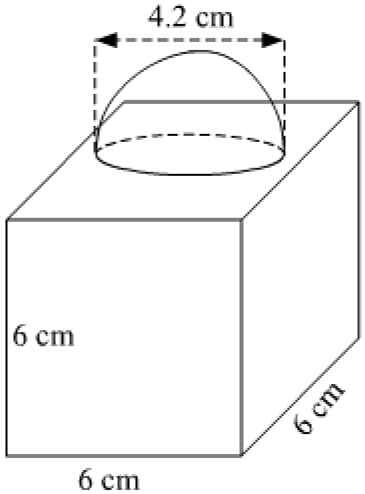
$\Rightarrow r^2=\frac{1}{2500}$

$\Rightarrow r=\frac{1}{50}\text{m}=2\text{ cm}$

Diameter is twice of radius

Therefore, internal diameter = $2\times 2=4\text{ cm}$.

### Question 23




In Figure 3, a decorative block is shown which is made of two solids, a cube and a hemisphere. The base of the block is a cube with edge 6 cm and the hemisphere fixed on the top has a diameter of 4·2 cm. Find

(a) the total surface area of the block.
(b) the volume of the block formed. (Take $\pi = \frac{22}{7}$)

**OR**

A bucket open at the top is in the form of a frustum of a cone with a capacity of 12308.8 $cm^3$. The radii of the top and bottom circular ends are 20 cm and 12 cm respectively. Find the height of the bucket and the area of metal sheet used in making the bucket. (Use $\pi = 3.14$)

**SOLUTION:**

(a) We need to find the total surface area of the block
Block excludes the base area of hemisphere
Surface area of block
= total surface area of cube + curved surface area of hemisphere – base area of hemisphere
Total surface area of cube = $6a^2$; a is side of cube
Total surface area of cube = $6 \times 6^2 = 216$
Base area of hemisphere = $\pi r^2$; base being circular in shape

And curved surface area of hemisphere = $2\pi r^2$
Total surface area of the block
$216 - \pi r^2 + 2\pi r^2$
$= 216 + \pi r^2$
$= 216 + \frac{22}{7} \times \left(\frac{4.2}{2}\right)^2$
$= 216 + \frac{22}{7} \times \frac{17.64}{4}$
$= 216 + \frac{388.08}{28}$
$= 229.86 \text{ cm}^2$
(b) Volume of the block formed = volume of hemisphere+ volume of cube
$= \frac{2}{3}\pi r^3 + a^3$
$= \frac{2}{3} \times \frac{22}{7} \times \left(\frac{4.2}{2}\right)^2 + 6^3$
$= \frac{2}{3} \times \frac{22}{7} \times 4.41 + 216$
$= \frac{194.04}{21} + 216$
$= 225.24 \text{ cm}^3$

**OR**

Capacity of the bucket = 12308.8 $\text{cm}^3$
Radius of lower end, $r$ =12 cm
Radius of upper end, $R$ = 20 cm
Height = $h$
$\text{Volume of bucket} = \frac{1}{3}\pi h\left(r^2 + R^2 + rR\right)$
$\Rightarrow 12308.8 = \frac{1}{3} \times 3.14h\left(12^2 + 20^2 + 12 \times 20\right)$
$\Rightarrow 11760 = h(144 + 400 + 240)$
$\Rightarrow 11760 = h(784)$
$\Rightarrow h = 15 \text{ cm}$
Thus, the height of the bucket is 15 cm.
Slant height of bucket,
$l = \sqrt{h^2 + (R - r)^2}$
$= \sqrt{(15)^2 + (20 - 12)^2}$
$= \sqrt{225 + 64}$
$= \sqrt{289}$
$= 17 \text{ cm}$

Curved surface area of bucket

$= \pi(r+R)l = 3.14(12+20) \times 17$

$= 1708.16 \text{ cm}^2$

Area of metal sheet used in making the bucket = Curved surface area of the bucket + area of the base of the bucket

$= 1708.16 + \pi r^2$

$= 1708.16 + 3.14 \times (12)^2$

$= 1708.16 + 452.16$

$= 2160.32 \text{ cm}^2$

Hence, the area of the metal sheet required is 2160.32 $\text{cm}^2$.

## Question 24

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio

**OR**

Prove that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

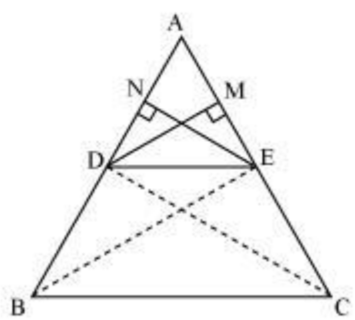
## SOLUTION:

Let ABC be a triangle in which a line DE is parallel to BC. It intersects the sides AB and AC at D and E respectively.

It has to be proved that,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Let us join BE and CD and draw perpendiculars DM and EN on AC and AB respectively.



Area of $\Delta ADE = \frac{1}{2} \times \text{Base} \times \text{Height}$

$= \frac{1}{2} \times AD \times EN = \frac{1}{2} \times AE \times DM$

Similarly, $ar(\Delta BDE) = \frac{1}{2} \times BD \times EN$

$ar(\Delta DEC) = \frac{1}{2} \times EC \times DM$

$\therefore \frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times BD \times EN} = \frac{AD}{BD}$ ... (1)

And, $\frac{ar(ADE)}{ar(DEC)} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC}$ ... (2)

ΔBDE and ΔDEC are on the same base and between the same parallels.

∴ ar (ΔBDE) = ar (ΔDEC)

From (1) and (2), we obtain

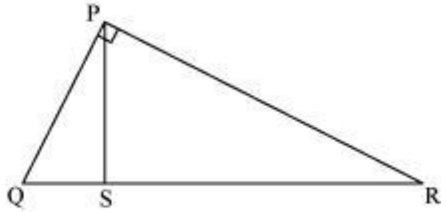
$\frac{AD}{BD} = \frac{AE}{EC}$

**OR**

To prove: In a right-angled triangle, the square of the hypotenuse is equal to the sum of squares of the other two sides of the triangle.

Proof: Let PQR be a triangle, right-angled at P.

Draw $PS \perp QR$



Now, we know that if a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, then the triangles on the both sides of the perpendicular are similar to the whole triangle and to each other.

∴ΔQSP ~ ΔQPR

Therefore, $\frac{QS}{QP}=\frac{QP}{QR}$ (Since the sides of similar triangles are proportional)

⇒ $QS.QR = QP^2$ … (1)

Also, we have

ΔPSR ~ ΔQPR

Therefore, $\frac{RS}{RP}=\frac{RP}{RQ}$ (Since the sides of similar triangles are proportional)

⇒ $RS.RQ = RP^2$ … (2)

Adding equations (1) and (2), we obtain

$QS.QR + RS.RQ = RP^2 + QP^2$

⇒ $QR.(QS + RS) = RP^2 + QP^2$

⇒ $QR.QR = RP^2 + QP^2$

⇒ $QR^2 = RP^2 + QP^2$

Thus, the square of the hypotenuse is equal to the sum of squares of the other two sides of the triangle.

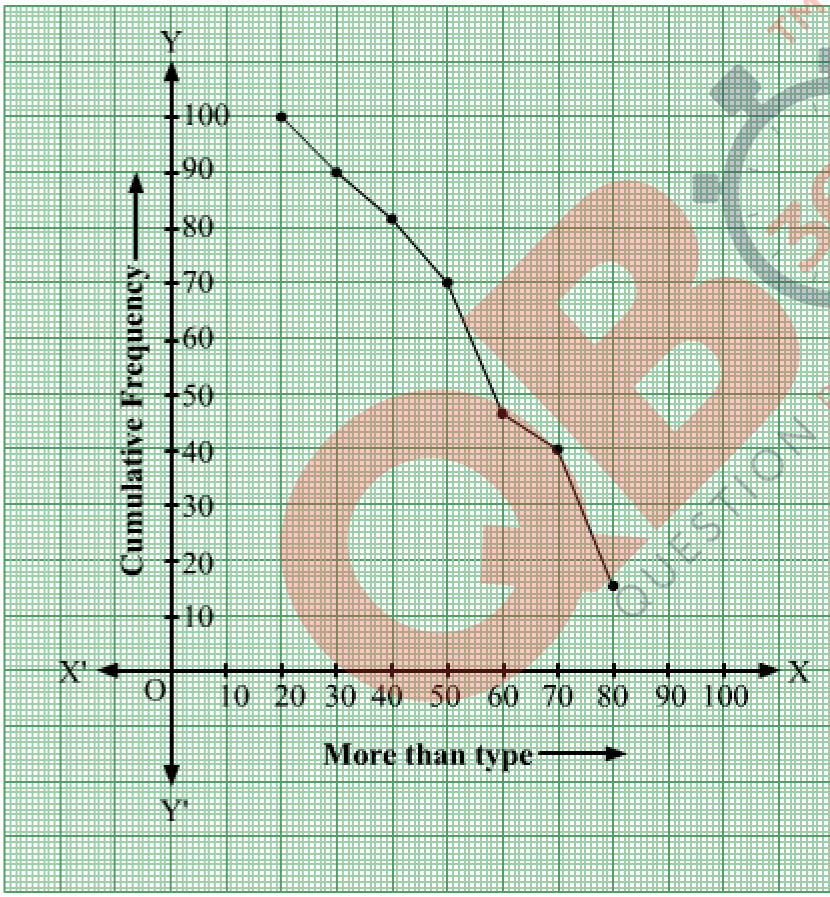
**Question 25**

Change the following distribution to a 'more than type' distribution. Hence draw the 'more than type' ogive for this distribution.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Class interval : | 20−30 | 30−40 | 40−50 | 50−60 | 60−70 | 70−80 | 80−90 |
| Frequency : | 10 | 8 | 12 | 24 | 6 | 25 | 15 |

**SOLUTION:**

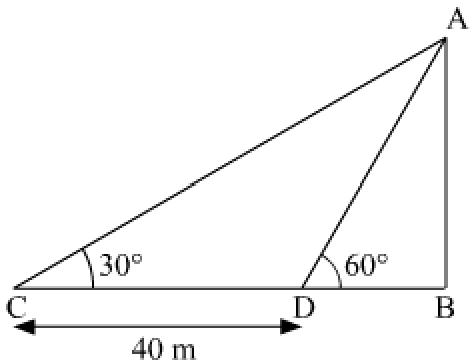
| Class interval | Cumulative Frequency |
|---|---|
| More than 20 | 100 |
| More than 30 | 90 |
| More than 40 | 82 |
| More than 50 | 70 |
| More than 60 | 46 |
| More than 70 | 40 |
| More than 80 | 15 |




**Question 26**

The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30° than when it was 60°. Find the height of the tower.
(Given $\sqrt{3}=1\cdot732$)

**SOLUTION:**




Let AB is the tower and BD is the length of the shadow when sun's altitude is 60°°.
Let, AB be *x* m and BD be *y* m.

So, CB=$(40+y)$ m
Now, we have two right angled triangles $\Delta$ABD and $\Delta$ABC
In ABD
$\tan 60^\circ = \frac{AB}{BD}$
$\sqrt{3} = \frac{x}{y}$

In ABC
$\tan 30^\circ = \frac{AB}{BD}$
$\frac{1}{\sqrt{3}} = \frac{x}{y+40} \ldots (1)$
We have $x = y\sqrt{3}$
substituting the value in $(1)$
$(y\sqrt{3})\sqrt{3} = y + 40$
$\Rightarrow 3y = y + 40$
$\Rightarrow y = 20$
$\Rightarrow x = 20\sqrt{3}\text{m} = 34.64\text{m}$
Hence, the height of the tower is $20\sqrt{3}$m.

**Question 27**
If *m* times the $m^{th}$ term of an Arithmetic Progression is equal to *n* times its $n^{th}$ term and $m \neq n$, show that the $(m + n)^{th}$ term of the A.P. is zero.

**OR**

The sum of the first three numbers in an Arithmetic Progression is 18. If the product of the first and the third term is 5 times the common difference, find the three numbers.

**SOLUTION:**

Let *a* be the first term and *d* is the common difference of an A.P

$a_m$ and $a_n$ be the *m*th and *n*th term respectively.

We have given *m* times the *m*th term is equal to *n* times the *n*th term

So, equation becomes

$m \times a_m = n \times a_n$

We know that $a_m = a + (m-1)d$

Similarly, $a_n = a + (n-1)d$

$m[a + (m-1)d] = n[a + (n-1)d]$

$m[a + (m-1)d] - n[a + (n-1)d] = 0$

$\Rightarrow am + m(m-1)d - an - n(n-1)d = 0$

$\Rightarrow a(m-n) + \left[d\left(m^2 - n^2\right) - d(m-n)\right] = 0$

$\Rightarrow a(m-n) + d[(m+n)(m-n) - (m-n)] = 0$

$\Rightarrow (m-n)[a + d((m+n) - 1)] = 0$

$\Rightarrow a + [(m+n) - 1]d = 0$

$\Rightarrow a_{m+n} = 0$

**OR**

Let the first three terms be *a-d*, *a*, *a+d*

We have been given that the sum of first three terms of an A.P is 18

Equation becomes

$a - d + a + a + d = 18$

$3a = 18$

$\Rightarrow a = 6$

Also, we have given the product of first and third term is 5 times the common difference

$(a-d)(a+d) = 5d$

$a^2 - d^2 = 5d$

$\Rightarrow a^2 = 5d + d^2 \quad (\because a = 6)$

$\Rightarrow d^2 + 5d = 36$

$\Rightarrow d^2 + 5d - 36 = 0$
$d^2 + 9d - 4d - 36 = 0$
$\Rightarrow d(d+9) - 4(d+9) = 0$
$\Rightarrow (d-4)(d+9) = 0$
$\Rightarrow d = 4, -9$

When $d$=4

First three numbers will be $6-4, 6, 6+4$

$\Rightarrow 2, 6, 10$

When $d$= - 9

First three numbers will be $6-(-9), 6, 6+(-9)$

$\Rightarrow 15, 6, -3$

**Question 28**

A shopkeeper buys a number of books for ₹ 80. If he had bought 4 more books for the same amount, each book would have cost ₹ 1 less. How many books did he buy?

**SOLUTION:**

Let the number of books purchased by the shopkeeper be *x*.

Cost price of *x* books = Rs 80

$\therefore$ Original cost price of one book = Rs $\frac{80}{x}$

If the shopkeeper had purchased 4 more books, then the number of books purchased by him would be (*x* + 4).

$\therefore$ New cost price of one book = Rs $\frac{80}{x+4}$

Given, Original cost price of one book − New cost price of one book = Rs 1

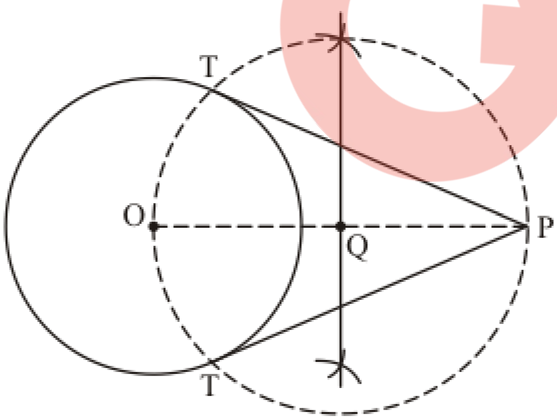
$$\therefore \frac{80}{x} - \frac{80}{x+4} = 1$$
$$\Rightarrow \frac{80(x+4) - 80x}{x(x+4)} = 1$$
$$\Rightarrow 80x + 320 - 80x = x(x+4)$$
$$\Rightarrow x^2 + 4x = 320$$
$$\Rightarrow x^2 + 4x - 320 = 0$$
$$\Rightarrow x^2 + 20x - 16x - 320 = 0$$
$$\Rightarrow x(x+20) - 16(x+20) = 0$$
$$\Rightarrow (x-16)(x+20) = 0$$
$$\Rightarrow x - 16 = 0 \text{ or } x + 20 = 0$$
$$\Rightarrow x - 16 = 0 \text{ or } x + 20 = 0$$
$$\Rightarrow x = 16 \text{ or } x = -20$$
$$\therefore x = 16 \qquad (\because \text{Number of books cannot be negative})$$

Thus, the number of books purchased by the shopkeeper is 16.

**Question 29**

Construct a pair of tangents to a circle of radius 4 cm from a point which is at a distance of 6 cm from its centre.

**SOLUTION:**




**Step of construction**

**Step: I-** First of all we draw a circle of radius AB = 4 cm.

**Step: II-** Mark a point P from the centre at a distance of 6 cm from the point O.

**Step: III -**Draw a right bisector of OP, intersecting OP at Q.

**Step: IV-** Taking Q as centre and radius OQ = PQ, draw a circle to intersect the given circle at T and T'.

**Step: V-** Join PT and PT' to obtain the required tangents.

Thus, PT and PT' are the required tangents.

## Question 30

Prove the following:

$$\frac{1}{1+\sin^2\theta}+\frac{1}{1+\cos^2\theta}+\frac{1}{1+\sec^2\theta}+\frac{1}{1+\mathrm{cosec}^2\theta}=2$$

## SOLUTION:

$$\frac{1}{1+\sin^2\theta}+\frac{1}{1+\cos^2\theta}+\frac{1}{1+sec^2\theta}+\frac{1}{1+\cos ec^2\theta}$$

$$=\frac{1}{1+\sin^2\theta}+\frac{1}{1+\cos^2\theta}+\frac{1}{1+\frac{1}{\cos^2\theta}}+\frac{1}{1+\frac{1}{\sin^2\theta}} \qquad \left(\because \cos\theta=\frac{1}{\sec\theta} \text{ and } \sin\theta=\frac{1}{\cos ec\theta}\right)$$

$$=\frac{1}{1+\sin^2\theta}+\frac{1}{1+\cos^2\theta}+\frac{\cos^2\theta}{1+\cos^2\theta}+\frac{\sin^2\theta}{1+\sin^2\theta}$$

$\text{Taking L.C.M}$

$$=\frac{(1+\cos^2\theta)+(1+\sin^2\theta)+(1+\sin^2\theta)(\cos^2\theta)+(\sin^2\theta)(1+\cos^2\theta)}{(1+\sin^2\theta)(1+\cos^2\theta)}$$

$$=\frac{1+\cos^2\theta+1+\sin^2\theta+\cos^2\theta+\sin^2\theta\cos^2\theta+\sin^2\theta+\sin^2\theta\cos^2\theta}{(1+\sin^2\theta)(1+\cos^2\theta)} \qquad (\because \sin^2\theta+\cos^2\theta=1)$$

$$=\frac{4+2\sin^2\theta\cos^2\theta}{1+\sin^2\theta+\cos^2\theta+\sin^2\theta\cos^2\theta}$$

$$=\frac{4+2\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}$$

$\text{Taking 2 as common factor}$

$$=\frac{2(2+\sin^2\theta\cos^2\theta)}{2+\sin^2\theta\cos^2\theta}=2.$$

$\text{R.H.S}$

Hence, proved