Class X
(CBSE 2019)
Mathematics
Abroad (Set-3)

General Instructions:

(i) **All** questions are compulsory.

(ii) The question paper consists of **30** questions divided into four sections **– A, B, C** and **D**.

(iii) Section **A** comprises **6** questions of **1** mark each. Section **B** contains **6** questions of **2** marks each. Section **C** contains **10** questions of **3** marks each. Section **D** contains **8** questions of 4 marks each.

(iv) There is no overall choice. However, an internal choice has been provided in **two** questions of **1** mark, **two** questions of **2** marks, **four** questions of **3** marks each and **three** questions of **4** marks each. You have to attempt only **one** of the alternative in all such questions.

(v) Use of calculators is **not** permitted.

Question 1

Which term of the A.P. −4, −1, 2, ... is 101?

SOLUTION:

We have been given an arithmetic progression where
$a = -4,\ d = -1 - (-4) = 3$ and $a_n = 101$
We need to find which term of the given AP is 101 so, we need to find n.
Using $a_n = a + (n - 1)d$
Substituting the values in the formula we get

$$101 = -4 + (n - 1)3$$
$$101 + 7 = 3n$$
$$3n = 108$$
$$n = 36$$

Therefore, 36th term of given A.P is 101.

Question 2

Evaluate:

$\frac{\tan 65^{\circ}}{\cot 25^{\circ}}$

OR

Express (sin 67° + cos 75°) in terms of trigonometric ratios of the angle between 0° and 45°.

SOLUTION:

$\frac{\tan 65^{\circ}}{\cot 25^{\circ}}$

$= \frac{\tan(90^{\circ}-25^{\circ})}{\cot 25^{\circ}} \quad (\because \ \tan(90^{\circ}-\theta) = \cot\theta)$

$= \frac{\cot 25^{\circ}}{\cot 25^{\circ}}$

$= 1$

OR

(sin67°+cos75°)

=(sin(90°−23°) + cos(90°−25°)) (∵ sin(90°−θ) =cosθ and cos(90°−θ) = sinθ) = (cos23°+sin25°)

Question 3

Find the value of k for which the quadratic equation $kx(x-2)+6=0$ has two equal roots.

SOLUTION:

Given quadratic equation is:

$kx(x-2)+6=0$

$\Rightarrow kx^2 - 2kx + 6 = 0$

For a quadratic equation to have equal roots,

$b^2 - 4ac = 0$

Comparing the given equation with general equation $ax^2 + bx + c = 0$

We get $a = k$, $b = -2k$ and $c = 6$

$(-2k)^2 - 4(k)(6) = 0$

$\Rightarrow 4k^2 - 24k = 0$

$\Rightarrow 4k(k-6) = 0$

Therefore, $k = 0$ and $k = 6$

Question 4

Find a rational number between √2 and √7.

OR

Write the number of zeroes in the end of a number whose prime factorization is $2^2 \times 5^3 \times 3^2 \times 17$.

SOLUTION:

We know

$\sqrt{2} = 1.414$

$\sqrt{7} = 1.732$

So, rational number between $\sqrt{2}$ and $\sqrt{7}$ will be $1.5 = \frac{3}{2}$.

OR

Given prime factorisation is $2^2 \times 5^3 \times 3^2 \times 17$.

A number will have zero at the end when we have 2×5.

In $2^2 \times 5^3 \times 3^2 \times 17$ we will have 2 zeroes as $(2^2 \times 5^2) \times 5 \times 3^2 \times 17$.

Question 5

Find the distance between the points (*a*, *b*) and (−*a*, −*b*).

SOLUTION:

Using distance formula:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Here, $x_1 = a,\ y_1 = b,\ x_2 = -a$ and $y_2 = -b$

On substituting the values in the formula we get

$\sqrt{(-a-a)^2 + (-b-b)^2}$

$= \sqrt{(-2a)^2 + (-2b)^2}$

$= \sqrt{4a^2 + 4b^2}$

$= 2\sqrt{a^2 + b^2}$

Therefore, the distance between $(a,\ b)$ and $(-a, -b)$ is $2\sqrt{(a)^2 + (b)^2}$

Question 6

Let ∆ ABC ∽ ∆ DEF and their areas be respectively, 64 cm^2 and 121 cm^2. If EF = 15·4 cm, find BC.

SOLUTION:

Given: $\triangle ABC \sim \triangle DEF$
We know ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$\frac{ar\triangle ABC}{ar\triangle DEF} = \left(\frac{BC}{EF}\right)^2$

$\Rightarrow \frac{64}{121} = \left(\frac{BC}{15.4}\right)^2$

$\Rightarrow \left(\frac{8}{11}\right)^2 = \left(\frac{BC}{15.4}\right)^2$

$\Rightarrow \frac{8}{11} = \frac{BC}{15.4}$

$\Rightarrow BC = \frac{8\times15.4}{11} = 11.2 \text{ cm}$

Thus, BC = 11.2 cm.

Question 7

Find the solution of the pair of equation :

$\frac{3}{x} + \frac{8}{y} = -1;\ \frac{1}{x} - \frac{2}{y} = 2,\ x,\ y \neq 0$

OR

Find the value(s) of *k* for which the pair of equations $\begin{cases} kx + 2y = 3 \\ 3x + 6y = 10 \end{cases}$ has a unique solution.

SOLUTION:

The given equations are

$\frac{3}{x} + \frac{8}{y} = -1 \qquad \ldots\ldots(1)$

$\frac{1}{x} - \frac{2}{y} = 2 \qquad \ldots\ldots(2)$

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$

(1) and (2) will become

$3u + 8v = -1 \qquad \ldots\ldots(3)$

$u - 2v = 2 \qquad \ldots\ldots(4)$

Multiply (4) with 4

$4u - 8v = 8 \qquad \ldots\ldots(5)$

Adding (3) and (5) we get

$7u = 7$

$\Rightarrow u = 1$

Putting this value in (4)

$1 - 2v = 2$

$\Rightarrow v = \frac{-1}{2}$

Now

$\frac{1}{x} = u$

$\Rightarrow \frac{1}{x} = 1$

$\Rightarrow x = 1$

And

$\frac{1}{y} = v$

$\Rightarrow \frac{1}{y} = \frac{-1}{2}$

$\Rightarrow y = -2$

OR

The given equations are

$kx + 2y = 3$

$3x + 6y = 10$

For a unique solution,

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

where $a_1 = k,\ a_2 = 3,\ b_1 = 2,\ b_2 = 6$

$\frac{k}{3} \neq \frac{2}{6}$

$\Rightarrow k \neq 1$

For all values of k except 1, the given linear equations will have unique solution.

Question 8

Use Euclid's division algorithm to find the HCF of 255 and 867.

SOLUTION:

The given numbers are 255 and 867.

Now 867 > 255. So, on applying Euclid's algorithm we get

867=255×3+102

Now the remainder is not 0 so, we repeat the process again on 255 and 102

255=102×2+51

The algorithm is applied again but this time on the numbers 102 and 51

102=51×2+0

Thus, the HCF obtained is 51.

Question 9

The point *R* divides the line segment AB, where A (−4, 0) and B (0, 6) such that $AR = \frac{3}{4} AB$. Find the coordinates of *R*.

SOLUTION:

We have given that R divides the line segment AB

AR+ RB= AB

$\frac{3}{4}\text{AB} + \text{RB} = \text{AB}$

$\Rightarrow \text{RB} = \frac{\text{AB}}{4}$

$\Rightarrow \text{AR} : \text{RB} = 3 : 1$

Using section formula:

$x = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}\right),\ y = \left(\frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

$m_1 = 3,\ m_2 = 1$

$x_1 = -4,\ y_1 = 0$

$x_2 = 0,\ y_2 = 6$

Plugging values in the formula we get

$x = \frac{3\times0+1\times(-4)}{3+1},\ y = \frac{3\times6+1\times0}{3+1}$

$x = \frac{-4}{4},\ y = \frac{18}{4}$

$\Rightarrow x = -1,\ y = \frac{9}{2}$

Therefore, the coordinates of R$\left(-1, \frac{9}{2}\right)$.

Question 10

How many multiples of 4 lie between 10 and 205 ?

OR

Determine the A.P. whose third term is 16 and 7th term exceeds the 5th by 12.

SOLUTION:

We need to find the number of multiples of 4 between 10 and 205.

So, multiples of 4 gives the sequence 12, 16, ..., 204

a = 12, d = 4 and $a_n = 204$

Using the formula $a_n = a + (n-1)d$

Plugging values in the formula we get

$$204 = 12 + (n - 1)4$$
$$204 = 12 + 4n - 4$$
$$4n = 196$$
$$n = 49$$

Thus, there are 49 multiples of 4 between 10 and 205.

OR

Given: 3rd term of the AP is 16.

$a_3 = 16$

$a + (3 - 1)d = 16$

$a + 2d = 16$ .....(1)

Also, 7th term exceeds the 5th term by 12.

$a_7 - a_5 = 12$

$[a + (7 - 1)d] - [a + (5 - 1)d] = 12$

$(a + 6d) - (a + 4d) = 12$

$2d = 12$

$d = 6$

From equation (1), we obtain

$a + 2(6) = 16$

$a + 12 = 16$

$a = 4$

Therefore, A.P. will be 4, 10, 16, 22, …

Question 11

Three different coins are tossed simultaneously. Find the probability of getting exactly one head.

SOLUTION:

Total possible outcomes of tossing three coins simultaneously are $\{TTT, TTH, THT, THH, HTT, HTH, HHT, HHH\}$

that is 8

We have to find the probability of getting exactly one head.

Cases of exactly one head are $\{TTH, THT, HTT\}$

that is 3

Probability of getting exactly on head is $\frac{3}{8}$.

Question 12

A die is thrown once. Find the probability of getting.
(a) a prime number.
(b) an odd number

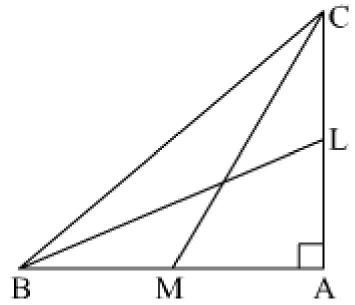
SOLUTION:

Total outcomes of throwing a dice once are 1, 2, 3, 4, 5 and 6

(1) Probability of getting a prime number
Prime numbers are 2, 3 and 5 in throwing a die once
Probability of getting a prime number $= \frac{3}{6} = \frac{1}{2}$
(2) Probability of getting an odd number
odd numbers are those that are not divisible by 2.
So, there three odd numbers in throwing a dice once which is 1, 3 and 5.
Probability of getting an odd number $= \frac{3}{6} = \frac{1}{2}$

Question 13

In Figure 1, BL and CM are medians of a ΔABC right-angled at A. Prove that $4(BL^2 + CM^2) = 5BC^2$.



OR

Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

SOLUTION:

To prove: $4\left(BL^2+CM^2\right)=5\,BC^2$

Proof: In $\triangle$CAB,

Applying Pythagoras theorem,

$AB^2+AC^2=BC^2$ .....(1)

In $\triangle$ABL,

$AL^2+AB^2=BL^2$

$\Rightarrow\left(\frac{AC}{2}\right)^2+AB^2=BL^2$

$\Rightarrow AC^2+4\,AB^2=4\,BL^2$ .....(2)

In $\triangle$CAM,

$CA^2+MA^2=CM^2$

$\Rightarrow\left(\frac{BA}{2}\right)^2+CA^2=CM^2$

$\Rightarrow BA^2+4\,CA^2=4\,CM^2$ .....(3)

Adding (2) and (3)

$AC^2+4\,AB^2+BA^2+4\,CA^2=4\,BL^2+4\,CM^2$

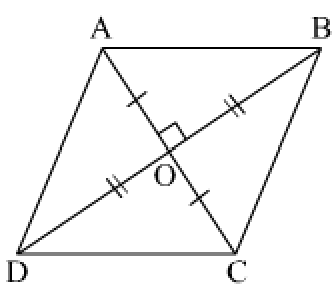
$\Rightarrow 5\,AC^2+5\,AB^2=4\left(BL^2+CM^2\right)$

$\Rightarrow 5\left(AC^2+AB^2\right)=4\left(BL^2+CM^2\right)$

$\Rightarrow 5\left(BC^2\right)=4\left(BL^2+CM^2\right)$ (From (1))

Hence Proved.

OR



In ΔAOB, ΔBOC, ΔCOD, ΔAOD,

Applying Pythagoras theorem, we obtain

$$AB^2 = AO^2 + OB^2 \quad ...(1)$$
$$BC^2 = BO^2 + OC^2 \quad ...(2)$$
$$CD^2 = CO^2 + OD^2 \quad ...(3)$$
$$AD^2 = AO^2 + OD^2 \quad ...(4)$$

Adding all these equations, we obtain

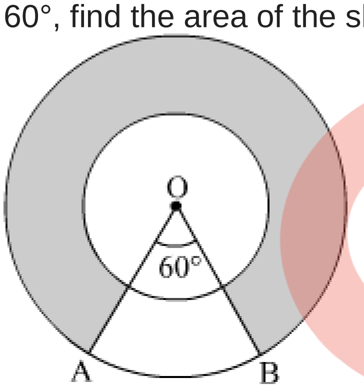
$$AB^2 + BC^2 + CD^2 + AD^2 = 2\left(AO^2 + OB^2 + OC^2 + OD^2\right)$$
$$= 2\left(\left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2 + \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2\right)$$

(Diagonals bisect each other)

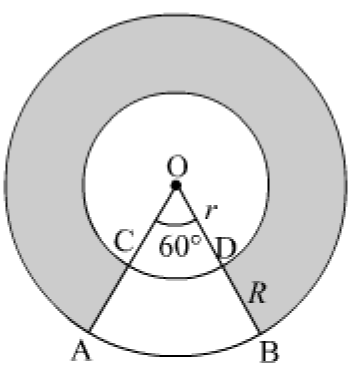
$$= 2\left(\frac{(AC)^2}{2} + \frac{(BD)^2}{2}\right)$$
$$= (AC)^2 + (BD)^2$$

Question 14

In Figure 2, two concentric circles with centre O, have radii 21 cm and 42 cm. If ∠AOB = 60°, find the area of the shaded region.



SOLUTION:



Radius of inner circle, OC = 21 cm

Radius of outer circle, OA = 42 cm

Area of circle with radius $R = \pi R^2 = \pi(42)^2$
Area of circle with radius $r = \pi r^2 = \pi(21)^2$
Area of sector AOB$= \frac{\theta}{360} \times \pi R^2 = \frac{60}{360} \times \pi(42)^2 = \frac{\pi(42)^2}{6}$
Area of sector COD$= \frac{\theta}{360} \times \pi r^2 = \frac{60}{360} \times \pi(21)^2 = \frac{\pi(21)^2}{6}$
Area of shaded portion = Area of circle with radius R − Area of circle with radius *r* − [Area of sector AOB − Area of sector COD]

$$= \pi(42)^2 - \pi(21)^2 - \left[\frac{\pi(42)^2}{6} - \frac{\pi(21)^2}{6}\right]$$
$$= \pi\left[(42)^2 - (21)^2 - \frac{1}{6}\left[(42)^2 - (21)^2\right]\right]$$
$$= \pi\left[\left((42)^2 - (21)^2\right)\left(1 - \frac{1}{6}\right)\right]$$
$$= \pi\left[(42 - 21)(42 + 21)\frac{5}{6}\right]$$
$$= \frac{22}{7} \times \frac{5}{6} \times 21 \times 63$$
$$= 3465 \text{ cm}^2$$

Question 15
A cone of height 24 cm and radius of base 6 cm is made up of modelling clay. A child reshapes it in the form of a sphere. Find the radius of the sphere and hence find the surface area of this sphere.

OR

A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in his field which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/hr, how much time will the tank be filled ?

SOLUTION:

A cone has been reshaped in sphere
Height of cone is 24 cm and radius of base is 6 cm
Volume of sphere = volume of cone
Volume of cone = $\frac{1}{3}\pi r^2 h$
Plugging the values in the formula we get
volume of cone = $\frac{1}{3}\pi(6)^2 24$
$$= 288\pi \text{ cm}^3$$
Let the radius of sphere be *r*
Volume of sphere = $\frac{4}{3}\pi r^3$

Since, volume of cone = volume of sphere

Volume of sphere = $288\pi \text{ cm}^3$

So,

$288\pi = \frac{4}{3}\pi r^3$

$\Rightarrow 288 = \frac{4}{3}r^3$

$\Rightarrow r^3 = 216$

$\Rightarrow r = 6 \text{ cm}$

Hence, radius of reshaped sphere is 6 cm

Now, surface area of sphere = $4\pi r^2$

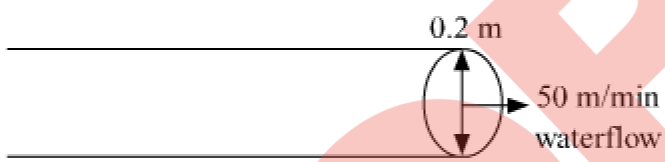
$= 4\pi(6)^2$

$= 144 \times \frac{22}{7}$

$= 452.5 \text{ cm}^2$

Therefore, surface area of sphere is 452.57 cm^2.

OR



Consider an area of cross-section of pipe as shown in the figure.

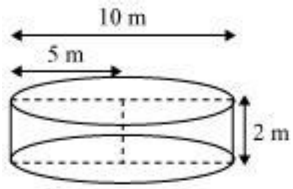
Radius (r_1) of circular end of pipe = $\frac{20}{200} = 0.1\text{m}$

Area of cross-section = $\pi \times r_1^2 = \pi \times (0.1)^2 = 0.01\pi \text{ m}^2$

Speed of water = 3 km/h = $\frac{3000}{60} = 50 \text{ metre/min}$

Volume of water that flows in 1 minute from pipe = 50 × 0.01π = 0.5π m^3

Volume of water that flows in t minutes from pipe = t × 0.5π m^3



Radius (r_2) of circular end of cylindrical tank $= \frac{10}{2} = 5$m

Depth (h_2) of cylindrical tank = 2 m

Let the tank be filled completely in *t* minutes.

Volume of water filled in tank in *t* minutes is equal to the volume of water flowed in *t* minutes from the pipe.

Volume of water that flows in *t* minutes from pipe = Volume of water in tank

$t \times 0.5\pi = \pi \times (r_2)^2 \times h_2$

$t \times 0.5 = 5^2 \times 2$

$t = 100$

Therefore, the cylindrical tank will be filled in 100 minutes.

Question 16

Calculate the mode of the following distribution :

Class :	10 − 15	15 − 20	20 − 25	25 − 30	30 − 35
Frequency :	4	7	20	8	1

SOLUTION:

Modal class is the class with highest frequency
modal class is 20 - 25
lower limit of modal class i.e l = 20
class size i.e h = 5
frequency of modal class $f_1 = 20$
frequency of preceding class $f_0 = 7$
frequency of succeeding class $f_2 = 8$
Using the formula

$$\text{mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

Plugging the values in the formula we get

$$\text{mode} = 20 + \left(\frac{20-7}{2\times20-7-8}\right) \times 5$$
$$\text{mode} = 20 + \left(\frac{13}{25}\right) \times 5$$
$$\text{mode} = 20 + \frac{13}{5}$$
$$\text{mode} = \frac{113}{5} = 22.6$$

Question 17

Show that $\frac{2+3\sqrt{2}}{7}$ is not a rational number, given that $\sqrt{2}$ is an irrational number.

SOLUTION:

To prove: $\frac{2+3\sqrt{2}}{7}$ is irrational, let us assume that $\frac{2+3\sqrt{2}}{7}$ is rational.

$\frac{2+3\sqrt{2}}{7} = \frac{a}{b}$; $b \neq 0$ and a and b are integers.

$\Rightarrow 2b + 3\sqrt{2}b = 7a$

$\Rightarrow 3\sqrt{2}b = 7a - 2b$

$\Rightarrow \sqrt{2} = \frac{7a-2b}{3b}$

Since a and b are integers so, $7a - 2b$ will also be an integer.

So, $\frac{7a-2b}{3b}$ will be rational which means $\sqrt{2}$ is also rational.

But we know $\sqrt{2}$ is irrational(given).

Thus, a contradiction has risen because of incorrect assumption.

Thus, $\frac{2+3\sqrt{2}}{7}$ is irrational.

Question 18

Obtain all the zeroes of the polynomial $2x^4 - 5x^3 - 11x^2 + 20x + 12$ when 2 and − 2 are two zeroes of the above polynomial

SOLUTION:

We know that if $x=\alpha$ is a zero of a polynomial, and then $x-\alpha$ is a factor of $f(x)$.

Since 2 and -2 are zeros of $f(x)$.

Therefore

$(x-2)(x+2) = x^2 - 4$

$(x^2 - 4)$ is a factor of $f(x)$. Now, we divide $2x^4 - 5x^3 - 11x^2 + 20x + 12$ by $g(x) = (x^2 - 4)$ to find the zero of $f(x)$.

$$\begin{array}{r|l} & 2x^2-5x-3 \\ x^2-4 & 2x^4-5x^3-11x^2+20x+12 \\ & 2x^4+0x^3-8x^2 \\ & -\quad -\quad + \\ \hline & -5x^3-3x^2+20x+12 \\ & -5x^3+0x^2+20x \\ & +\quad -\quad - \\ \hline & -3x^2 \quad +12 \\ & -3x^2 \quad +12 \\ & + \quad - \\ \hline & 0 \\ \hline \end{array}$$

By using division algorithm we have $f(x)=g(x)\times q(x)-r(x)$

$$2x^4-5x^3-11x^2+20x+12=(x^2-4)(2x^2-5x-3)$$
$$=(x-2)(x+2)[2x(x-3)+(x-3)]$$
$$=(x-2)(x+2)(x-3)(2x+1)$$

Hence, the zeros of the given polynomial are 2, $-2,\ 3,\ \frac{-1}{2}$.

Question 19

A motorboat whose speed is 18 km/hr in still water takes on hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

SOLUTION:

Speed of the motorboat is 18 km/h

Let us assume the speed of th stream be *x* km/h

speed of motorboat upstream is $(18-x)\ \mathrm{km/h}$

speed of motorboat downstream is $(18+x)\ \mathrm{km/h}$

Time taken to go downstream is $\left(\frac{24}{18+x}\right)$ hr

Time taken to go upstream is $\left(\frac{24}{18-x}\right)$ hr

Equation becomes:

$$\frac{24}{18-x}-\frac{24}{18+x}=1$$

Solving the above equation

$$\frac{24(18+x)-24(18-x)}{(18-x)(18+x)}=1$$

$$\frac{432+24x-432+24x}{324-x^2}=1$$

$$\frac{48x}{324-x^2}=1$$

$$48x = 324 - x^2$$
$$x^2 + 48x - 324 = 0$$

Solving for the value of x

using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-48 \pm \sqrt{(48)^2 - 4(1)(-324)}}{2}$$
$$x = \frac{-48 \pm \sqrt{2304 + 1296}}{2}$$
$$x = \frac{-48 \pm \sqrt{3600}}{2}$$
$$x = \frac{-48 \pm 60}{2}$$
$$x = \frac{12}{2}, \frac{-108}{2}$$
$$x = 6, -54$$

Since, speed can not be negative
So, the speed of the stream is 6 km/h.

Question 20

Prove that:

(sin θ + 1 + cos θ) (sin θ - 1 + cos θ) . sec θ cosec θ = 2

OR

Prove that :

$$\sqrt{\frac{\sec\ \theta - 1}{\sec\ \theta + 1}} + \sqrt{\frac{\sec\ \theta + 1}{\sec\ \theta - 1}} = 2\,\text{cosec}\ \theta$$

SOLUTION:

$\text{LHS} = (\sin\theta + 1 + \cos\theta)\,(\sin\theta - 1 + \cos\theta).\sec\theta\text{cosec}\theta$

$= [\sin^2\theta - \sin\theta + \sin\theta\cos\theta + \sin\theta - 1 + \cos\theta + \sin\theta\cos\theta - \cos\theta + \cos^2\theta]\,\frac{1}{\cos\theta}\,\frac{1}{\sin\theta} \quad (\because\ \sec\theta = \frac{1}{\cos\theta}\text{ and cosec}\theta = \frac{1}{\sin\theta})$

$= [1 + 2\sin\theta\cos\theta - 1]\frac{1}{\cos\theta}\,\frac{1}{\sin\theta}$

$= [2\sin\theta\cos\theta]\frac{1}{\cos\theta}\,\frac{1}{\sin\theta}$

$= 2 = \text{RHS}$

Hence proved

OR

$$\sqrt{\frac{\sec\theta-1}{\sec\theta+1}}+\sqrt{\frac{\sec\theta+1}{\sec\theta-1}}=\frac{\sqrt{\sec\theta-1}}{\sqrt{\sec\theta+1}}+\frac{\sqrt{\sec\theta+1}}{\sqrt{\sec\theta-1}}$$

$$=\frac{\sqrt{\sec\theta-1}\sqrt{\sec\theta-1}+\sqrt{\sec\theta+1}\sqrt{\sec\theta+1}}{\sqrt{\sec\theta+1}\sqrt{\sec\theta-1}}$$

$$=\frac{\left(\sqrt{\sec\theta-1}\right)^2+\left(\sqrt{\sec\theta+1}\right)^2}{\sqrt{(\sec\theta-1)(\sec\theta+1)}}$$

$$=\frac{\sec\theta-1+\sec\theta+1}{\sqrt{\sec^2\theta-1}}$$

$$=\frac{2\sec\theta}{\sqrt{\tan^2\theta}}$$

$$=\frac{2\sec\theta}{\tan\theta}$$

$$=\frac{2\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}$$

$$=2\frac{1}{\sin\theta}$$

$$=2\operatorname{cosec}\theta$$

Question 21

In what ratio does the point P(−4, y) divide the line segment joining the points A(−6, 10) and B(3, −8) ? Hence find the value of y.

OR

Find the value of p for which the points (−5, 1), (1, p) and (4, −2) are collinear.

SOLUTION:

Let P divides the line segment AB in the ratio *k* : 1
Using section formula

$$x=\frac{m_1x_2+m_2x_1}{m_1+m_2}, y=\frac{m_1y_2+m_2y_1}{m_1+m_2}$$

$$\text{A}(-6,\ 10)\text{ and B}(3,\ -8)$$
$$m_1 : m_2 = k : 1$$

plugging values in the formula we get

$-4 = \frac{k\times3+1\times(-6)}{k+1}, \; y = \frac{k\times(-8)+1\times10}{k+1}$

$-4 = \frac{3k-6}{k+1}, \; y = \frac{-8k+10}{k+1}$

Considering only *x* coordinate to find the value of *k*

$-4k - 4 = 3k - 6$

$-7k = -2$

$k = \frac{2}{7}$

$k : 1 = 2 : 7$

Now, we have to find the value of *y*

so, we will use section formula only in *y* coordinate to find the value of *y*

$y = \frac{2\times(-8)+7\times10}{2+7}$

$y = \frac{-16+70}{9}$

$y = 6$

Therefore, P divides the line segment AB in 2 : 7 ratio

And value of *y* is 6.

OR

Points are collinear means the area of triangle formed by the collinear points is 0.

Using

area of triangle = $\frac{1}{2}\left[x_1\left(y_2-y_3\right)+x_2\left(y_3-y_1\right)+x_3\left(y_1-y_2\right)\right]$

$= \frac{1}{2}\left[-5\left(p-(-2)\right)+1\left(-2-1\right)+4\left(1-p\right)\right]$

$= \frac{1}{2}\left[-5\left(p+2\right)+1\left(-3\right)+4\left(1-p\right)\right]$

$= \frac{1}{2}\left[-5p-10-3+4-4p\right]$

$= \frac{1}{2}\left[-5p-9-4p\right]$

Area of triangle will be zero points being collinear

$\frac{1}{2}\left[-5p-4p-9\right]=0$

$\frac{1}{2}\left[-9p-9\right]=0$

$9p+9=0$

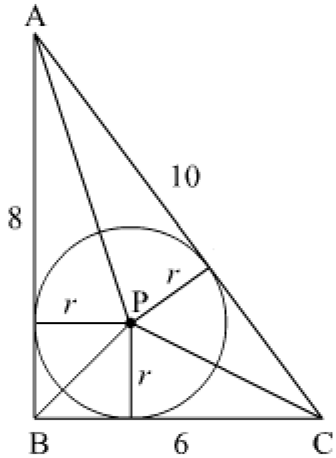
$p=-1$

Therefore, the value of $p = -1$.

Question 22

ABC is a right triangle in which ∠B = 90°. If AB = 8 cm and BC = 6 cm, find the diameter of the circle inscribed in the triangle.

SOLUTION:



We have given that a circle is inscribed in a triangle

Using pythagoras theorem

$(AC)^2 = (AB)^2 + (BC)^2$

$(AC)^2 = (8)^2 + (6)^2$

$(AC)^2 = 64 + 36$

$(AC)^2 = 100$

$\Rightarrow AC = 10$

Area of $\triangle$ ABC $=$ area of $\triangle$ APB $+$ area of $\triangle$ BPC $+$ area of $\triangle$ APC

$\frac{1}{2} \times b \times h = \frac{1}{2} \times b_1 \times h_1 + \frac{1}{2} \times b_2 \times h_2 + \frac{1}{2} \times b_3 \times h_3$

$\frac{1}{2} \times 6 \times 8 = \frac{1}{2} \times 8 \times r + \frac{1}{2} \times 6 \times r + \frac{1}{2} \times 10 \times r$

$24 = 4r + 3r + 5r$

$24 = 12r$

$\Rightarrow r = 2$

$\because \ d = 2r$

$\Rightarrow d = 2 \times 2$

$\Rightarrow d = 4 \text{ cm}$

Question 23

In an A.P., the first term is −4, the last term is 29 and the sum of all its terms is 150. Find its common difference

SOLUTION:

$a = -4,\ l = 29$ and $S_n = 150$

Using

$S_n = \frac{n}{2}[a + l]$

$150 = \frac{n}{2}[-4 + 29]$

$300 = 25n$

$n = 12$

Now, we will find d

using $a_n = a + (n-1)d$

Plugging the values in the formula we get

$29 = -4 + (12 - 1)d$

$33 = 11d$

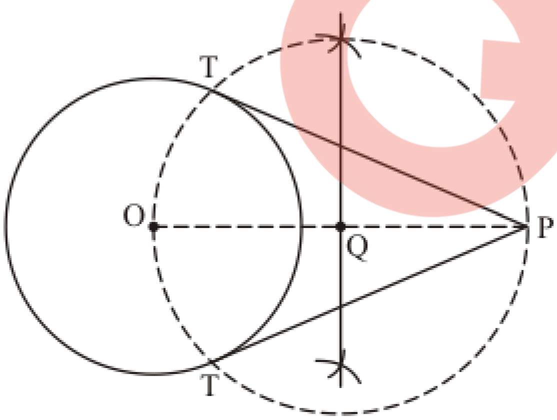
$d = 3$

Therefore, the common difference is 3.

Question 24

Draw a circle of radius 4 cm. From a point 6 cm away from its centre, construct a pair of tangents to the circle and measure their lengths

SOLUTION:



Step of construction

Step: I- First of all we draw a circle of radius AB = 4 cm.

Step: II- Mark a point P from the centre at a distance of 6 cm from the point O.

Step: III -Draw a perpendicular bisector of OP, intersecting OP at Q.

Step: IV- Taking Q as centre and radius OQ = PQ, draw a circle to intersect the given circle at T and T’.

Step: V- Join PT and PT’ to obtain the required tangents.

Thus, PT and PT' are the required tangents.

The length of PT = PT' ≈ 4.5 cm

Question 26

Solve for x: $\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x};\ x \neq 0,\ x \neq \frac{-2a-b}{2},\ a,\ b \neq 0$

OR

The sum of the areas of two squares is 640 m^2. If the difference of their perimeters is 64 m, find the sides of the square.

SOLUTION:

$$\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$$
$$\frac{1}{2a+b+2x} - \frac{1}{2x} = \frac{1}{2a} + \frac{1}{b}$$
$$\frac{2x-2a-b-2x}{4ax+2bx+4x^2} = \frac{b+2a}{2ab}$$
$$(-2a-b)(2ab) = (b+2a)\left(4ax+2bx+4x^2\right)$$
$$\frac{-(b+2a)(2ab)}{(b+2a)} = \left(4ax+2bx+4x^2\right)$$
$$-(2ab) = \left(4ax+2bx+4x^2\right)$$
$$4x^2+2bx+4ax+2ab=0$$
$$2x(2x+b)+2a(2x+b)=0$$
$$(2x+2a)(2x+b)=0$$
$$\Rightarrow (2x+2a)=0$$
$$\Rightarrow x=-a$$
or
$$(2x+b)=0$$
$$\Rightarrow x=\frac{-b}{2}$$

Therefore, values of x are $-a$ and $\frac{-b}{2}$

OR

Let the side of one square be *x*
And the side of other square be *y*
Sum of area of two square is 640
Equation becomes

$x^2 + y^2 = 640 \quad (\because \text{ area of square is side}^2) \quad .....(1)$

Now, difference of their perimeters is 64
Equation becomes

$4x - 4y = 64 \quad (\because \text{ perimeter of square is } 4 \times \text{side})$

$x - y = 16$

$\Rightarrow x = y + 16 \quad .....(2)$

Solving the two equation by substitution method
Substitute (2) in (1)

$(16 + y)^2 + y^2 = 640$

$256 + y^2 + 32y + y^2 = 640$

$2y^2 + 32y - 384 = 0$

$y^2 + 16y - 192 = 0$

Using $y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Plugging the values in the formula we get

$y = \frac{-16 \pm \sqrt{256 - 4(-192)}}{2}$

$y = \frac{-16 \pm \sqrt{1024}}{2}$

$y = \frac{-16 \pm 32}{2}$

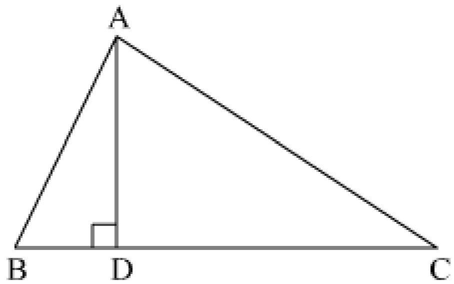
$y = \frac{-48}{2}, \frac{16}{2}$

$y = -24,\ 8$

Since, sides can not be negative
Therefore, *y* = 8
Put *y* = 8 in (2)
x = 8+ 16
x = 24
Therefore, the sides of square are 24 m and 8 m.

Question 27

In Δ ABC (Figure 3), AD $\perp$ BC. Prove that
$AC^2 = AB^2 + BC^2 - 2BC \times BD$



SOLUTION:

Applying Pythagoras theorem in ΔADB, we obtain

$AD^2 + DB^2 = AB^2$

$\Rightarrow AD^2 = AB^2 - DB^2$ .....(1)

Applying Pythagoras theorem in ΔADC, we obtain

$AD^2 + DC^2 = AC^2$

$AB^2 - BD^2 + DC^2 = AC^2$ [Using equation (1)]

$AB^2 - BD^2 + (BC - BD)^2 = AC^2$

$AC^2 = AB^2 - BD^2 + BC^2 + BD^2 - 2BC \times BD$

$AC^2 = AB^2 + BC^2 - 2BC \times BD$

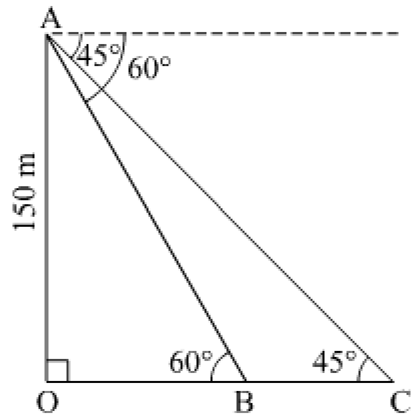
Question 28

A moving boat is observed from the top of a 150 m high cliff moving away from the cliff. The angle of depression of the boat changes from 60° to 45° in 2 minutes. Find the speed of the boat in m/min.

OR

There are two poles, one each on either bank of a river just opposite to each other. One pole is 60 m high. From the top of this pole, the angle of depression of the top and foot of the other pole are 30° and 60° respectively. Find the width of the river and height of the other pole.

SOLUTION:



Let AO be the cliff of height 150 m.
Let the speed of boat be *x* metres per minute.
And BC be the distance which man travelled.

So, $BC = 2x \quad [\because \text{Distance} = \text{Speed} \times \text{Time}]$

$\tan(60^\circ) = \frac{AO}{OB}$

$\sqrt{3} = \frac{150}{OB}$

$\Rightarrow OB = \frac{150\sqrt{3}}{3} = 50\sqrt{3}$

$\tan(45^\circ) = \frac{AO}{OC}$

$\Rightarrow 1 = \frac{150}{OC}$

$\Rightarrow OC = 150$

Now OC = OB + BC

$\Rightarrow 150 = 50\sqrt{3} + 2x$

$\Rightarrow x = \frac{150-50\sqrt{3}}{2}$

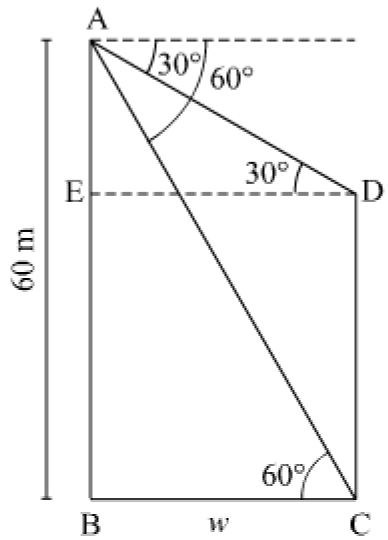
$\Rightarrow x = 75 - 25\sqrt{3}$

Using $\sqrt{3} = 1.73$

$x = 75 - 25 \times 1.732 \approx 32 \text{ m/min}$

Hence, the speed of the boat is 32 metres per minute.

OR



Let the width of the river be *w*.
In △ABC,

$\tan 60^\circ = \frac{AB}{BC}$

$\Rightarrow \sqrt{3} = \frac{60}{w}$

$\Rightarrow w = \frac{60}{\sqrt{3}} = \frac{60\sqrt{3}}{3} = 20\sqrt{3}$

In △AED,

$\tan 30^\circ = \frac{AE}{ED}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AE}{w}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AE}{20\sqrt{3}}$

$\Rightarrow AE = 20$

Height of pole CD = AB − AE

$= 60 - 20 = 40 \text{ m}$

Thus, width of river is $20\sqrt{3} = 20 \times 1.732 = 34.64 \text{ m}$

Height of pole = 40 m

Question 29

Calculate the mean of the following frequency distribution :

Class :	10−30	30−50	50−70	70−90	90−110	110−130
Frequency :	5	8	12	20	3	2

OR

The following table gives production yield in kg per hectare of wheat of 100 farms of a village :

Production yield (kg/hectare) :	40−45	45−50	50−55	55−60	60−65	65−70
Number of farms	4	6	16	20	30	24

Change the distribution to a 'more than type' distribution, and draw its ogive.

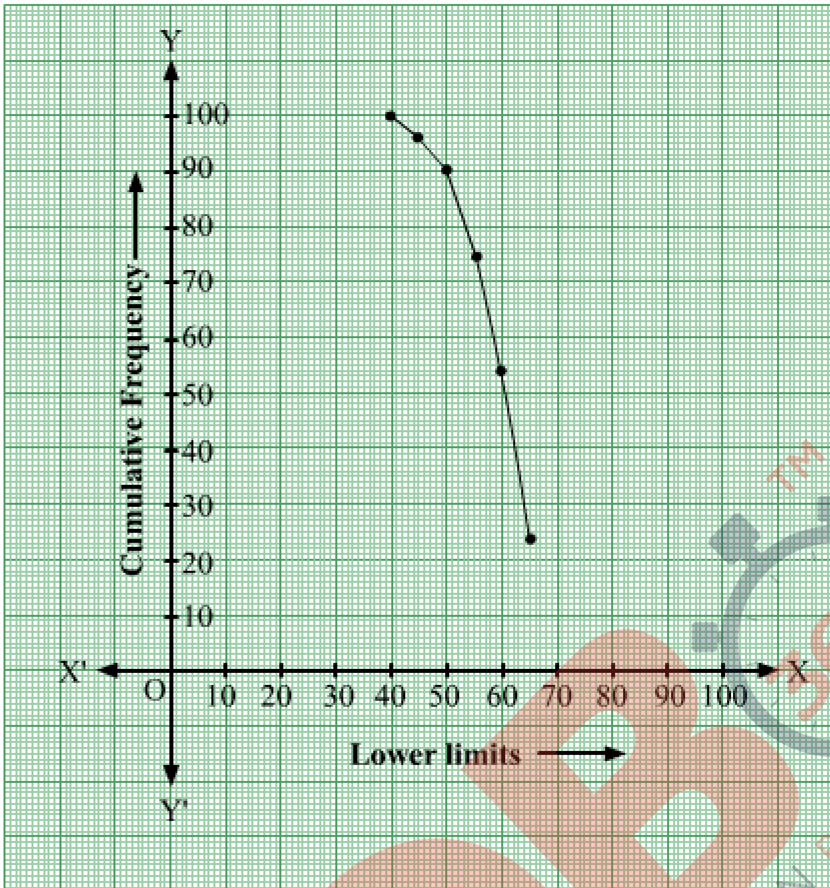
SOLUTION:

Class	frequency (f_i)	Class mark (x_i)	$f_i x_i$
10-30	5	$\frac{10+30}{2} = 20$	100
30-50	8	$\frac{30+50}{2} = 40$	320
50-70	12	$\frac{50+70}{2} = 60$	720
70-90	20	$\frac{70+90}{2} = 80$	1600
90-110	3	$\frac{90+110}{2} = 100$	300
110-130	2	$\frac{110+130}{2} = 120$	240
	$\sum f_i = 50$		$\sum f_i x_i = 3280$

Using: mean = $\frac{\sum f_i x_i}{\sum f_i}$
substituting the values in the formula
mean= $\frac{3280}{50} = 65.6$

OR

Production yield	Cumulative frequency
more than 40	100
more than 45	96
more than 50	90
more than 55	74
more than 60	54
more than 65	24



Question 30

A container opened at the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the container, at the rate of ₹ 50 per litre. Also find the cost of metal sheet used to make the container, if it costs ₹ 10 per 100 cm^2. (Take π = 3·14)

SOLUTION:

We have to find the cost of milk which can completely fill the container
Volume of container = Volume of frustum

$$=\frac{1}{3}\pi h\left(r_1{}^2+r_2{}^2+r_1 r_2\right)$$

Here,

height $= 16$ cm

radius of upper end $= 20$ cm

And radius of lower end $= 8$ cm

Plugging the values in the formula we get

$$\text{Volume of container} = \frac{1}{3} \times 3.14 \times 16\left((20)^2 + (8)^2 + 20 \times 8\right)$$
$$= \frac{1}{3} \times 50.24\,(400 + 64 + 160)$$
$$= \frac{1}{3} \times 50.24\,(624)$$
$$= 10449.92 \text{ cm}^3$$
$$= 10.449 \text{ litre} \qquad (\because 1 \text{ litre} = 1000 \text{ cm}^3)$$

Cost of 1 litre milk is Rs 50
Cost of 10.449 litre milk = 50×10.449 = Rs 522.45
We will find the cost of metal sheet to make the container
Firstly, we will find the area of container
Area of container = Curved surface area of the frustum + area of bottom circle ($\because$ container is closed from bottom)
Area of container = $\pi\,(r_1 + r_2)l + \pi r^2$
Now, we will find l

$$l = \sqrt{h^2 + (r_1 - r_2)^2}$$
$$l = \sqrt{(16)^2 + (20 - 8)^2}$$
$$l = \sqrt{(16)^2 + (12)^2}$$
$$l = \sqrt{256 + 144}$$
$$l = \sqrt{400}$$
$$l = 20 \text{ cm}$$

$$\text{Area of frustum} = 3.14 \times 20\,(20 + 8)$$
$$= 1758.4 \text{ cm}^2$$

$$\text{Area of bottom circle} = 3.14 \times 8^2 = 200.96 \text{ cm}^2$$
$$\text{Area of container} = 1758.4 + 200.96$$
$$= 1959.36 \text{ cm}^2$$

Cost of making $100 \text{ cm}^2 = \text{Rs } 10$
Cost of making $1 \text{ cm}^2 = \frac{10}{100} = \text{Rs } \frac{1}{10}$
Cost of making $1959.36 \text{ cm}^2 = \frac{1}{10} \times 1959.36 = 195.936$

Hence, cost of milk is Rs 522.45
And cost of metal sheet is Rs 195.936