

**Class: XII Session: 2020-21**

**Subject: Mathematics**

**Value points, Practice Paper 3**

S. No.	Solutions	Marks
1	Yes Or R is transitive but not symmetric	1
2	Yes	1
3	A and $\emptyset$ or 2	1
4	$m \times n$	1
5	3	1
6	405	1
7	$\frac{e^{x^3}}{3} + c$ , where c is arbitrary constant	1
8	32/3 sq units	1
9	order is 2 and degree is 1 Or C = 5	1
10	$\frac{5\sqrt{6}}{3}$ sq unit	1
11	$\sqrt{21}$	1
12	K=4	1

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13	$\frac{1}{p^2}$	1
14	$(\frac{17}{3}, 0, \frac{23}{3})$	1
15	0.12	1
16	False	1
17	(i) c (ii) b (iii) d (iv) a (v) b	1 1 1 1 1
18	(i) b (ii) a (iii) c (iv) c (v) b	1 1 1 1 1
19	$\tan^{-1}(1) + \cos^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right)$ $= \frac{\pi}{4} + (\pi - \cos^{-1}\left(\frac{1}{2}\right)) - \sin^{-1}\left(\frac{1}{2}\right)$ $= \frac{\pi}{4} + (\pi - \frac{\pi}{3}) - \frac{\pi}{6} = \frac{3\pi}{4}$	1 1

20	$(x \ -5 \ -1) \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix} \begin{pmatrix} x \\ 4 \\ 1 \end{pmatrix} = 0$ $\Rightarrow (x - 2 \ -10 \ 2x - 8) \begin{pmatrix} x \\ 4 \\ 1 \end{pmatrix} = 0$ $\Rightarrow x(x - 2) - 40 + 2x - 8 = 0$ $\Rightarrow x = \pm 4\sqrt{3}$ <p align="center">OR</p> $2 \begin{pmatrix} 3 & 4 \\ 5 & x \end{pmatrix} + \begin{pmatrix} 1 & y \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 0 \\ 10 & 5 \end{pmatrix}$ $\Rightarrow \begin{pmatrix} 7 & 8 + y \\ 10 & 2x + 1 \end{pmatrix} = \begin{pmatrix} 7 & 0 \\ 10 & 5 \end{pmatrix}$ $\Rightarrow x = 2 \text{ and } y = -8 \text{ and } x - y = 10$	1  1  1 1
21	$f(x) = \begin{cases} kx + 1, & \text{if } x \leq \pi \\ \cos x, & \text{if } x > \pi \end{cases} \text{ is continuous at } x = \pi$ <p>If LHL = RHL = f(<math>\pi</math>)</p> $\Rightarrow k\pi + 1 = \cos \pi$ $\Rightarrow k = \frac{-2}{\pi}$	1 1
22	$x = 1 - a \sin \theta, y = b \cos^2 \theta$ $\frac{dx}{d\theta} = -a \cos \theta, \quad \frac{dy}{d\theta} = 2b \cos \theta (-\sin \theta)$ <p>So, slope of the normal = <math>-\frac{dx}{dy} = \frac{-a}{2b \sin \theta}</math> at <math>\theta = \frac{\pi}{2}</math></p> $= \frac{-a}{2b}$	1  1
23	$\int e^x \frac{\sin^4 x - \cos^4 x}{\sin x - \cos x} dx$	

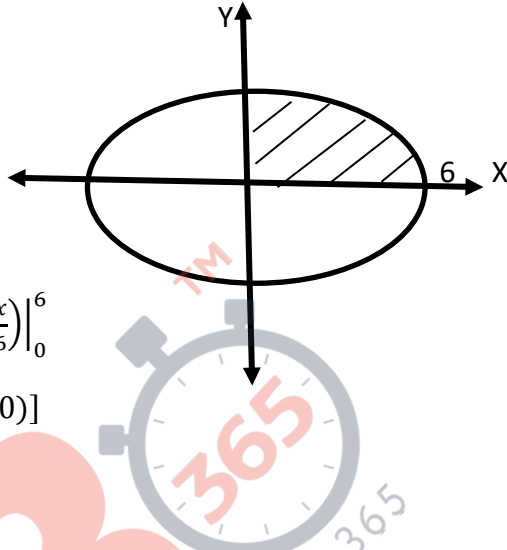
	$\sin^4 x - \cos^4 x = (\sin^2 x + \cos^2 x) (\sin^2 x - \cos^2 x)$ $= (\sin x + \cos x) (\sin x - \cos x)$ $(\sin^4 x - \cos^4 x) / (\sin x - \cos x) = (\sin x + \cos x)$ <p>So, <math>\int e^x \frac{\sin^4 x - \cos^4 x}{\sin x - \cos x} dx = \int e^x (\sin x + \cos x) dx = e^x \sin x + c</math></p> <p align="center">OR</p> $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x dx = 0 \text{ (being an odd function)}$	<p>1</p> <p>1</p> <p>1 + 1</p>
24	<p>Area = <math>\int_0^2 \sqrt{4 - x^2} dx =</math></p> $= \frac{x}{2} \sqrt{4 - x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2}$ <p>Putting the upper and lower limits, we get, Area = <math>\pi</math> sq units</p>	<p>1</p> <p>1</p>
25	<p><math>\frac{dy}{dx} = x^3 \operatorname{cosec} y</math>, given that <math>f(0) = 0</math></p> $\int \sin y dy = \int x^3 dx + c$ $-\cos y = \frac{x^4}{4} + c$ <p>As, <math>f(0) = 0</math> so <math>c = -1</math></p> $\frac{x^4}{4} + \cos y = 1$	<p>1</p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>
26	<p>The vector equation of a plane passing through A(2, 5, -3), B(-2, -3, 5) and C(5, 3, -3) is given by</p> $(\vec{r} - \vec{a}) [(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})] = 0$ $(\vec{r} - (2\hat{i} + 5\hat{j} - 3\hat{k})) [(-4\hat{i} - 8\hat{j} + 8\hat{k}) \times (3\hat{i} - 2\hat{j})] = 0$	<p>1</p> <p>1</p>
27	$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ <p>And <math>\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})</math>.</p> <p>Since, the drs of the above lines are in same ratios</p>	

	<p>So, lines are parallel</p> $S.D = \left  \frac{(\vec{a}_2 - \vec{a}_1) \times \vec{b}}{ \vec{b} } \right $ <p>Here <math>(\vec{a}_2 - \vec{a}_1) = 2\hat{i} + \hat{j} - \hat{k}</math>, <math> \vec{b}  = 7</math></p> $= \frac{\sqrt{293}}{7} \text{ units}$	<p><math>\frac{1}{2} + 1</math></p> <p><math>\frac{1}{2}</math></p>
28	$\sum p_i = 1$ $\Rightarrow 6k = 1$ $\Rightarrow k = 1/6$ $P(X \leq 2) = 6k = 1$	<p>1</p> <p>1</p>
29	<p><math>A = \{x \in Z : 0 \leq x \leq 12\}</math>, given by</p> <p><math>R = \{(a, b) :  a - b  \text{ is a multiple of } 4\}</math></p> <p><b>R is reflexive</b></p> <p>Let <math>a \in A</math>, <math>a R a \forall a \in A</math> as <math> a - a  = 0 = 0 \times 4</math> clearly a multiple of 4.</p> <p><b>R is symmetric</b></p> <p>Let <math>a, b \in A</math> such that <math>a R b</math></p> <p>i.e. <math> a - b </math> is a multiple of 4.</p> <p>Since <math> a - b  =  b - a </math></p> <p>So, <math> b - a </math> is a multiple of 4</p> <p>So, <math>b R a</math></p> <p><b>R is transitive</b></p> <p>Let <math>a, b, c \in A</math> such that <math>a R b</math> and <math>b R c</math></p> $\Rightarrow  a - b  = 4m \text{ and }  b - c  = 4n \text{ where } m, n \in N$ $\Rightarrow a - b = \pm 4m \text{ and } b - c = \pm 4n$ $\Rightarrow a - c = (a - b) + (b - c) = \pm 4(m + n)$ $\Rightarrow  a - c  = 4(m + n), \text{ clearly a multiple of } 4.$ <p>So, <math>a R c</math></p> <p>Hence R is an equivalence relation.</p>	<p><math>\frac{1}{2}</math></p> <p>1</p> <p>1</p>

		½
30	$y = x^a + x^{\sin x} = u + v$ $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ <p>Here, <math>\frac{du}{dx} = a x^{a-1}</math></p> $V = x^{\sin x}$ $\log v = \sin x \log x$ $\frac{1}{v} \frac{dv}{dx} = \frac{\sin x}{x} + \log x \cos x$ $\frac{dv}{dx} = x^{\sin x} \left( \frac{\sin x}{x} + \log x \cos x \right)$ <p>So, <math>\frac{dy}{dx} = a x^{a-1} + x^{\sin x} \left( \frac{\sin x}{x} + \log x \cos x \right)</math></p>	 ½ ½    1 ½  ½
31	$y = 3e^{2x} + 2e^{3x}$ $\frac{dy}{dx} = 6e^{2x} + 6e^{3x}$ $\frac{d^2y}{dx^2} = 12e^{2x} + 18e^{3x}$ $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 12e^{2x} + 18e^{3x} - 5(6e^{2x} + 6e^{3x}) + 6(3e^{2x} + 2e^{3x}) = 0$ <p align="center"><b>OR</b></p> <p>We have <math>x = a(\cos\theta + \theta\sin\theta)</math>, <math>y = a(\sin\theta - \theta\cos\theta)</math></p> $\frac{dy}{d\theta} = a(\cos\theta + \theta\sin\theta - \cos\theta) = a\theta\sin\theta$ $\frac{dx}{d\theta} = a(-\sin\theta + \theta\cos\theta + \sin\theta) = a\theta\cos\theta$ $\frac{dy}{dx} = \frac{a\theta\sin\theta}{a\theta\cos\theta} = \tan\theta$	 1  1  1        1  1

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	$\frac{dy}{dx} = \frac{d}{dx} \left( \frac{dy}{dx} \right) = \frac{d}{dx} (\tan \theta) = \sec^2 \theta \frac{d\theta}{dx} = \sec^2 \theta \times \frac{1}{a\theta \cos \theta}$ $= \frac{\sec^3 \theta}{a\theta}$	1												
32	<p><math>f(x) = 4x^3 - 6x^2 - 72x + 30</math></p> <p><math>f'(x) = 12x^2 - 12x - 72</math></p> <p><math>12x^2 - 12x - 72 = 0</math></p> <p><math>12(x^2 - x - 6) = 0</math></p> <p><math>x = -2, 3</math></p> <table border="0"> <thead> <tr> <th align="left">Interval</th> <th align="center">Sign of <math>f'(x)</math></th> <th align="left">Nature of <math>f(x)</math></th> </tr> </thead> <tbody> <tr> <td><math>(-\infty, -2)</math></td> <td align="center">+</td> <td><math>f</math> is strictly increasing</td> </tr> <tr> <td><math>(-2, 3)</math></td> <td align="center">-</td> <td><math>f</math> is strictly decreasing</td> </tr> <tr> <td><math>(3, \infty)</math></td> <td align="center">+</td> <td><math>f</math> is strictly increasing</td> </tr> </tbody> </table>	Interval	Sign of $f'(x)$	Nature of $f(x)$	$(-\infty, -2)$	+	$f$ is strictly increasing	$(-2, 3)$	-	$f$ is strictly decreasing	$(3, \infty)$	+	$f$ is strictly increasing	<p>1</p> <p><math>\frac{1}{2}</math></p> <p>1 <math>\frac{1}{2}</math></p>
Interval	Sign of $f'(x)$	Nature of $f(x)$												
$(-\infty, -2)$	+	$f$ is strictly increasing												
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33	<p>Let <math>I = \int \frac{x^2}{(x^2+1)(x^2+4)} dx</math></p> <p>Put <math>x^2 = t</math></p> $\frac{x^2}{(x^2+1)(x^2+4)} = \frac{t}{(t+1)(t+4)} = \frac{A}{t+1} + \frac{B}{t+4}$ <p><math>A = -1/3</math> , <math>B = 4/3</math></p> $= \frac{-1}{3(x^2+1)} + \frac{4}{3(x^2+4)}$ <p><math>I = \int \frac{-1}{3(x^2+1)} dx + \int \frac{4}{3(x^2+4)} dx</math></p>	<p>1</p> <p>1</p>												

	$I = \frac{-1}{3} \tan^{-1} x + \frac{2}{3} \tan^{-1} \frac{x}{2} + c$	1
34	$x^2 + 9y^2 = 36$ $\frac{x^2}{36} + \frac{y^2}{4} = 1$ $A = 4 \int_0^6 y \, dx$ $A = \frac{4}{3} \int_0^6 \sqrt{6^2 - x^2} \, dx$ $A = \left[ \frac{4}{3} \left( \frac{x}{2} \sqrt{36 - x^2} + \frac{36}{2} \sin^{-1} \frac{x}{6} \right) \right]_0^6$ $A = \frac{4}{3} [18 \sin^{-1}(1) - 18 \sin^{-1}(0)]$ $A = \frac{4}{3} \times 18 \times \frac{\pi}{2} = 12\pi$ 	1 1 1 1
35	$e^x \tan y \, dx + (1 - e^x) \sec^2 y \, dy = 0$ $\frac{e^x}{1 - e^x} dx + \frac{\sec^2 y}{\tan y} dy = 0$ $\int \frac{e^x}{1 - e^x} dx + \int \frac{\sec^2 y}{\tan y} dy = 0$ $-\log  1 - e^x  + \log  \tan y  = c$ $\tan y = c(1 - e^x)$ <p><b>OR</b></p> $x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x$	1 1 ½ ½

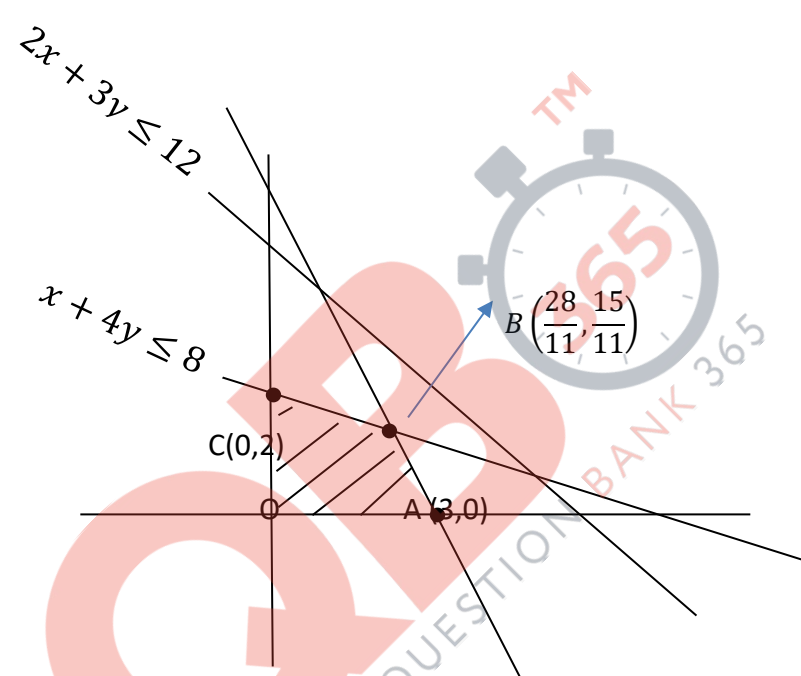


	$\frac{dy}{dx} + \frac{1}{x \log x} y = \frac{2}{x^2}$ $\text{I.F.} = e^{\int \frac{1}{x \log x} dx} = e^{\log(\log x)} = \log x$ $y \log x = \int \frac{2}{x^2} \log x dx = 2 \int (\log x) x^{-2} dx + c$ $= 2 \left[ \frac{-\log x}{x} + \int x^{-2} dx \right] + c$ $y \log x = \frac{-2}{x} (1 + \log x) + c$	
<p>36</p>	$x + 2y - 3z = -4$ $2x + 3y + 2z = 2$ $3x - 3y - 4z = 11$ $\begin{pmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -4 \\ 2 \\ 11 \end{pmatrix}$ <p>A X = B</p> $X = A^{-1}B$ $A^{-1} = \frac{\text{adj } A}{ A }$ $ A  = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{vmatrix}$ $= 1(-12+6) - 2(-8-6) - 3(-6-9)$ $= 67 \neq 0$ $\text{Adj } A = \begin{pmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{pmatrix}$ $A^{-1} = \frac{1}{67} \begin{pmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{pmatrix}$	<p>1/2</p> <p>1</p> <p>1/2</p> <p>1 1/2</p>

$X = A^{-1}B$ $= \frac{1}{67} \begin{pmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{pmatrix} \begin{pmatrix} -4 \\ 2 \\ 11 \end{pmatrix}$ $= \frac{1}{67} \begin{pmatrix} 201 \\ -134 \\ 67 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$ <p><math>X=3, y=-2, z=1</math></p> <p><b>OR</b></p> $A = \begin{pmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{pmatrix} \quad B = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix}$ $BA = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{pmatrix}$ $= \begin{pmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{pmatrix} = 6I$ $y+2z=7$ $x-y=3$ $2x+3y+4z=17$ <p>Let's rearrange the equations</p> $x-y=3$ $2x+3y+4z=17$ $Y+2z=7$	<p>1 ½</p> <p>1 ½</p> <p>1</p>
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	$= \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 17 \\ 7 \end{pmatrix}$ $= \frac{1}{6} \begin{pmatrix} 6 + 34 - 28 \\ -12 + 34 - 28 \\ 6 - 17 + 35 \end{pmatrix}$ $= \frac{1}{6} \begin{pmatrix} 12 \\ -6 \\ 24 \end{pmatrix}$ $= \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$ <p align="center"><math>\therefore x = 2, \quad y = -1, \quad z = 4</math></p>	<p align="center">2</p> <p align="center"><math>\frac{1}{2}</math></p>
<p>37</p>	$\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$ $\frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3} = k \text{ (say)}$ <p><math>X = -2k+4, \quad y = 6k, \quad z = -3k+1</math></p> <p>D.R.'s are <math>(-2k+4-2, 6k-3, -3k+1+8)</math></p> <p align="center"><math>= (-2k+2, 6k-3, -3k+9)</math></p> <p><math>(-2k+2)(-2) + (6k-3)(6) + (-3k+9)(-3) = 0</math></p> <p><math>4k-4+36k-18+9k-27=0</math></p> <p><math>49k-49=0</math></p> <p><math>K=1</math></p> <p><math>X = -2 \times 1 + 4, \quad y = 6 \times 1, \quad z = -3 \times 1 + 1</math></p> <p><math>X = 2, \quad y = 6, \quad z = -2</math></p>	<p align="center">1</p> <p align="center"><math>\frac{1}{2}</math></p> <p align="center"><math>\frac{1}{2}</math></p> <p align="center"><math>1 \frac{1}{2}</math></p> <p align="center"><math>\frac{1}{2}</math></p> <p align="center">1</p>

<p>Distance <math>=\sqrt{(2-2)^2 + (6-3)^2 + (-2+8)^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}</math></p> <p>OR</p> <p>The equation of plane passing through the intersection of two given planes is,</p> $(x+3y-6)+t(3x-y-4z)=0$ $(1+3t)x + (3-t)y - 4tz - 6 = 0$ <p>Distance of this plane from the origin (0,0,0) is unity.</p> <p>So,</p> $1 = \left  \frac{-6}{\sqrt{(1+3t)^2 + (3-t)^2 + (-4t)^2}} \right $ $1 = \left  \frac{-6}{\sqrt{10+26t^2}} \right $ $10 + 26t^2 = 36$ $t = \pm 1$ <p>putting <math>t=1</math> in (A) we get,</p> $4x+2y-4z-6=0$ <p>putting <math>t=-1</math> in (A) we get,</p> $-2x+4y+4z-6=0$	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>
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<p>38</p>	<p>Max <math>z = x + y</math></p> <p>s,t     <math>x + 4y \leq 8</math></p> <p>          <math>2x + 3y \leq 12</math></p> <p>          <math>3x + y \leq 9</math></p> <p><math>x, y \geq 0</math></p>  <p>Region OABC is the feasible region</p> <p>At O(0,0)    <math>Z=0+0=0</math></p> <p>At A(3,0) ,   <math>Z=3+0=3</math></p> <p>At B(<math>\frac{28}{11}, \frac{15}{11}</math>) , <math>Z=\frac{28}{11} + \frac{15}{11} = \frac{43}{11}</math>,</p> <p>At C(0,2) ,   <math>Z=0+2=2</math></p> <p>Therefore, Optimal solution is (<math>\frac{28}{11}, \frac{15}{11}</math>) and maximum value of the function is <math>\frac{43}{11}</math></p> <p>OR</p> <p>1) Points <span style="float: right;"><math>z = x + 2y</math></span></p>	<p>3</p> <p>1</p> <p>1</p>
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$P\left(\frac{3}{13}, \frac{24}{13}\right)$ $Q\left(\frac{3}{2}, \frac{15}{4}\right)$ $R\left(\frac{7}{2}, \frac{15}{4}\right)$ $S\left(\frac{18}{7}, \frac{2}{7}\right)$	$\frac{51}{13}$ 9 max 5 $\frac{22}{7}$ min	1 ½ 1
Max $z = 9$ at $Q\left(\frac{3}{2}, \frac{15}{4}\right)$ and min $z$ is $\frac{22}{7}$ at $S\left(\frac{18}{7}, \frac{2}{7}\right)$		
2) $Z = px + qy$		
If max $Z$ occurs at $Q\left(\frac{3}{2}, \frac{15}{4}\right)$ and $R\left(\frac{7}{2}, \frac{3}{4}\right)$ , then		
$\frac{3p}{2} + \frac{15q}{4} = \frac{7p}{2} + \frac{3q}{4}$		
Simplifying, we get, $2p = 3q$		
This is the required condition.		
Also, the number of optimal solutions in this case will be infinite solutions lying on the line segment QR.		
		2 ½