PRACTICE PAPER 4 (2020-21)
CLASS XII MATHEMATICS

TIME ALLOWED: 3 HOURS MAX MARKS: 80

GENERAL INSTRUCTIONS

(i) This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks.

(ii) Part A has Objective type Questions and Part B has descriptive type Questions.

Part- A

(a) It consists of two sections- I and II.

(b) Section I comprises of 16 very short answer type questions.

(c) Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part-B

(a) It consists of three sections- III, IV and V.

(b) Section III comprises of 10 questions of 2 marks each.

(c) Section IV comprises of 7 questions of 3 marks each.

(d) Section V comprises of 3 questions of 5 marks each.

(e) Internal choice is provided in three questions of section-III, 2 questions of section IV and 3 questions of section V .

SECTION-1

All questions are compulsory. In case of internal choices attempt any one.

Q1 Evaluate : $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$

Q2 State the reason for the relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} not to be transitive.

Q3 Write the values of x – y + z from the following equation :

$$\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}.$$

OR

If $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$, then write the value of k

Q4 Write A^{-1} for A = $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

Q5 If A is a square matrix of order 3 and | A|= 7. Write the value of | adj. A|.

Q6 If $\vec{a}$ is a unit vector and $(\vec{x} - \vec{a})(\vec{x} + \vec{a}) = 80$, then find $|\vec{x}|$

OR

Find the scalar components of the vector $\overrightarrow{AB}$ with initial point A (2,1) and terminal point B (–5, 7).

Q7 Write the value of $(\hat{i} \times \hat{j}).\hat{k} + \hat{\mathbf{i}} . \hat{j}$

Q8 What is the range of the function f(x) = $\frac{|x-1|}{(x-1)}$?

Q9 What is the degree of the following differential equation?

$$5x\left(\frac{dy}{dx}\right)^2 - \frac{d^2y}{dx^2} - 6y = \log x$$

Q10 Find the value of p, if $(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+3\hat{j}+p\hat{k})=\vec{0}$

Q11 Write the direction cosines of a line equally inclined to the three coordinate axes.

Q12 If the equation of a line AB is $\frac{x-3}{1}=\frac{y+2}{-2}=\frac{z-5}{4}$, find the direction ratios of a line parallel to AB.

Q13 In a college, 30% students fail in Physics, 25%fail in Mathematics and 10% fail in both. One student is chosen at random, find the probability that the student fails in Physics if he/she failed in Mathematics.

Q14 Find the area enclosed by y = sinx and x-axis from x=0 to $x=2\pi$

Q15 If P(A)= 0.4, P(B)= 0.8 and P(B|A)= 0.6, then find P(A|B).

Q16 Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let f = {(1, 4), (2, 5), (3 , 6)} be a function from A to B. State whether f is one-one or not.

SECTION-II

Both the case study based questions are compulsory. Attempt 4 sub parts from each question. Each question carries 1 mark.

Q17 In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random.

(i) What is the probability that she reads neither Hindi nor English Newspaper?

(a) $\frac{2}{5}$

(b) $\frac{3}{5}$

(c) $\frac{1}{5}$

(d) 0

(ii) If she reads Hindi newspaper, what is the probability that she reads English Newspaper?

(a) 0

(b) $\frac{1}{3}$

(c) $\frac{2}{3}$

(d) None of these

(iii) If she reads English Newspaper, what is the probability that she reads Hindi Newspaper?

(a) $\frac{1}{5}$

(b) $\frac{1}{2}$

(c) $\frac{1}{4}$

(d) $\frac{1}{3}$

(iv) What is the probability that she reads only Hindi Newspaper?

(a) $\frac{2}{5}$

(b) $\frac{4}{5}$

(c) 1

(d) $\frac{3}{5}$

(v) What is the probability that she reads either Hindi or English Newspaper?

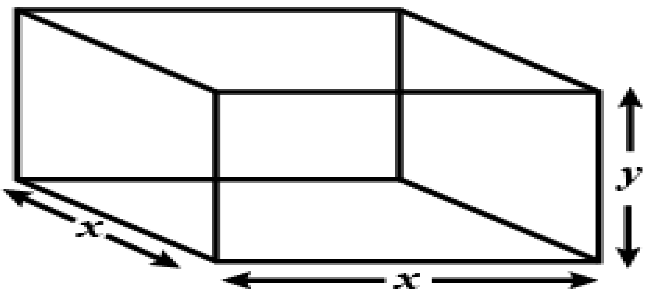
(a) $\frac{1}{3}$

(b) $\frac{2}{3}$

(c) $\frac{4}{5}$

(d) $\frac{3}{5}$

Q 18 A metal box with square base and vertical sides is to contain 1024cm^3 of water. The material for the top and bottom costs ₹ 5 per cm^2 and the material for the sides costs ₹ 2.50 per cm^2



(i) What will be the relation between x and y?

(a) $xy^2 = 1024$

(b) $x^2+4xy=1024$

(c) $x^2y=1024$

(d) $2x^2+4xy=1024$

(ii) What will be the total cost(C) of the material used to construct the box?

(a) $C= 5x^2+20xy$

(b) $C=x^2+4xy$

(c) $C=10x^2+10xy$

(d) None of these

(iii) What will be the total cost(C) of the box in terms of x?

(a) $C=5x^2+\frac{10240}{x}$

(b) $C=10x^2+\frac{10240}{x}$

(c) $C=x^2+\frac{1024}{x}$

(d) $C=20x-\frac{1024}{x}$

(iv) What should be the dimensions of the box to minimize the cost?

(a) x=16, y=8
(b) x=8, y=16
(c) x=8, y=8
(d) x=8, y=4

(v) What is the least cost of the box?

(a) ₹1620
(b) ₹1024
(c) ₹1920
(d) ₹1780

PART-B
SECTION-III

Q19 Solve: $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}$

Q20 If siny = x sin (a + y), prove that $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$

OR

Find the value of k so that the function f defined by

$$f(x)=\begin{cases}\frac{k\cos x}{\pi-2x}, & \text{if } x\neq \pi/2\\ 3, & \text{if } x=\pi/2\end{cases}$$ is continuous at x = π/2.

Q21Find the points on the curve $y = x^3$ at which the slope of the tangent is equal to the y-coordinate of the point.

OR

Show that $y=\log(1+x)-\frac{2x}{2+x}, -1<x<1$ is an increasing function of x throughout its domain.

Q22Evaluate: $\int_0^{\pi}\frac{4x}{1+\cos^2 x}dx$ **OR** $\int\frac{x+2}{\sqrt{x^2+5x+6}}$

Q23 Probabilities of solving a specific problem independently by A and B are 1/2 and 1/3 respectively. If both try to solve the problem independently, find the probability that

(i) The problem is solved
(ii) Exactly one of them solves the problem.

Q24 Find the shortest distance between the lines
$\vec{r}=3\hat{\imath}+2\hat{\jmath}-4\hat{k}+\lambda(\hat{\imath}+2\hat{\jmath}+2\hat{k})$ and $\vec{r}=5\hat{\imath}-2\hat{\jmath}+\mu(3\hat{\imath}+2\hat{\jmath}+6\hat{k})$.

Q25 Find a unit vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a}=3\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$.

Q26 Solve the following differential equation:

$$(1+x^2)\frac{dy}{dx}+y=\tan^{-1}x$$

OR

Solve the following differential equation:

$$(x^2-1)\frac{dy}{dx}+2xy=\frac{2}{x^2-1}.$$

Q 27 Find the area of the region bounded by $y^2=9x$, $x=2$, $x=4$ and the x-axis in the first quadrant.

Q28 If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then find $A - B^T$.

SECTION –IV

Q29 Show that the function in $\mathbf{A} = \mathbf{R} - \left\{\frac{2}{3}\right\}$ **defined as** $\mathbf{f(x)} = \frac{\mathbf{4x+3}}{\mathbf{6x-4}}$ is one-one and onto.

Q30 $If \;\; y = \log\left[x + \sqrt{x^2 + a^2}\right], showthat (x^2 + a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$

Q31 Differentiate the following function w.r.t. x:

$x^{\sin x} + (\sin x)^{\cos x}$.

OR

If $x = a(\theta - \sin\theta),\; y = a(1 + \cos\theta)$, find $\frac{d^2y}{dx^2}$

Q32 Solve the following differential equation:

$$(3xy + y^2)dx + (x^2 + xy)dy = 0$$

OR

Solve the following differential equation:

$(1 + x^2)\, dy + 2xy\, dx = \cot x\, dx;\; x \neq 0$

Q33 Find the intervals in which the function f given by

$f(x) = \sin x + \cos x,\; 0 \leq x \leq 2\pi$, is strictly increasing or strictly decreasing.

OR

Prove that the curves $x = y^2$ and $xy = k$ intersect at right angles if $8k^2 = 1$.

Q34 Evaluate $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

Q35 Find the area of the region bounded by the ellipse
$\frac{x^2}{16}+\frac{y^2}{9}=1$

SECTION V

Q36 Use product $\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$ to solve the system of equations:

x –y + 2z = 1
2y – 3z = 1
3x – 2y + 4z = 2

OR

Using matrices, solve the following system of linear equation:

x-y+2z =7
3x+4y-5z =-5
2x-y+3z = 12

Q37 Find the distance between the point (7 ,2,4) and the plane determined by the points A(2,5, –3), B(–2, –3, 5) and C(5, 3, –3).

OR

Find the distance of the point (−1, −5, −10) from the point of intersection of the line $\vec{r} = (2\hat{i} - 1\hat{j} + -2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r}.(\hat{i} - \hat{j} + \hat{k}) = 5$

Q38 Solve the following linear programming problem graphically:

Maximize Z= 12x+16y

Subject to constraints: $x+y \leq 1200$

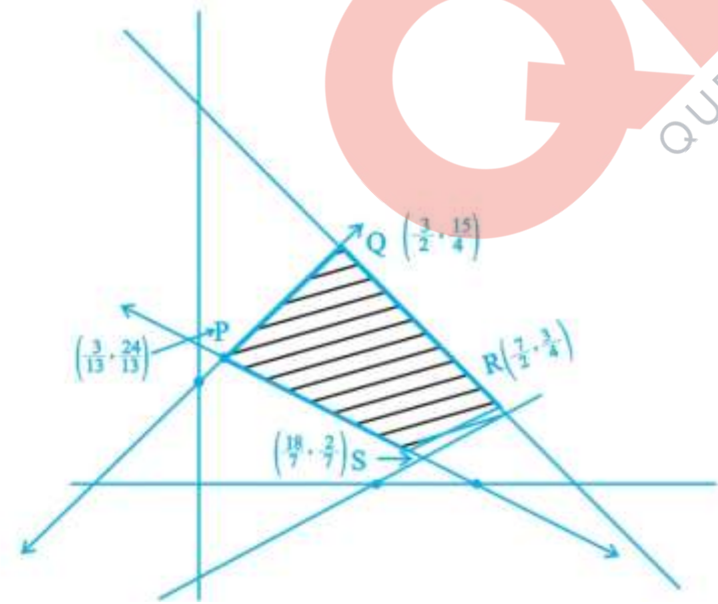
$y \leq \frac{x}{2}$

$x-3y \leq 600$

$x \geq 0, y \geq 0$

OR

The corner points of the feasible region determined by the system of linear constraints are shown below:



(i) Let z= x+2y be the objective function. Find the maximum and minimum value of z and also the corresponding points at which the maximum and minimum value occurs.

(ii) Let z= px+qy, where p, q>0 be the objective function. Find the conditions on p and q so that the maximum occurs at Q$\left(\frac{3}{2},\frac{15}{4}\right)$ and R$\left(\frac{7}{2},\frac{3}{4}\right)$.