

QB365 - Question Bank Software

CLASS XII SESSION 2020-21

PRACTICE PAPER IV

SUBJECT : MATHEMATICS

MARKING SCHEME/VALUE POINTS (THEORY)

Sr No	Objective type Question Section I Value points	Marks
1	$\int \frac{\cos\sqrt{x}}{\sqrt{x}} \cdot dx$ Let $\sqrt{x} = t \Rightarrow I = 2\sin\sqrt{x} + C$	1
2	$(1,2) \in R, (2,1) \in R$ but $(1,1) \notin R$ therefore R is not transitive	1
3	1 OR $k=17$	1
4	$A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$	1
5	49	1
6	9 OR Scalar component of $\vec{AB} = -7,6$	1
7	1	1
8	Range = $\{-1,1\}$	1
9	1	1
10	$27/2$	1
11	Reqd d'cs are, $\langle \pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}} \rangle$	1
12	Dr's of a line parallel to AB = 1, -2.4	1
13	$2/5$	1
14	4 sq units	1
15	0.3	1
16	One- one	1
17	$P(H) = \frac{60}{100} = 0.60, P(E) = 0.40, P(H \cap E) = 0.20$	

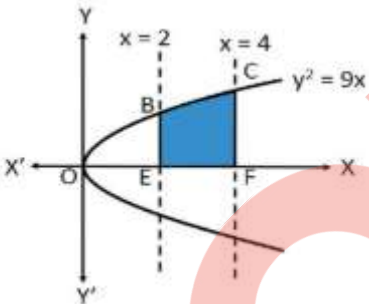
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	<p>$P(H \cup E) = P(H) + P(E) - P(H \cap E) = 0.60 + 0.40 - 0.20 = 0.80$</p> <p>(i) $P(\bar{H} \cap \bar{E}) = 1 - P(H \cup E) = 1 - 0.80 = 0.20 = \frac{1}{5}$</p> <p align="center">ANS(C)</p> <p>(ii) $P\left(\frac{E}{H}\right) = \frac{P(E \cap H)}{P(H)} = \frac{0.20}{0.60} = \frac{2}{6} = \frac{1}{3}$</p> <p align="center">Ans(B)</p> <p>(iii) $P\left(\frac{H}{E}\right) = \frac{P(H \cap E)}{P(E)} = \frac{0.20}{0.40} = \frac{1}{2}$</p> <p align="center">Ans(B)</p> <p>(iv) $P(H \cap \bar{E}) = P(H) - P(H \cap E) = 0.60 - 0.20 = 0.40 = \frac{2}{5}$</p> <p align="center">ANS (A)</p> <p>(v) $P(\text{Either Hindi or English}) = P(H \cup E) = 0.80 = \frac{4}{5}$</p> <p align="center">Ans (c)</p>	<p align="center">1</p> <p align="center">1</p> <p align="center">1</p> <p align="center">1</p> <p align="center">1</p>
18	<p align="center">(i) C</p> <p align="center">(ii) C</p> <p align="center">(iii) b</p> <p align="center">(iv) b</p> <p align="center">(v) c</p>	<p align="center">1</p> <p align="center">1</p> <p align="center">1</p> <p align="center">1</p> <p align="center">1</p>
19	<p align="center">$\tan^{-1}(x + 1) + \tan^{-1}(x - 1) = \tan^{-1} \frac{8}{31}$</p> <p align="center">Solution- $\tan^{-1} \left[\frac{x+1+x-1}{1-(x^2-1)} \right] = \tan^{-1} \frac{8}{31}$</p> <p align="center">$x^2 - 1 < 1 \Rightarrow x^2 < 2 \dots \dots \dots (1)$</p> <p align="center">$\frac{2x}{2-x^2} = \frac{8}{31}$</p> <p align="center">$\Rightarrow 31x = 8 - 4x^2$</p> <p align="center">$\Rightarrow 4x^2 + 31x - 8 = 0$</p> <p align="center">$\Rightarrow (4x-1)(x+8) = 0$</p> <p align="center">$\Rightarrow x = 1/4 \text{ or } x = -8$</p>	<p align="center">$\frac{1}{2}$</p> <p align="center">1</p>

	$asx^2 < 2 \text{ from (1)} \Rightarrow x = 1/4$	1/2
20	$\sin y = x \sin(a+y) \Rightarrow \frac{\sin y}{\sin(a+y)} = x$ <p>Differentiating both sides with respect to x we have</p> $\frac{\sin(a+y) \cos y \frac{dy}{dx} - \sin y \cos(a+y) \frac{dy}{dx}}{\sin^2(a+y)} = 1$ $\Leftrightarrow \frac{dy}{dx} \left[\frac{\sin(a+y-y)}{\sin^2(a+y)} \right] = 1$ $\Leftrightarrow \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$ <p align="center">OR</p> $f\left(\frac{\pi}{2}\right) = 3$ $\lim_{x \rightarrow \frac{\pi}{2}} f(x) = \lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{k \cos x}{\pi - 2x} \right)$ <p>Let $x = \frac{\pi}{2} + t$ as $x \rightarrow \frac{\pi}{2} \Rightarrow t \rightarrow 0$</p> $\Rightarrow \lim_{t \rightarrow 0} \frac{\cos\left(\frac{\pi}{2} + t\right)}{\pi - \pi - 2t} = \lim_{t \rightarrow 0} \frac{-\sin t}{-2t} = \frac{k}{2} \times 1 = \frac{k}{2}$ $\Rightarrow \frac{k}{2} = 3 \Rightarrow k = 6$	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1</p> <p>1/2</p>
21	$Y = x^3 \Rightarrow \frac{dy}{dx} = 3x^2$ <p>But $\frac{dy}{dx} = y \Rightarrow x^3 = 3x^2$</p> $\Rightarrow x^2(x - 3) = 0 \Rightarrow x = 0, 3$ <p>(0,0) and (3,27) are the required points</p> <p align="center">OR</p> $f(x) = \log(1+x) \cdot \frac{2x}{2+x}$ <p>clearly f(x) is defined in (-1,1)</p> $f'(x) = \frac{1}{1+x} \cdot \frac{4}{(2+x)^2} = \left(\frac{x}{2+x}\right)^2 \left(\frac{1}{1+x}\right)$ $f'(x) > 0 \Rightarrow \frac{1}{1+x} > 0 \Rightarrow x+1 > 0 \Rightarrow x > -1$ <p>$\therefore f(x)$ is an increasing function in (-1,1)</p>	<p>1/2</p> <p>1/2</p> <p>1</p> <p>1</p>

22	$I = \int_0^{\pi} \frac{4x}{1+\cos^2 x} dx$ $= \int_0^{\pi} \frac{4(\pi-x)}{1+\cos^2(\pi-x)} dx = 4\pi \int \frac{dx}{1+\cos^2 x} I$ $\Rightarrow 2I = 4\pi \int \frac{dx}{1+\cos^2 x} = 4\pi \cdot 2 \int_0^{\frac{\pi}{2}} \frac{dx}{1+\cos^2 x} =$ $\therefore f(\pi-x) = f(x)$ $\Rightarrow I = 4\pi \cdot \int_0^{\frac{\pi}{2}} \frac{dx}{1+\cos^2 x}$ <p>Dividing numerator and denominator by $\cos^2 x$</p> $I = 4\pi \cdot \int_0^{\frac{\pi}{2}} \frac{\sec^2 x dx}{\sec^2 x + 1} = 4\pi \int_0^{\frac{\pi}{2}} \frac{\sec^2 x dx}{\tan^2 x + 2}$ <p>put $\tan x = t \Rightarrow \sec^2 x dx = dt$</p> $= 4\pi \int_0^{\infty} \frac{dt}{2+t^2} = 4\pi \cdot \frac{1}{\sqrt{2}} \left[\tan^{-1} \frac{t}{\sqrt{2}} \right]_0^{\infty} = \frac{4\pi}{\sqrt{2}} \left[\tan^{-1} \infty - \tan^{-1} 0 \right] = \frac{4\pi}{\sqrt{2}} \cdot \frac{\pi}{2} = \sqrt{2} \pi^2$ <p>OR</p> $I = \int \frac{x+2}{\sqrt{x^2+5x+6}} dx$ $\Rightarrow x+2 = A \frac{d}{dx} (x^2 + 5x + 6) + B \Rightarrow A=1/2, B=-1/2$ $\therefore I = \frac{1}{2} \int \frac{2x+5}{\sqrt{x^2+5x+6}} dx - \frac{1}{2} \int \frac{dx}{\sqrt{x^2+5x+6}}$ $= \sqrt{x^2 + 5x + 6} - \frac{1}{2} \log \left \left(x + \frac{5}{2}\right) + \sqrt{x^2 + 5x + 6} \right + C$	<p style="text-align: center;">1</p> <p style="text-align: center;">1</p> <p style="text-align: center;">1</p> <p style="text-align: center;">1</p>
23	$P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$ <p>(i) $P(\text{Problem is solved}) = 1 - P(\bar{A} \cap \bar{B}) = 1 - P(\bar{A})P(\bar{B}) = 1 - \frac{1}{2} \times \frac{2}{3} = \frac{2}{3}$</p> <p>(ii) $P(\text{Exactly one can solve the problem}) = P(\bar{A} \cap B) + P(A \cap \bar{B})$</p> $= P(\bar{A})P(B) + P(A)P(\bar{B}) = \frac{1}{2} \times \frac{1}{3} + \frac{1}{2} \times \frac{2}{3} = \frac{1}{2}$	<p style="text-align: center;">1</p> <p style="text-align: center;">1</p>
24	$\vec{a}_1 = 3\hat{i} + 2\hat{j} - 4\hat{k}, \vec{b}_1 = \hat{i} + 2\hat{j} + 2\hat{k}$ $\vec{a}_2 = 5\hat{i} - 2\hat{j}, \vec{b}_2 = 3\hat{i} + 2\hat{j} + 6\hat{k}$	

	$\vec{a}_2 - \vec{a}_1 = 2\hat{i} - 4\hat{j} + 4\hat{k}$ $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 3 & 2 & 6 \end{vmatrix} = \hat{i}(12-4) - \hat{j}(6-6) + \hat{k}(2-6) = 8\hat{i} - 4\hat{k}$ $SD = \frac{ (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) }{ \vec{b}_1 \times \vec{b}_2 }$ $= \frac{ 2 \times 8 - 0 - 4 \times 4 }{\sqrt{(8)^2 + (-4)^2}} = 0$	1
25	$\vec{a} = 3\hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ $\therefore \vec{a} + \vec{b} = 4\hat{i} + 4\hat{j} + \hat{k}$ $\vec{a} - \vec{b} = 2\hat{i} + 5\hat{k}$ $\text{Perpendicular vector} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 1 \\ 2 & 0 & 5 \end{vmatrix} = 20\hat{i} - 18\hat{j} - 8\hat{k}$ $ \vec{r} = \sqrt{400 + 324 + 64} = \sqrt{788} = 2\sqrt{197}$ $\text{unit vector } \hat{r} = \frac{10}{\sqrt{197}}\hat{i} - \frac{9}{\sqrt{197}}\hat{j} - \frac{4}{\sqrt{197}}\hat{k}$	1 1
26	$(1+x^2) \frac{dy}{dx} + y = \tan^{-1} x$ $\Rightarrow \frac{dy}{dx} + \frac{1}{1+x^2} y = \frac{\tan^{-1} x}{1+x^2}$ $P = \frac{1}{1+x^2}, Q = \frac{\tan^{-1} x}{1+x^2}$ $I.F = e^{\int \frac{dx}{1+x^2}} = e^{\tan^{-1} x}$ $y \cdot e^{\tan^{-1} x} = \int \frac{\tan^{-1} x}{1+x^2} e^{\tan^{-1} x} dx$ <p>Put $\tan^{-1} x = t$</p> $\frac{1}{1+x^2} dx = dt$ $\Rightarrow y \cdot e^{\tan^{-1} x} = \int e^t \cdot t \cdot dt$ $\Rightarrow y \cdot e^{\tan^{-1} x} = t \cdot e^t - \int e^t \cdot dt = te^t - e^t + C$ $\Rightarrow y \cdot e^{\tan^{-1} x} = (\tan^{-1} x - 1) \cdot e^{\tan^{-1} x} + C$ $\Rightarrow y = \tan^{-1} x - 1 + C e^{-\tan^{-1} x}$	$\frac{1}{2}$ 1/2 1

	<p align="center">OR</p> $(x^2 - 1) \frac{dy}{dx} + 2xy = \frac{2}{x^2 - 1}$ $\frac{dy}{dx} + \frac{2x}{x^2 - 1} \cdot y = \frac{2}{(x^2 - 1)^2}$ $P = \frac{2x}{x^2 - 1} \quad Q = \frac{2}{(x^2 - 1)^2}$ $I.F. = e^{\int \frac{2x}{x^2 - 1} dx} = e^{\log(x^2 - 1)} = x^2 - 1$ $y(x^2 - 1) = \int \frac{2}{(x^2 - 1)^2} \cdot (x^2 - 1) dx = 2 \int \frac{dx}{x^2 - 1} = 2 \cdot \frac{1}{2} \log \frac{x-1}{x+1} + C$ $\Rightarrow y(x^2 - 1) = \log \left \frac{x-1}{x+1} \right + C$	
27	 <p>Area bounded by $y^2 = 9x, x = 2,$ $x = 4$ and x axis in first quadrant</p> <p>Required area = $\int_2^4 3\sqrt{x} dx$</p> $= 3 \left[\frac{2}{3} x^{3/2} \right]_2^4 = 2 \times 8 - 2 \times 2\sqrt{2} = (16 - 4\sqrt{2}) \text{ Sq units}$	1

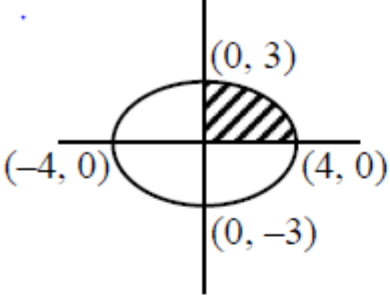
28	$A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$ $B^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ $A - B^T = \begin{bmatrix} 4 & -3 & -1 \\ 3 & 0 & -2 \end{bmatrix}$	$\frac{1}{2}$ $1\frac{1}{2}$
SECTION IV		
29	<p>$A = \mathbb{R} - \left\{ \frac{2}{3} \right\}$</p> <p>$f(x) = \frac{4x+3}{6x-4}$, $f(x)$ is defined for $x \neq \frac{2}{3}$</p> <p>For one-one let $x_1, x_2 \in A$ such that $f(x_1) = f(x_2) \Rightarrow \frac{4x_1+3}{6x_1-4} = \frac{4x_2+3}{6x_2-4}$</p> $24x_1x_2 - 16x_1 + 18x_2 - 12 = 24x_1x_2 + 18x_1 - 16x_2 - 12$ <p>$\Rightarrow x_1 = x_2$ hence $f(x)$ is one- one</p> <p><u>ONTO :</u></p> <p>Let $y = \frac{4x+3}{6x-4} \Rightarrow x = \frac{4y+3}{6y-4}$</p> <p>Here $x \neq \frac{2}{3}$</p> <p>If $x = \frac{2}{3}$ then $\frac{2}{3} = \frac{4y+3}{6y-4}$</p> $\Rightarrow 12y+9 = 12y-8 \Rightarrow 9 = -8$ which is wrong <p>Thus every y has its pre image in $A = \mathbb{R} - \left\{ \frac{2}{3} \right\}$</p> <p>for every $x \in A$</p> <p style="text-align: center;">$\therefore f(x)$ is Onto</p>	$1\frac{1}{2}$ $1\frac{1}{2}$

30	$Y = \log(x + \sqrt{x^2 + a^2})$ $\Rightarrow \frac{dy}{dx} = \frac{1}{x + \sqrt{x^2 + a^2}} \times \left[1 + \frac{2x}{2\sqrt{x^2 + a^2}} \right] = \frac{1}{\sqrt{x^2 + a^2}}$ $\Rightarrow \sqrt{x^2 + a^2} \frac{dy}{dx} = 1$ $\Rightarrow \sqrt{x^2 + a^2} \frac{d^2y}{dx^2} + \frac{2x}{2\sqrt{x^2 + a^2}} \frac{dy}{dx} = 0$ $\Rightarrow (x^2 + a^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = 0$	<p align="center">$1\frac{1}{2}$</p> <p align="center">$1\frac{1}{2}$</p>
31	<p>Let $y = x^{\sin x} + (\sin x)^{\cos x}$</p> <p>$Y = u + v$ where</p> <p>$u = x^{\sin x}$, $\Rightarrow \log u = \sin x \log x$ $v = (\sin x)^{\cos x}$</p> $\Rightarrow \frac{1}{u} \frac{du}{dx} = \cos x \log x + \frac{\sin x}{x}$ $\Rightarrow \frac{du}{dx} = x^{\sin x} \left(\cos x \log x + \frac{\sin x}{x} \right)$ <p>Now $v = (\sin x)^{\cos x}$</p> $\Rightarrow \log v = \cos x \cdot \log \sin x$ $\Rightarrow \frac{1}{v} \frac{dv}{dx} = -\sin x \cdot \log \sin x + \frac{\cos x}{\sin x} \cdot \cos x$ $\Rightarrow \frac{dv}{dx} = (\sin x)^{\cos x} [-\sin x \cdot \log \sin x + \cot x \cdot \cos x]$ <p>Now $y = u + v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$</p> $= (x)^{\sin x} \left[\cos x \cdot \log x + \frac{\sin x}{x} \right] + (\sin x)^{\cos x} [-\sin x \cdot \log \sin x + \cot x \cdot \cos x]$ <p align="center">OR</p> <p>$x = a(\theta - \sin \theta)$, $y = a(1 + \cos \theta)$</p> $\frac{dx}{d\theta} = a(1 - \cos \theta), \frac{dy}{d\theta} = -a \sin \theta$ $\Rightarrow \frac{dy}{dx} = \frac{-a \sin \theta}{a(1 - \cos \theta)} = \frac{-2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \sin^2 \frac{\theta}{2}} = -\cot \frac{\theta}{2}$ $\frac{d^2y}{dx^2} = \frac{1}{2} \operatorname{cosec}^2 \frac{\theta}{2} \cdot \frac{d\theta}{dx} = \frac{1}{2a(1 - \cos \theta)} \operatorname{cosec}^2 \frac{\theta}{2}$	<p align="center">$1\frac{1}{2}$</p> <p align="center">$1\frac{1}{2}$</p> <p align="center">$1\frac{1}{2}$</p>

	$\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{4a} \frac{\operatorname{cosec}^2 \frac{\theta}{2}}{\sin^2 \frac{\theta}{2}} = \frac{1}{4a} \operatorname{cosec}^4 \frac{\theta}{2}$	$1\frac{1}{2}$
<p>32</p>	<p>$(3xy+y^2)dx + (x^2 + xy)dy = 0$</p> <p>$\Rightarrow \frac{dy}{dx} = -\frac{(3xy+y^2)}{(x^2+xy)}$</p> <p>Put $y=vx$</p> <p>$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$</p> <p>$\Rightarrow v + x \frac{dv}{dx} = -\frac{(3x.vx+v^2x^2)}{(x^2+x.vx)} \Rightarrow x \frac{dv}{dx} = \frac{-3v-v^2}{1+v} - v$</p> <p>$= \frac{-3v-v^2-v-v^2}{1+v} = \frac{-4v-2v^2}{1+v}$</p> <p>$\frac{1+v}{2v^2+4v} dv = \frac{-1}{x} dx$</p> <p>Integrating both sides we get</p> <p>$\frac{1}{4} \log v^2+2v = -\log x + c$</p> <p>$\log v^2+2v = -\log x^4 + 4c$</p> <p>$\log v^2+2v + \log x^4 = 4c$</p> <p>$\frac{y^2+2xy}{x^2} \cdot x^4 = C$</p> <p>$(y^2+2xy)x^2 = C$</p> <p align="center">OR</p> <p>$(1+x^2)dy + 2xydx = \cot x dx, \quad x \neq 0$</p>	<p>$1\frac{1}{2}$</p> <p>$1\frac{1}{2}$</p>

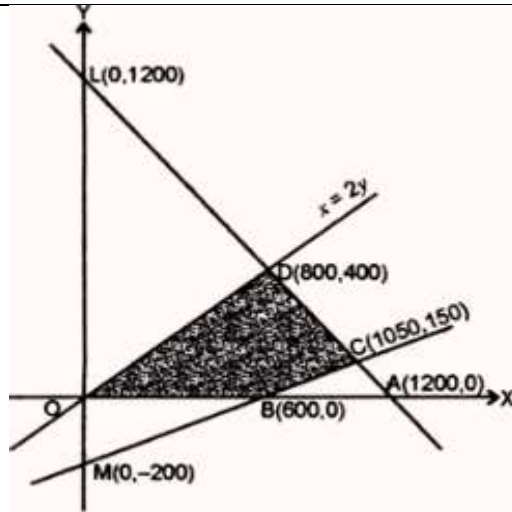
	$\frac{dy}{dx} = \frac{\cot x}{1+x^2} \cdot \frac{2x}{1+x^2} y \Rightarrow \frac{dy}{dx} + \frac{2x}{1+x^2} y = \frac{\cot x}{1+x^2}$ $P = \frac{2x}{1+x^2}, Q = \frac{\cot x}{1+x^2}$ $I.F = e^{\int \frac{2x}{1+x^2} dx} = e^{\log(1+x^2)} = (1+x^2)$ $y \cdot (1+x^2) = \int \frac{\cot x}{1+x^2} (1+x^2) dx = \int \cot x dx = \log \sin x + C$ $\Rightarrow y \cdot (1+x^2) = \log \sin x + C$	<p align="center">$1\frac{1}{2}$</p> <p align="center">$1\frac{1}{2}$</p>
<p>33</p>	<p>$f(x) = \sin x + \cos x, 0 \leq x \leq 2\pi$</p> <p>$f'(x) = \cos x - \sin x = \sqrt{2}(\cos x \sin \frac{\pi}{4} - \sin x \cos \frac{\pi}{4}) = -\sqrt{2} \sin(x - \frac{\pi}{4})$</p> <p>$0 < x < 2\pi \Rightarrow 0 - \frac{\pi}{4} < x - \frac{\pi}{4} < 2\pi - \frac{\pi}{4}$</p> <p>$\frac{\pi}{4} < x - \frac{\pi}{4} < \frac{7\pi}{4}$</p> <p><u>$f(x)$ is strictly increasing if $f'(x) > 0$</u></p> <p>$\Rightarrow -\sqrt{2} \sin(x - \frac{\pi}{4}) > 0$</p> <p>$\Rightarrow \sin(x - \frac{\pi}{4}) < 0 \Rightarrow \frac{-\pi}{4} < x - \frac{\pi}{4} < 0$</p> <p><u>Or</u> $\pi < x - \frac{\pi}{4} < \frac{7\pi}{4}$</p> <p>Or $0 < x < \frac{\pi}{4}$ or $5\frac{\pi}{4} < x < 2\pi$</p> <p>$\Rightarrow f(x)$ is strictly increasing when $x \in [0, \frac{\pi}{4}) \cup (\frac{5\pi}{4}, 2\pi]$</p> <p><u>For strictly decreasing:</u></p> <p>$-\sqrt{2} \sin(x - \frac{\pi}{4}) < 0$</p> <p>$\sin(x - \frac{\pi}{4}) > 0$</p> <p>$\Rightarrow 0 < x - \frac{\pi}{4} < \pi \Rightarrow \frac{\pi}{4} < x < \frac{5\pi}{4} \quad x \in (\frac{\pi}{4}, \frac{5\pi}{4})$</p> <p>$f(x)$ is strictly decreasing when $x \in (\frac{\pi}{4}, \frac{5\pi}{4})$</p> <p><u>OR</u></p> <p>$x = y^2$</p>	<p align="center">$1\frac{1}{2}$</p> <p align="center">$1\frac{1}{2}$</p>

	$1=2y\frac{dy}{dx}$ $\frac{dy}{dx} = \frac{1}{2y}$ $xy=k \Rightarrow x\frac{dy}{dx} + y=0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x}$ <p>curve intersect at right angle if $m_1 m_2 = -1$</p> $\Rightarrow \left(\frac{1}{2y}\right)\left(-\frac{y}{x}\right) = -1 \Rightarrow \frac{1}{2x} = 1 \Rightarrow x = \frac{1}{2}$ $x = y^2 = \frac{1}{2}$ $xy = k \Rightarrow x^2 y^2 = k^2$ $\left(\frac{1}{4}\right)\left(\frac{1}{2}\right) = k^2 \therefore 8k^2 = 1$	<p align="center">1</p> <p align="center">1</p> <p align="center">1</p>
<p>34</p>	$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1 + \sqrt{\tan x}}$ $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \dots\dots\dots (i)$ $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos(\frac{\pi}{2} - x)}}{\sqrt{\cos(\frac{\pi}{2} - x)} + \sqrt{\sin(\frac{\pi}{2} - x)}} dx$ $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \dots\dots\dots (ii)$ <p>Adding (i) & (ii) we get</p> $2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$ $2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1 \cdot dx = x \Big _{\frac{\pi}{6}}^{\frac{\pi}{3}} \Rightarrow 2I = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6} \Rightarrow I = \frac{\pi}{12}$	<p align="center">2</p> <p align="center">1</p>

<p>35</p>	 $\frac{x^2}{16} + \frac{y^2}{9} = 1 \Rightarrow y^2 = 9\left(1 - \frac{x^2}{16}\right)$ <p>Required Area</p> $= 4 \int_0^4 3 \sqrt{1 - \frac{x^2}{16}} dx$ $= 3 \int_0^4 \sqrt{16 - x^2} dx = 3 \left[\frac{x}{2} \sqrt{16 - x^2} + 8 \sin^{-1} \frac{x}{4} \right]_0^4$ $= 3 \left[8 \times \frac{\pi}{2} - 0 \right] = 12\pi \text{ sq. units}$	<p>1</p> <p>1</p> <p>1</p>
<p>36</p>	<p>Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$, $X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$</p> <p>$\Rightarrow A = -1$ $A^{-1} = \frac{1}{ A } \text{Adj } A$</p> <p>$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1}B = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$</p> <p>$x=0, y=5, z=3$</p> <p align="center">OR</p> <p>Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$</p>	<p>2</p> <p>$\frac{1}{2}$</p> <p>2</p> <p>$\frac{1}{2}$</p>

	$\text{adj}A = \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$ $ A = 4, \quad A^{-1} = \frac{1}{ A } \text{Adj} A$ $X = A^{-1}B = \frac{1}{4} \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix} \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ <p>$x=2, y=1, z=3$</p>	<p align="center">2</p> <p align="center">$\frac{1}{2} + \frac{1}{2}$</p> <p align="center">$1\frac{1}{2}$</p> <p align="center">$\frac{1}{2}$</p>
<p>37</p>	<p>Equation of plane through given 3 points $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$</p> $\Rightarrow \begin{vmatrix} x - 2 & y - 5 & z + 3 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix} = 0$ $\Rightarrow 2x + 3y + 4z = 7$ <p>Distance = $\frac{ ax_1 + by_1 + cz_1 + d }{\sqrt{a^2 + b^2 + c^2}} = \frac{2 \times 7 + 3 \times 2 + 4 \times 4 - 7}{\sqrt{4 + 9 + 16}} = \frac{29}{\sqrt{29}} = \sqrt{29}$</p> <p align="center">OR</p> <p>Solving the equation of line and plane, we get</p> $(2 + 3\lambda) \cdot 1 - (-1 + 4\lambda) + (-2 + 2\lambda) = 5$ $\Rightarrow \lambda = 4$ <p>Therefore point of intersection of line and plane is (14, 15, 6)</p> <p>Thus Distance between the point (-1, -5, -10) and (14, 15, 6) is</p> $\sqrt{225 + 400 + 256} = \sqrt{881}$	<p align="center">2</p> <p align="center">1</p> <p align="center">2</p> <p align="center">$2\frac{1}{2}$</p> <p align="center">1</p> <p align="center">$1\frac{1}{2}$</p>

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2

Let x is the number of dolls of type A and y is the number of dolls of type B that are manufactured.

Now according to question

Maximize $Z = 12x + 16y$

subject to constraints

$x + y \leq 1200$ 1

$y \leq x/2$

$\Rightarrow 2y \leq x$

$\Rightarrow x - 2y \geq 0$ 2

$x \leq 3y + 600$

$\Rightarrow x - 3y \leq 600$ 3

$x \geq 0, y \geq 0$ 4

Now draw the graph using equation 1 to 4 as shown in figure.

The shaded portion in the graph depicts the feasible region.

Point $Z = 12x + 16y$

A(800, 400) 7200

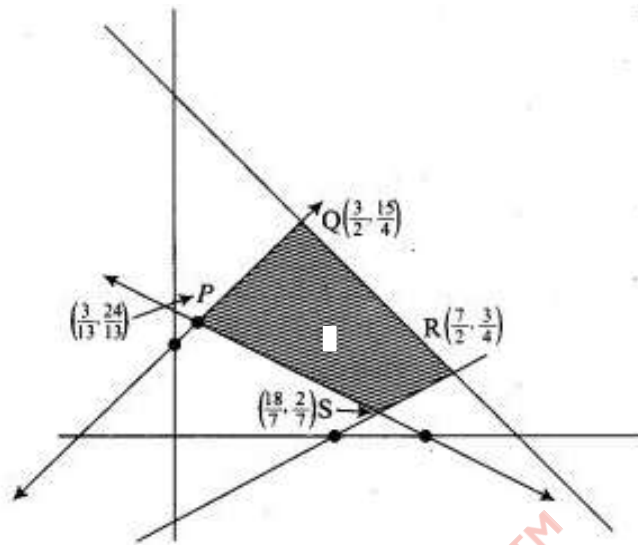
B(1050, 150) 15000

C(600, 0) 16000

The maximum profit is 16000.

OR

3



Sol. From the figure we have bounded region, with corner points as

$$P\left(\frac{3}{13}, \frac{24}{13}\right), Q\left(\frac{3}{2}, \frac{15}{4}\right), R\left(\frac{7}{2}, \frac{3}{4}\right), S\left(\frac{18}{7}, \frac{2}{7}\right)$$

Also $Z = x + 2y$.

Corner points	Corresponding value of Z
$\left(\frac{3}{13}, \frac{24}{13}\right)$	$\frac{51}{13} = 3\frac{12}{13}$
$\left(\frac{18}{7}, \frac{2}{7}\right)$	$\frac{22}{7} = 3\frac{1}{7}$ (Minimum)
$\left(\frac{7}{2}, \frac{3}{4}\right)$	$\frac{20}{4} = 5$
$\left(\frac{3}{2}, \frac{15}{4}\right)$	$\frac{36}{4} = 9$ (Maximum)

Hence, the maximum and minimum value of Z are 9 and $3\frac{1}{7}$, respectively

3

(ii) $Z = px + qy$

$$Z = \frac{3p}{2} + \frac{15q}{4} \text{ at } \left(\frac{3}{2}, \frac{15}{4}\right)$$

$$Z = \frac{7p}{2} + \frac{3q}{4} \text{ at } \left(\frac{7}{2}, \frac{3}{4}\right)$$

Both values of Z are maximum. $\therefore \frac{3p}{2} + \frac{15q}{4} = \frac{7p}{2} + \frac{3q}{4}$

$$\frac{4p}{2} = \frac{12q}{4} \Rightarrow 2p = 3q$$

No of maximum solutions are infinite lying in the line joining Q and R.

2

