1. Real Numbers

Exercise 1.1

1 A. Question

Using Euclid's division algorithm, find the HCF of

156 and 504

Answer

Given numbers are 156 and 504

Here, $504 > 156$

So, we divide 504 by 156

By using **Euclid's division lemma**, we get

$504 = 156 \times 3 + 36$

Here, $r = 36 \neq 0$.

On taking 156 as dividend and 36 as the divisor and we apply Euclid's division lemma, we get

$156 = 36 \times 4 + 12$

Here, $r = 12 \neq 0$

So, on taking 36 as dividend and 12 as the divisor and again we apply Euclid's division lemma, we get

$36 = 12 \times 3 + 0$

The remainder has now become 0, so our procedure stops. Since the divisor at this last stage is 12, the **HCF of 156 and 504 is 12.**

1 B. Question

Using Euclid's division algorithm, find the HCF of

135 and 225

Answer

Given numbers are 135 and 225

Here, 225 > 135

So, we divide 225 by 135

By using **Euclid's division lemma**, we get

225 = 135 × 1 + 90

Here, r = 90 ≠ 0.

On taking 135 as dividend and 90 as the divisor and we apply Euclid's division lemma, we get

135 = 90 × 1 + 45

Here, r = 45 ≠ 0

So, on taking 90 as dividend and 45 as the divisor and again we apply Euclid's division lemma, we get

90 = 45 × 2 + 0

The remainder has now become 0, so our procedure stops. Since the divisor at this last stage is 45, the **HCF of 135 and 225 is 45.**

1 C. Question

Using Euclid's division algorithm, find the HCF of

455 and 42

Answer

Given numbers are 455 and 42

Here, 455 > 42

So, we divide 455 by 42

By using **Euclid's division lemma**, we get

455 = 42 × 10 + 35

Here, r = 35 ≠ 0.

On taking 42 as dividend and 35 as the divisor and we apply Euclid's division lemma, we get

42 = 35 × 1 + 7

Here, r = 7 ≠ 0

So, on taking 35 as dividend and 7 as the divisor and again we apply Euclid's division lemma, we get

35 = 7 × 5 + 0

The remainder has now become 0, so our procedure stops. Since the divisor at this last stage is 7, the **HCF of 455 and 42 is 7.**

1 D. Question

Using Euclid's division algorithm, find the HCF of

8840 and 23120

Answer

Given numbers are 8840 and 23120

Here, 23120 > 8840

So, we divide 23120 by 8840

By using **Euclid's division lemma**, we get

23120 = 8840 × 2 + 5440

Here, r = 5440 ≠ 0.

On taking 8840 as dividend and 5440 as the divisor and we apply Euclid's division lemma, we get

8840 = 5440 × 1 + 3400

Here, r = 3400 ≠ 0

On taking 5440 as dividend and 3400 as the divisor and again we apply Euclid's division lemma, we get

5440 = 3400 × 1 + 2040

Here, r = 2040 ≠ 0.

On taking 3400 as dividend and 2040 as the divisor and we apply Euclid's division lemma, we get

3400 = 2040 × 1 + 1360

Here, r = 1360 ≠ 0

So, on taking 2040 as dividend and 1360 as the divisor and again we apply Euclid's division lemma, we get

2040 = 1360 × 1 + 680

Here, r = 680 ≠ 0

So, on taking 1360 as dividend and 680 as the divisor and again we apply Euclid's division lemma, we get

1360 = 680 × 2 + 0

The remainder has now become 0, so our procedure stops. Since the divisor at this last stage is 680, the **HCF of 8840 and 23120 is 680.**

1 E. Question

Using Euclid's division algorithm, find the HCF of

4052 and 12576

Answer

Given numbers are 4052 and 12576

Here, 12576 > 4052

So, we divide 12576 by 4052

By using **Euclid's division lemma**, we get

12576 = 4052 × 3 + 420

Here, r = 420 ≠ 0.

On taking 4052 as dividend and 420 as the divisor and we apply Euclid's division lemma, we get

4052 = 420 × 9 + 272

Here, r = 272 ≠ 0

On taking 420 as dividend and 272 as the divisor and again we apply Euclid's division lemma, we get

420 = 272 × 1 + 148

Here, r = 148 ≠ 0

On taking 272 as dividend and 148 as the divisor and again we apply Euclid's division lemma, we get

272 = 148 × 1 + 124

Here, r = 124 ≠ 0.

On taking 148 as dividend and 124 as the divisor and we apply Euclid's division lemma, we get

148 = 124 × 1 + 24

Here, r = 24 ≠ 0

So, on taking 124 as dividend and 24 as the divisor and again we apply Euclid's division lemma, we get

124 = 24 × 5 + 4

Here, r = 4 ≠ 0

So, on taking 24 as dividend and 4 as the divisor and again we apply

Euclid's division lemma, we get

24 = 4 × 6 + 0

The remainder has now become 0, so our procedure stops. Since the divisor at this last stage is 4, the **HCF of 4052 and 12576 is 4.**

1 F. Question

Using Euclid's division algorithm, find the HCF of

3318 and 4661

Answer

Given numbers are 3318 and 4661

Here, 4661 > 3318

So, we divide 4661 by 3318

By using **Euclid's division lemma**, we get

4661 = 3318 × 1 + 1343

Here, r = 1343 ≠ 0.

On taking 3318 as dividend and 1343 as the divisor and we apply Euclid's division lemma, we get

3318 = 1343 × 2 + 632

Here, r = 632 ≠ 0

So, on taking 1343 as dividend and 632 as the divisor and again we apply Euclid's division lemma, we get

1343 = 632 × 2 + 79

Here, r = 79 ≠ 0

So, on taking 632 as dividend and 79 as the divisor and again we apply Euclid's division lemma, we get

632 = 79 × 8 + 0

The remainder has now become 0, so our procedure stops. Since the divisor at this last stage is 79, the **HCF of 3318 and 4661 is 79.**

1 G. Question

Using Euclid's division algorithm, find the HCF of

250, 175 and 425

Answer

Given numbers are 250, 175 and 425

∴ 425 > 250 > 175

On applying **Euclid's division lemma** for 425 and 250, we get

425 = 250 × 1 + 175

Here, r = 175 ≠ 0.

So, again applying Euclid's division lemma with new dividend 250 and new divisor 175, we get

250 = 175 × 1 + 75

Here, r = 75 ≠ 0

So, on taking 175 as dividend and 75 as the divisor and again we apply Euclid's division lemma, we get

175 = 75 × 2 + 25

Here, r = 25 ≠ 0.

So, again applying Euclid's division lemma with new dividend 75 and new divisor 25, we get

75 = 25 × 3 + 0

Here, r = 0 and divisor is 25.

So, HCF of 425 and 225 is 25.

Now, applying Euclid's division lemma for 175 and 25, we get

175 = 25 × 7 + 0

Here, remainder = 0

So, **HCF of 250, 175 and 425 is 25.**

1 H. Question

Using Euclid's division algorithm, find the HCF of

4407, 2938 and 1469

Answer

Given numbers are 4407, 2938 and 1469

∴ 4407 > 2938 > 1469

On applying **Euclid's division lemma** for 4407 and 2938, we get

4407 = 2938 × 1 + 1469

Here, r = 1469 ≠ 0.

So, again applying Euclid's division lemma with new dividend 2938 and new divisor 1469, we get

2938 = 1469 × 2 + 0

The remainder has now become 0, so our procedure stops. Since the divisor at this stage is 1469, the HCF of 4407 and 2938 is 1469**.**

Now, applying Euclid's division lemma for 1469 and 1469, we get

1469 = 1469 × 1 + 0

Here, remainder = 0

So, **HCF of 4407, 2938 and 1469 is 1469.**

2. Question

Show that every positive even integer is of the form 2q and that every positive odd integer is of the form 2q + 1, where q is some integer.

Answer

Let a and b be any two positive integers, such that a > b.

Then, a = bq + r, 0 ≤ r < b …(i) [by Euclid's division lemma]

On putting b = 2 in Eq. (i), we get

a = 2q + r, 0 ≤ r < 2 …(ii)

$\Rightarrow$ r = 0 or 1

When r = 0, then from Eq. (ii), a = 2q, which is divisible by 2

When r = 1, then from Eq. (ii), a = 2q + 1, which is not divisible by 2.

Thus, every positive integer is either of the form 2q or 2q + 1.

That means every positive integer is either even or odd. So, if a is a positive even integer, then a is of the form 2q and if a, is a positive odd integer, then a is of the form 2q + 1.

3. Question

Show that any positive odd integer is of the form 4q + 1 or 4q + 3, where q is some integer.

Answer

Let a be any positive odd integer. We apply the division algorithm with a and b = 4.

Since $0 \leq r < 4$, the possible remainders are 0,1,2 and 3.

i.e. a can be 4q, or 4q + 1, or 4q + 2, or 4q + 3, where q is the quotient.

As we know a is odd, a can't be 4q or 4q + 2 because they both are divisible by 2.

Therefore, any positive odd integer is of the form 4q + 1 or 4q + 3.

4. Question

There are 250 and 425 liters of milk in two containers. What is the maximum capacity of the container which can measure completely the quantity of milk in the two containers?

Answer

Given the capacities of the two containers are 250 L and 425 L.

Here, 425 > 250

Now, we divide 425 by 250.

We used **Euclid's division lemma.**

$425 = 250 \times 1 + 175$

Here, remainder $r = 175 \neq 0$

So, the new dividend is 250 and the new divisor is 175, again we apply Euclid division algorithm.

250 = 175 × 1 + 75

Here, remainder r = 75 ≠ 0

On taking the new dividend is 175 and the new divisor is 75, we apply Euclid division algorithm.

175 = 75 × 2 + 25

Here, remainder r = 25 ≠ 0

On taking new dividend is 75 and the new divisor is 25, again we apply Euclid division algorithm.

75 = 25 × 3 + 0

Here, remainder is zero and divisor is 25.

So, the HCF of 425 and 250 is 25.

Hence, **the maximum capacity of the required container is 25 L.**

5. Question

A rectangular surface has length 4661 meters and breadth 3318 meters. On this area, square tiles are to be put. Find the maximum length of such tiles.

Answer

Given length and breadth are 4661 m and 3318 m respectively.

Here, 4661 > 3318

So, we divide 4661 by 3318

By using **Euclid's division lemma**, we get

4661 = 3318 × 1 + 1343

Here, r = 1343 ≠ 0.

On taking 3318 as dividend and 1343 as the divisor and we apply Euclid's division lemma, we get

3318 = 1343 × 2 + 632

Here, r = 632 ≠ 0

So, on taking 1343 as dividend and 632 as the divisor and again we apply Euclid's division lemma, we get

1343 = 632 × 2 + 79

Here, r = 79 ≠ 0

So, on taking 632 as dividend and 79 as the divisor and again we apply Euclid's division lemma, we get

632 = 79 × 8 + 0

The remainder has now become 0, so our procedure stops. Since the divisor at this last stage is 79, the HCF of 3318 and 4661 is 79.

Hence, the maximum length of such tiles is 79 meters.

6. Question

Find the least number of square tiles which can the floor of a rectangular shape having length and breadth 16 meters 58 centimeters and 8 meters 32.

Answer

Firstly, we find the length of the largest tile so for that we have to find the HCF of 1658 and 832.

Here, 1658 > 832

So, we divide 1658 by 832

By using **Euclid's division lemma**, we get

1658 = 832 × 1 + 826

Here, r = 826 ≠ 0.

On taking 832 as dividend and 826 as the divisor and we apply Euclid's division lemma, we get

832 = 826 × 1 + 6

Here, r = 6 ≠ 0

So, on taking 826 as dividend and 6 as the divisor and again we apply Euclid's division lemma, we get

826 = 6 × 137 + 4

Here, r = 4 ≠ 0

So, on taking 6 as dividend and 4 as the divisor and again we apply Euclid's division lemma, we get

6 = 4 × 1 + 2

Here, r = 2 ≠ 0

So, on taking 4 as dividend and 2 as the divisor and again we apply Euclid's division lemma, we get

4 = 2 × 2 + 0

The remainder has now become 0, so our procedure stops. Since the divisor at this last stage is 79, the HCF of 1658 and 832 is 2.

So, the length of the largest tile is 2 cm

Area of each tile = 2 × 2 = $4cm^2$

The required number of tiles = $\frac{\text{Area of floor}}{\text{Area of tiles}}$

$= \frac{1658 \times 832}{2 \times 2}$

= 344864

Least number of square tiles are required are 344864

Exercise 1.2

1 A. Question

Express each of the following numbers as a product of its prime factors:

4320

Answer

Given number is 4320

Factorization of 4320 is

2	4320
2	2160
2	1080
2	540
2	270
3	135
3	45
3	15
5	5
	1

Hence, 4320 = $2^5 \times 3^3 \times 5$ (Product of its prime factors)

1 B. Question

Express each of the following numbers as a product of its prime factors:

7560

Answer

Given number is 7560

Factorization of 7560 is

2	7560
2	3780
2	1890
3	945
3	315
3	105
5	35
7	7
	1

Hence, 7560 = $2^3 \times 3^3 \times 5 \times 7$ (Product of its prime factors)

1 C. Question

Express each of the following numbers as a product of its prime factors:

140

Answer

Given number is 140

Factorization of 140 is

2	140
2	70
5	35
7	7
	1

Hence, 140 = $2^2 \times 5 \times 7$ (Product of its prime factors)

1 D. Question

Express each of the following numbers as a product of its prime factors:

5005

Answer

Given number is 5005

Factorization of 5005 is

5	5005
7	1001
11	143
13	13
	1

Hence, 5005 = 5 × 7 × 11 × 13 (Product of its prime factors)

1 E. Question

Express each of the following numbers as a product of its prime factors:

32760

Answer

Given number is 32760

Factorization of 32760 is

2	32760
2	16380
2	8190
3	4095
3	1365
5	455
7	91
13	13
	1

Hence, 32760 = $2^3 \times 3^2 \times 5 \times 7 \times 13$ (Product of its prime factors)

1 F. Question

Express each of the following numbers as a product of its prime factors:

156

Answer

Given number is 156

Factorization of 156 is

2	156
2	78
3	39
13	13
	1

Hence, $156 = 2^2 \times 3 \times 13$ (Product of its prime factors)

1 G. Question

Express each of the following numbers as a product of its prime factors:

729

Answer

Given number is 729

Factorization of 729 is

3	729
3	243
3	81
3	27
3	9
3	3
	1

Hence, $729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$ (Product of its prime factors)

2. Question

Find the highest power of 5 in 23750.

Answer

To find the highest power of 5 in 23750, we have to factorize 23570

2	23750
5	11875
5	2375
5	475
5	95
19	19
	1

Hence, $23750 = 2 \times 5^4 \times 19$

So, the highest power of 5 in 23750 is **4.**

3. Question

Find the highest power of 2 in 1440.

Answer

Given number is 1440

Factorization of 1440 is

2	1440
2	720
2	360
2	180
2	90
3	45
3	15
5	5
	1

Factors of 1440 = $2^5 \times 3^2 \times 5$

So, the highest power of 2 is **5.**

4. Question

If 6370 = $2^m .5^n.7^k.13^p$, then find m + n + k + p.

Answer

We have to factorize the 6370 to find the value of m, n, k and p

Factorization of 6370 is

2	6370
5	3185
7	637
7	91
13	13
	1

6370 = $2 \times 5 \times 7^2 \times 13$

On Comparing, we get

6370 = $2^m .5^n.7^k.13^p = 2^1 \times 5^1 \times 7^2 \times 13^1$

m = 1

n = 1

k = 2

p = 1

So, m + n + k + p = 5

5 A. Question

Which of the following is a pair of co-primes?

(32,62)

Answer

Given numbers are 32 and 62

For pairs to be co-primes there should be no common factor except 1

Factorization of 32 and 62 are

2	32
2	16
2	8
2	4
2	2
	1

Factors of $32 = \textcircled{2} \times 2 \times 2 \times 2 \times 2 = 2^5$

2	62
31	31
	1

Factors of $62 = \textcircled{2} \times 31$

Here, we can see that 2 is the common factor. So, (32,62) is not co-prime.

5 B. Question

Which of the following is a pair of co-primes?

(18,25)

Answer

Given numbers are 18 and 25

For pairs to be co-primes there should be no common factor except 1

Factorization of 18 and 25 are

2	18
3	9
3	3
	1

Factors of $32 = 2 \times 3 \times 3$

5	25
5	5
	1

Factors of 62 = 5 × 5

Therefore, there is no common factor. So, (18,25) is co-prime.

5 C. Question

Which of the following is a pair of co-primes?

(31, 93)

Answer

Given numbers are 31 and 93

For pairs to be co-primes there should be no common factor except 1

Factorization of 31 and 93 are

3	93
31	31
2	1

Factors of $93 = \textcircled{31} \times 3$

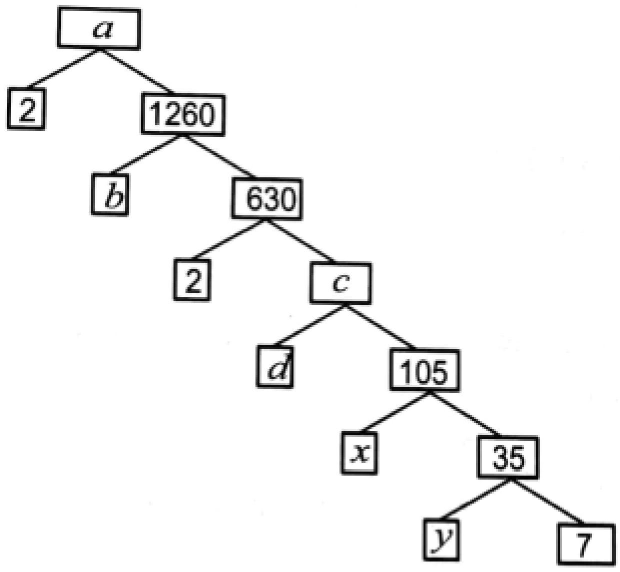
31	31
	1

Factors of $31 = \textcircled{31} \times 1$

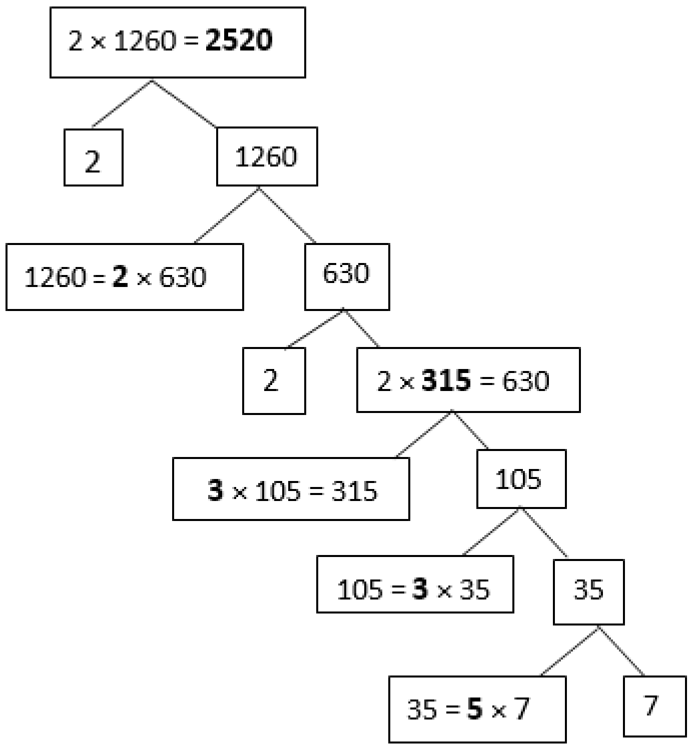
Here, we can see that 31 is the common factor. So, (31, 93) is not co-prime.

6 A. Question

Write down the missing numbers a, b, c, d, x, y in the following factor tree :



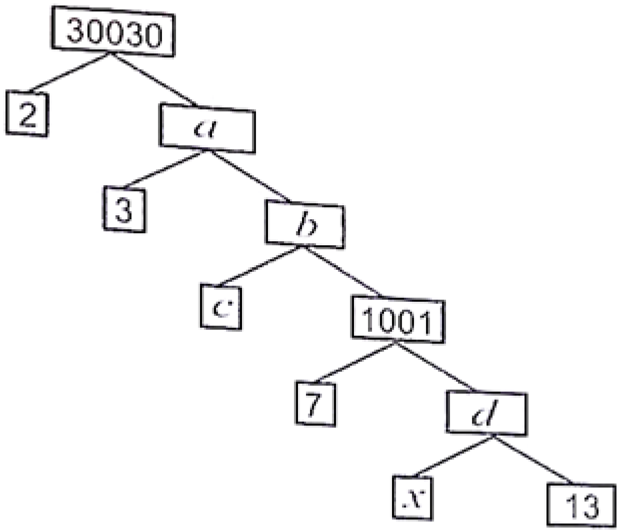
Answer



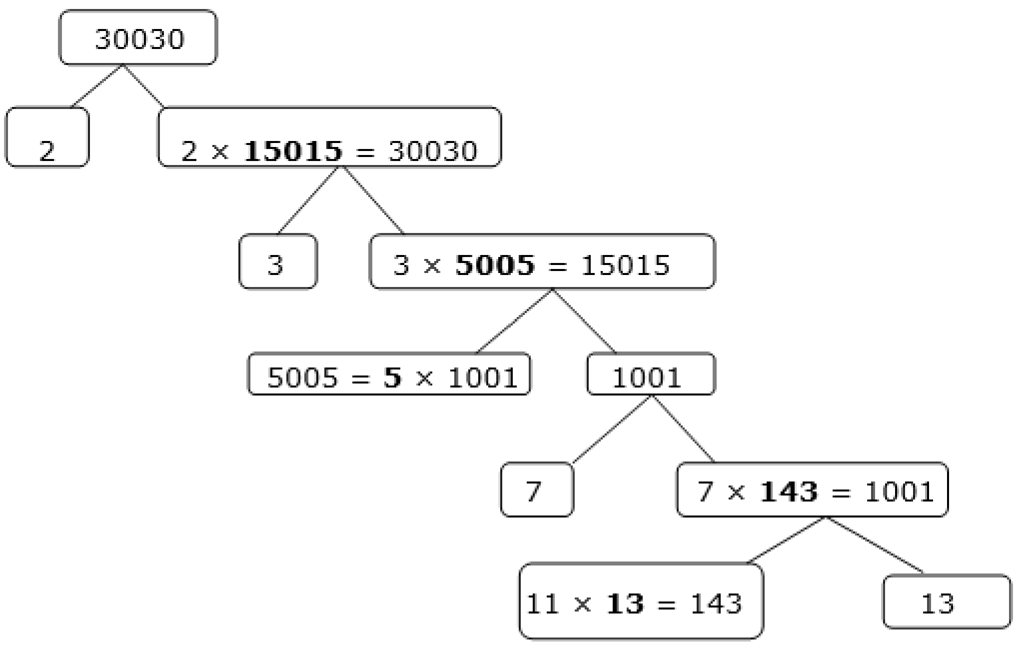
Here, a = 2520; b = 2; c = 315; d = 3; x = 3; y = 5

6 B. Question

Write down the missing numbers a, b, c, d, x, y in the following factor tree :



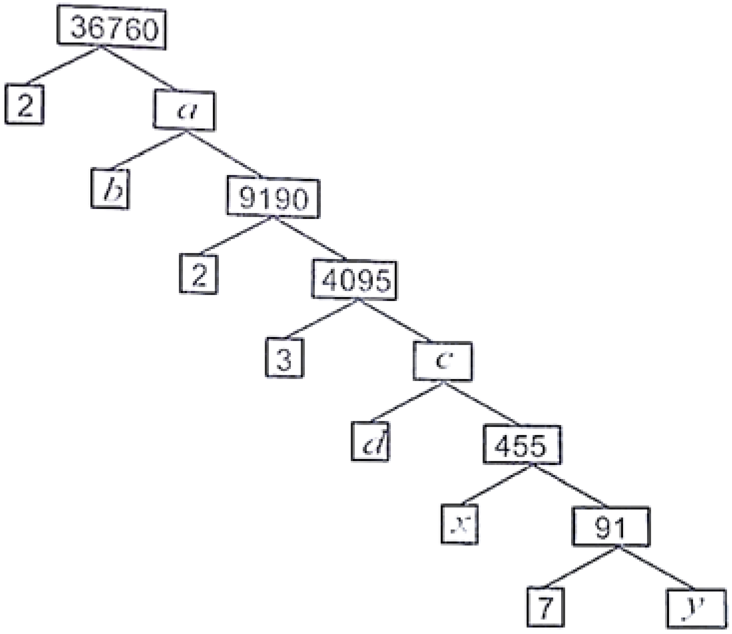
Answer



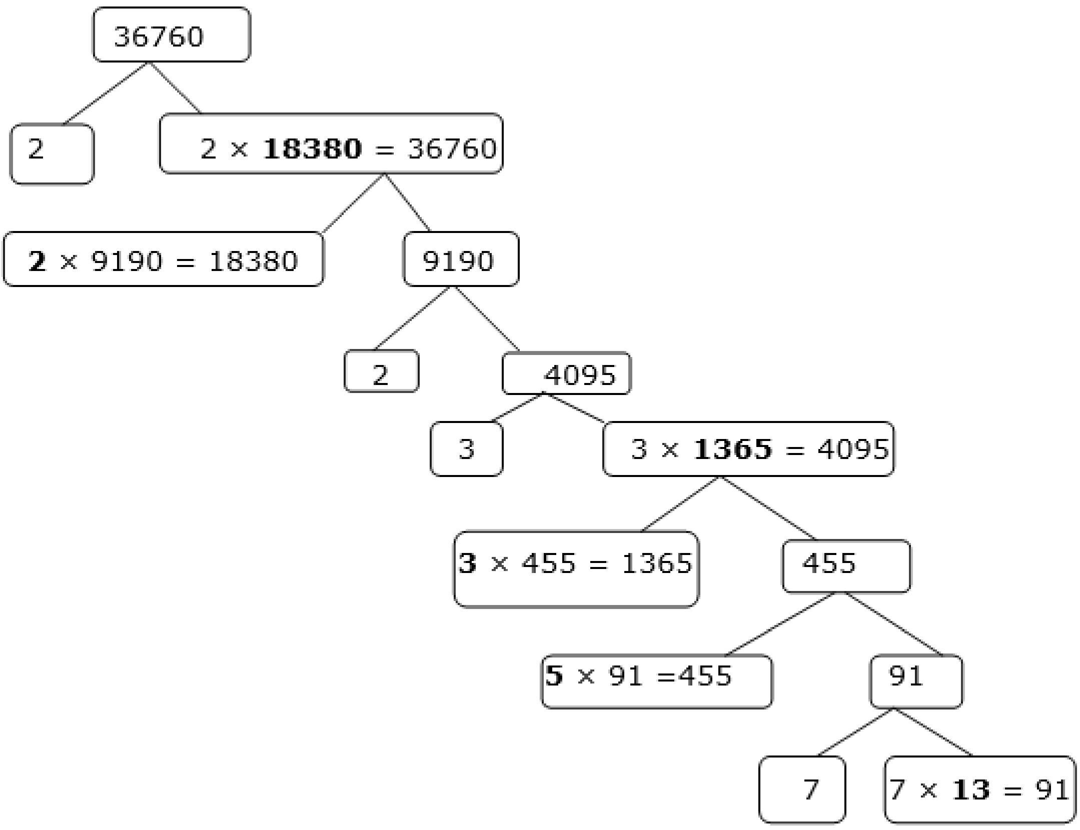
Here, a = 15015; b = 5005; c = 5; d = 143; x = 13

6 C. Question

Write down the missing numbers a, b, c, d, x, y in the following factor tree :



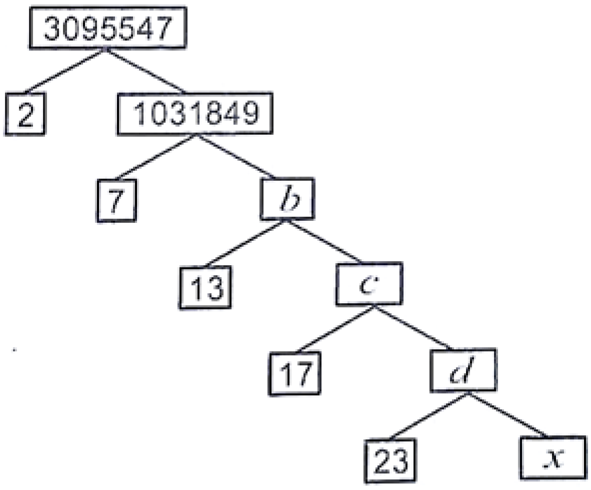
Answer



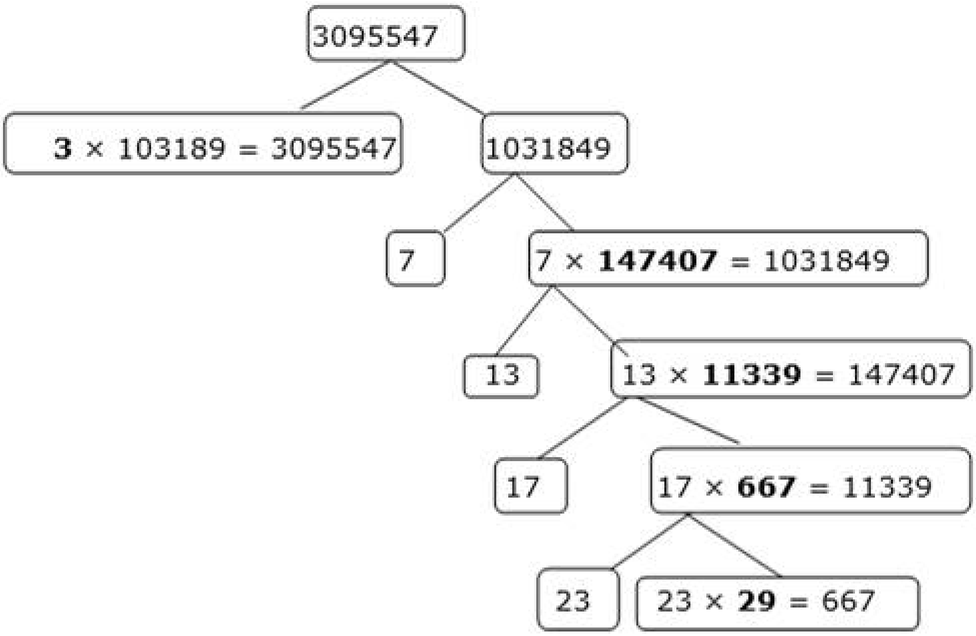
Here, a = 18380; b = 2; c = 1365; d = 3; x = 5; y = 13

6 D. Question

Write down the missing numbers a, b, c, d, x, y in the following factor tree :



Answer



Here, a = 3; b = 147407; c = 11339; d = 667; x = 29

7 A. Question

Find the LCM and HCF of the following integers by applying the prime factorization method :

96 and 404

Answer

Given numbers are 96 and 404.

The prime factorization of 96 and 404 gives:

2	96
2	48
2	24
2	12
2	6
3	3
	1

$96 = 2^5 \times 3 = (2 \times 2) \times 2 \times 2 \times 2 \times 3$

2	404
2	202
101	101
	1

$404 = 2^2 \times 101 = (2 \times 2) \times 101$

Here, 2^2 is the smallest power of the common factor 2.

Therefore, the **H.C.F** of these two integers is 2 × 2 = **4**

$2^5 \times 3^1 \times 101^1$ are the greatest powers of the prime factors 2, 3 and 101 respectively involved in the given numbers.

Now, **L.C.M** of 96 and 404 is 2 × 2 × 2 × 2 × 2 × 3 × 101 = **9696**

7 B. Question

Find the LCM and HCF of the following integers by applying the prime factorization method :

6 and 20

Answer

Given numbers are 6 and 20

The prime factorization of 6 and 20 gives:

2	6
3	3
	1

$6 = (2) \times 3$

2	20
2	10
5	5
	1

$20 = 2 \times 2 \times 5$

Here, 2^1 is the smallest power of the common factor 2.

Therefore, the **H.C.F** of these two integers = **2**

$2^2 \times 3^1 \times 5^1$ are the greatest powers of the prime factors 2, 3 and 5 respectively involved in the given numbers.

Now, **L.C.M** of 6 and 20 = 2 × 2 × 3 × 5 = **60**

7 C. Question

Find the LCM and HCF of the following integers by applying the prime factorization method :

26 and 91

Answer

Given numbers are 26 and 91.

The prime factorization of 26 and 91 gives:

2	26
13	13
	1

$26 = 2 \times 13$

7	91
13	13
	1

$91 = 7 \times 13$

Here, 13^1 is the smallest power of the common factor 13.

Therefore, the **H.C.F** of these two integers = **13**

$2^1 \times 7^1 \times 13^1$ are the greatest powers of the prime factors 2, 7 and 13 respectively involved in the given numbers.

Now, **L.C.M** of 6 and 21 is 2 × 7 × 13 = **182**

7 D. Question

Find the LCM and HCF of the following integers by applying the prime factorization method :

87 and 145

Answer

Given numbers are 87 and 145.

The prime factorization of 87 and 145 gives:

3	87
29	29
	1

$87 = 3 \times \textcircled{29}$

5	145
29	29
	1

$145 = 5 \times \textcircled{29}$

Here, 29^1 is the smallest power of the common factor 29.

Therefore, the **H.C.F** of these two integers = **29**

$3^1 \times 5^1 \times 29^1$ are the greatest powers of the prime factors 3, 5 and 29 respectively involved in the given numbers.

Now, **L.C.M** of 87 and 145 = 3 × 5 × 29 = **435**

7 E. Question

Find the LCM and HCF of the following integers by applying the prime factorization method :

1485 and 4356

Answer

Given numbers are 1485 and 4356.

The prime factorization of 1485 and 4356 gives:

3	1485
3	495
3	165
5	55
11	11
	1

$1485 = \boxed{3 \times 3} \times 3 \times 5 \times \textcircled{11}$

2	4356
2	2178
3	1089
3	363
11	121
11	11
	1

$4356 = 2 \times 2 \times 3 \times 3 \times 11 \times 11$

Here, $3^2 \times 11$ is the smallest power of the common factors 3 and 11.

Therefore, the **H.C.F** of these two integers = 3 × 3 × 11 = **99**

$2^2 \times 3^3 \times 5^1 \times 11^2$ are the greatest powers of the prime factors 2, 3 and 7 respectively involved in the given numbers.

Now, **L.C.M** of 1485 and 4356 = 2 × 2 × 3 × 3 × 3 × 5 × 11 × 11 = **65430**

7 F. Question

Find the LCM and HCF of the following integers by applying the prime factorization method :

1095 and 1168

Answer

Given numbers are 1095 and 1168.

The prime factorization of 1095 and 1168 gives:

3	1095
5	365
73	73
	1

$1485 = 3 \times 5 \times 73$

2	1168
2	584
2	292
2	146
73	73
	1

$4356 = 2 \times 2 \times 2 \times 2 \times 73$

Here, 73^1 is the smallest power of the common factor 73.

Therefore, the **H.C.F** of these two integers = **73**

$2^4 \times 3^1 \times 5^1 \times 73^1$ are the greatest powers of the prime factors 2, 3, 5 and 73 respectively involved in the given numbers.

Now, **L.C.M** of 1485 and 4356 = 2 × 2 × 2 × 2 × 3 × 5 × 73= **17520**

7 G. Question

Find the LCM and HCF of the following integers by applying the prime factorization method :

6 and 21

Answer

Given numbers are 6 and 21.

The prime factorization of 6 and 21 gives:

2	6
3	3
	1

6 = 2 ×(3)

3	21
7	7
	1

21 =(3)× 7

Here, 3^1 is the smallest power of the common factor 3.

Therefore, the **H.C.F** of these two integers = **3**

$2^1 \times 3^1 \times 7^1$ are the greatest powers of the prime factors 2, 3 and 7 respectively involved in the given numbers.

Now, **L.C.M** of 6 and 21 is 2 ×3 × 7 = **42**

8 A. Question

Find the LCM and HCF of the following pair of integers and verify that LCM X HCF = Product of two numbers :

96 and 404

Answer

Given numbers are 96 and 404

The prime factorization of 96 and 404 gives:

$96 = 2^5 \times 3$ and $404 = 2^2 \times 101$

Therefore, the H.C.F of these two integers = 2^2 = **4**

Now, the L.C.M of 96 and 404 = 2 × 2 × 2 × 2 × 2 × 3 × 101 = **9696**

Now, we have to verify

L.C.M (a, b) × H.C.F (a, b) = Product of two numbers (a × b)

L.H.S = L.C.M × H.C.F = 9696 × 4 = 38784

R.H.S = Product of two numbers = 96 × 404 = 38784

Hence, L.H.S = R.H.S

So, the product of two numbers is equal to the product of their HCF and LCM.

8 B. Question

Find the LCM and HCF of the following pair of integers and verify that LCM X HCF = Product of two numbers :

852 and 1491

Answer

Given numbers are 852 and 1491

The prime factorization of 852 and 1491 gives:

852 = 2 × 2 × 3 × 71 and 1491 = 3 × 7 × 71

Therefore, the H.C.F of these two integers = 3 × 71 = **213**

Now, the L.C.M of 96 and 404 = 2 × 2 × 3 × 7 × 71 = **5964**

Now, we have to verify

L.C.M (a, b) × H.C.F (a, b) = Product of two numbers (a × b)

L.H.S = L.C.M × H.C.F = 5964 × 213 = 1270332

R.H.S = Product of two numbers = 852 × 1491 = 1270332

Hence, L.H.S = R.H.S

So, the product of two numbers is equal to the product of their HCF and LCM.

8 C. Question

Find the LCM and HCF of the following pair of integers and verify that LCM X HCF = Product of two numbers :

777 and 1147

Answer

Given numbers are 777 and 1147

The prime factorization of 777 and 1147 gives:

$777 = 3 \times 7 \times 37$ and $1147 = 31 \times 37$

Therefore, the H.C.F of these two integers = **37**

Now, the L.C.M of 96 and 404 = $3 \times 7 \times 31 \times 37$ = **24087**

Now, we have to verify

L.C.M (a, b) × H.C.F (a, b) = Product of two numbers (a × b)

L.H.S = L.C.M × H.C.F = $24087 \times 37 = 891219$

R.H.S = Product of two numbers = $777 \times 1147 = 891219$

Hence, L.H.S = R.H.S

So, the product of two numbers is equal to the product of their HCF and LCM.

8 D. Question

Find the LCM and HCF of the following pair of integers and verify that LCM X HCF = Product of two numbers :

36 and 64

Answer

Given numbers are 36 and 64

The prime factorization of 36 and 64 gives:

$36 = 2 \times 2 \times 3 \times 3$ and $64 = 2^6$

Therefore, the H.C.F of these two integers = 2×2 = **4**

Now, the L.C.M of 36 and 64 = $3 \times 3 \times 2^6$ = **576**

Now, we have to verify

L.C.M (a, b) × H.C.F (a, b) = Product of two numbers (a × b)

L.H.S = L.C.M × H.C.F = 576 × 4 = 2304

R.H.S = Product of two numbers = 36 × 64 = 2304

Hence, L.H.S = R.H.S

So, the product of two numbers is equal to the product of their HCF and LCM.

8 E. Question

Find the LCM and HCF of the following pair of integers and verify that LCM X HCF = Product of two numbers :

32 and 80

Answer

Given numbers are 32 and 80

The prime factorization of 32 and 80 gives:

$32 = 2^5$ and $80 = 2^4 \times 5$

Therefore, the H.C.F of these two integers = 2^4 = **16**

Now, the L.C.M of 32 and 80 = 5×2^5 = **160**

Now, we have to verify

L.C.M (a, b) × H.C.F (a, b) = Product of two numbers (a × b)

L.H.S = L.C.M × H.C.F = 160 × 16 = 2560

R.H.S = Product of two numbers = 32 × 80 = 2560

Hence, L.H.S = R.H.S

So, the product of two numbers is equal to the product of their HCF and LCM.

8 F. Question

Find the LCM and HCF of the following pair of integers and verify that LCM X HCF = Product of two numbers :

902 and 1517

Answer

Given numbers are 902 and 1517

The prime factorization of 902 and 1517 gives:

902 = 2 × 11 × 41 and 1517 = 37 × 41

Therefore, the H.C.F of these two integers = **41**

Now, the L.C.M of 902 and 1517 = 2 ×

11 × 37 × 41 = **33374**

Now, we have to verify

L.C.M (a, b) × H.C.F (a, b) = Product of two numbers (a × b)

L.H.S = L.C.M × H.C.F = 33374 × 41 = 1368334

R.H.S = Product of two numbers = 902 × 1517 = 1368334

Hence, L.H.S = R.H.S

So, the product of two numbers is equal to the product of their HCF and LCM.

9 A. Question

Find LCM and HCF of the following integers by using prime factorization method:

6, 72 and 120

Answer

Given numbers are 6, 72 and 120

Factorization of 6, 72 and 120

2	6
3	3
	1

2	72
2	36
2	18
3	9
3	3
	1

2	120
2	60
2	30
3	15
5	5
	1

$$6 = 2 \times 3 = 2^1 \times 3^1$$
$$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$$
$$120 = 2 \times 2 \times 2 \times 3 \times 5 = 2^3 \times 3^1 \times 5^1$$

Here, $2^1 \times 3^1$ are the smallest powers of the common factors 2 and 3, respectively.

So, **HCF (6, 72, 120)** = 2 × 3 = **6**

$2^3 \times 3^2 \times 5^1$ are the greatest powers of the prime factors 2, 3 and 5 respectively involved in the given three numbers .

LCM of these three integers = 2 × 2 × 2 × 3 × 3 × 5 = **360**

9 B. Question

Find LCM and HCF of the following integers by using prime factorization method:

8, 9, and 25

Answer

Given numbers are 8, 9 and 25

Factorization of 8, 9 and 25

2	8
2	4
2	2
	1

3	9
3	3
	1

5	25
5	5
	1

8 = 2 × 2 × 2 × 1 = 2^3 × 1

9 = 3 × 3 × 1 = 3^2 × 1

25 = 5 × 5 × 1 = 5^2 × 1

Here, 1^1 is the smallest power of the common factor 1.

So, **HCF (8, 9, 25) = 1**

$2^3 \times 3^2 \times 5^2$ are the greatest powers of the prime factors 2, 3 and 5 respectively involved in the given three numbers .

LCM of these three integers = 2 × 2 × 2 × 3 × 3 × 5 × 5 = **1800**

9 C. Question

Find LCM and HCF of the following integers by using prime factorization method:

12, 15, and 21

Answer

Given numbers are 12, 15 and 21

Factorization of 12, 15 and 21

2	12
2	6
3	3
	1

3	15
5	5
	1

3	21
7	7
	1

$12 = 2 \times 2 \times 3 = 2^2 \times 3^1$
$15 = 3 \times 5 = 3^1 \times 5^1$
$21 = 3 \times 7 = 3^1 \times 7^1$

Here, 3^1 is the smallest power of the common factor 3.

So, **HCF (12, 15, 21) = 3**

$2^2 \times 3^1 \times 5^1 \times 7^1$ are the greatest powers of the prime factors 2, 3, 5 and 7 respectively involved in the given three numbers .

LCM of these three integers = 2 × 2 × 3 × 5 × 7 = **420**

9 D. Question

Find LCM and HCF of the following integers by using prime factorization method:

36, 45, and 72

Answer

Given numbers are 36, 45 and 72

Factorization of 36, 45 and 72

2	36
2	18
3	9
3	3
	1

3	45
3	15
5	5
	1

2	72
2	36
2	18
3	9
3	3
	1

$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

$45 = 3 \times 3 \times 5 = 3^2 \times 5^1$

$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$

Here, 3^2 is the smallest power of the common factor 3.

So, **HCF (36, 45, 72)** = 3 × 3 = **9**

$2^3 \times 3^2 \times 5^1$ are the greatest powers of the prime factors 2, 3 and 5 respectively involved in the given three numbers .

LCM of these three integers = 2 × 2 × 2 × 3 × 3 × 5 = **360**

9 E. Question

Find LCM and HCF of the following integers by using prime factorization method:

42, 63 and 140

Answer

Given numbers are 42, 63 and 140

Factorization of 42, 63 and 140

2	42
3	21
7	7
	1

3	63
3	21
7	7
	1

2	140
2	70
5	35
7	7
	1

$$42 = \boxed{2} \times \boxed{3} \times \textcircled{7} = 2^1 \times 3^1 \times 7^1$$

$$63 = 3 \times \boxed{3} \times \textcircled{7} = 3^2 \times 7^1$$

$$140 = \boxed{2} \times 2 \times 5 \times \textcircled{7} = 2^2 \times 5^1 \times 7^1$$

Here, 7^1 is the smallest power of the common factor 7.

So, **HCF (42, 63, 140) = 7**

$2^2 \times 3^2 \times 5^1 \times 7^1$ are the greatest powers of the prime factors 2, 3, 5 and 7 respectively involved in the given three numbers .

LCM of these three integers = 2 × 2 × 3 × 3 × 5 × 7 = **1260**

9 F. Question

Find LCM and HCF of the following integers by using prime factorization method:

48, 72 and 108

Answer

Given numbers are 48, 72 and 108

Factorization of 48, 72 and 108

2	48
2	24
2	12
2	6
3	3
	1

2	72
2	36
2	18
3	9
3	3
	1

2	108
2	54
3	27
3	9
3	3
	1

$$48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$$

$$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$$

$$108 = 2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^3$$

Here, $2^2 \times 3^1$ are the smallest powers of the common factors 2 and 3 respectively.

So, **HCF (48, 72, 108)** = 2 × 2 × 3 = **12**

$2^4 \times 3^3$ are the greatest powers of the prime factors 2 and 3 respectively involved in the given three numbers .

LCM of these three integers = 2 × 2 × 2 × 2 × 3 × 3 × 3 = **432**

10 A. Question

If HCF (96, 404) and 4, then, find LCM (96, 404)

Answer

Given: HCF (96 , 404) = 4

To Find: LCM (96, 404)

We use the formula

L.C.M (a,b) × H.C.F (a,b) = Product of two numbers (a×b)

LCM (96, 404) × HCF (96, 404) = 96 × 404

⇒ LCM (96, 404) × 4 = 96 × 404 [∵HCF (96, 404) = 4]

⇒ LCM (96, 404) = $\frac{96 \times 404}{4}$

⇒ **LCM (96, 404) = 9696**

10 B. Question

If LCM (72, 126) = 504, find HCF (72, 126)

Answer

Given: LCM (72, 126) = 504

To Find: HCF (72, 126)

We use the formula

L.C.M (a,b) × H.C.F (a,b) = Product of two numbers (a×b)

LCM (72, 126) × HCF (72, 126) = 72 × 126

⇒ 504 × HCF (72, 126) = 72 × 126 [∵LCM(72,126)=504]

⇒ HCF (72, 126) = $\frac{72 \times 126}{504}$

⇒ HCF (72 , 126) = 18

10 C. Question

If HCF (18, 504) = 18, find LCM (18, 504)

Answer

Given: HCF (18, 504) = 18

To Find: LCM (18, 504)

We use the formula

L.C.M (a,b) × H.C.F (a,b) = Product of two numbers (a×b)

LCM (18, 504) × HCF (18, 504) = 18 × 504

⇒ LCM (18, 504) × 18 = 18 × 504 [∵HCF(18, 504) = 18]

⇒ LCM (18, 504) = $\frac{18 \times 504}{18}$

⇒ LCM (18, 504) = 504

10 D. Question

If LCM (96, 168) = 672, find HCF (96, 168)

Answer

Given: LCM (96, 168) = 672

To Find: HCF (96, 168)

We use the formula

L.C.M (a,b) × H.C.F (a,b) = Product of two numbers (a×b)

LCM (96, 168) × HCF (96, 168) = 96 × 168

⇒ 672 × HCF (96, 168) = 96 × 168 [∵LCM(96, 168)=672]

$\Rightarrow$ HCF (96, 168) = $\frac{96 \times 168}{672}$

$\Rightarrow$ HCF (96, 168) = 24

10 E. Question

If HCF (306, 657) = 9, find LCM (306, 657)

Answer

Given: HCF (306, 657) = 9

To Find: LCM (306, 657)

We use the formula

L.C.M (a,b) × H.C.F (a,b) = Product of two numbers (a×b)

LCM (306, 657) × HCF (306, 657) = 306 × 657

$\Rightarrow$ LCM (306, 657) × 9 = 306 × 657 [∵HCF(306,657)= 9]

$\Rightarrow$ LCM (306, 657) = $\frac{306 \times 657}{9}$

$\Rightarrow$ LCM (306, 657) = 22338

10 F. Question

If HCF (36, 64) = 4, find LCM (36, 64)

Answer

Given: HCF (36, 64) = 4

To Find: LCM (36, 64)

We use the formula

L.C.M (a,b) × H.C.F (a,b) = Product of two numbers (a×b)

LCM (36, 64) × HCF (36, 64) = 36 × 64

$\Rightarrow$ LCM (36, 64) × 4 = 36 × 64 [∵HCF (36, 64)= 4]

$\Rightarrow$ LCM (36, 64) = $\frac{36 \times 64}{4}$

$\Rightarrow$ LCM (36, 64) = 576

11 A. Question

Examine whether $(15)^n$ can end with the digit 0 for any n ϵ N.

Answer

If $(15)^n$ end with the digit 0, then the number should be divisible by 2 and 5.

As 2 × 5 = 10

⇒This means the prime factorization of 15^n should contain prime factors 2 and 5.

But $(15)^n = (3 \times 5)^n$ and it does not have the prime factor 2 but have 3 and 5.

∵, 2 is not present in the prime factorization, there is no natural number nor which 15^n ends with digit zero.

So, 15^n cannot end with digit zero.

11 B. Question

Examine whether $(24)^n$ can end with the digit 5 for any n ϵ N.

Answer

If $(24)^n$ end with the digit 5, then the number should be divisible by 5.

⇒This means the prime factorization of 24^n should contain prime factors 5.

But $(24)^n = (2^3 \times 3)^n$ and it does not have the prime factor 5 but have 3 and 2.

∵, 5 is not present in the prime factorization, there is no natural number nor which 24^n ends with digit 5.

So, 24^n cannot end with digit 5.

11 C. Question

Examine whether $(21)^n$ can end with the digit 0 for any n ϵ N.

Answer

If $(21)^n$ end with the digit 0, then the number should be divisible by 2 and 5.

As 2 × 5 = 10

⇒This means the prime factorization of 21^n should contain prime factors 2 and 5.

But $(21)^n = (3 \times 7)^n$ and it does not have the prime factor 2 and 5 but have 3 and 7.

∵, 2 and 5 is not present in the prime factorization, there is no natural number nor which 21^n ends with digit zero.

So, 21^n cannot end with digit zero.

11 D. Question

Examine whether $(8)^n$ can end with the digit 5 for any $n \in N$.

Answer

If $(8)^n$ end with the digit 5, then the number should be divisible by 5.

⇒This means the prime factorization of 8^n should contain prime factor 5.

But $(8)^n = (2^3)^n$ and it does not have the prime factor 5 but have 2. So, the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of 8^n.

∵, 5 is not present in the prime factorization, there is no natural number nor which 8^n ends with digit 5.

So, 8^n cannot end with digit 5.

11 E. Question

Examine whether $(4)^n$ can end with the digit 0 for any $n \in N$.

Answer

If $(4)^n$ end with the digit 0, then the number should be divisible by 5.

As $2 \times 5 = 10$

⇒This means the prime factorization of 4^n should contain prime factor 5.

This is not possible because $(4)^n = (2^{2n})$, so the only prime in the factorization of 4^n is 2. So, the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of 4^n.

∵, 5 is not present in the prime factorization, there is no natural number nor which 4^n ends with digit zero.

So, 4^n cannot end with digit zero.

11 F. Question

Examine whether $(7)^n$ can end with the digit 5 for any $n \in N$.

Answer

If $(7)^n$ end with the digit 5, then the number should be divisible by 5.

⇒This means the prime factorization of 7^n should contain prime factor 5.

But $(7)^n$ does not have the prime factor 5. So, the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of 7^n.

∵, 5 is not present in the prime factorization, there is no natural number nor which 7^n ends with digit 5.

So, 7^n cannot end with digit 5.

12 A. Question

Explain why 7 x 11 x 13 x 17 +17 is a composite number.

Answer

Composite number: A whole number that can be divided exactly by numbers other than 1 or itself.

Given 7 x 11 x 13 x 17 +17⇒17 (7 x 11 x 13 x 17 +1)

⇒17 (7 x 11 x 13 x 17 +1)

⇒17 (17017 + 1)

⇒17 (17018)

⇒17 (2 × 8509)

⇒17 × 2 × 8509

So, given number is the composite number because it is the product of more than one prime numbers.

12 B. Question

Explain why 5 x 7 x 13 + 5 is a composite number.

Answer

Composite number: A whole number that can be divided exactly by numbers other than 1 or itself.

5 x 7 x 13 + 5

⇒5 (1 x 7 x 13 +1)

$\Rightarrow 5\ (91 + 1)$

$\Rightarrow 5\ (92)$

$\Rightarrow 5\ (2^2 \times 23)$

$\Rightarrow 5 \times 2 \times 2 \times 23$

So, given number is the composite number because it is the product of more than one prime numbers.

12 C. Question

Show that 5 x 7 x 11 x 13 + 55 is a composite number.

Answer

Composite number: A whole number that can be divided exactly by numbers other than 1 or itself.

5 x 7 x 11 x 13 + 55

$\Rightarrow 5\ (1 \times 7 \times 11 \times 13 + 11)$

$\Rightarrow 5 \times 11\ (91 + 1)$

$\Rightarrow 5 \times 11\ (92)$

$\Rightarrow 5 \times 11\ (2^2 \times 23)$

$\Rightarrow 5 \times 11 \times 2 \times 2 \times 23$

So, given number is the composite number because it is the product of more than one prime numbers.

13. Question

Three measuring rods 64 cm, 80 cm and 96 cm in length. Find the least length of cloth that can be measured exact number of times using anyone of the above rods.

Answer

Lengths of three measuring rods = 64cm, 80cm and 96cm

Least Length of cloth that can be measured = LCM (64, 80, 96)

2	64
2	32
2	16
2	8
2	4
2	2
	1

2	80
2	40
2	20
3	10
5	5
	1

2	96
2	48
2	24
2	12
2	6
3	3
	1

$64 = 2^6$

$80 = 2^3 \times 3 \times 5$

$96 = 2^5 \times 3$

So, $2^6 \times 3 \times 5$ are the greatest powers of the prime factors 2, 3 and 5

LCM (64, 80, 96) = $2^6 \times 3 \times 5 = 960$

Least Length of cloth that can be measured is 960 cm

14. Question

Three containers contain 27 litres, 36 litres and 72 litres of milk. What biggest measure can measure exactly the milk in the three containers?

Answer

Milk in three containers = 27L, 36L, 72L

Biggest measure which can exactly measure the milk = HCF (27, 36, 72)

2	36
2	18
3	9
3	3
	1

2	72
2	36
2	18
3	9
3	3
	1

3	27
3	9
3	3
	1

$27 = 3^3$

$36 = 2^2 \times 3^2$

$72 = 2^3 \times 3^2$

Here, 3^2 is the smallest power of the common factor of the prime 3

HCF (27, 36, 72) = 9

So, biggest measure which can exactly measure the milk = 9L

15. Question

Three different containers contain different quantities of mixtures of milk and water, whose measurements are 403 kg, 434 kg and 465 kg, what biggest measure can measure all the different quantities exactly.

Answer

Mixtures of milk and water in three containers = 403kg, 434kg,

465kg

Biggest measure which can exactly measure different quantities = HCF (403, 434, 465)

13	403
31	31
	1

2	434
7	217
31	31
	1

3	465
5	155
31	31
	1

$403 = 13 \times 31$

$434 = 2 \times 7 \times 31$

$465 = 3 \times 5 \times 31$

Here, 31^1 is the smallest power of the common factor.

HCF (403, 434, 465) = 31

So, biggest measure which can exactly measure the milk = 31L

Exercise 1.3

1. Question

Prove that $\sqrt{2}$ is irrational.

Answer

Let us assume that $\sqrt{2}$ is rational. So, we can find integers p and q ($\neq 0$) such that $\sqrt{2} = \frac{p}{q}$.

Suppose p and q have a common factor other than 1.

Then, we divide by the common factor to get $\sqrt{2} = \frac{a}{b}$, where a and b are coprime.

So, $b\sqrt{2} = a$.

Squaring on both sides, we get

$2b^2 = a^2$

Therefore, 2 divides a^2.

Now, by Theorem which states that **Let p be a prime number. If p divides a^2 , then p divides a, where a is a positive integer**,

⇒ 2 divides a^2.

So, we can write a = 2c for some integer c

Substituting for a, we get $2b^2 = 4c^2$,i.e. $b^2 = 2c^2$.

This means that 2 divides b^2, and so 2 divides b (again using the above Theorem with p = 2). Therefore, a and b have at least 2 as a common factor.

But this contradicts the fact that a and b have no common factors other than 1.

This contradiction has arisen because of our incorrect assumption that 2 is rational.

So, we conclude that √2 is irrational.

2. Question

Prove that √3 is irrational.

Answer

Let us assume that √3 is rational.

Hence, √3 can be written in the form $\frac{a}{b}$

where a and b (≠ 0) are co-prime (no common factor other than 1).

Hence, $\sqrt{3} = \frac{a}{b}$

So, b√3 = a.

Squaring on both sides, we get

$3b^2 = a^2$

$\frac{a^2}{3} = b^2$

Hence, 3 divides a^2.

By theorem: **Let p is a prime number and p divides a^2 , then p divides a, where a is a positive integer**,

⇒ 3 divides a also ...(1)

Hence, we can say a = 3c for some integer c

Now, we know that $3b^2 = a^2$

Putting a = 3c

$3b^2 = (3c)^2$

$3b^2 = 9c^2$

$b^2 = 3c^2$

Hence, 3 divides b^2

By theorem: **Let p is a prime number and p divides a^2, then p divides a, where a is a positive integer**,

So, 3 divides b also ...(2)

By (1) and (2)

3 divides both a and b

Hence, 3 is a factor of a and b

So, a and b have a factor 3

Therefore, a and b are not co-prime.

Hence, our assumption is wrong

Therefore, by contradiction **$\sqrt{3}$ is irrational**.

3. Question

Prove that $\frac{1}{\sqrt{5}}$ is irrational.

Answer

Let us assume that $\frac{1}{\sqrt{5}}$ be a rational number.

Then, it will be of the form $\frac{a}{b}$ where a and b are co-prime and b≠0.

Now, $\frac{a}{b} = \frac{1}{\sqrt{5}}$

$\Rightarrow \frac{a}{b} = \frac{1 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}}$

$\Rightarrow \frac{5a}{b} = \sqrt{5}$

Since, 5a is an integer and b is also an integer

So, $\frac{5a}{b}$ is a rational number

$\Rightarrow$ $\sqrt{5}$ is a rational number

But this contradicts to the fact that $\sqrt{5}$ is an irrational number.

Therefore, our assumption is wrong.

Hence, $\frac{1}{\sqrt{5}}$ is an irrational number.

4 A. Question

Prove that following numbers are not rational :

$(6)^{1/3}$

Answer

Suppose $6^{1/3}$ is rational.

Then, $6^{1/3} = \frac{n}{m}$ for some integers n and m which are co-prime.

So, $6 = \frac{n^3}{m^3}$

$\Rightarrow 6m^3 = n^3$

So, n^3 must be divisible by 6

$\Rightarrow$ n must be divisible by 6.

Let n = 6p for some integer p

This gives

$6 = \frac{(6p)^3}{m^3}$

$1 = \frac{6^2p^3}{m^3}$

$\Rightarrow m^3$ is divisible by 6

Hence, m must be divisible by 6.

But n and m where co-prime.

So, we have a contradiction.

Hence, $(6)^{1/3}$ is irrational

4 B. Question

Prove that following numbers are not rational :

$3\sqrt{3}$

Answer

Let us assume that $3\sqrt{3}$ be a rational number.

Then, it will be of the form $\frac{a}{b}$ where a and b are co-prime and b≠0.

Now, $\frac{a}{b}=3\sqrt{3}$

$\Rightarrow \frac{a}{3b}=\sqrt{3}$

Since, a is an integer and 3b is also an integer (3b ≠ 0)

So, $\frac{a}{3b}$ is a rational number

$\Rightarrow \sqrt{3}$ is a rational number

But this contradicts to the fact that $\sqrt{3}$ is an irrational number.

Therefore, our assumption is wrong.

Hence, $3\sqrt{3}$ is an irrational number.

4 C. Question

Prove that following numbers are not rational :

$5\sqrt{3}$

Answer

Let us assume that $5\sqrt{3}$ be a rational number.

Then, it will be of the form $\frac{a}{b}$ where a and b are co-prime and b≠0.

Now, $\frac{a}{b}=5\sqrt{3}$

$\Rightarrow \frac{a}{5b}=\sqrt{3}$

Since, a is an integer and 3b is also an integer (5b ≠ 0)

So, $\frac{a}{5b}$ is a rational number

⇒ $\sqrt{3}$ is a rational number

But this contradicts to the fact that $\sqrt{3}$ is an irrational number.

Therefore, our assumption is wrong.

Hence, $5\sqrt{3}$ is an irrational number.

5 A. Question

Prove that following numbers are irrational :

$6 + \sqrt{2}$

Answer

Let us assume $6 + \sqrt{2}$ is rational

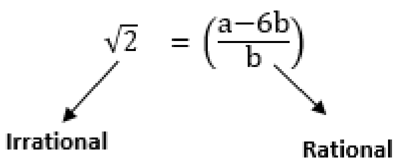
⇒ $6 + \sqrt{2}$ can be written in the form $\frac{a}{b}$ where a and b are co-prime.

Hence, $6 + \sqrt{2} = \frac{a}{b}$

$\sqrt{2} = \frac{a}{b} - 6$

$\sqrt{2} = \frac{a-6b}{b}$

$\sqrt{2} = \left(\frac{a-6b}{b}\right)$



Since, rational ≠ irrational

This is a contradiction.

∴, Our assumption is incorrect.

Hence, $6 + \sqrt{2}$ is irrational.

5 B. Question

Prove that following numbers are irrational :

$5 - \sqrt{3}$

Answer

Let us assume 5 - $\sqrt{3}$ is rational

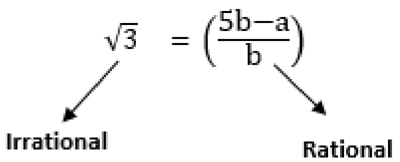
⇒ 5 - $\sqrt{3}$ can be written in the form $\frac{a}{b}$ where a and b are co-prime.

Hence, 5 - $\sqrt{3}$ = $\frac{a}{b}$

-$\sqrt{3}$ = $\frac{a}{b}$ – 5

-$\sqrt{3}$ = $\frac{a-5b}{b}$

$\sqrt{3}$ =- $\left(\frac{a-5b}{b}\right)$



Since, rational ≠ irrational

This is a contradiction.

∴, Our assumption is incorrect.

Hence, 5 - $\sqrt{3}$ is irrational.

5 C. Question

Prove that following numbers are irrational :

2 + √2

Answer

Let us assume 2 + $\sqrt{2}$ is rational

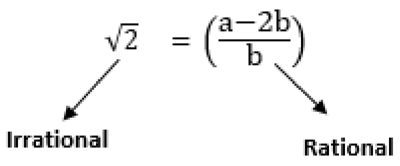
⇒ 2 + $\sqrt{2}$ can be written in the form $\frac{a}{b}$ where a and b are co-prime.

Hence, 2 + $\sqrt{2}$ = $\frac{a}{b}$

$\sqrt{2}$ = $\frac{a}{b}$ – 2

$\sqrt{2}$ = $\frac{a-2b}{b}$

$\sqrt{2}$ = $\left(\frac{a-2b}{b}\right)$



Since, rational ≠ irrational

This is a contradiction.

∴, Our assumption is incorrect.

Hence, $2 + \sqrt{2}$ is irrational.

5 D. Question

Prove that following numbers are irrational :

$3 + \sqrt{5}$

Answer

Let us assume $3 + \sqrt{5}$ is rational

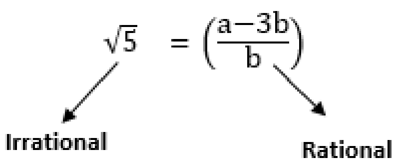
⇒ $3 + \sqrt{5}$ can be written in the form $\frac{a}{b}$ where a and b are co-prime.

Hence, $3 + \sqrt{5} = \frac{a}{b}$

$\sqrt{5} = \frac{a}{b} - 3$

$\sqrt{5} = \frac{a-3b}{b}$

$\sqrt{5} = \left(\frac{a-3b}{b}\right)$



Since, rational ≠ irrational

This is a contradiction.

∴, Our assumption is incorrect.

Hence, $3 + \sqrt{5}$ is irrational.

5 E. Question

Prove that following numbers are irrational :

$\sqrt{3} - \sqrt{2}$

Answer

Let us assume $\sqrt{3}$ - $\sqrt{2}$ is rational

Let, $\sqrt{3} - \sqrt{2} = \frac{a}{b}$

Squaring both sides, we get

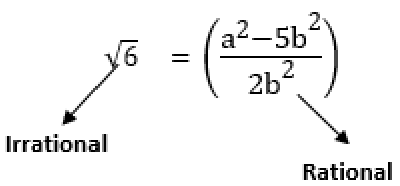
$(\sqrt{3} - \sqrt{2})^2 = \frac{a^2}{b^2}$

$5 - 2\sqrt{6} = \frac{a^2}{b^2}$

$2\sqrt{6} = \frac{a^2}{b^2} - 5$

$2\sqrt{6} = \frac{a^2 - 5b^2}{b^2}$

$\sqrt{6} = \left(\frac{a^2 - 5b^2}{2b^2}\right)$



Since, rational ≠ irrational

This is a contradiction.

∴, Our assumption is incorrect.

Hence, $\sqrt{3} - \sqrt{2}$ is irrational.

5 F. Question

Prove that following numbers are irrational :

$\sqrt{7} - \sqrt{5}$

Answer

Let us assume $\sqrt{7}$ - $\sqrt{5}$ is rational

Let, $\sqrt{7} - \sqrt{5} = \frac{a}{b}$

Squaring both sides, we get

$$\left(\sqrt{7} - \sqrt{5}\right)^2 = \frac{a^2}{b^2}$$

$$12 - 2\sqrt{35} = \frac{a^2}{b^2}$$

$$2\sqrt{35} = \frac{a^2}{b^2} - 12$$

$$2\sqrt{35} = \left(\frac{a^2 - 12b^2}{b^2}\right)$$

$$\sqrt{35} = \left(\frac{a^2 - 12b^2}{2b^2}\right)$$

$$\sqrt{35} = \left(\frac{a^2 - 12b^2}{2b^2}\right)$$

Irrational

Rational

Since, rational ≠ irrational

This is a contradiction.

∴, Our assumption is incorrect.

Hence, $\sqrt{7} - \sqrt{5}$ is irrational.

Exercise 1.4

1 A. Question

Without actually performing the long division, state whether the following rational numbers have terminating or non-terminating repeating (recurring) decimal expansion.

$$\frac{17}{8}$$

Answer

Given rational number is $\frac{17}{8}$

$\frac{p}{q}$ is terminating if

a) p and q are co-prime &

b) q is of the form of $2^n\ 5^m$ where n and m are non-negative integers.

Firstly, we check co-prime

17 = 17 × 1

8 = 2 × 2 × 2

⇒17 and 8 have no common factors

Therefore, 17 and 8 are co-prime.

Now, we have to check that q is in the form of 2^n5^m

8 = 2^3

= 1 × 2^3

= $5^0 \times 2^3$

So, denominator is of the form 2^n5^m where n = 3 and m = 0

Thus, $\frac{17}{8}$ is a **terminating** decimal.

1 B. Question

Without actually performing the long division, state whether the following rational numbers have terminating or non-terminating repeating (recurring) decimal expansion.

$$\frac{3}{8}$$

Answer

Given rational number is $\frac{3}{8}$

$\frac{p}{q}$ is terminating if

a) p and q are co-prime &

b) q is of the form of $2^n\ 5^m$ where n and m are non-negative integers.

Firstly, we check co-prime

3 = 3 × 1

8 = 2 × 2 × 2

⇒3 and 8 have no common factors

Therefore, 3 and 8 are co-prime.

Now, we have to check that q is in the form of 2^n5^m

$8 = 2^3$

$= 1 \times 2^3$

$= 5^0 \times 2^3$

So, denominator is of the form 2^n5^m where n = 3 and m = 0

Thus, $\frac{3}{8}$ is a **terminating** decimal.

1 C. Question

Without actually performing the long division, state whether the following rational numbers have terminating or non-terminating repeating (recurring) decimal expansion.

$$\frac{29}{343}$$

Answer

Given rational number is $\frac{29}{343}$

$\frac{p}{q}$ is terminating if

a) p and q are co-prime &

b) q is of the form of 2^n 5^m where n and m are non-negative integers.

<u>Firstly, we check co-prime</u>

$29 = 29 \times 1$

$343 = 7 \times 7 \times 7$

$\Rightarrow$29 and 343 have no common factors

Therefore, 29 and 343 are co-prime.

Now, we have to check that q is in the form of 2^n5^m

$343 = 7^3$

So, denominator is not of the form 2^n5^m

Thus, $\frac{29}{343}$ is a **non-terminating repeating** decimal.

1 D. Question

Without actually performing the long division, state whether the following rational numbers have terminating or non-terminating repeating (recurring) decimal expansion.

$$\frac{13}{125}$$

Answer

Given rational number is $\frac{13}{125}$

$\frac{p}{q}$ is terminating if

a) p and q are co-prime &

b) q is of the form of 2^n 5^m where n and m are non-negative integers.

Firstly, we check co-prime

13 = 13 × 1

125 = 5 × 5 × 5

⇒13 and 125 have no common factors

Therefore, 13 and 125 are co-prime.

Now, we have to check that q is in the form of $2^n 5^m$

$125 = 5^3$

$= 1 \times 2^3$

$= 2^0 \times 5^3$

So, denominator is of the form $2^n 5^m$ where n = 0 and m = 3

Thus, $\frac{13}{125}$ is a **terminating** decimal.

1 E. Question

Without actually performing the long division, state whether the following rational numbers have terminating or non-terminating repeating (recurring) decimal expansion.

$$\frac{27}{8}$$

Answer

Given rational number is $\frac{27}{8}$

$\frac{p}{q}$ is terminating if

a) p and q are co-prime &

b) q is of the form of 2^n 5^m where n and m are non-negative integers.

Firstly, we check co-prime

27 = 3 × 3 × 3

8 = 2 × 2 × 2

⇒27 and 8 have no common factors

Therefore, 27 and 8 are co-prime.

Now, we have to check that q is in the form of 2^n5^m

$8 = 2^3$

$= 1 \times 2^3$

$= 5^0 \times 2^3$

So, denominator is of the form 2^n5^m where n = 3 and m = 0

Thus, $\frac{27}{8}$ is a **terminating** decimal.

1 F. Question

Without actually performing the long division, state whether the following rational numbers have terminating or non-terminating repeating (recurring) decimal expansion.

$$\frac{7}{80}$$

Answer

Given rational number is $\frac{7}{80}$

$\frac{p}{q}$ is terminating if

a) p and q are co-prime &

b) q is of the form of 2^n 5^m where n and m are non-negative integers.

Firstly, we check co-prime

7 = 7 × 1

80 = 2 × 2 × 2 × 2 × 5

⇒7 and 80 have no common factors

Therefore, 7 and 80 are co-prime.

Now, we have to check that q is in the form of 2^n5^m

80 = 2^4 × 5

So, denominator is of the form 2^n5^m where n = 4 and m = 1

Thus, $\frac{7}{80}$ is a **terminating** decimal.

1 G. Question

Without actually performing the long division, state whether the following rational numbers have terminating or non-terminating repeating (recurring) decimal expansion.

$$\frac{64}{455}$$

Answer

Given rational number is $\frac{64}{455}$

$\frac{p}{q}$ is terminating if

a) p and q are co-prime &

b) q is of the form of 2^n 5^m where n and m are non-negative integers.

Firstly, we check co-prime

64 = 2^6

455 = 5 × 7 × 13

⇒64 and 455 have no common factors

Therefore, 64 and 455 are co-prime.

Now, we have to check that q is in the form of 2^n5^m

455 = 5 × 7 × 13

So, denominator is not of the form 2^n5^m

Thus, $\frac{64}{455}$ is a **non-terminating repeating** decimal.

1 H. Question

Without actually performing the long division, state whether the following rational numbers have terminating or non-terminating repeating (recurring) decimal expansion.

$$\frac{6}{15}$$

Answer

Given rational number is $\frac{6}{15}$

$\frac{6}{15} = \frac{2}{5}$

$\frac{p}{q}$ is terminating if

a) p and q are co-prime &

b) q is of the form of 2^n 5^m where n and m are non-negative integers.

Firstly, we check co-prime

⇒ 2 and 5 have no common factor

Therefore, 2 and 5 are co-prime.

Now, we have to check that q is in the form of 2^n5^m

$5 = 5^1 \times 1$

$= 5^1 \times 2^0$

So, denominator is of the form 2^n5^m where n = 0 and m = 1

Thus, $\frac{6}{15}$ is a **terminating** decimal.

1 I. Question

Without actually performing the long division, state whether the following rational numbers have terminating or non-terminating repeating (recurring) decimal expansion.

$$\frac{35}{50}$$

Answer

Given rational number is $\frac{35}{50}$

$\frac{35}{50} = \frac{7}{10}$

$\frac{p}{q}$ is terminating if

a) p and q are co-prime &

b) q is of the form of 2^n 5^m where n and m are non-negative integers.

<u>Firstly, we check co-prime</u>

$7 = 1 \times 7$

$10 = 2 \times 5$

$\Rightarrow$ 7 and 10 have no common factor

Therefore, 7 and 10 are co-prime.

Now, we have to check that q is in the form of 2^n5^m

$10 = 5^1 \times 2^1$

So, denominator is of the form 2^n5^m where n = 1 and m = 1

Thus, $\frac{35}{50}$ is a **terminating** decimal.

1 J. Question

Without actually performing the long division, state whether the following rational numbers have terminating or non-terminating repeating (recurring) decimal expansion.

$$\frac{129}{2^2 5^7 7^5}$$

Answer

Given rational number is $\frac{129}{2^2 \times 5^7 \times 7^5}$

$\frac{p}{q}$ is terminating if

a) p and q are co-prime &

b) q is of the form of 2^n 5^m where n and m are non-negative integers.

Firstly, we check co-prime

129 = 3 × 43

Denominator = $2^2 \times 5^7 \times 7^5$

$\Rightarrow$129 and $2^2 \times 5^7 \times 7^5$ have no common factors

Therefore, 129 and $2^2 \times 5^7 \times 7^5$ are co-prime.

Now, we have to check that q is in the form of $2^n 5^m$

Denominator = $2^2 \times 5^7 \times 7^5$

So, denominator is not of the form $2^n 5^m$

Thus, $\frac{129}{2^2 \times 5^7 \times 7^5}$ is a **non-terminating repeating** decimal.

1 K. Question

Without actually performing the long division, state whether the following rational numbers have terminating or non-terminating repeating (recurring) decimal expansion.

$$\frac{2^2 \times 7}{5^4}$$

Answer

Given rational number is $\frac{2^2 \times 7}{5^4}$

$\frac{p}{q}$ is terminating if

a) p and q are co-prime &

b) q is of the form of 2^n 5^m where n and m are non-negative integers.

Firstly, we check co-prime

$28 = 7 \times 2^2$

$625 = 5^4$

$\Rightarrow$ 28 and 625 have no common factors

Therefore, 28 and 625 are co-prime.

Now, we have to check that q is in the form of 2^n5^m

$625 = 5^4 \times 1$

$= 5^4 \times 2^0$

So, denominator is of the form 2^n5^m where n = 0 and m = 4

Thus, $\frac{2^2 \times 7}{5^4}$ is a **terminating** decimal.

1 L. Question

Without actually performing the long division, state whether the following rational numbers have terminating or non-terminating repeating (recurring) decimal expansion.

$$\frac{29}{243}$$

Answer

Given rational number is $\frac{29}{243}$

$\frac{p}{q}$ is terminating if

a) p and q are co-prime &

b) q is of the form of $2^n\ 5^m$ where n and m are non-negative integers.

Firstly, we check co-prime

$29 = 29 \times 1$

$243 = 3^5$

$\Rightarrow$29 and 243 have no common factors

Therefore, 29 and 243 are co-prime.

Now, we have to check that q is in the form of 2^n5^m

$243 = 3^5$

So, the denominator is not of the form 2^n5^m

Thus, $\frac{29}{243}$ is a **non- terminating repeating** decimal.

2 A. Question

Write down the decimal expansions of the following numbers which have terminating decimal expansions.

$$\frac{17}{8}$$

Answer

We know, $\frac{17}{8} = \frac{17}{2^3 \times 5^0}$

Multiplying and dividing by 5^3

$= \frac{17 \times 5^3}{2^3 \times 5^0 \times 5^3}$

$= \frac{17 \times 125}{2^3 \times 1 \times 5^3}$

$= \frac{2125}{(2 \times 5)^3}$

$= \frac{2125}{(10)^3}$

$= \frac{2125}{1000}$

= **2.125**

2 B. Question

Write down the decimal expansions of the following numbers which have terminating decimal expansions.

$$\frac{3}{8}$$

Answer

We know, $\frac{3}{8} = \frac{3}{2^3 \times 5^0}$

Multiplying and dividing by 5^3

$= \frac{3 \times 5^3}{2^3 \times 5^0 \times 5^3}$

$= \frac{3 \times 125}{2^3 \times 1 \times 5^3}$

$= \frac{375}{(2 \times 5)^3}$

$= \frac{375}{(10)^3}$

$= \frac{375}{1000}$

= **0.375**

2 C. Question

Write down the decimal expansions of the following numbers which have terminating decimal expansions.

$\frac{29}{343}$

Answer

We know, $\frac{29}{343} = \frac{29}{7^3}$

Given rational number is $\frac{29}{343}$

$\frac{p}{q}$ is terminating if

a) p and q are co-prime &

b) q is of the form of $2^n \, 5^m$ where n and m are non-negative integers.

Firstly we check co-prime

29 = 29 × 1

343 = 7 × 7 × 7

⇒29 and 343 have no common factors

Therefore, 29 and 343 are co-prime.

Now, we have to check that q is in the form of $2^n 5^m$

$343 = 7^3$

So, the denominator is not of the form 2^n5^m

Thus, $\frac{29}{343}$ is a **non-terminating repeating** decimal.

2 D. Question

Write down the decimal expansions of the following numbers which have terminating decimal expansions.

$$\frac{13}{125}$$

Answer

We know, $\frac{13}{125} = \frac{13}{2^0 \times 5^3}$

Multiplying and dividing by 2^3

$= \frac{13 \times 2^3}{2^0 \times 5^3 \times 2^3}$

$= \frac{13 \times 8}{1 \times 2^3 \times 5^3}$

$= \frac{104}{(2 \times 5)^3}$

$= \frac{104}{(10)^3}$

$= \frac{104}{1000}$

= **0.104**

2 E. Question

Write down the decimal expansions of the following numbers which have terminating decimal expansions.

$$\frac{27}{8}$$

Answer

We know, $\frac{27}{8} = \frac{3^3}{2^3 \times 5^0}$

Multiplying and dividing by 5^3

$= \frac{27 \times 5^3}{2^0 \times 2^3 \times 5^3}$

$= \frac{27 \times 125}{1 \times 2^3 \times 5^3}$

$= \frac{3375}{(2 \times 5)^3}$

$= \frac{3375}{(10)^3}$

$= \frac{3375}{1000}$

= **3.375**

2 F. Question

Write down the decimal expansions of the following numbers which have terminating decimal expansions.

$$\frac{7}{80}$$

Answer

We know, $\frac{7}{80} = \frac{7}{2^4 \times 5^1}$

Multiplying and dividing by 5^3

$= \frac{7 \times 5^3}{2^4 \times 5^1 \times 5^3}$

$= \frac{7 \times 125}{2^4 \times 5^4}$

$= \frac{875}{(2 \times 5)^4}$

$= \frac{875}{(10)^4}$

$= \frac{875}{10000}$

= **0.0875**

2 G. Question

Write down the decimal expansions of the following numbers which have terminating decimal expansions.

$$\frac{64}{455}$$

Answer

We know, $\frac{64}{455} = \frac{2^6}{5 \times 7 \times 13}$

Since the denominator is not of the form $2^n 5^m$

$\frac{64}{455}$ has a non-terminating repeating decimal expansion.

2 H. Question

Write down the decimal expansions of the following numbers which have terminating decimal expansions.

$$\frac{6}{15}$$

Answer

We know, $\frac{6}{15} = \frac{2}{5} = \frac{2}{2^0 \times 5}$

Multiplying and dividing by 2^1

$= \frac{2 \times 2}{2^0 \times 5^1 \times 2}$

$= \frac{4}{1 \times 10}$

$= \frac{4}{10}$

= **0.4**

2 I. Question

Write down the decimal expansions of the following numbers which have terminating decimal expansions.

$$\frac{35}{50}$$

Answer

We know, $\frac{35}{50}$

$= \frac{7}{10}$

$= \frac{7}{2 \times 5}$

$= \frac{7}{10}$

= **0.7**

2 J. Question

Write down the decimal expansions of the following numbers which have terminating decimal expansions.

$$\frac{129}{2^2 5^7 7^5}$$

Answer

Given rational number is $\frac{129}{2^2 \times 5^7 \times 7^5}$

Since the denominator is not of the form $2^n 5^m$

$\frac{129}{2^2 \times 5^7 \times 7^5}$ has a non-terminating repeating decimal expansion.

2 K. Question

Write down the decimal expansions of the following numbers which have terminating decimal expansions.

$$\frac{2^2 \times 7}{5^4}$$

Answer

We know, $\frac{2^2 \times 7}{5^4} = \frac{2^2 \times 7}{2^0 \times 5^4}$

Multiplying and dividing by 2^6

$= \frac{2^2 \times 7 \times 2^6}{2^0 \times 5^4 \times 2^6}$

$= \frac{2^2 \times 7 \times 2^6}{1 \times 5^4 \times 2^6}$

$= \frac{7 \times 2^6}{(2 \times 5)^4}$

$= \frac{448}{(10)^4}$

$= \frac{448}{10000}$

= **0.0448**

2 L. Question

Write down the decimal expansions of the following numbers which have terminating decimal expansions.

$$\frac{29}{243}$$

Answer

Given rational number is $\frac{29}{243}$

$\frac{29}{243} = \frac{29}{3^5}$

Since the denominator is not of the form 2^n5^m.

$\frac{29}{243}$ has a non-terminating repeating decimal expansion.

3. Question

The following real numbers have decimal expansions as given below. In each case examine whether they are rational or not. If they are a rational number of the form p/q, what can be said about q?

(i) 7.2345

(ii) $5.\overline{234}$

(iii) 23.245789

(iv) $7.\overline{3427}$

(v) 0.120120012000120000…

(vi) 23.142857

(vii) 2.313313313331…

(viii) 0.02002000220002…

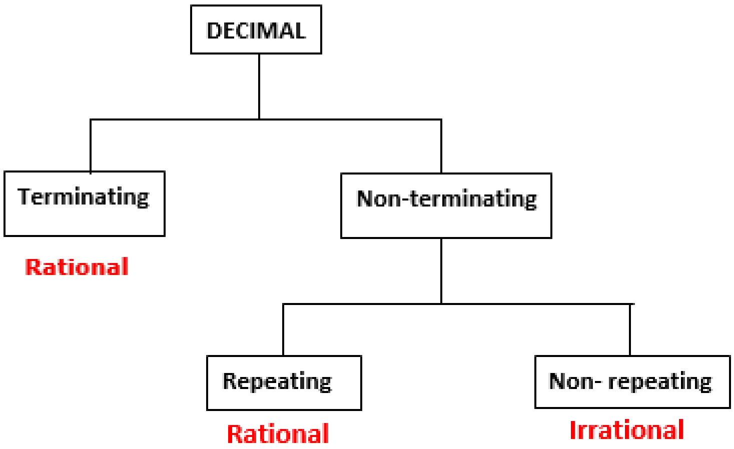
(ix) 3.300030000300003…

(x) 1.7320508…

(xi) 2.645713

(xii) 2.8284271…

Answer



(i) 7.2345

Here, 7.2345 has terminating decimal expansion.

So, it represents a rational number.

i.e. $7.2345 = \frac{7.2345}{10000} = \frac{p}{q}$

Thus, $q = 10^4$, those factors are $2^3 \times 5^3$

(ii) $5.\overline{234}$

$5.\overline{234}$ is non-terminating but repeating.

So, it would be a rational number.

In a non-terminating repeating expansion of $\frac{p}{q}$,

q will have factors other than 2 or 5.

(iii) 23.245789

23.245789 is terminating decimal expansion

So, it would be a rational number.

i.e. $23.245789 = \frac{23.245789}{1000000} = \frac{p}{q}$

Thus, $q = 10^6$, those factors are $2^5 \times 5^5$

In a terminating expansion of $\frac{p}{q}$, q is of the form 2^n5^m

So, prime factors of q will be either 2 or 5 or both.

(iv) $7.\overline{3427}$

$7.\overline{3427}$ is non-terminating but repeating.

So, it would be a rational number.

In a non-terminating repeating expansion of $\frac{p}{q}$,

q will have factors other than 2 or 5.

(v) 0.120120012000120000…

0.120120012000120000… is non-terminating and non-repeating.

So, it is not a rational number as we see in the chart.

(vi) 23.142857

23.142857 is terminating expansion.

So, it would be a rational number.

i.e. $23.142857 = \frac{23.142857}{1000000} = \frac{p}{q}$

Thus, $q = 10^6$, whose factors are $2^5 \times 5^5$

In a terminating expansion of $\frac{p}{q}$, q is of the form 2^n5^m

So, prime factors of q will be either 2 or 5 or both.

(vii) 2.313313313331…

2.313313313331… is non-terminating and non-repeating.

So, it is not a rational number as we see in the chart.

(viii) 0.02002000220002…

0.02002000220002… is non-terminating and non-repeating.

So, it is not a rational number as we see in the chart.

(ix) 3.300030000300003…

3.300030000300003… is non-terminating and non-repeating.

So, it is not a rational number as we see in the chart.

(x) 1.7320508…

1.7320508… is non-terminating and non-repeating.

So, it is not a rational number as we see in the chart.

(xi) 2.645713

2.645713 is terminating expansion

So, it would be a rational number.

i.e. 2.645713 = $\frac{2.645713}{1000000} = \frac{p}{q}$

Thus, q = 10^6, those factors are $2^5 \times 5^5$

In a terminating expansion of $\frac{p}{q}$, q is of the form 2^n5^m

So, prime factors of q will be either 2 or 5 or both.

(xii) 2.8284271…

2.8284271… is non-terminating and non-repeating.

So, it is not a rational number as we see in the chart.