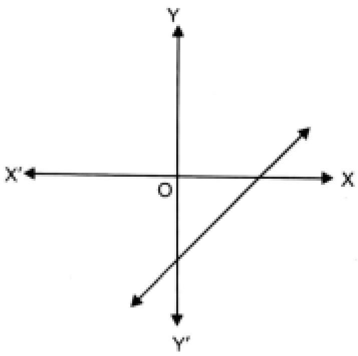
2. Polynomials

Exercise 2.1

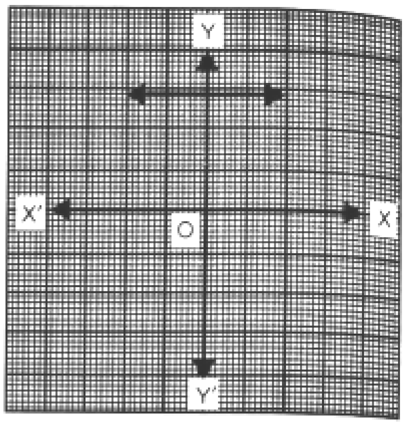
1. Question

Examine, seeing the graph of the polynomials given below, whether they are a linear or quadratic polynomial or neither linear nor quadratic polynomial:

(i)



(ii)



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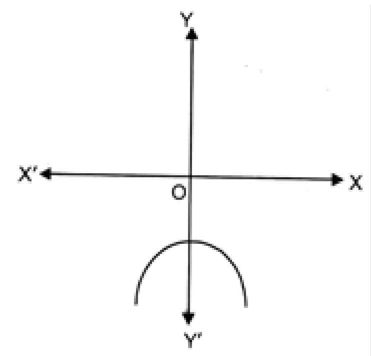
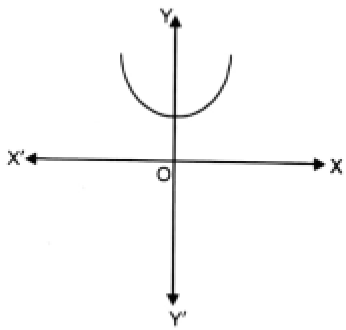
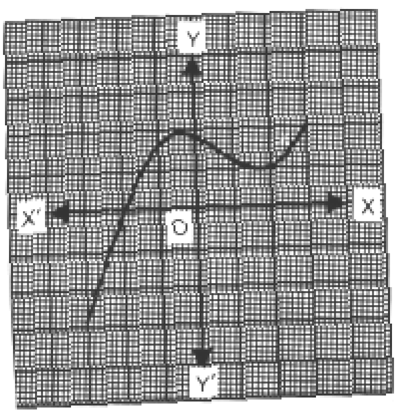


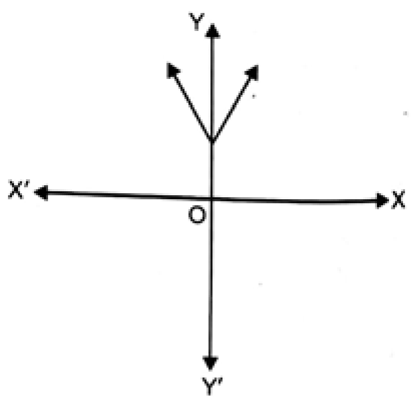
(iv)

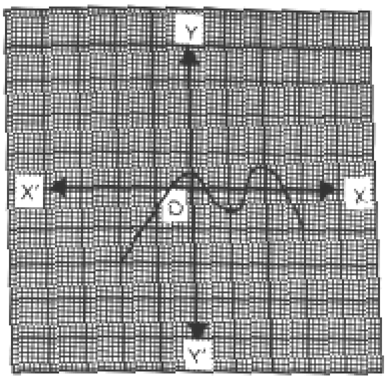


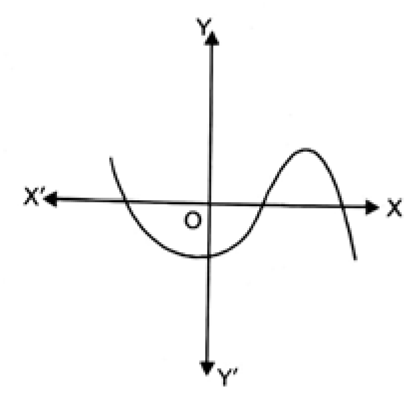
(v)



(vi)



(vii) 

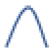
(viii) 

Answer

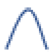
(i) In general, we know that for a linear polynomial ax + b, a≠0, the graph of y = ax + b is a straight line which intersects the x – axis at exactly one point.

And here, we can see that the graph of y = p(x) is a straight line and intersects the x – axis at exactly one point. Therefore, the given graph is of a **Linear Polynomial.**

(ii) Here, the graph of y = p(x) is a straight line and parallel to the x – axis . Therefore, the given graph is of a **Linear Polynomial.**

(iii) For any quadratic polynomial $ax^2 + bx + c$, a ≠ 0, the graph of the corresponding equation $y = a\,x^2 + bx + c$ has one of the two shapes either open upwards like  or open downwards like  depending on whether a > 0 or a < 0. (These curves are called parabolas.)

Here, we can see that the shape of the graph is a parabola. Therefore, the given graph is of a **Quadratic Polynomial.**

(iv) For any quadratic polynomial $ax^2 + bx + c$, a ≠ 0, the graph of the corresponding equation $y = ax^2 + bx + c$ has one of the two shapes either open upwards like  or open downwards like  depending on whether a > 0 or a < 0. (These curves are called parabolas.)

Here, we can see that the shape of the graph is parabola. Therefore, the given graph is of a **Quadratic Polynomial.**

(v) The shape of the graph is neither a straight line nor a parabola. So, the graph is not of a linear polynomial nor a quadratic polynomial.

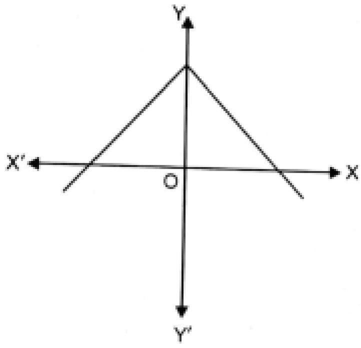
(vi) The given graph have a straight line but it doesn't intersect at x – axis and the shape of the graph is also not a parabola. So, it is not a graph of a quadratic polynomial. Therefore, it is not a graph of linear polynomial or quadratic polynomial.

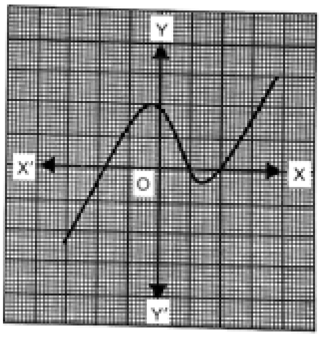
(vii) The shape of the graph is neither a straight line nor a parabola. So, the graph is not of a linear polynomial or a quadratic polynomial.

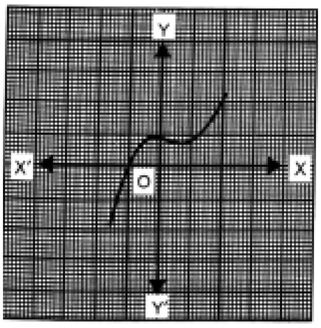
(viii) The shape of the graph is neither a straight line nor a parabola. So, the graph is not of a linear polynomial or a quadratic polynomial.

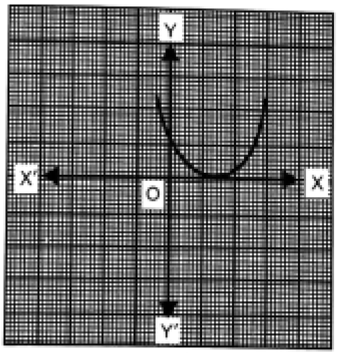
2. Question

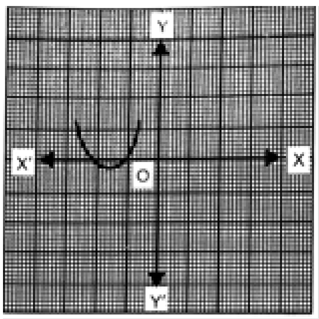
The graphs of y – p(x) are given in the figures below, where p(x) is a polynomial. Find the number of zeros in each case.

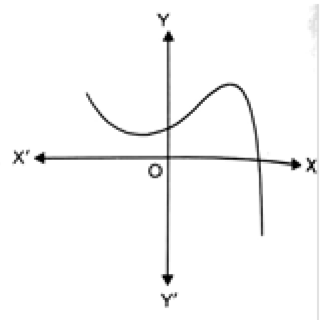
(i)


(ii)


(iii)


(iv)


(v)


(vi)


Answer

(i) Here, the graph of y = p(x) intersects the x – axis at two points. So, the number of zeroes is 2.

(ii) Here, the graph of y = p(x) intersects the x – axis at three points. So, the number of zeroes is 3.

(iii) Here, the graph of y = p(x) intersects the x – axis at one point only. So, the number of zeroes is 1.

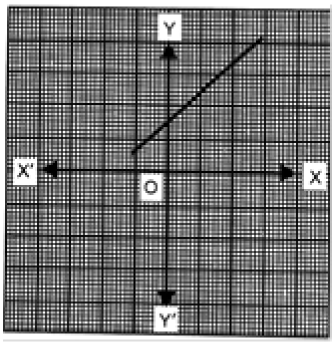
(iv) Here, the graph of y = p(x) intersects the x – axis at exactly one point. So, the number of zeroes is 1.

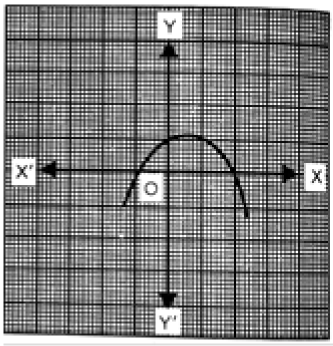
(v) Here, the graph of y = p(x) intersects the x – axis at two points. So, the number of zeroes is 2.

(vi) Here, the graph of y = p(x) intersects the x – axis at exactly one point. So, the number of zeroes is 1.

3. Question

The graphs of y = p(x) are given in the figures below, where p(x) is a polynomial Find the number of zeroes in each case.

(i)


(ii)


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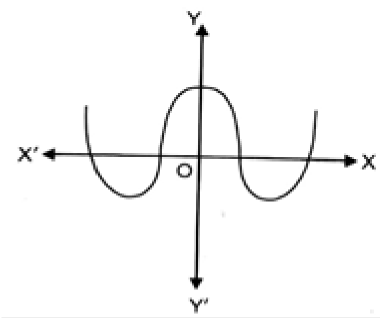
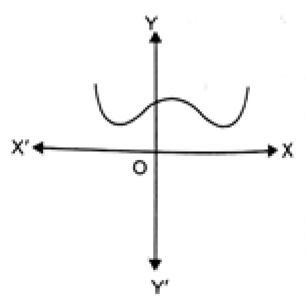
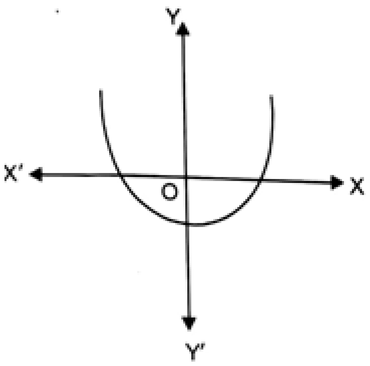
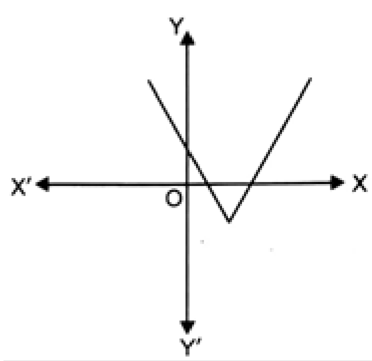


(iv)


(v)


(vi)


Answer

(i) Here, the graph of y = p(x) intersect the x – axis at zero points. So, the number of zeroes is 0.

(ii) Here, the graph of y = p(x) intersects the x – axis at two points. So, the number of zeroes is 2.

(iii) Here, the graph of y = p(x) intersects the x – axis at four points. So, the number of zeroes is 4.

(iv) Here, the graph of y = p(x) does not intersects the x – axis. So, the number of zeroes is 0.

(v) Here, the graph of y = p(x) intersects the x – axis at two points. So, the number of zeroes is 2.

(vi) Here, the graph of y = p(x) intersects the x – axis at two points. So, the number of zeroes is 2.

Exercise 2.2

1 A. Question

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients:

$x^2 - 3$

Answer

Let $f(x) = x^2 - 3$

Now, if we recall the identity

$(a^2 - b^2) = (a - b)(a + b)$

Using this identity, we can write

$x^2 - 3 = (x - \sqrt{3})(x + \sqrt{3})$

So, the value of $x^2 - 3$ is zero when $x = \sqrt{3}$ or $x = -\sqrt{3}$

Therefore, the zeroes of $x^2 - 3$ are $\sqrt{3}$ and $-\sqrt{3}$.

Verification

Now,

Sum of zeroes = $\alpha + \beta = \sqrt{3} + (-\sqrt{3}) = 0$ **or**

$$= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{0}{1} = 0$$

Product of zeroes = $\alpha\beta = (\sqrt{3})(-\sqrt{3}) = -3$ **or**

$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{-3}{1} = -3$$

So, the relationship between the zeroes and the coefficients is verified.

1 B. Question

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients:

$2x^2 - 8x + 6$

Answer

Let $f(x) = 2x^2 - 8x + 6$

By splitting the middle term, we get

$f(x) = 2x^2 - (2 + 6)x + 6$ [$\because -8 = -(2 + 6)$ and $2\times6 = 12$]

$= 2x^2 - 2x - 6x + 6$

$= 2x(x - 1) - 6(x - 1)$

$= (2x - 6)(x - 1)$

On putting $f(x) = 0$, we get

$(2x - 6)(x - 1) = 0$

$\Rightarrow 2x - 6 = 0$ or $x - 1 = 0$

$\Rightarrow x = 3$ or $x = 1$

Thus, the zeroes of the given polynomial $2x^2 - 8x + 6$ are 1 and 3

Verification

Sum of zeroes = $\alpha + \beta = 3 + 1 = 4$ **or**

$$= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{(-8)}{2} = 4$$

Product of zeroes = $\alpha\beta = (3)(1) = 3$ **or**

$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{6}{2} = 3$$

So, the relationship between the zeroes and the coefficients is verified.

1 C. Question

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients:

$x^2 - 2x - 8$

Answer

Let $f(x) = x^2 - 2x - 8$

By splitting the middle term, we get

$f(x) = x^2 - 4x + 2x - 8$ [$\because -2 = 2 - 4$ and $2 \times 4 = 8$]

$= x(x - 4) + 2(x - 4)$

$= (x + 2)(x - 4)$

On putting $f(x) = 0$, we get

$(x + 2)(x - 4) = 0$

$\Rightarrow x + 2 = 0$ or $x - 4 = 0$

$\Rightarrow x = -2$ or $x = 4$

Thus, the zeroes of the given polynomial $x^2 - 2x - 8$ are -2 and 4

Verification

Sum of zeroes $= \alpha + \beta = -2 + 4 = 2$ **or**

$$= -\frac{\text{Coefficient of x}}{\text{Coefficient of x}^2} = -\frac{(-2)}{1} = 2$$

Product of zeroes $= \alpha\beta = (-2)(4) = -8$ **or**

$$= \frac{\text{Constant term}}{\text{Coefficient of x}^2} = \frac{-8}{1} = -8$$

So, the relationship between the zeroes and the coefficients is verified.

1 D. Question

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients:

$3x^2 + 5x - 2$

Answer

Let $f(x) = 3x^2 + 5x - 2$

By splitting the middle term, we get

$f(x) = 3x^2 + (6 - 1)x - 2$ [$\because 5 = 6 - 1$ and $2 \times 3 = 6$]

$= 3x^2 + 6x - x - 2$

$= 3x(x + 2) - 1(x + 2)$

$= (3x - 1)(x + 2)$

On putting $f(x) = 0$, we get

$(3x - 1)(x + 2) = 0$

$\Rightarrow 3x - 1 = 0$ or $x + 2 = 0$

$\Rightarrow x = \frac{1}{3}$ or $x = -2$

Thus, the zeroes of the given polynomial $3x^2 + 5x - 2$ are -2 and $\frac{1}{3}$

Verification

Sum of zeroes $= \alpha + \beta = \frac{1}{3} + (-2) = \frac{1-6}{3} = -\frac{5}{3}$ **or**

$$= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{5}{3}$$

Product of zeroes $= \alpha\beta = \left(\frac{1}{3}\right)(-2) = \frac{-2}{3}$ **or**

$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{-2}{3}$$

So, the relationship between the zeroes and the coefficients is verified.

1 E. Question

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients:

$3x^2 - x - 4$

Answer

Let $f(x) = 3x^2 - x - 4$

By splitting the middle term, we get

$f(x) = 3x^2 - (4 - 3)x - 4$ [$\because -1 = 3 - 4$ and $4 \times 3 = 12$]

$= 3x^2 + 3x - 4x - 4$

$= 3x(x + 1) - 4(x + 1)$

$= (3x - 4)(x + 1)$

On putting $f(x) = 0$, we get

$(3x - 4)(x + 1) = 0$

$\Rightarrow$ 3x – 4 = 0 or x + 1 = 0

$\Rightarrow x = \frac{4}{3}$ or x = – 1

Thus, the zeroes of the given polynomial $3x^2 - x - 4$ are – 1 and $\frac{4}{3}$

Verification

Sum of zeroes $= \alpha + \beta = \frac{4}{3} + (-1) = \frac{4-3}{3} = \frac{1}{3}$ **or**

$$= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{(-1)}{3} = \frac{1}{3}$$

Product of zeroes $= \alpha\beta = \left(\frac{4}{3}\right)(-1) = \frac{-4}{3}$ **or**

$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{-4}{3}$$

So, the relationship between the zeroes and the coefficients is verified.

1 F. Question

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients:

$x^2 + 7x + 10$

Answer

Let $f(x) = x^2 + 7x + 10$

By splitting the middle term, we get

$f(x) = x^2 + 5x + 2x + 10$ [$\because$ 7 = 2 + 5 and 2×5 = 10]

= x(x + 5) + 2(x + 5)

= (x + 2) (x + 5)

On putting f(x) = 0 , we get

(x + 2) (x + 5) = 0

$\Rightarrow$ x + 2 = 0 or x + 5 = 0

$\Rightarrow$x = – 2 or x = – 5

Thus, the zeroes of the given polynomial $x^2 + 7x + 10$ are – 2 and – 5

Verification

Sum of zeroes = $\alpha + \beta = -2 + (-5) = -7$ **or**

$$= -\frac{\text{Coefficient of x}}{\text{Coefficient of x}^2} = -\frac{7}{1} = -7$$

Product of zeroes = $\alpha\beta = (-2)(-5) = 10$ **or**

$$= \frac{\text{Constant term}}{\text{Coefficient of x}^2} = \frac{10}{1} = 10$$

So, the relationship between the zeroes and the coefficients is verified.

1 G. Question

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients:

$t^2 - 15$

Answer

Let $f(x) = t^2 - 15$

Now, if we recall the identity

$(a^2 - b^2) = (a - b)(a + b)$

Using this identity, we can write

$t^2 - 15 = (t - \sqrt{15})(x + \sqrt{15})$

So, the value of $t^2 - 15$ is zero when $t = \sqrt{15}$ or $t = -\sqrt{15}$

Therefore, the zeroes of $t^2 - 15$ are $\sqrt{15}$ and $-\sqrt{15}$.

<u>Verification</u>

Now,

Sum of zeroes = $\alpha + \beta = \sqrt{15} + (-\sqrt{15}) = 0$ **or**

$$= -\frac{\text{Coefficient of t}}{\text{Coefficient of t}^2} = -\frac{0}{1} = 0$$

Product of zeroes = $\alpha\beta = (\sqrt{15})(-\sqrt{15}) = -15$ **or**

$$= \frac{\text{Constant term}}{\text{Coefficient of t}^2} = \frac{-15}{1} = -15$$

So, the relationship between the zeroes and the coefficients is verified.

1 H. Question

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients:

$4\ s^2 - 4s + 1$

Answer

Let $f(x) = 4\ s^2 - 4s + 1$

By splitting the middle term, we get

$f(x) = 4\ s^2 - (2 - 2)s + 1$ [$\because -4 = -(2 + 2)$ and $2\times2 = 4$]

$= 4\ s^2 - 2s - 2s + 1$

$= 2s(2s - 1) - 1(2s - 1)$

$= (2s - 1)(2s - 1)$

On putting $f(x) = 0$, we get

$(2s - 1)(2s - 1) = 0$

$\Rightarrow 2s - 1 = 0$ or $2s - 1 = 0$

$\Rightarrow s = \frac{1}{2}$ or $s = \frac{1}{2}$

Thus, the zeroes of the given polynomial $4\ s^2 - 4s + 1$ are $\frac{1}{2}$ and $\frac{1}{2}$

Verification

Sum of zeroes $= \alpha + \beta = \frac{1}{2} + \frac{1}{2} = 1$ **or**

$$= -\frac{\text{Coefficient of s}}{\text{Coefficient of s}^2} = -\frac{(-4)}{4} = 1$$

Product of zeroes $= \alpha\beta = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ **or**

$$= \frac{\text{Constant term}}{\text{Coefficient of s}^2} = \frac{1}{4}$$

So, the relationship between the zeroes and the coefficients is verified.

2 A. Question

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients:

$8x^2 - 22x - 21$

Answer

Let $f(x) = 8x^2 - 22x - 21$

By splitting the middle term, we get

$f(x) = 8x^2 - 28x + 6x - 21$

$= 4x(2x - 7) + 3(2x - 7)$

$= (4x + 3)(2x - 7)$

On putting f(x) = 0 , we get

$(4x + 3)(2x - 7) = 0$

$\Rightarrow 4x + 3 = 0$ or $2x - 7 = 0$

$\Rightarrow x = \frac{-3}{4}$ or $x = \frac{7}{2}$

Thus, the zeroes of the given polynomial $8x^2 - 22x - 21$ are $\frac{-3}{4}$ and $\frac{7}{2}$

Verification

Sum of zeroes $= \alpha + \beta = \frac{-3}{4} + \frac{7}{2} = \frac{-3+14}{4} = \frac{11}{4}$ **or**

$$= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{(-22)}{8} = \frac{11}{4}$$

The product of zeroes $= \alpha\beta = \frac{-3}{4} \times \frac{7}{2} = \frac{-21}{8}$ **or**

$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{-21}{8}$$

So, the relationship between the zeroes and the coefficients is verified.

2 B. Question

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients:

$2x^2 - 7x$

Answer

Let $f(x) = 2x^2 - 7x$

In this the constant term is zero.

$f(x) = 2x^2 - 7x$

= x(2x – 7)

On putting f(x) = 0 , we get

x(2x – 7) = 0

⇒ 2x – 7 = 0 or x = 0

⇒ $x = \frac{7}{2}$ or x = 0

Thus, the zeroes of the given polynomial $2x^2 - 7x$ are 0 and $\frac{7}{2}$

Verification

Sum of zeroes $= \alpha + \beta = 0 + \frac{7}{2} = \frac{7}{2}$ **or**

$$= -\frac{\text{Coefficient of x}}{\text{Coefficient of x}^2} = -\frac{(-7)}{2} = \frac{7}{2}$$

Product of zeroes $= \alpha\beta = 0 \times \frac{7}{2} = 0$ **or**

$$= \frac{\text{Constant term}}{\text{Coefficient of x}^2} = \frac{0}{2} = 0$$

So, the relationship between the zeroes and the coefficients is verified.

2 C. Question

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients:

$10x^2 + 3x - 1$

Answer

Let $f(x) = 10x^2 + 3x - 1$

By splitting the middle term, we get

$f(x) = 10x^2 - 2x + 5x - 1$

= 2x(5x – 1) + 1(5x – 1)

= (2x + 1) (5x – 1)

On putting f(x) = 0, we get

(2x + 1) (5x – 1) = 0

⇒ 2x + 1 = 0 or 5x – 1 = 0

$\Rightarrow x = \frac{-1}{2}$ or $x = \frac{1}{5}$

Thus, the zeroes of the given polynomial $10x^2 + 3x - 1$ are $\frac{-1}{2}$ and $\frac{1}{5}$

Verification

Sum of zeroes $= \alpha + \beta = \frac{-1}{2} + \frac{1}{5} = \frac{-5+2}{10} = \frac{-3}{10}$ **or**

$$= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{3}{10} = -\frac{3}{10}$$

Product of zeroes $= \alpha\beta = \frac{-1}{2} \times \frac{1}{5} = \frac{-1}{10}$ **or**

$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{-1}{10}$$

So, the relationship between the zeroes and the coefficients is verified.

2 D. Question

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients:

$px^2 + (2q - p^2)x - 2pq, p \neq 0$

Answer

Let $f(x) = px^2 + (2q - p^2)x - 2pq$

$f(x) = px^2 + 2qx - p^2 x - 2pq$

$= x(px + 2q) - p(px + 2q)$

$= (x - p)(px + 2q)$

On putting $f(x) = 0$, we get

$(x - p)(px + 2q) = 0$

$\Rightarrow x - p = 0$ or $px + 2q = 0$

$\Rightarrow x = p$ or $x = \frac{-2q}{p}$

Thus, the zeroes of the given polynomial $px^2 + (2q - p^2)x - 2pq$ are p and $\frac{-2q}{p}$

Verification

Sum of zeroes $= \alpha + \beta = p + \frac{(-2q)}{p} = \frac{p^2 - 2q}{p}$ **or**

$$= -\frac{\text{Coefficient of x}}{\text{Coefficient of } x^2} = -\frac{(-p^2 + 2q)}{p} = \frac{p^2 - 2q}{p}$$

Product of zeroes = $\alpha\beta = p \times \frac{-2q}{p} = -2q$ **or**

$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{-2pq}{p} = -2q$$

So, the relationship between the zeroes and the coefficients is verified.

2 E. Question

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients:

$x^2 - (2a + b)x + 2ab$

Answer

Let $f(x) = x^2 - (2a + b)x + 2ab$

$f(x) = x^2 - 2ax - bx + 2ab$

$= x(x - 2a) - b(x - 2a)$

$= (x - 2a)(x - b)$

On putting f(x) = 0 , we get

$(x - 2a)(x - b) = 0$

$\Rightarrow x - 2a = 0$ or $x - b = 0$

$\Rightarrow x = 2a$ or $x = b$

Thus, the zeroes of the given polynomial $x^2 - (2a + b)x + 2ab$ are 2a and b

Verification

Sum of zeroes = $\alpha + \beta = 2a + b$ **or**

$$= -\frac{\text{Coefficient of x}}{\text{Coefficient of } x^2} = -\frac{(-2a - b)}{1} = 2a + b$$

Product of zeroes = $\alpha\beta = 2a \times b = 2ab$ **or**

$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{2ab}{1} = 2ab$$

So, the relationship between the zeroes and the coefficients is verified.

2 F. Question

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients:

$r^2s^2x^2 + 6rstx + 9t^2$

Answer

Let $f(x) = r^2s^2x^2 + 6rstx + 9t^2$

Now, if we recall the identity

$(a + b)^2 = a^2 + b^2 + 2ab$

Using this identity, we can write

$r^2s^2x^2 + 6rstx + 9t^2 = (rsx + 3t)^2$

On putting f(x) = 0 , we get

$(rsx + 3t)^2 = 0$

$$\Rightarrow x = \frac{-3t}{rs}, \frac{-3t}{rs}$$

Thus, the zeroes of the given polynomial $r^2s^2x^2 + 6rstx + 9t^2$ are $\frac{-3t}{rs}$ and $\frac{-3t}{rs}$

Verification

Sum of zeroes $= \alpha + \beta = \frac{-3t}{rs} + \frac{-3t}{rs} = -\frac{6t}{rs}$ **or**

$$= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{6rst}{r^2s^2} = -\frac{6t}{rs}$$

Product of zeroes $= \alpha\beta = \frac{-3t}{rs} \times \frac{-3t}{rs} = \frac{9t^2}{r^2s^2}$ **or**

$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{9t^2}{r^2s^2}$$

So, the relationship between the zeroes and the coefficients is verified.

3 A. Question

Find the zeroes of the quadratic polynomial $5x^2 – 8x – 4$ and verify the relationship between the zeroes and the coefficients of the polynomial.

Answer

Let $f(x) = 5x^2 – 8x – 4$

By splitting the middle term, we get

$f(x) = 5x^2 - 10x + 2x - 4$

$= 5x(x - 2) + 2(x - 2)$

$= (5x + 2)(x - 2)$

On putting f(x) = 0 we get

$(5x + 2)(x - 2) = 0$

$\Rightarrow 5x + 2 = 0$ or $x - 2 = 0$

$\Rightarrow x = \frac{-2}{5}$ or $x = 2$

Thus, the zeroes of the given polynomial $5x^2 - 8x - 4$ are $\frac{-2}{5}$ and 2

Verification

Sum of zeroes $= \alpha + \beta = \frac{-2}{5} + 2 = \frac{-2+10}{5} = \frac{8}{5}$ **or**

$$= -\frac{\text{Coefficient of x}}{\text{Coefficient of } x^2} = -\frac{(-8)}{5} = \frac{8}{5}$$

Product of zeroes $= \alpha\beta = \frac{-2}{5} \times 2 = \frac{-4}{5}$ **or**

$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{-4}{5}$$

So, the relationship between the zeroes and the coefficients is verified.

3 B. Question

Find the zeroes of the quadratic polynomial $4x^2 - 4x - 3$ and verify the relationship between the zeroes and the coefficients of the polynomial.

Answer

Let $f(x) = 4x^2 - 4x - 3$

By splitting the middle term, we get

$f(x) = 4x^2 - 6x + 2x - 3$

$= 2x(2x - 3) + 1(2x - 3)$

$= (2x + 1)(2x - 3)$

On putting f(x) = 0, we get

$(2x + 1)(2x - 3) = 0$

$\Rightarrow 2x + 1 = 0$ or $2x - 3 = 0$

$\Rightarrow x = \frac{-1}{2}$ or $x = \frac{3}{2}$

Thus, the zeroes of the given polynomial $4x^2 - 4x - 3$ are $\frac{-1}{2}$ and $\frac{3}{2}$

Verification

Sum of zeroes $= \alpha + \beta = \frac{-1}{2} + \frac{3}{2} = \frac{-1+3}{2} = 1$ **or**

$$= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{(-4)}{4} = 1$$

Product of zeroes $= \alpha\beta = \frac{-1}{2} \times \frac{3}{2} = \frac{-3}{4}$ **or**

$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{-3}{4}$$

So, the relationship between the zeroes and the coefficients is verified.

3 C. Question

Find the zeroes of the quadratic polynomial $\sqrt{3}x^2 - 8x + 4\sqrt{3}$.

Answer

Let $f(x) = \sqrt{3}x^2 - 8x + 4\sqrt{3}$

By splitting the middle term, we get

$f(x) = \sqrt{3}x^2 - 6x - 2x + 4\sqrt{3}$

$= \sqrt{3}x(x - 2\sqrt{3}) - 2(x - 2\sqrt{3})$

$= (\sqrt{3}x - 2)(x - 2\sqrt{3})$

On putting $f(x) = 0$, we get

$(\sqrt{3}x - 2)(x - 2\sqrt{3}) = 0$

$\Rightarrow \sqrt{3}x - 2 = 0$ or $x - 2\sqrt{(3 = 0)}$

$\Rightarrow x = \frac{2}{\sqrt{3}}$ or $x = 2\sqrt{3}$

Thus, the zeroes of the given polynomial $\sqrt{3}x^2 - 8x + 4\sqrt{3}$ are $\frac{2}{\sqrt{3}}$ and $2\sqrt{3}$

Verification

Sum of zeroes = $\alpha + \beta = \frac{2}{\sqrt{3}} + 2\sqrt{3} = \frac{2+6}{\sqrt{3}} = \frac{8}{\sqrt{3}}$ **or**

$$= -\frac{\text{Coefficient of x}}{\text{Coefficient of } x^2} = -\frac{(-8)}{\sqrt{3}} = \frac{8}{\sqrt{3}}$$

Product of zeroes = $\alpha\beta = \frac{2}{\sqrt{3}} \times 2\sqrt{3} = 4$ **or**

$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{4\sqrt{3}}{\sqrt{3}} = 4$$

So, the relationship between the zeroes and the coefficients is verified.

4. Question

If α and β be the zeroes of the polynomial $2x^2 + 3x - 6$, find the values of

(i) $\alpha^2 + \beta^2$ (ii) $\alpha^2 + \beta^2 + \alpha\beta$

(iii) $\alpha^2\beta + \alpha\beta^2$ (iv) $\frac{1}{\alpha} + \frac{1}{\beta}$

(v) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ (vi) $\alpha - \beta$

(vii) $\alpha^3 + \beta^3$ (viii) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

Answer

Let the quadratic polynomial be $2x^2 + 3x - 6$, and its zeroes are α and β.

We have

$$\alpha + \beta = \frac{-b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

Here, a = 2 , b = 3 and c = – 6

$\alpha + \beta = \frac{-b}{a} = \frac{-3}{2}$ ....(1)

$\alpha\beta = \frac{c}{a} = \frac{-6}{2} = -3$ ....(2)

(i) $\alpha^2 + \beta^2$

We have to find the value of $\alpha^2 + \beta^2$

Now, if we recall the identity

$(a + b)^2 = a^2 + b^2 + 2ab$

Using the identity, we get

$(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$\Rightarrow \left(\frac{-3}{2}\right)^2 = \alpha^2 + \beta^2 + 2(-3)$ {from eq^n (1) & (2)} $\Rightarrow \frac{9}{4} = \alpha^2 + \beta^2 - 6$

$\Rightarrow \alpha^2 + \beta^2 = \frac{9}{4} + 6$

$\Rightarrow \alpha^2 + \beta^2 = \frac{9 + 24}{4} = \frac{33}{4}$

(ii) $\alpha^2 + \beta^2 + \alpha\beta$

$\alpha^2 + \beta^2 = \frac{33}{4}$ { from part (i)}

and, we have $\alpha\beta = -3$

So, $\alpha^2 + \beta^2 + \alpha\beta = \frac{33}{4} + (-3)$

$= \frac{33-12}{4}$

$= \frac{21}{4}$

(iii) $\alpha^2\beta + \alpha\,\beta^2$

Firstly, take $\alpha\beta$ common, we get

$\alpha\beta(\alpha + \beta)$

and we already know the value of $\alpha\beta$ and $\alpha + \beta$.

So, $\alpha^2\beta + \alpha\,\beta^2 = \alpha\beta(\alpha + \beta)$

$= (-3)\left(\frac{-3}{2}\right)$ {from eq^n (1) and (2)}

$= \frac{9}{2}$

(iv) $\frac{1}{\alpha} + \frac{1}{\beta}$

Let's take the LCM first then we get,

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta}$$

$$= \frac{\left(\frac{-3}{2}\right)}{-3}$$

$$= \frac{1}{2}$$

(v) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

Let's take the LCM first then we get,

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\beta\alpha}$$

$= \frac{\left(\frac{33}{4}\right)}{-3}$ {from part(i) and eq^n (2)}

$$= \frac{-11}{4}$$

(vi) $\alpha - \beta$

Now, recall the identity

$(a - b)^2 = a^2 + b^2 - 2ab$

Using the identity , we get

$(\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\,\alpha\beta$

$\Rightarrow (\alpha - \beta)^2 = \left(\frac{33}{4}\right) - 2(-3)$ {from part(i) and eq^n (2)}

$$\Rightarrow (\alpha - \beta)^2 = \frac{33}{4} + 6$$

$$\Rightarrow (\alpha - \beta)^2 = \frac{33 + 24}{4} = \frac{57}{4}$$

$$\Rightarrow (\alpha - \beta) = \pm \frac{\sqrt{57}}{2}$$

(vii) $\alpha^3 + \beta^3$

Now, recall the identity

$(a + b)^3 = a^3 + b^3 + 3a^2 b + 3ab^2$

Using the identity, we get

$\Rightarrow (\alpha + \beta)^3 = \alpha^3 + \beta^3 + 3\alpha^2 \beta + 3\alpha\beta^2$

$$\Rightarrow \left(\frac{-3}{2}\right)^3 = \alpha^3 + \beta^3 + 3\left(\alpha^2\beta + \alpha\beta^2\right)$$

$$\Rightarrow \frac{-27}{8} = \alpha^3 + \beta^3 + 3 \times \frac{9}{2}$$

$$\Rightarrow \alpha^3 + \beta^3 = \frac{-27 - 108}{8}$$

$$\Rightarrow \alpha^3 + \beta^3 = -\frac{135}{4}$$

(viii) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

Let's take the LCM first then we get,

$$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\beta\alpha}$$

$= \frac{\left(\frac{-135}{8}\right)}{-3}$ {from part(vii) and eq^n (2)}

$$= \frac{45}{8}$$

5. Question

If α and β be the zeroes of the polynomial $ax^2 + bx + c$, find the values of

(i) $a^2 + \beta^2$

(ii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

(ii) $\alpha^3 + \beta^3$

Answer

Let the quadratic poynomial be $ax^2 + bx + c$, and its zeroes be α and β.

We have

$\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$

(i) $a^2 + \beta^2$

We have to find the value of $\alpha^2 + \beta^2$

Now, if we recall the identity

$(a + b)^2 = a^2 + b^2 + 2ab$

Using the identity, we get $(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$\Rightarrow \left(\frac{-b}{a}\right)^2 = \alpha^2 + \beta^2 + 2 \times \frac{c}{a}$ {from eq^n (1) & (2)}

$\Rightarrow \frac{b^2}{a^2} = \alpha^2 + \beta^2 + \frac{2c}{a}$

$\Rightarrow \alpha^2 + \beta^2 = \frac{b^2}{a^2} - \frac{2c}{a}$

$\Rightarrow \alpha^2 + \beta^2 = \frac{b^2 - 2ca}{a^2}$

(ii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

Let's take the LCM first then we get,

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\beta\alpha}$

$= \frac{\left(\frac{b^2 - 2ca}{a^2}\right)}{\frac{c}{a}}$ $\{\because\ \alpha\beta = \frac{c}{a}\}$

$= \frac{b^2 - 2ca}{ca}$

(iii) $\alpha^3 + \beta^3$

Now, recall the identity

$(a + b)^3 = a^3 + b^3 + 3a^2 b + 3ab^2$

Using the identity, we get

$\Rightarrow (\alpha + \beta)^3 = \alpha^3 + \beta^3 + 3\alpha^2 \beta + 3\alpha\beta^2$

$$\Rightarrow \left(\frac{-b}{a}\right)^3 = \alpha^3 + \beta^3 + 3\,(\alpha^2\beta + \alpha\beta^2)$$

$$\Rightarrow \left(\frac{-b^3}{a^3}\right) = \alpha^3 + \beta^3 + 3\alpha\beta(\alpha + \beta)$$

$$\Rightarrow \alpha^3 + \beta^3 = \left(\frac{-b^3}{a^3}\right) + 3 \times \frac{c}{a} \times \left(\frac{-b}{a}\right)$$

$$\Rightarrow \alpha^3 + \beta^3 = \frac{3abc - b^3}{a^3}$$

6. Question

If α, β are the zeroes of the quadratic polynomial $x^2 + kx = 12$, such that $\alpha - \beta = 1$, find the value of k.

Answer

The given quadratic polynomial is $x^2 + kx = 12$ and $\alpha - \beta = 1$

If we rearrange the polynomial then we get

$p(x) = x^2 + kx - 12$

We have,

$\alpha + \beta = \frac{-b}{a}$ and $\alpha\,\beta = \frac{c}{a}$

So,

$\alpha + \beta = \frac{-b}{a} = \frac{-k}{1} = -k$ ...(1)

$\alpha\,\beta = \frac{c}{a} = \frac{-12}{1} = -12$ ...(2)

Now, if we recall the identities

$(a + b)^2 = a^2 + b^2 + 2ab$

Using the identity, we get

$(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\,\alpha\beta$

$(-k)^2 = \alpha^2 + \beta^2 + 2(-12)$

$\Rightarrow \alpha^2 + \beta^2 = k^2 + 24$ ...(3)

Again, using the identity

$(a - b)^2 = a^2 + b^2 - 2ab$

Using the identity, we get $(\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$

$(1)^2 = \alpha^2 + \beta^2 - 2(-12)$ $\{\because (\alpha - \beta) = 1\}$

$\Rightarrow \alpha^2 + \beta^2 = 1 - 24$

$\Rightarrow \alpha^2 + \beta^2 = -23$ ...(4)

From eq^n (3) and (4), we get

$k^2 + 24 = -23$

$\Rightarrow k^2 = -23 - 24$

$\Rightarrow k^2 = -47$

Now the square can never be negative, so the value of k is imaginary.

7. Question

If the sum of squares of the zeroes of the quadratic polynomial $x^2 - 8x + k$ is 40, find k.

Answer

Given : $p(x) = x^2 - 8x + k$

$\alpha^2 + \beta^2 = 40$

We have,

$$\alpha + \beta = \frac{-b}{a}$$

and $\alpha\beta = \frac{c}{a}$

So,

$\alpha + \beta = \frac{-b}{a} = \frac{-(-8)}{1} = 8$ ...(1)

$\alpha\beta = \frac{c}{a} = \frac{k}{1} = k$ ...(2)

Now, if we recall the identities

$(a + b)^2 = a^2 + b^2 + 2ab$

Using the identity, we get

$(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\,\alpha\beta$

$(8)^2 = 40 + 2(k)$

$\Rightarrow 2k = 64 - 40$

$\Rightarrow k = \frac{24}{2} = 12$

8 A. Question

If one zero of the polynomial $(\alpha^2 + 9)\,x^2 + 13x + 6\alpha$ is reciprocal of the other, find the value of a.

Answer

Let one zero of the given polynomial is α

According to the given condition,

The other zero of the polynomial is $\frac{1}{\alpha}$

We have,

Product of zeroes, $\alpha\,\beta = \frac{c}{a} = \frac{6\alpha}{\alpha^2 + 9}$

$\Rightarrow \alpha \times \frac{1}{\alpha} = \frac{6\alpha}{\alpha^2 + 9}$

$\Rightarrow \alpha^2 - 6\alpha + 9 = 0$

$\Rightarrow \alpha^2 - 3\alpha - 3\alpha + 9 = 0$

$\Rightarrow \alpha(\alpha - 3) - 3(\alpha - 3) = 0$

$\Rightarrow (\alpha - 3)(\alpha - 3) = 0$

$\Rightarrow (\alpha - 3) = 0\ \&\ (\alpha - 3) = 0$

$\Rightarrow \alpha = 3, 3$

8 B. Question

If the product of zeroes of the polynomial $\alpha^2 - 6x - 6$ is 4, find the value of a.

Answer

Given Product of zeroes, $\alpha\,\beta = 4$

$p(x) = \alpha\,x^2 - 6x - 6$

to find: value of α

We know,

Product of zeroes , $\alpha\beta = \frac{c}{a} = \frac{-6}{\alpha}$

$\Rightarrow 4 = \frac{-6}{\alpha}$

$\Rightarrow \alpha = \frac{-6}{4} = \frac{-3}{2}$

8 C. Question

If (x + a) is a factor $2x^2 + 2ax + 5x + 10$, find a.

Answer

Given x + a is a factor of $2x^2 + 2ax + 5x + 10,$

So, g(x) = x + a

x + a = 0

⇒x = – a

Putting the value x = – a in the given polynomial, we get

$2(-a)^2 + 2a(-a) + 5(-a) + 10 = 0$

$2a^2 - 2a^2 - 5a + 10 = 0$

$-5a + 10 = 0$

$a = \frac{-10}{-5}$

a = 2

9 A. Question

Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively:

1,1

Answer

Given: Sum of zeroes = α + β = 1

Product of zeroes = αβ = 1

Then, the quadratic polynomial

= x^2 – (sum of zeroes)x + product of zeroes

= x^2 – (1)x + 1

= x^2 – x + 1

9 B. Question

Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively:

0,3

Answer

Given: Sum of zeroes = α + β = 0

Product of zeroes = αβ = 3

Then, the quadratic polynomial

= x^2 – (sum of zeroes)x + product of zeroes

= x^2 – (0)x + 3

= x^2 + 3

9 C. Question

Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively:

$\frac{1}{4}, -1$

Answer

Given: Sum of zeroes = α + β = 1/4

Product of zeroes = αβ = – 1

Then, the quadratic polynomial

= x^2 – (sum of zeroes)x + product of zeroes

= $x^2 - \left(\frac{1}{4}\right)x + (-1)$

= $x^2 - \frac{x}{4} - 1$

$= \frac{4x^2 - x - 4}{4}$

We can consider $4x^2 - x - 4$ as required quadratic polynomial because it will also satisfy the given conditions.

9 D. Question

Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively:

4,1

Answer

Given: Sum of zeroes = $\alpha + \beta = 4$

Product of zeroes = $\alpha\beta = 1$

Then, the quadratic polynomial

= x^2 – (sum of zeroes)x + product of zeroes

$= x^2 - (4)x + 1$

$= x^2 - 4x + 1$

9 E. Question

Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively:

$\frac{10}{3}, -1$

Answer

Given: Sum of zeroes $= \alpha + \beta = \frac{10}{3}$

Product of zeroes = $\alpha\beta = -1$

Then, the quadratic polynomial

= x^2 – (sum of zeroes)x + product of zeroes

$= x^2 - \left(\frac{10}{3}\right)x + (-1)$

$= x^2 - \frac{10x}{3} - 1$

$= \frac{3x^2-10x-3}{3}$

We can consider $3x^2 - 10x - 3$ as required quadratic polynomial because it will also satisfy the given conditions.

9 F. Question

Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively:

$$-\frac{1}{2}, -\frac{1}{2}$$

Answer

Given: Sum of zeroes $= \alpha + \beta = -\frac{1}{2}$

Product of zeroes $= \alpha\beta = -\frac{1}{2}$

Then, the quadratic polynomial

$= x^2 -$ (sum of zeroes)x + product of zeroes

$= x^2 - \left(-\frac{1}{2}\right)x + \left(-\frac{1}{2}\right)$

$= x^2 + \frac{x}{2} - \frac{1}{2}$

$= \frac{2x^2+x-1}{2}$

We can consider $2x^2 + x - 1$ as required quadratic polynomial because it will also satisfy the given conditions.

10 A. Question

Find quadratic polynomial whose zeroes are :

3, – 3

Answer

Let the zeroes of the quadratic polynomial be

$\alpha = 3$, $\beta = -3$

Then, $\alpha + \beta = 3 + (-3) = 0$

$\alpha\beta = 3 \times (-3) = -9$

Sum of zeroes = $\alpha + \beta = 0$

Product of zeroes = $\alpha\beta = -9$

Then, the quadratic polynomial

= x^2 – (sum of zeroes)x + product of zeroes

= $x^2 - (0)x + (-9)$

= $x^2 - 9$

10 B. Question

Find quadratic polynomial whose zeroes are :

$$\frac{2+\sqrt{5}}{2}, \frac{2-\sqrt{5}}{2}$$

Answer

Let the zeroes of the quadratic polynomial be

$$\alpha = \frac{2 + \sqrt{5}}{2}, \beta = \frac{2-\sqrt{5}}{2}$$

Then, $\alpha + \beta = \frac{2+\sqrt{5}}{2} + \frac{2-\sqrt{5}}{2} = \frac{2+\sqrt{5}+2-\sqrt{5}}{2} = 2$

$$\alpha\beta = \frac{2+\sqrt{5}}{2} \times \frac{2-\sqrt{5}}{2} = \frac{(2+\sqrt{5})(2-\sqrt{5})}{4} = \frac{4-5}{4} = \frac{-1}{4}$$

Sum of zeroes = $\alpha + \beta = 2$

Product of zeroes = $\alpha\beta = -\frac{1}{4}$

Then, the quadratic polynomial

= x^2 – (sum of zeroes)x + product of zeroes

= $x^2 - (2)x + (-\frac{1}{4})$

= $x^2 - 2x - \frac{1}{4}$

= $\frac{4x^2-8x-1}{2}$

We can consider $4x^2 - 8x - 1$ as required quadratic polynomial because it will also satisfy the given conditions.

10 C. Question

Find quadratic polynomial whose zeroes are :

$3+\sqrt{7}, 3-\sqrt{7}$

Answer

Let the zeroes of the quadratic polynomial be

$\alpha = 3 + \sqrt{7}, \beta = 3 - \sqrt{7}$

Then, $\alpha + \beta = 3 + \sqrt{7} + 3 - \sqrt{7} = 6$

$\alpha\beta = (3 + \sqrt{(7)}) \times (3 - \sqrt{7}) = 9 - 7 = 2$

Sum of zeroes = $\alpha + \beta = 6$

Product of zeroes = $\alpha\beta = 2$

Then, the quadratic polynomial

= x^2 – (sum of zeroes)x + product of zeroes

= $x^2 - (6)x + 2$

= $x^2 - 6x + 2$

10 D. Question

Find quadratic polynomial whose zeroes are :

$1+2\sqrt{3}, 1-2\sqrt{3}$

Answer

Let the zeroes of the quadratic polynomial be

$\alpha = 1 + 2\sqrt{3}, \beta = 1 - 2\sqrt{3}$

Then, $\alpha + \beta = 1 + 2\sqrt{3} + 1 - 2\sqrt{3} = 2$

$\alpha\beta = (1 + 2\sqrt{3}) \times (1 - 2\sqrt{3}) = 1 - 12 = -11$

Sum of zeroes = $\alpha + \beta = 2$

Product of zeroes = $\alpha\beta = -11$

Then, the quadratic polynomial

= x^2 – (sum of zeroes)x + product of zeroes

$= x^2 - (2)x + (-11)$

$= x^2 - 2x - 11$

10 E. Question

Find quadratic polynomial whose zeroes are :

$$\frac{2-\sqrt{3}}{3}, \frac{2+\sqrt{3}}{3}$$

Answer

Let the zeroes of the quadratic polynomial be

$$\alpha = \frac{2-\sqrt{3}}{3}, \beta = \frac{2+\sqrt{3}}{3}$$

Then, $\alpha + \beta = \frac{2-\sqrt{3}}{3} + \frac{2+\sqrt{3}}{3} = \frac{2-\sqrt{3}+2+\sqrt{3}}{3} = \frac{4}{3}$

$$\alpha\beta = \frac{2-\sqrt{3}}{3} \times \frac{2+\sqrt{3}}{3} = \frac{(2-\sqrt{3})(2+\sqrt{3})}{9} = \frac{4-3}{9} = \frac{1}{9}$$

Sum of zeroes $= \alpha + \beta = \frac{4}{3}$

Product of zeroes $= \alpha\beta = \frac{1}{9}$

Then, the quadratic polynomial

$= x^2 -$ (sum of zeroes)x + product of zeroes

$= x^2 - \left(\frac{4}{3}\right)x + \left(\frac{1}{9}\right)$

$= x^2 - \frac{4}{3}x + \frac{1}{9}$

$= \frac{9x^2 - 12x + 1}{2}$

We can consider $9x^2 - 12x + 1$ as required quadratic polynomial because it will also satisfy the given conditions.

10 F. Question

Find quadratic polynomial whose zeroes are :

$$\sqrt{2}, 2\sqrt{2}$$

Answer

Let the zeroes of the quadratic polynomial be

$\alpha = \sqrt{2}, \beta = 2\sqrt{2}$

Then, $\alpha + \beta = \sqrt{2} + 2\sqrt{2} = \sqrt{2}\ (1 + 2) = 3\sqrt{2}$

$\alpha\beta = \sqrt{2} \times 2\sqrt{2} = 4$

Sum of zeroes = $\alpha + \beta = 3\sqrt{2}$

Product of zeroes = $\alpha\beta = 4$

Then, the quadratic polynomial

$= x^2 - (\text{sum of zeroes})x + \text{product of zeroes}$

$= x^2 - (3\sqrt{2})x + 4$

$= x^2 - 3\sqrt{2}\,x + 4$

11. Question

Find the quadratic polynomial whose zeroes are square of the zeroes of the polynomial $x^2 - x - 1$.

Answer

et the zeroes of the polynomial $x^2 - x - 1$ be α and β

We have,

$$\alpha + \beta = \frac{-b}{a}$$

and $\alpha\beta = \frac{c}{a}$

So,

$$\alpha + \beta = \frac{-b}{a} = -\frac{-1}{1} = 1$$

$$\alpha\beta = \frac{c}{a} = \frac{-1}{1} = -1$$

Now, according to the given condition,

$\alpha^2\beta^2 = (-1)^2 = 1$

& $(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$\Rightarrow \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$\Rightarrow \alpha^2 + \beta^2 = (1)^2 - 2(-1)$

$\Rightarrow \alpha^2 + \beta^2 = 3$

So, the quadratic polynomial

= x^2 – (sum of zeroes)x + product of zeroes

= $x^2 - (3)x + 1$

= $x^2 - 3x + 1$

12 A. Question

If α and β be the zeroes of the polynomial $x^2 + 10x + 30$, then find the quadratic polynomial whose zeroes are α + 2β and 2α + β.

Answer

Given : $p(x) = x^2 + 10x + 30$

So, Sum of zeroes $= \alpha + \beta = \frac{-b}{a} = \frac{-10}{1} = -10$ ...(1)

Product of zeroes $= \alpha\beta = \frac{c}{a} = \frac{30}{1} = 30$ ...(2)

Now,

Let the zeroes of the quadratic polynomial be

α^' = α + 2β , β' = 2α + β

Then, $\alpha' + \beta' = \alpha + 2\beta + 2\alpha + \beta = 3\alpha + 3\beta = 3(\alpha + \beta)$

$\alpha'\beta' = (\alpha + 2\beta) \times (2\alpha + \beta) = 2\alpha^2 + 2\beta^2 + 5\alpha\beta$

Sum of zeroes = $3(\alpha + \beta)$

Product of zeroes = $2\alpha^2 + 2\beta^2 + 5\alpha\beta$

Then, the quadratic polynomial

= x^2 – (sum of zeroes)x + product of zeroes

= $x^2 - (3(\alpha + \beta))x + 2\alpha^2 + 2\beta^2 + 5\alpha\beta$

= $x^2 - 3(-10)x + 2(\alpha^2 + \beta^2) + 5(30)$ {from eq^n (1) & (2)}

= $x^2 + 30x + 2(\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta) + 150$

$= x^2 + 30x + 2(\alpha + \beta)^2 - 4\alpha\beta + 150$

$= x^2 + 30x + 2(-10)^2 - 4(30) + 150$

$= x^2 + 30x + 200 - 120 + 150$

$= x^2 + 30x + 230$

So, the required quadratic polynomial is $x^2 + 30x + 230$

12 B. Question

If α and β be the zeroes of the polynomial $x^2 + 4x + 3$, find the quadratic polynomial whose zeroes are $1+\frac{\alpha}{\beta}$ and $1+\frac{\beta}{\alpha}$.

Answer

Given : $p(x) = x^2 + 4x + 3$

So, Sum of zeroes $= \alpha + \beta = \frac{-b}{a} = \frac{-4}{1} = -4$ ...(1)

Product of zeroes $= \alpha\beta = \frac{c}{a} = \frac{3}{1} = 3$ ...(2)

Now,

Let the zeroes of the quadratic polynomial be

$\alpha' = 1 + \frac{\alpha}{\beta}, \beta' = 1 + \frac{\beta}{\alpha}$

Then, $\alpha' + \beta' = 1 + \frac{\alpha}{\beta} + 1 + \frac{\beta}{\alpha} = \frac{2\alpha\beta + \alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2}{\alpha\beta}$

$$\alpha'\beta' = (1 + \frac{\alpha}{\beta}) \times (1 + \frac{\beta}{\alpha}) = 1 + \frac{\beta}{\alpha} + \frac{\alpha}{\beta} + 1 = \frac{2\alpha\beta + \alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2}{\alpha\beta}$$

Sum of zeroes $= \frac{(\alpha + \beta)^2}{\alpha\beta}$

Product of zeroes $= \frac{(\alpha + \beta)^2}{\alpha\beta}$

Then, the quadratic polynomial

$= x^2 -$ (sum of zeroes)x + product of zeroes

$= x^2 - \frac{(\alpha + \beta)^2}{\alpha\beta}x + \frac{(\alpha + \beta)^2}{\alpha\beta}$

$= x^2 - \frac{(4)^2}{3}x + \frac{(4)^2}{3}$ {from eq^n (1) & (2)}

$= x^2 - \frac{16}{3}x + \frac{16}{3}$

$= \frac{3x^2 - 16x + 16}{3}$

So, the required quadratic polynomial is $3x^2 - 16x + 16$

13 A. Question

Find a quadratic polynomial whose zeroes are 1 and – 3. Verify the relation between the coefficients and zeroes of the polynomial.

Answer

Let the zeroes of the quadratic polynomial be

$\alpha = 1$, $\beta = -3$

Then, $\alpha + \beta = 1 + (-3) = -2$

$\alpha\beta = 1 \times (-3) = -3$

Sum of zeroes = $\alpha + \beta = -2$

Product of zeroes = $\alpha\beta = -3$

Then, the quadratic polynomial

$= x^2$ – (sum of zeroes)x + product of zeroes

$= x^2 - (-2)x + (-3)$

$= x^2 + 2x - 3$

__**Verification**__

Sum of zeroes = $\alpha + \beta = 1 + (-3) = -2$ **or**

$$= -\frac{\text{Coefficient of x}}{\text{Coefficient of x}^2} = -\frac{(2)}{1} = -2$$

Product of zeroes = $\alpha\beta = (1)(-3) = -3$ **or**

$$= \frac{\text{Constant term}}{\text{Coefficient of x}^2} = \frac{-3}{1} = -3$$

So, the relationship between the zeroes and the coefficients is verified.

13 B. Question

Find the quadratic polynomial sum of whose zeroes in 8 and their product is 12. Hence find the zeroes of the polynomial.

Answer

Given: Sum of zeroes = $\alpha + \beta = 8$

Product of zeroes = $\alpha\beta = 12$

Then, the quadratic polynomial

= x^2 – (sum of zeroes)x + product of zeroes

= $x^2 - (8)x + 12$

= $x^2 - 8x + 12$

Now, we have

$\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$

So,

$$\alpha + \beta = \frac{-b}{a} = -\frac{-8}{1} = 8$$

$$\alpha\beta = \frac{c}{a} = \frac{12}{1} = 12$$

Exercise 2.3

1. Question

Divide $2x^3 + 3x + 1$ by $x + 2$ and find the quotient and the reminder. Is q(x) a factor of $2x^3 + 3x + 1$?

Answer

Here, dividend and divisor both are in the standard form.

Now, on dividing p(x) by g(x) we get the following division process

$$
\begin{array}{l}
\quad\quad\quad 2x^2 - 4x + 11 \\
x+2 \,\big)\, 2x^3 + 3x + 1 \\
\quad\quad\quad 2x^3 \quad\quad\quad\quad + 4x^2 \\
\quad\quad\quad - \quad\quad\quad\quad\quad - \\
\hline
\quad\quad\quad 3x \quad +1 \quad - 4x^2 \\
\quad\quad\quad -8x \quad\quad\quad - 4x^2 \\
\quad\quad\quad + \quad\quad\quad\quad + \\
\hline
\quad\quad\quad 11x \quad +1 \\
\quad\quad\quad 11x \quad +22 \\
\quad\quad\quad - \quad\quad - \\
\hline
\quad\quad\quad\quad -21 \\
\hline
\end{array}
$$

Quotient = $2x^2 - 4x + 11$

Remainder = – 21

No, $2x^2 - 4x + 11$ is not a factor of $2x^3 + 3x + 1$ because remainder ≠ 0

2. Question

Divide $3x^3 + x^2 + 2x + 5$ by $1 + 2x + x^2$ and find the quotient and the remainder. Is $1 + 2x + x^2$ a factor of $3x^3 + x^2 + 2x + 5$?

Answer

Dividend = $3x^3 + x^2 + 2x + 5$

Divisor = $x^2 + 2x + 1$

Now, dividend and divisor both are in the standard form.

Now, on dividing p(x) by g(x) we get the following division process

$$
\begin{array}{l}
\quad\quad\quad\quad 3x - 5 \\
x^2+2x+1 \,\big)\, 3x^3 + x^2 + 2x + 5 \\
\quad\quad\quad\quad 3x^3 + 6x^2 + 3x \\
\quad\quad\quad\quad - \quad - \quad - \\
\hline
\quad\quad\quad\quad -5x^2 - x + 5 \\
\quad\quad\quad\quad -5x^2 - 10x - 5 \\
\quad\quad\quad\quad + \quad + \quad + \\
\hline
\quad\quad\quad\quad\quad 9x + 10 \\
\hline
\end{array}
$$

Quotient = $3x - 5$

Remainder = $9x + 10$

No, $x^2 + 2x + 1$ is not a factor of $3x^3 + x^2 + 2x + 5$ because remainder ≠ 0

3 A. Question

Divide the polynomial p(x) by the polynomial g(x) and find the quotient q(x) and remainder r(x) in each case :

$p(x) = x^3 - 3x^2 + 4x + 2$, $g(x) = x - 1$

Answer

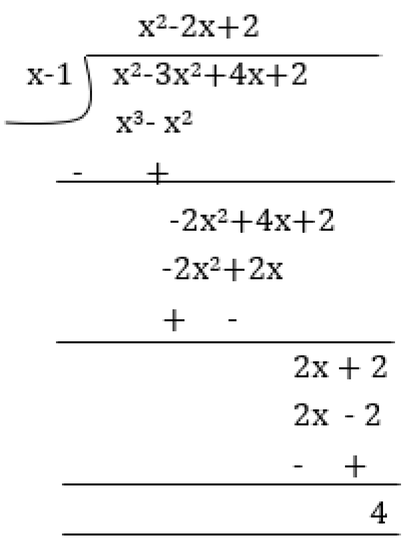
$p(x) = x^3 - 3x^2 + 4x + 2, g(x) = x - 1$

Dividend = $x^3 - 3x^2 + 4x + 2$

Divisor = $x - 1$

Here, dividend and divisor both are in the standard form.

Now, on dividing p(x) by g(x) we get the following division process



Quotient = $x^2 - 2x + 2$

Remainder = 4

3 B. Question

Divide the polynomial p(x) by the polynomial g(x) and find the quotient q(x) and remainder r(x) in each case :

$p(x) = 4x^3 - 3x^2 + 2x + 3$, $g(x) = x + 4$

Answer

$p(x) = 4x^3 - 3x^2 + 2x + 3, g(x) = x + 4$

Dividend = $4x^3 - 3x^2 + 2x + 3$

Divisor = x + 4

Here, dividend and divisor both are in the standard form.

Now, on dividing p(x) by g(x) we get the following division process

$$
\begin{array}{r|l}
 & 4x^2-13x+54 \\
x+4 & 4x^3 \quad +3x^2 \quad +2x \quad +3 \\
 & 4x^3 \quad +16x^2 \\
 & - \qquad - \\
\hline
 & -13x^2+2x+3 \\
 & -13x^2-52x \\
 & + \qquad + \\
\hline
 & 54x+3 \\
 & 54x+216 \\
 & - \qquad - \\
\hline
 & -213 \\
\hline
\end{array}
$$

Quotient = $4x^2 - 13x + 54$

Remainder = – 213

3 C. Question

Divide the polynomial p(x) by the polynomial g(x) and find the quotient q(x) and remainder r(x) in each case :

$p(x) = 2x^4 + 3x^3 + 4x^2 + 19x + 45, g(x) = x - 2$

Answer

$p(x) = 2x^4 + 3x^3 + 4x^2 + 19x + 45, g(x) = x - 2$

Dividend = $2x^4 + 3x^3 + 4x^2 + 19x + 45$

Divisor = x – 2

Here, dividend and divisor both are in the standard form.

Now, on dividing p(x) by g(x) we get the following division process we get,

$$
\begin{array}{r|l}
 & 2x^3 + 7x^2 + 18x + 55 \\
\hline
x - 2 & 2x^4 + 3x^3 + 4x^2 + 19x + 45 \\
 & 2x^4 - 4x^3 \\
 & - \quad + \\
\hline
 & 7x^3 + 4x^2 \\
 & 7x^3 - 14x^2 \\
 & - \quad + \\
\hline
 & 18x^2 + 19x \\
 & 18x^2 - 36x \\
 & - \quad + \\
\hline
 & 55x + 45 \\
 & 55x - 110 \\
 & - \quad + \\
\hline
 & 155 \\
\hline
\end{array}
$$

q(x) = $2x^3+7x^2+18x+55$

r(x) = 155

3 D. Question

Divide the polynomial p(x) by the polynomial g(x) and find the quotient q(x) and remainder r(x) in each case :

p(x) = $x^4 + 2x^3 – 3x^2 + x – 1$, g(x) = x – 2

Answer

p(x) = $x^4 + 2x^3 – 3x^2 + x – 1$, g(x) = x – 2

Dividend = $x^4 + 2x^3 – 3x^2 + x – 1$

Divisor = x – 2

Here, dividend and divisor both are in the standard form.

Now, on dividing p(x) by g(x) we get the following division process we get,

$$
\begin{array}{r|l}
 & x^3 + 4x^2 + 5x + 11 \\
\hline
x - 2 & x^4 + 2x^3 - 3x^2 + x - 1 \\
 & x^4 - 2x^3 \\
 & - \quad + \\
\hline
 & 4x^3 - 3x^2 \\
 & 4x^3 - 8x^2 \\
 & - \quad + \\
\hline
 & 5x^2 + x \\
 & 5x^2 - 10x \\
 & - \quad + \\
\hline
 & 11x - 1 \\
 & 11x - 22 \\
 & - \quad + \\
\hline
 & 21 \\
\hline
\end{array}
$$

q(x) = $x^3+4x^2+5x+11$

r(x) = 21

3 E. Question

Divide the polynomial p(x) by the polynomial g(x) and find the quotient q(x) and remainder r(x) in each case :

p(x) = $x^3 - 3x^2 - x + 3$, g(x) = $x^2 - 4x + 3$

Answer

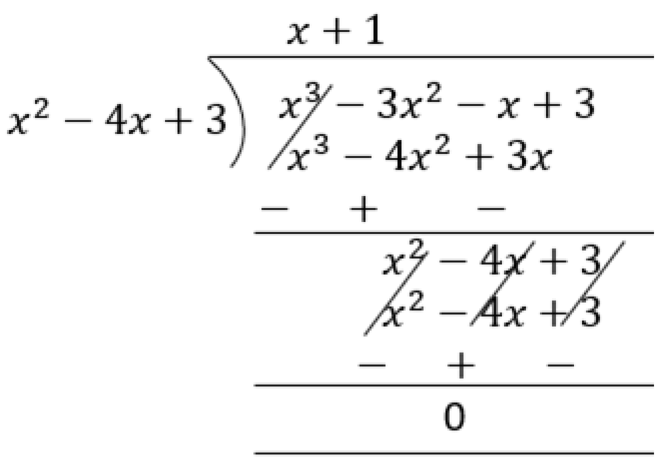
p(x) = $x^3 - 3x^2 - x + 3$, g(x) = $x^2 - 4x + 3$

Dividend = $x^3 - 3x^2 - x + 3$

Divisor = $x^2 - 4x + 3$

Here, dividend and divisor both are in the standard form.

Now, on dividing p(x) by g(x) we get the following division process we get,



q(x)= x + 1

r(x) = 0

3 F. Question

Divide the polynomial p(x) by the polynomial g(x) and find the quotient q(x) and remainder r(x) in each case :

p(x) = $x^6 + x^4 + x^3 + x^2 + 2x + 2$, g(x) = $x^3 + 1$

Answer

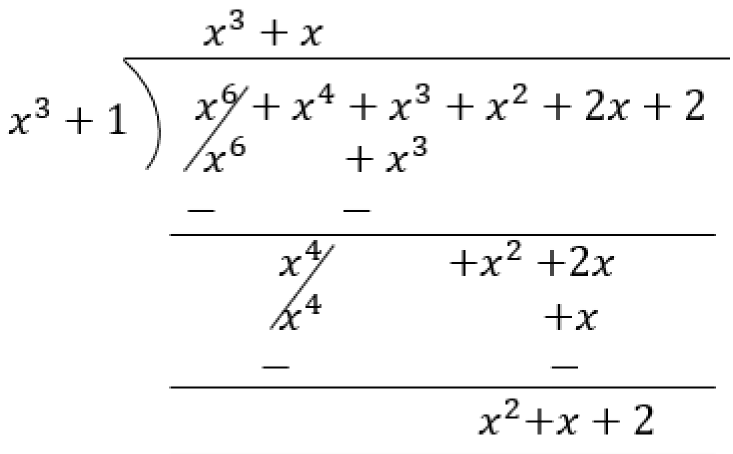
p(x) = $x^6 + x^4 + x^3 + x^2 + 2x + 2$, g(x) = $x^3 + 1$

Dividend = $x^6 + x^4 + x^3 + x^2 + 2x + 2$

Divisor = $x^3 + 1$

Here, dividend and divisor both are in the standard form.

Now, on dividing p(x) by g(x) we get the following division process we get,



q(x) = $x^3 + x$

r(x) = $x^2 + x + 2$

3 G. Question

Divide the polynomial p(x) by the polynomial g(x) and find the quotient q(x) and remainder r(x) in each case :

$p(x) = x^6 + 3x^2 + 10$ and $g(x) = x^3 + 1$

Answer

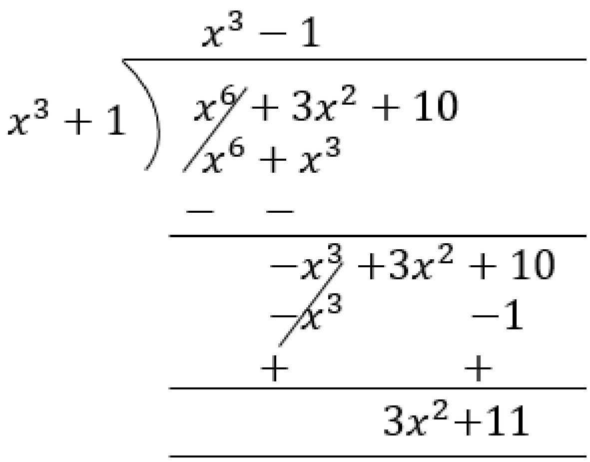
$p(x) = x^6 + 3x^2 + 10$, $g(x) = x^3 + 1$

Dividend = $x^6 + 3x^2 + 10$

Divisor = $x^3 + 1$

Here, dividend and divisor both are in the standard form.

Now, on dividing p(x) by g(x) we get the following division process we get,



$q(x) = x^3 – 1$

$r(x) = 3x^2 + 11$

3 H. Question

Divide the polynomial p(x) by the polynomial g(x) and find the quotient q(x) and remainder r(x) in each case :

$p(x) = x^4 + 1$, $g(x) = x + 1$

Answer

$p(x) = x^4 + 1$, $g(x) = x + 1$

Dividend = $x^4 + 1$,

Divisor = x + 1

$$
\begin{array}{r|l}
 & x^3 - x^2 + x - 1 \\
\hline
x + 1 & x^4 \qquad\qquad + 1 \\
 & x^4 + x^3 \\
 & - \quad - \\
\hline
 & -x^3 \qquad + 1 \\
 & -x^3 - x^2 \\
 & + \quad + \\
\hline
 & x^2 \qquad + 1 \\
 & x^2 + x \\
 & - \quad - \\
\hline
 & -x + 1 \\
 & -x + 1 \\
 & + \quad - \\
\hline
 & 0 \\
\hline
\end{array}
$$

$q(x) = x^3 - x^2 + x - 1$

$r(x) = 0$

4. Question

By division process, find the value of k for which x – 1 is a factor of $x^3 - 6x^2 + 11x + k$.

Answer

On dividing $x^3 - 6x^2 + 11x + k$ by x – 1 we get,

$$
\begin{array}{r|l}
 & x^2 - 5x \\
\hline
x - 1 & x^3 - 6x^2 + 11x + k \\
 & x^3 - x^2 \\
 & - \quad + \\
\hline
 & -5x^2 + 11x \\
 & -5x^2 + 5x \\
 & + \qquad - \\
\hline
 & 6x + k \\
\hline
\end{array}
$$

Since x – 1 is a factor of $x^3 - 6x^2 + 11x + k$,

This means x – 1 divides the given polynomial completely.

→ 6 x + k = 0

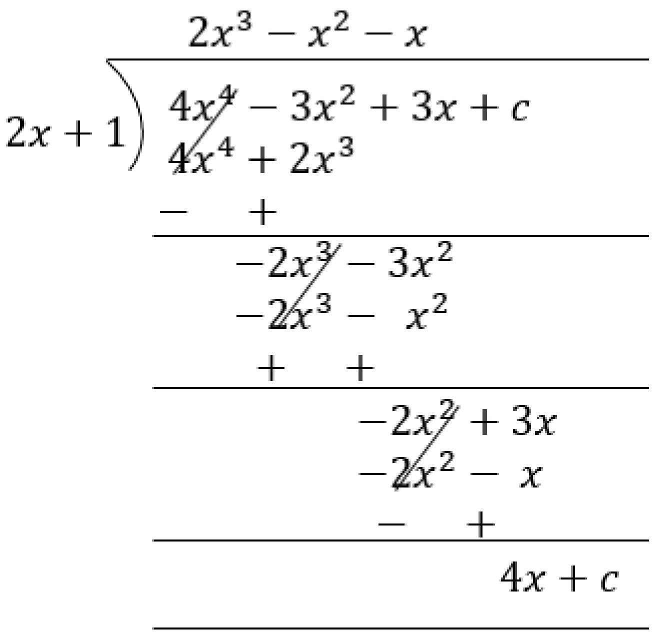
→ k = – 6x

5. Question

By division process, find the value of c for which 2x + 1 is a factor of $4x^4 - 3x^2 + 3x + c$.

Answer

On dividing $4x^4 - 3x^2 + 3x + c$ by 2x + 1 we get,



Since 2x + 1 is a factor of $4x^4 - 3x^2 + 3x + c$,

This means 2x + 1 divides the given polynomial completely,

→ 4x + c = o

→ c = – 4x

6 A. Question

Apply Division Algorithm to find the quotient q(x) and remainder r(x) on dividing p(x) by g(x) as given below:

$p(x) = 2x^2 + 3x + 1$, g(x) = x + 2

Answer

$p(x) = 2x^2 + 3x + 1$, g(x) = x + 2

Dividend = $2x^2 + 3x + 1$,

Divisor = x + 2

Apply the division algorithm we get,

$$
\begin{array}{r|l}
 & 2x - 1 \\ \hline
x + 2 & 2x^2 + 3x + 1 \\
 & 2x^2 + 4x \\
 & - \quad - \\ \hline
 & \quad -x + 1 \\
 & \quad -x - 2 \\
 & \quad + \quad + \\ \hline
 & \qquad 3 \\ \hline
\end{array}
$$

q(x) = 2x – 1

r(x) = 3

6 B. Question

Apply Division Algorithm to find the quotient q(x) and remainder r(x) on dividing p(x) by g(x) as given below:

p(x) = $x^3 - 3x^2 + 5x - 3$, g(x) = $x^2 - 2$

Answer

p(x) = $x^3 - 3x^2 + 5x - 3$, g(x) = $x^2 - 2$

Dividend = $x^3 - 3x^2 + 5x - 3$,

Divisor = $x^2 - 2$

On applying division algorithm, we get,

$$
\begin{array}{r|l}
 & x - 3 \\ \hline
x^2 - 2 & x^3 - 3x^2 + 5x - 3 \\
 & x^3 \qquad\quad - 2x \\
 & - \qquad\quad + \\ \hline
 & \quad -3x^2 + 7x - 3 \\
 & \quad -3x^2 \qquad + 6 \\
 & \quad + \qquad\quad - \\ \hline
 & \qquad\quad 7x - 9 \\ \hline
\end{array}
$$

q(x) = x – 3

r(x) = 7x – 9

6 C. Question

Apply Division Algorithm to find the quotient q(x) and remainder r(x) on dividing p(x) by g(x) as given below:

$p(x) = x^4 - 1$, $g(x) = x + 1$

Answer

$p(x) = x^4 - 1, g(x) = x + 1$

Dividend = $x^4 - 1$

Divisor = $x + 1$

Here, dividend and divisor both are in the standard form.

Now, on dividing p(x) by g(x) we get the following division process

$x^3 - x^2 + x - 1$

$x+1 \;\big)\; x^4 + 0x^3 + 0x^2 + 0x - 1$

$x^4 + x^3$

$- \quad -$

$-x^3 + 0x^2 + 0x - 1$

$-x^3 - x^2$

$+ \quad +$

$x^2 + 0x - 1$

$x^2 + x$

$- \quad -$

$-x - 1$

$-x - 1$

$+ \quad +$

0

Quotient = $x^3 - x^2 + x - 1$

Remainder = 0

6 D. Question

Apply Division Algorithm to find the quotient q(x) and remainder r(x) on dividing p(x) by g(x) as given below:

$p(x) = x^3 - 3x^2 + 4x + 2$, $g(x) = x - 1$

Answer

$p(x) = x^3 - 3x^2 + 4x + 2, g(x) = x - 1$

Dividend = $x^3 - 3x^2 + 4x + 2$

Divisor = x – 1

Here, dividend and divisor both are in the standard form.

Now, on dividing p(x) by g(x) we get the following division process

$$
\begin{array}{r|l}
 & x - 5 \\
\hline
x^2+2x+1 & x^3 - 3x^2 + 4x + 2 \\
 & x^3 + 2x^2 + x \\
 & - \quad - \quad - \\
\hline
 & -5x^2 + 3x + 2 \\
 & -5x^2 - 10x - 5 \\
 & + \quad + \quad + \\
\hline
 & 13x + 7 \\
\hline
\end{array}
$$

Quotient = x – 5

Remainder = 13x + 7

6 E. Question

Apply Division Algorithm to find the quotient q(x) and remainder r(x) on dividing p(x) by g(x) as given below:

p(x) = x^3 – 6x^2 + 11x – 6 , g(x) = x^2 – 5x + 6

Answer

p(x) = x^3 – 6x^2 + 11x – 6,g(x) = x^2 – 5x + 6

Dividend = x^3 – 6x^2 + 11x – 6

Divisor = x^2 – 5x + 6

Here, dividend and divisor both are in the standard form.

Now, on dividing p(x) by g(x) we get the following division process

$$
\begin{array}{r|l}
 & x - 1 \\
\hline
x^2-5x+6 & x^3 - 6x^2 + 11x - 6 \\
 & x^3 - 5x^2 \\
 & + \quad + \\
\hline
 & -x^2 + 11x - 6 \\
 & -x^2 + 5x - 6 \\
 & + \quad - \quad + \\
\hline
 & 6x \\
\hline
\end{array}
$$

Quotient = x – 1

Remainder = 6x

6 F. Question

Apply Division Algorithm to find the quotient q(x) and remainder r(x) on dividing p(x) by g(x) as given below:

p(x) = $6x^3 + 13x^2 + x - 2$, g(x) = 2x + 1

Answer

p(x) = $6x^3 + 13x^2 + x - 2$,g(x) = 2x + 1

Dividend = $6x^3 + 13x^2 + x - 2$

Divisor = 2x + 1

Here, dividend and divisor both are in the standard form.

Now, on dividing p(x) by g(x) we get the following division process

$$\begin{array}{r|l} & 3x^2+5x-2 \\ \hline 2x+1 & 6x^3+13x^2+x-2 \\ & 6x^3+3x^2 \\ & -\quad - \\ \hline & 10x^2+x-2 \\ & 10x^2+5x \\ & -\quad - \\ \hline & -4x-2 \\ & -4x-2 \\ & +\quad + \\ \hline & 0 \\ \hline \end{array}$$

Quotient = $3x^2 + 5x - 2$

Remainder = 0

7 A. Question

Applying the Division Algorithm, check whether the first polynomial is a factor of the second polynomial:

x – 2, $x^3 + 3x^3 - 12x + 4$

Answer

Let us divide $x^3 + 3x^2 - 12x + 4$ by x – 2

The division process is

$$
\begin{array}{r|l}
 & x^2+5x-2 \\
\hline
x-2 & x^3+3x^2-12x+4 \\
 & x^3-2x^2 \\
 & -\quad + \\
\hline
 & 5x^2-12x+4 \\
 & 5x^2-10x \\
 & -\quad + \\
\hline
 & -2x+4 \\
 & -2x+4 \\
 & +\quad - \\
\hline
 & 0
\end{array}
$$

Here, the remainder is 0, therefore x – 2 is a factor of $x^3 + 3x^2 - 12x + 4$

7 B. Question

Applying the Division Algorithm, check whether the first polynomial is a factor of the second polynomial:

$x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$

Answer

Let us divide $3x^4 + 5x^3 - 7x^2 + 2x + 2$ by $x^2 + 3x + 1$

The division process is

$$
\begin{array}{r|l}
 & 3x^2-4x+2 \\
\hline
x^2+3x+1 & 3x^4+5x^3-7x^2+2x+2 \\
 & 3x^4+9x^3+3x^2 \\
 & -\quad -\quad - \\
\hline
 & -4x^3-10x^2+2x+2 \\
 & -4x^3-12x^2-4x \\
 & +\quad +\quad + \\
\hline
 & 2x^2+6x+2 \\
 & 2x^2+6x+2 \\
 & -\quad -\quad - \\
\hline
 & 0
\end{array}
$$

Here, the remainder is 0, therefore $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$

7 C. Question

Applying the Division Algorithm, check whether the first polynomial is a factor of the second polynomial:

$x^2 - 3x + 4, 2x^4 - 11x^3 + 29x^2 - 30x + 29$

Answer

Let us divide $2x^4 - 11x^3 + 29x^2 - 30x + 29$ by $x^2 - 3x + 4$

The division process is

$$
\begin{array}{r|l}
 & 2x^2+5x+36 \\
x^2-3x+4 & 2x^4-11x^3+29x^2-30x+29 \\
 & 2x^4-6x^3+8x^2 \\
 & - \quad + \quad - \\
\hline
 & 5x^3+21x^2-30x+29 \\
 & 5x^3-15x^2+20x \\
 & - \quad + \quad - \\
\hline
 & 36x^2-50x+29 \\
 & 36x^2-108x+144 \\
 & - \quad + \quad - \\
\hline
 & 58x-115 \\
\hline
\end{array}
$$

Here, the remainder is $58x - 115$, therefore $x^2 - 3x + 4$ is not a factor of $2x^4 - 11x^3 + 29x^2 - 30x + 29$

7 D. Question

Applying the Division Algorithm, check whether the first polynomial is a factor of the second polynomial:

$x^2 - 4x + 3, x^3 - x^3 - 3x^4 - x + 3$

Answer

Let us divide $x^3 - 3x^2 - x + 3$ by $x^2 - 4x + 3$

The division process is

$$\begin{array}{r|l} & x+1 \\ \hline x^2-4x+3 & x^3-3x^2-x+3 \\ & x^3-4x^2+3x \\ & -\quad+\quad- \\ \hline & x^2-4x+3 \\ & x^2-4x+3 \\ & -\quad+\quad- \\ \hline & 0 \\ \hline \end{array}$$

Here, the remainder is 0, therefore $x^2 - 4x + 3$ is a factor of $x^3 - 3x^2 - x + 3$

7 E. Question

Applying the Division Algorithm, check whether the first polynomial is a factor of the second polynomial:

$t - 1, t^3 + t^2 - 2t + 1$

Answer

Let us divide $t^3 + t^2 - 2t + 1$ by $t - 1$

The division process is

$$\begin{array}{r|l} & t^2+2t \\ \hline t-1 & t^3+t^2-2t+1 \\ & t^3-t^2 \\ & -\quad+ \\ \hline & 2t^2-2t+1 \\ & 2t^2-2t \\ & -\quad+ \\ \hline & 1 \\ \hline \end{array}$$

Here, the remainder is 1, therefore $t - 1$ is not a factor of $t^3 + t^2 - 2t + 1$

7 F. Question

Applying the Division Algorithm, check whether the first polynomial is a factor of the second polynomial:

$t^2 - 5t + 6, t^2 + 11t - 6$

Answer

Let us divide $t^2 + 11t - 6$ by $t^2 - 5t + 6$

The division process is

$$\begin{array}{r|l} & \quad\quad 1 \\ t^2-5t+6 & t^2+11t-6 \\ & t^2-5t\ \ +6 \\ & -\quad +\quad - \\ \hline & \quad\quad 16t-12 \end{array}$$

Here, the remainder is 16t – 12,

Therefore, $t^2 - 5t + 6$ is not a factor of $t^2 + 11t - 6$

8. Question

Give examples of polynomials p(x), g(x), q(x) and r(x) satisfying the Division Algorithm

p(x) = g(x).q(x) + r(x),deg r(x)<deg g(x)

And also satisfying

(i) deg p(x) = deg q(x) + 1

(ii)deg q(x) = 1

(iii) deg q(x) = deg r(x) + 1

Answer

(i)Let $p(x) = 12x^2 + 8x + 25$, g(x) = 4,

$q(x) = 3x^2 + 2x + 6$, r(x) = 0

Here, degree p(x) = degree q(x) = 2

Now, $g(x).q(x) + r(x) = (3x^2 + 2x + 6)\times4 + 1$

= $12x^2 + 8x + 24 + 1$

$= 12x^2 + 8x + 25$

(ii) Let $p(x) = t^3 + t^2 - 2t$, $g(x) = t^2 + 2t$,

q(x) = t – 1 , r(x) = 0

Here, degree q(x) = 1

Now, $g(x).q(x) + r(x) = (t^2 + 2t)\times(t - 1) + 0$

= $t^3 - t^2 + 2t^2 - 2t$

$= t^3 + t^2 - 2t$

(iii) Let $p(x) = x^3 + x^2 + x + 1$, $g(x) = x^2 - 1$,

$q(x) = x + 1$, $r(x) = 2x + 2$

Here, degree q(x) = degree r(x) + 1 = 1

Now, $g(x).q(x) + r(x) = (x^2 - 1)\times(x + 1) + 2x + 2$

= $x^3 + x^2 - x - 1 + 2x + 2$

$= x^3 + x^2 + x + 1$

9 A. Question

Find all the zeroes of the polynomial given below having given numbers as its zeroes.

$x^3 - 6x^2 + 11x - 6;3$

Answer

Given zeroes is 3

So, $(x - 3)$ is the factor of $x^3 - 6x^2 + 11x - 6$

Let us divide $x^3 - 6x^2 + 11x - 6$ by $x - 3$

The division process is

$$
\begin{array}{r|l}
 & x^2-3x+2 \\
\hline
x-3 & x^3-6x^2+11x-6 \\
 & x^3-3x^2 \\
 & -\quad + \\
\hline
 & -3x^2+11x-6 \\
 & -3x^2+9x \\
 & +\quad - \\
\hline
 & 2x-6 \\
 & 2x-6 \\
 & -\quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

Here, quotient $= x^2 - 3x + 2$

$= x^2 - 2x - x + 2$

$= x(x - 2) - 1(x - 2)$

$= (x - 1)(x - 2)$

So, the zeroes are 1 and 2

Hence, all the zeroes of the given polynomial are 1, 2 and 3.

9 B. Question

Find all the zeroes of the polynomial given below having given numbers as its zeroes.

$x^4 - 8x^3 + 23x^2 - 28x + 12$;1,2

Answer

Given zeroes are 1 and 2

So, (x − 1)and (x − 2) are the factors of $x^4 - 8x^3 + 23x^2 - 28x + 12$

⟹ (x − 1)(x − 2) = $x^2 - 3x + 2$ is a factor of given polynomial.

Consequently, $x^2 - 3x + 2$ is also a factor of the given polynomial.

Now, let us divide $x^4 - 8x^3 + 23x^2 - 28x + 12$ by $x^2 - 3x + 2$

The division process is

x^2-5x+6

x^2-3x+2) $x^4-8x^3+23x^2-28x+12$

$x^4-3x^3+2x^2$

$-\quad+\quad-$

$-5x^3+21x^2-28x+12$

$-5x^3+15x^2-10x$

$+\quad-\quad+$

$6x^2-18x+12$

$6x^2-18x+12$

$-\quad+\quad-$

0

Here, quotient = $x^2 - 5x + 6$

= $x^2 - 2x - 3x + 6$

= x(x − 2) − 3(x − 2)

= (x − 3)(x − 2)

So, the zeroes are 3 and 2

Hence, all the zeroes of the given polynomial are 1, 2 ,2 and 3.

9 C. Question

Find all the zeroes of the polynomial given below having given numbers as its zeroes.

$x^3 + 2x^2 - 2; -2$

Answer

Given zeroes is − 2

So, (x + 2) is the factor of $x^3 + 2x^2 - x - 2$

Let us divide $x^3 + 2x^2 - x - 2$ by x + 2

The division process is

$$
\begin{array}{r|l}
 & x^2 - 1 \\
\hline
x+2 & x^3 + 2x^2 - x - 2 \\
 & x^3 + 2x^2 \\
 & - \quad - \\
\hline
 & \qquad -x - 2 \\
 & \qquad -x - 2 \\
\hline
 & \qquad\quad 0 \\
\hline
\end{array}
$$

Here, quotient = $x^2 - 1$

= (x − 1)(x + 1)

So, the zeroes are − 1 and 1

Hence, all the zeroes of the given polynomial are − 1, − 2 and 1.

9 D. Question

Find all the zeroes of the polynomial given below having given numbers as its zeroes.

$x^3 + 5x^2 + 7x + 3; -3$

Answer

Given zeroes is − 3

So, (x + 3) is the factor of $x^3 + 5x^2 + 7x + 3$

Let us divide $x^3 + 5x^2 + 7x + 3$ by x + 3

The division process is

$$
\begin{array}{r|l}
 & x^2+2x+1 \\
\hline
x+3 & x^3+5x^2+7x+3 \\
 & x^3+3x^2 \\
 & -\quad - \\
\hline
 & 2x^2+7x+3 \\
 & 2x^2+6x \\
 & -\quad - \\
\hline
 & x+3 \\
 & x+3 \\
 & -\quad - \\
\hline
 & 0 \\
\hline
\end{array}
$$

Here, quotient = $x^2 + 2x + 1$

= $(x + 1)^2$

So, the zeroes are – 1 and – 1

Hence, all the zeroes of the given polynomial are – 1, – 1 and – 3.

9 E. Question

Find all the zeroes of the polynomial given below having given numbers as its zeroes.

$x^4 - 6x^3 - 26x^2 + 138x - 35; 2\pm\sqrt{3}$

Answer

$x^4 - 6x^3 - 26x^2 + 138x - 35; 2\pm\sqrt{3}$

Given zeroes are $2 + \sqrt{3}$ and $2 - \sqrt{3}$

So, $(x - 2 - \sqrt{3})$and $(x - 2 + \sqrt{3})$ are the factors of $x^4 - 6x^3 - 26x^2 + 138x - 35$

$\Rightarrow (x - 2 - \sqrt{3})(x - 2 + \sqrt{3})$

$= x^2 - 2x + \sqrt{3}\,x - 2x + 4 - 2\sqrt{3} - \sqrt{3}\,x + 2\sqrt{3} - 3$

$= x^2 - 4x + 1$ is a factor of given polynomial.

Consequently, $x^2 - 4x + 1$ is also a factor of the given polynomial.

Now, let us divide $x^4 - 6x^3 - 26x^2 + 138x - 35$ by $x^2 - 4x + 1$

The division process is

$$
\begin{array}{l}
\qquad\qquad x^2-2x-35 \\
x^2-4x+1 \,\big)\, x^4-6x^3-26x^2+138x-35 \\
\qquad\qquad x^4-4x^3+x^2 \\
\qquad\qquad -\quad + \quad - \\
\hline
\qquad\qquad -2x^3-27x^2+138x-35 \\
\qquad\qquad -2x^3+8x^2-2x \\
\qquad\qquad +\quad -\quad + \\
\hline
\qquad\qquad\qquad -35x^2+140x-35 \\
\qquad\qquad\qquad -35x^2+140x-12 \\
\qquad\qquad\qquad +\quad -\quad + \\
\hline
\qquad\qquad\qquad\qquad\qquad 0 \\
\hline
\end{array}
$$

Here, quotient = $x^2 - 2x - 35$

$= x^2 - 7x + 5x - 35$

$= x(x - 7) + 5(x - 7)$

$= (x + 5)(x - 7)$

So, the zeroes are -5 and 7

Hence, all the zeroes of the given polynomial are $-5, 7, 2 + \sqrt{3}$ and $2 - \sqrt{3}$

9 F. Question

Find all the zeroes of the polynomial given below having given numbers as its zeroes.

$x^4 + x^3 - 34x^2 - 4x + 120; 2, -2.$

Answer

$x^4 + x^3 - 34x^2 - 4x + 120; 2, -2.$

Given zeroes are -2 and 2

So, $(x + 2)$and $(x - 2)$ are the factors of $x^4 + x^3 - 34x^2 - 4x + 120$

$\Rightarrow (x + 2)(x - 2) = x^2 - 4$ is a factor of given polynomial.

Consequently, $x^2 - 4$ is also a factor of the given polynomial.

Now, let us divide $x^4 + x^3 - 34x^2 - 4x + 120$ by $x^2 - 4$

The division process is

$$
\begin{array}{r|l}
 & x^2+x-30 \\
\hline
x^2-4 & x^4+x^3-34x^2-4x+120 \\
 & x^4 \quad -4x^2 \\
 & - \quad + \\
\hline
 & x^3-30x^2-4x+120 \\
 & x^3 \quad -4x \\
 & - \quad + \\
\hline
 & -30x^2+120 \\
 & -30x^2+120 \\
 & + \quad - \\
\hline
 & 0 \\
\hline
\end{array}
$$

Here, quotient = $x^2 + x - 30$

= $x^2 + 6x - 5x - 30$

= $x(x + 6) - 5(x + 6)$

= $(x + 6)(x - 5)$

So, the zeroes are -6 and 5

Hence, all the zeroes of the given polynomial are -2, -6, 2 and 5.

9 G. Question

Find all the zeroes of the polynomial given below having given numbers as its zeroes.

$2x^4 + 7x^3 - 19x^2 - 14x + 30$;√(2, – √2)

Answer

$2x^4 + 7x^3 - 19x^2 - 14x + 30$;√(2, – √2)

Given zeroes are $\sqrt{2}$ and $-\sqrt{2}$

So, $(x - \sqrt{2})$and $(x + \sqrt{2})$ are the factors of $2x^4 + 7x^3 - 19x^2 - 14x + 30$

$\Longrightarrow (x - \sqrt{2})(x + \sqrt{2}) = x^2 - 2$ is a factor of given polynomial.

Consequently, $x^2 - 2$ is also a factor of the given polynomial.

Now, let us divide $2x^4 + 7x^3 - 19x^2 - 14x + 30$ by $x^2 - 2$

The division process is

$$
\begin{array}{r|l}
 & 2x^2+7x-15 \\
\hline
x^2-2 & 2x^4+7x^3-19x^2-14x+30 \\
 & 2x^4 \qquad -4x^2 \\
 & - \qquad\quad + \\
\hline
 & 7x^3-15x^2-14x+30 \\
 & 7x^3-14x \\
 & - \quad + \\
\hline
 & -15x^2+30 \\
 & -15x^2+30 \\
 & + \qquad - \\
\hline
 & 0 \\
\hline
\end{array}
$$

Here, quotient = $2x^2 + 7x - 15$

$= 2x^2 + 10x - 3x - 15$

$= 2x(x + 5) - 3(x + 5)$

$= (2x - 3)(x + 5)$

So, the zeroes are -5 and $\frac{3}{2}$

Hence, all the zeroes of the given polynomial are $-5, -\sqrt{2}, \sqrt{2}$ and $\frac{3}{2}$

9 H. Question

Find all the zeroes of the polynomial given below having given numbers as its zeroes.

$2x^4 - 9x^3 + 5x^2 + 3x - 1; 2\pm\sqrt{3}$

Answer

$2x^4 - 9x^3 + 5x^2 + 3x - 1; 2\pm\sqrt{3}$

Given zeroes are $2 + \sqrt{3}$ and $2 - \sqrt{3}$

So, $(x - 2 - \sqrt{3})$and $(x - 2 + \sqrt{3})$ are the factors of $2x^4 - 9x^3 + 5x^2 + 3x - 1$

$\Rightarrow (x - 2 - \sqrt{3})(x - 2 + \sqrt{3})$

$= x^2 - 2x + \sqrt{3}x - 2x + 4 - 2\sqrt{3} - \sqrt{3}x + 2\sqrt{3} - 3$

$= x^2 - 4x + 1$ is a factor of given polynomial.

Consequently, $x^2 - 4x + 1$ is also a factor of the given polynomial.

Now, let us divide $2x^4 - 9x^3 + 5x^2 + 3x - 1$ by $x^2 - 4x + 1$

The division process is

$$
\begin{array}{r|l}
 & 2x^2-x-1 \\
\hline
x^2-4x+1 & 2x^4-9x^3+5x^2+3x-1 \\
 & 2x^4-8x^3+2x^2 \\
 & -\quad +\quad - \\
\hline
 & -x^3+3x^2+3x-1 \\
 & -x^3+4x^2-x \\
 & +\quad -\quad + \\
\hline
 & -x^2+4x-1 \\
 & -x^2+4x-1 \\
 & +\quad -\quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

Here, quotient = $2x^2 - x - 1$

$= 2x^2 - 2x + x - 1$

$= 2x(x - 1) + 1(x - 1)$

$= (2x + 1)(x - 1)$

So, the zeroes are $-\frac{1}{2}$ and 1

Hence, all the zeroes of the given polynomial are $-\frac{1}{2}$, 1, $2 + \sqrt{3}$ and $2 - \sqrt{3}$

9 I. Question

Find all the zeroes of the polynomial given below having given numbers as its zeroes.

$2x^3 - 4x - x^2 + 2; \sqrt{2}, -\sqrt{2}$

Answer

$2x^3 - 4x - x^2 + 2; \sqrt{2}, -\sqrt{2}$

Given zeroes are $\sqrt{2}$ and $-\sqrt{2}$

So, $(x - \sqrt{2})$ and $(x + \sqrt{2})$ are the factors of $2x^3 - 4x - x^2 + 2$

$\Longrightarrow (x - \sqrt{2})(x + \sqrt{2}) = x^2 - 2$ is a factor of given polynomial.

Consequently, $x^2 - 2$ is also a factor of the given polynomial.

Now, let us divide $2x^3 - 4x - x^2 + 2$by $x^2 - 2$

The division process is

$$\begin{array}{r|l} & 2x-1 \\ \hline x^2-2 & 2x^3-4x-x^2+2 \\ & 2x^3-4x \\ & -\quad + \\ \hline & \quad -x^2+2 \\ & \quad -x^2+2 \\ & \quad +\quad - \\ \hline & \qquad 0 \end{array}$$

Here, quotient = 2x – 1

So, the zeroes is $\frac{1}{2}$

Hence, all the zeroes of the given polynomial are $-\sqrt{2}, \sqrt{2}$ and $\frac{1}{2}$

10. Question

Verify that $3, -1, -\frac{1}{3}$ are the zeroes of the cubic polynomial p(x) = $3x^2 - 5x^2 - 11x - 3$ and then verify the relationship between the zeroes and the coefficients.

Answer

Let p(x) = $3x^3 - 5x^2 - 11x - 3$

Then, $p(-1) = 3(-1)^3 - 5(-1)^2 - 11(-1) - 3$

$= -3 - 5 + 11 - 3$

$= 0$

$$p\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right)^3 - 5\left(-\frac{1}{3}\right)^2 - 11\left(-\frac{1}{3}\right) - 3$$

$$= \left(-\frac{1}{9}\right) - \left(\frac{5}{9}\right) + \left(\frac{11}{3}\right) - 3$$

$$= \left(\frac{-1 - 5 + 33 - 27}{9}\right)$$

$= 0$

$p(3) = 3(3)^3 - 5(3)^2 - 11(3) - 3$

$= 81 - 45 - 33 - 3$

= 0

Hence, we verified that 3, – 1 and $-\frac{1}{3}$ are the zeroes of the given polynomial.

So, we take α = 3, β = – 1, $\gamma = -\frac{1}{3}$

<u>Verification</u>

$\alpha + \beta + \gamma = 3 + (-1) + \left(-\frac{1}{3}\right) = \left(\frac{5}{3}\right)$

$$= -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3} = -\frac{5}{3}$$

$\alpha\beta + \beta\gamma + \gamma\alpha = (3)(-1) + (-1)\left(-\frac{1}{3}\right) + \left(-\frac{1}{3}\right)(3)$

$$= \left(-\frac{11}{3}\right)$$

$$= \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3} = \frac{-11}{3} = \left(-\frac{11}{3}\right)$$

and $\alpha\beta\gamma = 3 \times -1 \times \left(-\frac{1}{3}\right)$

= 1

$$= -\frac{\text{Constant term}}{\text{Coefficient of } x^3} = -\frac{-3}{3} = 1$$

Thus, the relationship between the zeroes and the coefficients is verified.

11 A. Question

Verify that the numbers given alongside of the cubic polynomial are their zeros. Also verify the relationship between the zeroes and the coefficients in each case :

$x^3 - 4x^2 + 5x - 2$;2, 1, 1

Answer

Let $p(x) = x^3 - 4x^2 + 5x - 2$

Then, $p(2) = (2)^3 - 4(2)^2 + 5(2) - 2$

= 8 – 16 + 10 – 2

= 0

$p(1) = (1)^3 - 4(1)^2 + 5(1) - 2$

$= 1 - 4 + 5 - 2$

$= 0$

Hence, 2, 1 and 1 are the zeroes of the given polynomial $x^3 - 4x^2 + 5x - 2$.

Now, Let $\alpha = 2$, $\beta = 1$ and $\gamma = 1$

Then, $\alpha + \beta + \gamma = 2 + 1 + 1 = 4$

$$= -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3} = -\frac{-4}{1} = 4$$

$\alpha\beta + \beta\gamma + \gamma\alpha = (2)(1) + (1)(1) + (1)(2)$

$= 2 + 1 + 2$

$= 5$

$$= \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3} = \frac{5}{1} = 5$$

and $\alpha\beta\gamma = 2 \times 1 \times 1$

$= 2$

$$= -\frac{\text{Constant term}}{\text{Coefficient of } x^3} = -\frac{-2}{1} = 2$$

Thus, the relationship between the zeroes and the coefficients is verified.

11 B. Question

Verify that the numbers given alongside of the cubic polynomial are their zeros. Also verify the relationship between the zeroes and the coefficients in each case :

$x^3 - 6x^2 + 11x - 6$;1, 2, 3

Answer

Let $p(x) = x^3 - 6x^2 + 11x - 6$

Then, $p(1) = (1)^3 - 6(1)^2 + 11(1) - 6$

$= 1 - 6 + 11 - 6$

$= 0$

$p(2) = (2)^3 - 6(2)^2 + 11(2) - 6$

= 8 – 24 + 22 – 6

= 0p(3) = $(3)^3 - 6(3)^2 + 11(3) - 6$

= 27 – 54 + 33 – 6

= 0

Hence, 1, 2 and 3 are the zeroes of the given polynomial $x^3 - 6x^2 + 11x - 6$.

Now, Let α = 1 , β = 2 and γ = 3

Then, α + β + γ = 1 + 2 + 3 = 6

$$= -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3} = -\frac{-6}{1} = 6$$

αβ + βγ + γα = (1)(2) + (2)(3) + (3)(1)

= 2 + 6 + 3

= 11

$$= \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3} = \frac{11}{1} = 11$$

and αβγ = 1 × 2 × 3

= 6

$$= -\frac{\text{Constant term}}{\text{Coefficient of } x^3} = -\frac{-6}{1} = 6$$

Thus, the relationship between the zeroes and the coefficients is verified.

11 C. Question

Verify that the numbers given alongside of the cubic polynomial are their zeros. Also verify the relationship between the zeroes and the coefficients in each case :

$x^3 + 2x^2 - x - 2$; – 2 – 2, 1

Answer

Let p(x) = $x^3 + 2x^2 - x - 2$

Then, p(– 2) = $(-2)^3 + 2(-2)^2 - (-2) - 2$

= – 8 + 8 + 2 – 2

= 0p(1) = $(1)^3 + 2(1)^2 - (1) - 2$

= 1 + 2 – 1 – 2

= 0

Hence, – 2, – 2 and 1 are the zeroes of the given polynomial $x^3 + 2x^2 - x - 2$.

Now, Let α = – 2 , β = – 2 and γ = 1

Then, α + β + γ = – 2 + (– 2) + 1 = – 3

$$= -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3} = -\frac{2}{1} = -2$$

αβ + βγ + γα = (– 2)(– 2) + (– 2)(1) + (1)(– 2)

= 4 – 2 – 2

= 0

$$= \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3} = \frac{-1}{1} = -1$$

and αβγ = (– 2) × (– 2) × 1

= 4

$$= -\frac{\text{Constant term}}{\text{Coefficient of } x^3} = -\frac{-2}{1} = 2$$

Thus, the relationship between the zeroes and the coefficients is not verified.

11 D. Question

Verify that the numbers given alongside of the cubic polynomial are their zeros. Also verify the relationship between the zeroes and the coefficients in each case :

$x^3 + 5x^2 + 7x + 3$; – 3, 2 – 1, – 1

Answer

Let $p(x) = x^3 + 5x^2 + 7x + 3$.

Then, $p(-1) = (-1)^3 + 5(-1)^2 + 7(-1) + 3$

= – 1 + 5 – 7 + 3

= 0p(– 3) = $(-3)^3 + 5(-3)^2 + 7(-3) + 3$

= – 27 + 45 – 21 + 3

= 0

Hence, – 1, – 1 and – 3 are the zeroes of the given polynomial $x^3 + 5x^2 + 7x + 3$.

Now, Let α = – 1 , β = – 1 and γ = – 3

Then, α + β + γ = – 1 + (– 1) + (– 3) = – 5

$$= -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3} = -\frac{5}{1} = -5$$

αβ + βγ + γα = (– 1)(– 1) + (– 1)(– 3) + (– 3)(– 1)

= 1 + 3 + 3

= 7

$$= \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3} = \frac{7}{1} = 7$$

and αβγ = (– 1) × (– 1) × (– 3)

= – 3

$$= -\frac{\text{Constant term}}{\text{Coefficient of } x^3} = -\frac{3}{1} = -3$$

Thus, the relationship between the zeroes and the coefficients is verified.

12. Question

Find a cubic polynomial having 1, 2, 3 as its zeroes.

Answer

Let the zeroes of the cubic polynomial be

α = 1, β = 2 and γ = 3

Then, α + β + γ = 1 + 2 + 3 = 6

αβ + βγ + γα = (1)(2) + (2)(3) + (3)(1)

= 2 + 6 + 3

= 11

and αβγ = 1 × 2 × 3

= 6

Now, required cubic polynomial

$= x^3 - (\alpha + \beta + \gamma)\, x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$

$= x^3 - (6) x^2 + (11)x - 6$

$= x^3 - 6 x^2 + 11x - 6$

So, $x^3 - 6 x^2 + 11x - 6$ is the required cubic polynomial which satisfy the given conditions.

13. Question

Find a cubic polynomial having – 3, – 2, 2 as its zeroes.

Answer

Let the zeroes of the cubic polynomial be

$\alpha = -3, \beta = -2$ and $\gamma = 2$

Then, $\alpha + \beta + \gamma = -3 + (-2) + 2$

$= -3 - 2 + 2$

$= -3$

$\alpha\beta + \beta\gamma + \gamma\alpha = (-3)(-2) + (-2)(2) + (2)(-3)$

$= 6 - 4 - 6$

$= -4$

and $\alpha\beta\gamma = (-3) \times (-2) \times 2$

$= 6 \times 2$

$= 12$

Now, required cubic polynomial

$= x^3 - (\alpha + \beta + \gamma) x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$

$= x^3 - (-3) x^2 + (-4)x - 12$

$= x^3 + 3 x^2 - 4x - 12$

So, x^3 + 3x^2 – 4x – 12 is the required cubic polynomial which satisfy the given conditions.

14. Question

Find a cubic polynomial with the sum of its zeroes are 0, – 7 and – 6 respectively.

Answer

Let the zeroes be α, β and γ.

Then, we have

$\alpha + \beta + \gamma = 0$

$\alpha\beta + \beta\gamma + \gamma\alpha = -7$

and $\alpha\beta\gamma = -6$

Now, required cubic polynomial

$= x^3 - (\alpha + \beta + \gamma) x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$

$= x^3 - (0) x^2 + (-7)x - (-6)$

$= x^3 - 7x + 6$

So, $x^3 - 7x + 6$ is the required cubic polynomial.

15 A. Question

Find a cubic polynomial with the sum of its zeroes, sum of the products of its zeroes taken two at a time and product of its zeroes as the numbers given below:

2, – 7, – 14

Answer

Let the zeroes be α, β and γ.

Then, we have

$\alpha + \beta + \gamma = 2$

$\alpha\beta + \beta\gamma + \gamma\alpha = -7$

and $\alpha\beta\gamma = -14$

Now, required cubic polynomial

$= x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$

$= x^3 - (2)x^2 + (-7)x - (-14)$

$= x^3 - 2x^2 - 7x + 14$

So, $x^3 - 2x^2 - 7x + 14$ is the required cubic polynomial.

15 B. Question

Find a cubic polynomial with the sum of its zeroes, sum of the products of its zeroes taken two at a time and product of its zeroes as the numbers given below:

$$-4\frac{1}{2},\frac{1}{3}$$

Answer

Let the zeroes be α, β and γ.

Then, we have

$$\alpha + \beta + \gamma = -4$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{1}{2}$$

$$\alpha\beta\gamma = \frac{1}{3}$$

Now, required cubic polynomial

$$= x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$$

$$= x^3 - (-4)x^2 + (\frac{1}{2})x - (\frac{1}{3})$$

$$= \frac{6x^3 + 24x^2 + 3x - 2}{6}$$

So, $6x^3 + 24x^2 + 3x - 2$ is the required cubic polynomial which satisfy the given conditions.

15 C. Question

Find a cubic polynomial with the sum of its zeroes, sum of the products of its zeroes taken two at a time and product of its zeroes as the numbers given below:

$$\frac{5}{7},\frac{1}{7},\frac{1}{7}$$

Answer

Let the zeroes be α, β and γ.

Then, we have

$$\alpha + \beta + \gamma = \frac{5}{7}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{1}{7}$$

$$\alpha\beta\gamma = \frac{1}{7}$$

Now, required cubic polynomial

$$= x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$$

$$= x^3 - (\frac{5}{7})x^2 + (\frac{1}{7})x - (\frac{1}{7})$$

$$= \frac{7x^3 - 5x^2 + x - 1}{7}$$

So, $7x^3 - 5x^2 + x - 1$ is the required cubic polynomial which satisfy the given conditions.

15 D. Question

Find a cubic polynomial with the sum of its zeroes, sum of the products of its zeroes taken two at a time and product of its zeroes as the numbers given below:

$$\frac{2}{5}, \frac{1}{10}, \frac{1}{2}$$

Answer

Let the zeroes be α, β and γ.

Then, we have

$$\alpha + \beta + \gamma = \frac{2}{5}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{1}{10}$$

$$\alpha\beta\gamma = \frac{1}{2}$$

Now, required cubic polynomial

$$= x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$$

$$= x^3 - (\frac{2}{5})x^2 + (\frac{1}{10})x - (\frac{1}{2})$$

$$= \frac{10x^3 - 4x^2 + x - 5}{10}$$

So, $10x^3 - 4x^2 + x - 5$ is the required cubic polynomial which satisfy the given conditions.