3. Pair of Linear Equations in Two Variables

Exercise 3.1

1. Question

Sudha went to market with her friends. They wanted to eat `gol - gappa' as well as `dahi - bhalla'. The number of plates of gol - gappa taken by them is half that of dahi - bhalla. The cost of one plate of gol - gappa was Rs. 10 and cost of one plate of dahi - bhalla was Rs. 5. She spent Rs. 60. Represent the situation algebraically and graphically.

Answer

Let no. of plates of gol - gappa = x

and no. of plates of dhai - bhalla = y

Cost of 1 plate gol - gappa = Rs. 10

Cost of 1 plate dhai - bhalla = Rs. 5

Total money spent = Rs. 20

According to the question,

$x = \frac{1}{2}y$ ...(1)

10x + 5y = 60 ...(2)

From eq^n (1), we get

2x – y = 0 ...(3)

Now, table for 2x – y = 0

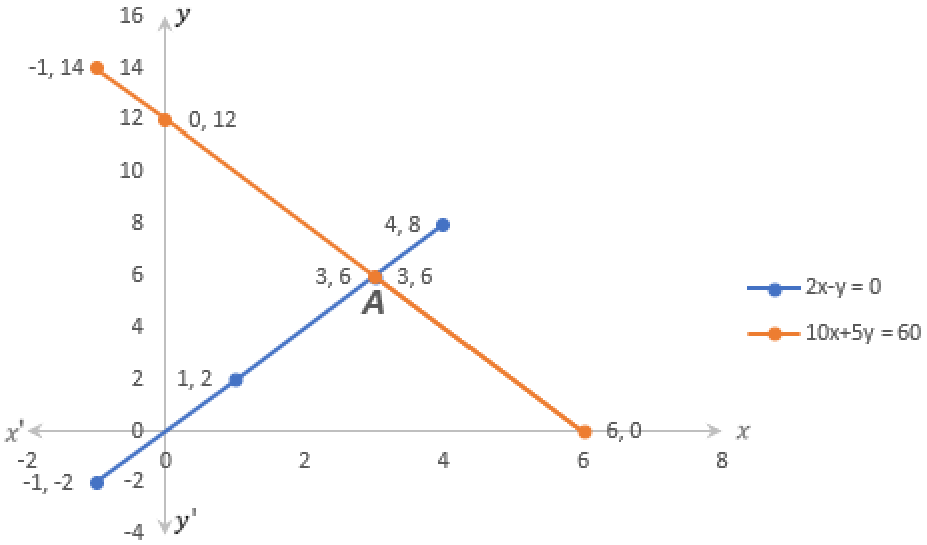
x	- 1	0	1	2
y = 2x	- 2	0	2	4

Now, table for 10x + 5y = 60

x	- 1	0	6	1
$y = \frac{60 - 10x}{5}$	14	12	0	10

On plotting points on a graph paper and join them to get a straight line representing $x = \frac{1}{2}y$.

Similarly, on plotting the points on the same graph paper and join them to get a straight line representing 10x + 5y = 60.



Here, the lines representing Eq. (1) and Eq. (2) intersecting at point A i.e. (3,6).

2. Question

Romila went to a stationary shop and purchased 2 pencils and 3 erasers for Rs. 9. Her friend Sonali saw the new variety of pencils and erasers with Romila and she also bought 4 pencils and 6 erasers of the same kind for Rs. 18. Represent this situation algebraically and graphically.

Answer

Let the cost of one pencil = Rs x

and cost of one eraser = Rs y

Romila spent = Rs. 9

Sonali spent = Rs. 18

According to the question

2x + 3y = 9 ...(1)

4x + 6y = 18 ...(2)

Now, table for 2x + 3y = 9

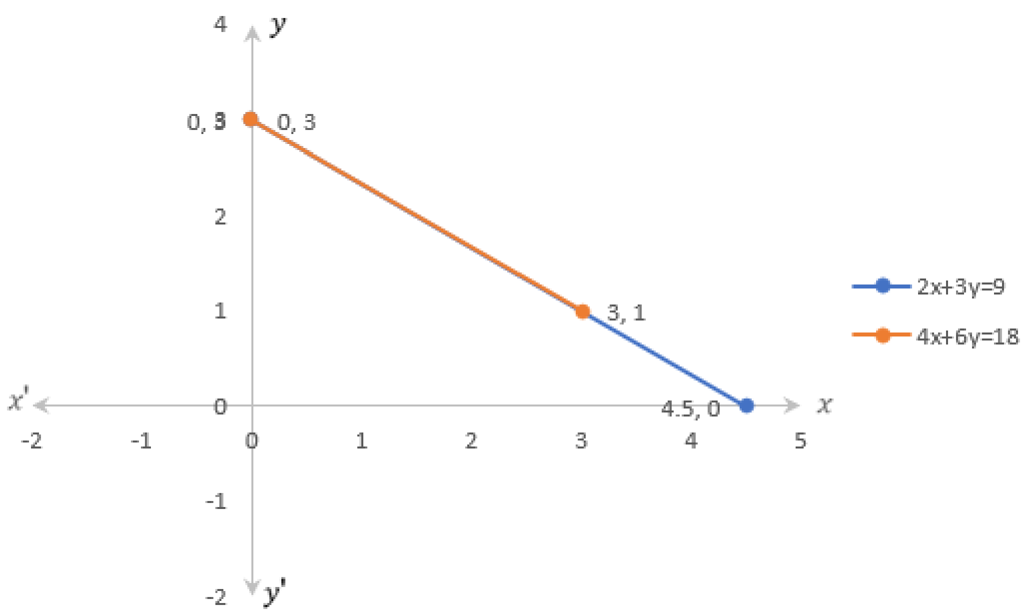
x	0	4.5
$y = \frac{9-2x}{3}$	3	0

Now, table for 4x + 6y = 18

x	0	3
$y = \frac{18 - 4x}{6}$	3	1

On plotting points on a graph paper and join them to get a straight line representing 2x + 3y = 9.

Similarly, on plotting the points on the same graph paper and join them to get a straight line representing 4x + 6y = 18.



Here, we can see that both the lines coincide. This is so, because, both the equations are equivalent, i.e.2(2x + 3y) = 2×9 equation (2) is derived from the other.

3. Question

Present age of father is 30 years more than twice that of his son. After 10 years, the age of father will be thrice the age of his son. Represent this situation algebraically and geometrically.

Answer

Let the present age of son = x year

and the age of his father = y year

According to the question

y = 2x + 30

or, 2x – y = – 30 ...(1)

After 10 years,

Age of son = (x + 10)year

Age of father = (y + 10)year

So, According to the question

y + 10 = 3(x + 10)

y + 10 = 3x + 30

y = 3x + 20

or, 3x – y = – 20 ...(2)

Now, table for y = 2x + 30

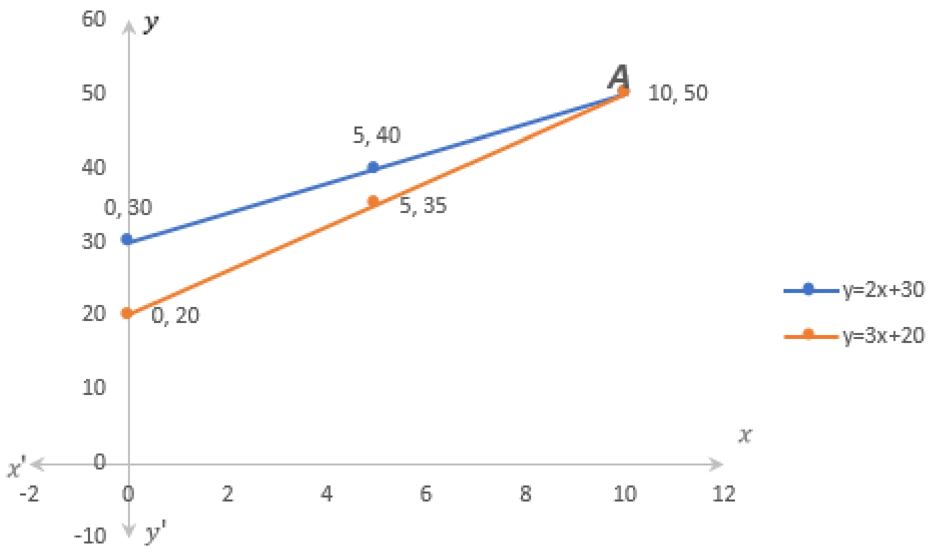
x	0	- 15	5
y = 2x + 30	30	0	40

Now, table for y = 3x + 20

x	0	5	10
y = 3x + 20	20	35	50

On plotting points on a graph paper and join them to get a straight line representing y = 2x + 30.

Similarly, on plotting the points on the same graph paper and join them to get a straight line representing y = 3x + 20.



Here, the lines representing Eq. (1) and Eq. (2) intersecting at point A i.e. (10,50).

So, the age of son is 10years and age of his father is 50years.

4. Question

The path of a wheel of train A is given by the equation x + 2y — 4 = 0 and the path of a wheel of another train B is given by the equation 2x + 4y — 12 = O. Represent this situation geometrically.

Answer

The given equation is

x + 2y – 4 = 0

and

2x + 4y – 12 = 0

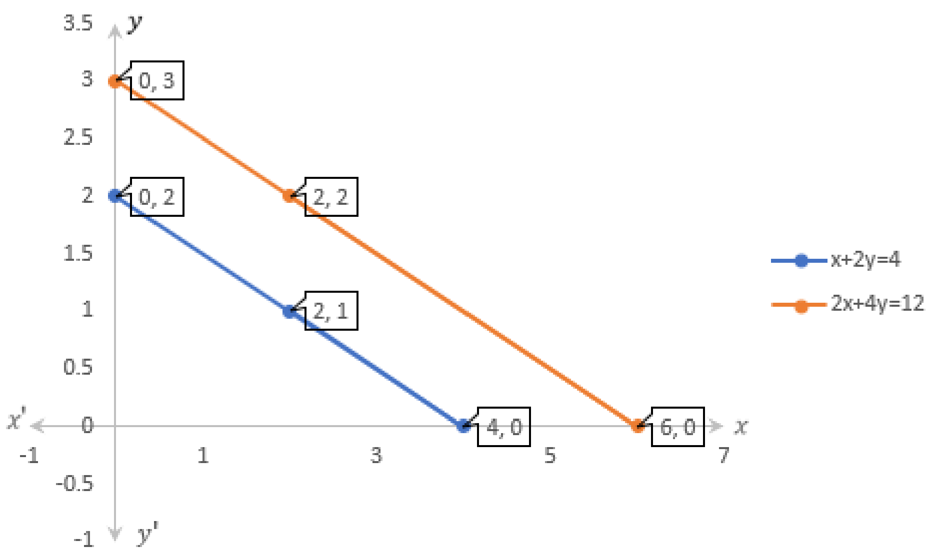
Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for x + 2y – 4 = 0 or $y = \frac{4-x}{2}$

x	0	4	2
$y = \frac{4-x}{2}$	2	0	1

Now, table for 2x + 4y – 12 = 0 or $y = \frac{12-2x}{4}$

x	0	6	2
$y = \frac{12-2x}{4}$	3	0	2



From the graph, it is clear that lines represented by the equations x + 2y – 4 = 0 and 2x + 4y – 12 = 0 are parallel.

5. Question

The path of highway number 1 and 2 are given by the equations x — y = 1 and 2x + 3y = 12 respectively. Represent these equations geometrically.

Answer

The given equation is

$x - y = 1$

and

$2x + 3y = 12$

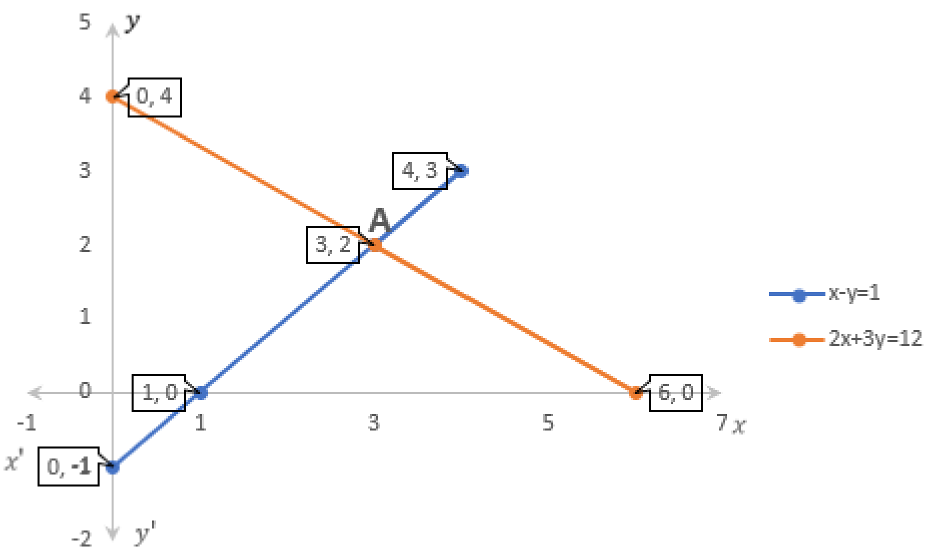
Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for $x - y = 1$ or $y = x - 1$

x	0	1	3	4
y = x - 1	- 1	0	2	3

Now, table for $2x + 3y = 12$ or $y = \frac{12-2x}{3}$

x	0	6	3
$y = \frac{12-2x}{3}$	4	0	2



From the graph, it is clear that lines represented by the equations $x - y = 1$ and $2x + 3y - 12 = 0$ are intersecting at a point A i.e. (3,2)

6. Question

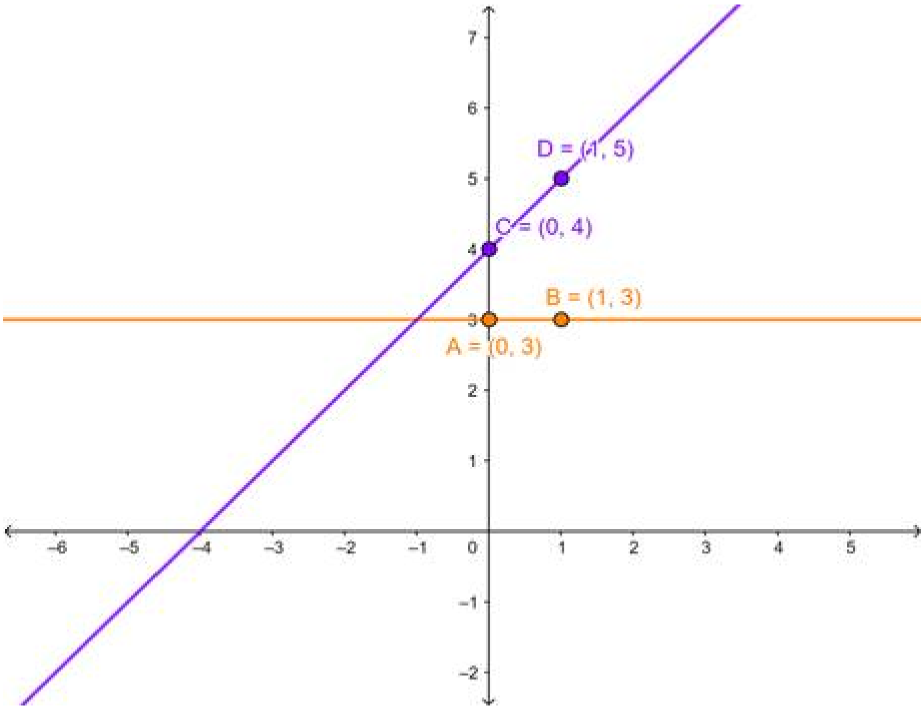
Person A walks along the path joining points (0, 3) and (1, 3) and person B walks along the path joining points (0, 4) and (1, 5). Represent this situation geometrically.

Answer

The given points are at which Person A walks (0,3) and (1,3)

and the points at which person B walks (0,4) and (1,5)

Now, we plot these points on a same graph as shown in the following figure.



7. Question

Examine which of the following pair of values of x and y is a solution of equation 4x — 3y + 24 = 0.

(i) x = 0, y = 8 (ii) x = — 6, y = 0

(iii) x = 1, y = — 2 (iv) x = – 3,y = 4

(v) x = 1, y = — 2 (vi) x = — 4, y = 2

Answer

Given equation is $4x - 3y + 24 = 0$

i) Justification

On substituting x = 0, y = 8 in LHS of given equation, we get

LHS = 4(0) – 3(8) + 24 = 0 – 24 + 24 = 0 = RHS

Hence, x = 0, y = 8 is a solution of the equation 4x – 3y + 24 = 0

ii) Justification

On substituting x = – 6, y = 0 in LHS of given equation, we get

LHS = 4(– 6) – 3(0) + 24 = – 24 + 24 = 0 = RHS

Hence, x = – 6, y = 0 is a solution of the equation 4x – 3y + 24 = 0

iii) Justification

On substituting x = 1, y = – 2 in LHS of given equation, we get

LHS = 4(1) – 3(– 2) + 24 = 4 + 6 + 24 = 34 ≠ RHS

Hence, x = 1, y = – 2 is not a solution of the equation 4x – 3y + 24 = 0

iv) Justification

On substituting x = – 3, y = 4 in LHS of given equation, we get

LHS = 4(– 3) – 3(4) + 24 = – 12 – 12 + 24 = 0 = RHS

Hence, x = – 3, y = 4 is a solution of the equation 4x – 3y + 24 = 0

v) Justification

On substituting x = 1, y = – 2 in LHS of given equation, we get

LHS = 4(1) – 3(– 2) + 24 = 4 + 6 + 24 = 34 ≠ RHS

Hence, x = 1, y = – 2 is not a solution of the equation 4x – 3y + 24 = 0

vi) Justification

On substituting x = – 4, y = 2 in LHS of given equation, we get

LHS = 4(– 4) – 3(2) + 24 = – 16 – 6 + 24 = – 22 + 24 = 2 ≠ RHS

Hence, x = – 4, y = 2 is not a solution of the equation 4x – 3y + 24 = 0

8. Question

Examine which of the following points lie on the graph of the linear equation 5x — 3y + 30 = 0.

(i) A (— 6, 0) (ii) B (0, 10)

(iii) C (3, — 5) (iv) D (4, 2)

(v) E (— 9, 5) (vi) F (— 3, 5)

(vii) G (— 9, — 5)

Answer

The given equation is 5x – 3y + 30 = 0

(i) Given A (– 6,0). Here x = – 6 and y = 0

On substituting x = – 6, y = 0 in LHS of given equation, we get

LHS = 5(– 6) – 3(0) + 30 = – 30 + 30 = 0 = RHS

So, x = – 6, y = 0 is a solution of the equation 5x – 3y + 30 = 0.

Hence, point A lies on the graph of the linear equation 5x – 3y + 30 = 0.

(ii) Given B (0,10). Here x = 0 and y = 10

On substituting x = 0, y = 10 in LHS of given equation, we get

LHS = 5(0) – 3(10) + 30 = – 30 + 30 = 0 = RHS

So, x = 0, y = 10 is a solution of the equation 5x – 3y + 30 = 0

Hence, point B lies on the graph of the linear equation 5x – 3y + 30 = 0.

(iii) Given C (3, – 5). Here x = 3 and y = – 5

On substituting x = 3, y = – 5 in LHS of given equation, we get

LHS = 5(3) – 3(– 5) + 30 = 15 + 15 + 30 = 60 ≠ RHS

So, x = 3, y = – 5 is not a solution of the equation 5x – 3y + 30 = 0

Hence, point C does not lie on the graph of the linear equation 5x – 3y + 30 = 0.

(iv) Given D (4,2). Here x = 4 and y = 2

On substituting x = 4, y = 2 in LHS of given equation, we get

LHS = 5(4) – 3(2) + 30 = 20 – 6 + 30 = 44 ≠ RHS

So, x = 4, y = 2 is not a solution of the equation 5x – 3y + 30 = 0

Hence, point D does not lie on the graph of the linear equation 5x – 3y + 30 = 0.

(v) Given E (– 9,5). Here x = – 9 and y = 5

On substituting x = – 9, y = 5 in LHS of given equation, we get

LHS = 5(– 9) – 3(5) + 30 = – 45 – 15 + 30 = – 30 ≠ RHS

So, x = – 9, y = 5 is not a solution of the equation 5x – 3y + 30 = 0

Hence, point E does not lie on the graph of the linear equation 5x – 3y + 30 = 0.

(vi) Given F (– 3,5). Here x = – 3 and y = 5

On substituting x = – 3, y = 5 in LHS of given equation, we get

LHS = 5(– 3) – 3(5) + 30 = – 15 + 15 + 30 = 0 = RHS

So, x = – 3, y = 5 is a solution of the equation 5x – 3y + 30 = 0

Hence, point F lies on the graph of the linear equation 5x – 3y + 30 = 0.

(vii) Given G (– 9, – 5). Here x = 3 and y = – 5

On substituting x = – 9, y = – 5 in LHS of given equation, we get

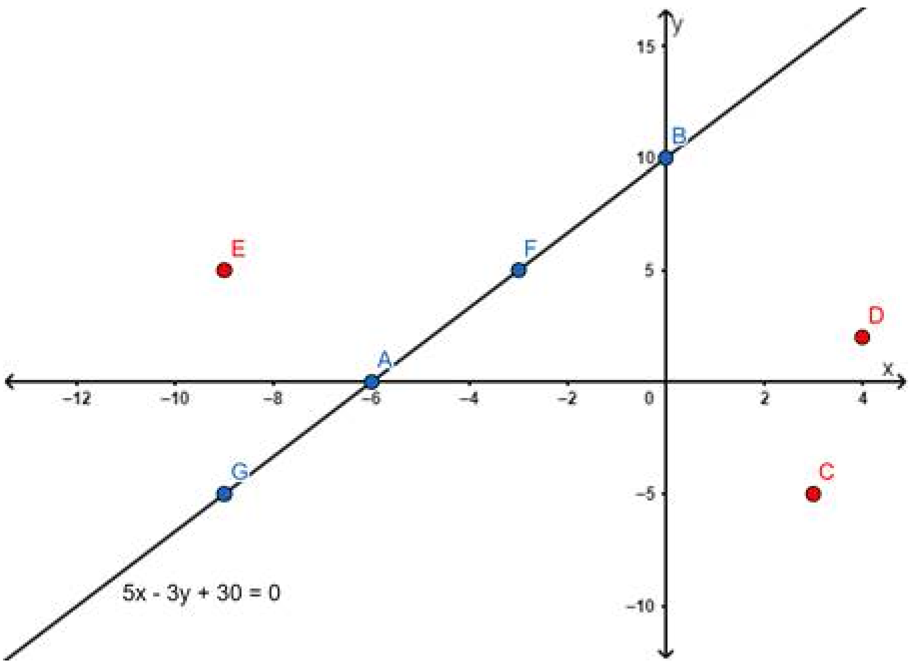
LHS = 5(– 9) – 3(– 5) + 30 = – 45 + 15 + 30 = 0 = RHS

So, x = – 9, y = – 5 is a solution of the equation 5x – 3y + 30 = 0

Hence, point G lies on the graph of the linear equation 5x – 3y + 30 = 0.

Or **Graphically**

Here, we can see through the graph also that Point A, B, F and G lie on the graph of the linear equation 5x – 3y + 30 = 0



9 A. Question

Solve graphically the following system of linear equations if it has unique solution:

3x + y = 2

6x + 2y = 1

Answer

The given pair of linear equations is

3x + y = 2 or 3x + y – 2 = 0

and 6x + 2y = 1 or 6x + 2y – 1 = 0

On comparing the given equations with standard form of pair of linear equations i.e. $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, we get

$a_1 = 3$, $b_1 = 1$ and $c_1 = -2$

and $a_2 = 6$, $b_2 = 2$ and $c_2 = -1$

$$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{1}{2} = 1 \text{ and } \frac{c_1}{c_2} = \frac{-2}{-1} = 2$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

The lines representing the given pair of linear equations are parallel.

9 B. Question

Solve graphically the following system of linear equations if it has unique solution:

2x — 3y + 13 = 0

3x — 2y + 12 = 0

Answer

The given pair of linear equations is

2x – 3y + 13 = 0

and 3x – 2y + 12 = 0

On comparing the given equations with standard form of pair of linear equations i.e. $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, we get

$a_1 = 2$, $b_1 = -3$ and $c_1 = 13$

and $a_2 = 3$, $b_2 = -2$ and $c_2 = 12$

$$\frac{a_1}{a_2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{-3}{-2} = \frac{3}{2} \text{ and } \frac{c_1}{c_2} = \frac{13}{12}$$

$$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

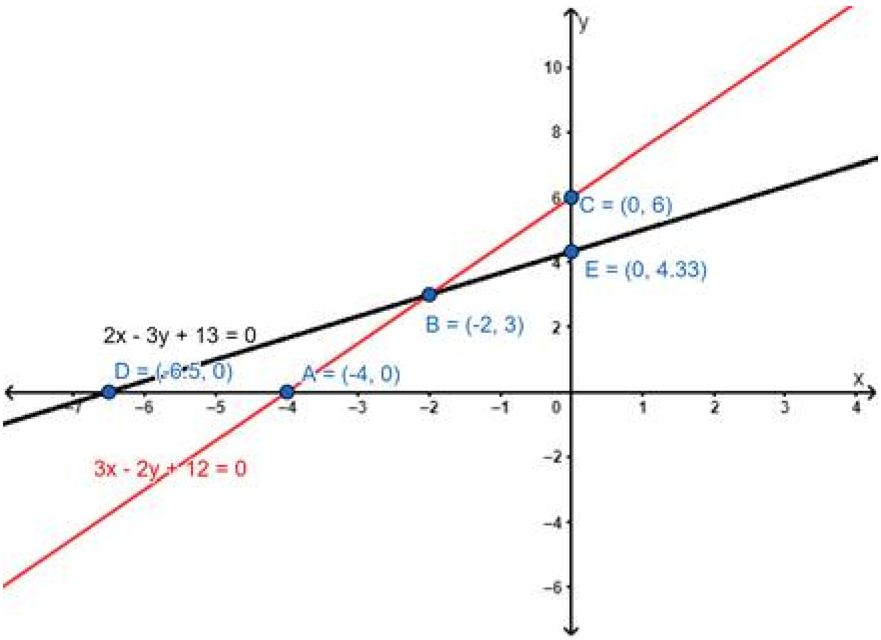
∴ The lines representing the given pair of linear equations will intersect at a point.

Now, table for $2x - 3y + 13 = 0$ or $y = \frac{2x+13}{3}$

x	- 6.5	- 2	0
$y = \frac{2x+13}{3}$	0	3	$\frac{13}{3}$

Now, table for $3x - 2y + 12 = 0$ or $y = \frac{3x+12}{2}$

x	0	4	- 2
$y = \frac{3x-12}{2}$	6	0	3



Here, the lines intersecting at point B i.e. (– 2,3)

Hence, the unique solution is x = – 2 and y = 3.

9 C. Question

Solve graphically the following system of linear equations if it has unique solution:

3x + 2y = 14

x — 4y = — 14

Answer

The given pair of linear equations is

3x + 2y = 14 or 3x + 2y – 14 = 0

and x – 4y = – 14 or x – 4y + 14 = 0

On comparing the given equations with standard form of pair of linear equations i.e. $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, we get

$a_1 = 3$, $b_1 = 2$ and $c_1 = -14$

and $a_2 = 1$, $b_2 = -4$ and $c_2 = 14$

$$\frac{a_1}{a_2} = \frac{3}{1}, \frac{b_1}{b_2} = \frac{2}{-4} = -\frac{1}{2} \text{ and } \frac{c_1}{c_2} = \frac{-14}{14} = -1$$

$$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

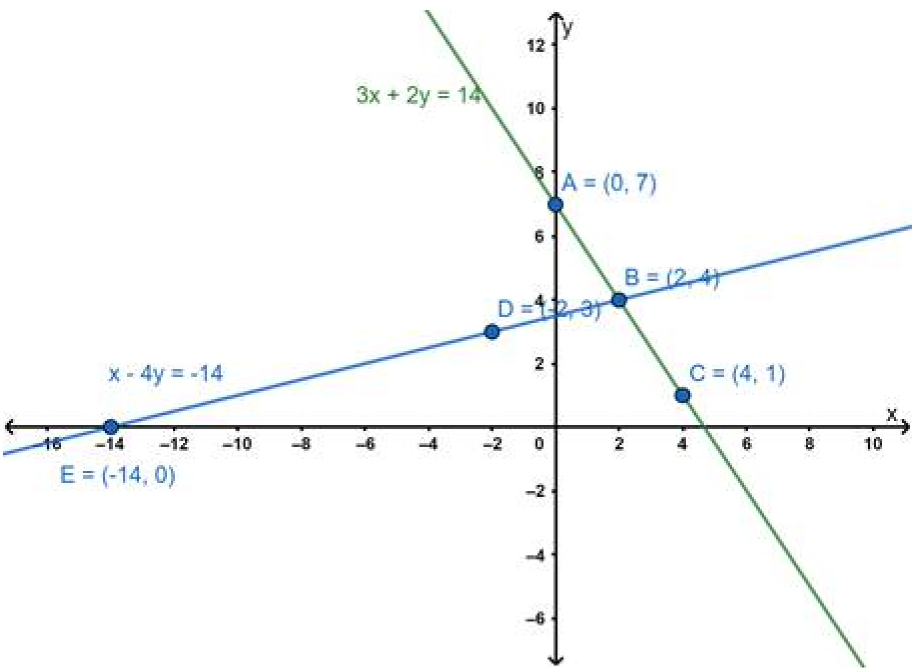
The lines representing the given pair of linear equations will intersect at a point.

Now, table for $3x + 2y = 14$ or $y = \frac{14-3x}{2}$

x	0	2	4
$y = \frac{14-3x}{2}$	7	4	1

Now, table for $x - 4y + 14 = 0$ or $y = \frac{x+14}{4}$

x	- 2	14	2
$y = \frac{x+14}{4}$	3	0	4



Here, the lines intersecting at point B i.e. (2,4)

Hence, the unique solution is x = 2 and y = 4.

9 D. Question

Solve graphically the following system of linear equations if it has unique solution:

$2x - 3y = 1$

$3x - 4y = 1$

Answer

The given pair of linear equations is

$2x - 3y = 1$ or $2x - 3y - 1 = 0$

and $3x - 4y = 1$ or $3x - 4y - 1 = 0$

On comparing the given equations with standard form of pair of linear equations i.e. $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, we get

$a_1 = 2, b_1 = -3$ and $c_1 = -1$

and $a_2 = 3, b_2 = -4$ and $c_2 = -1$

$$\frac{a_1}{a_2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{-3}{-4} = \frac{3}{4} \text{ and } \frac{c_1}{c_2} = \frac{-1}{-1} = 1$$

$$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

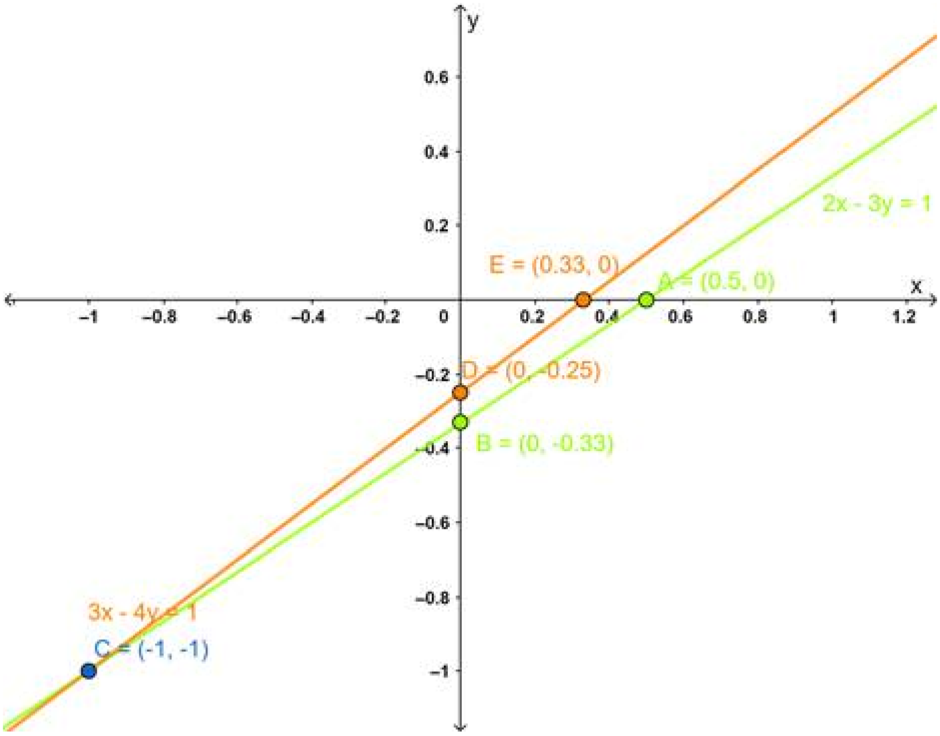
∴ The lines representing the given pair of linear equations will intersect at a point.

Now, table for $2x - 3y = 1$ or $y = \frac{2x-1}{3}$

x	0.5	0	- 1
$y = \frac{2x-1}{3}$	0	- 0.33	- 1

Now, table for $3x - 4y = 1$ or $y = \frac{3x-1}{4}$

x	0	0.33	- 1
$y = \frac{3x-1}{4}$	- 0.25	0	- 1



Here, the lines intersecting at point C i.e. (– 1, – 1)

Hence, the unique solution is x = – 1 and y = – 1.

9 E. Question

Solve graphically the following system of linear equations if it has unique solution:

2x — y = 9

5x + 2y = 27

Answer

The given pair of linear equations is

2x – y = 9 or 2x – y – 9 = 0

and 5x + 2y = 27 or 5x + 2y – 27 = 0

On comparing the given equations with standard form of pair of linear equations i.e. $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, we get

$a_1 = 2$, $b_1 = -1$ and $c_1 = -9$

and $a_2 = 5$, $b_2 = 2$ and $c_2 = -27$

$$\frac{a_1}{a_2} = \frac{2}{5}, \frac{b_1}{b_2} = \frac{-1}{2} \text{ and } \frac{c_1}{c_2} = \frac{-9}{-27} = \frac{1}{3}$$

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

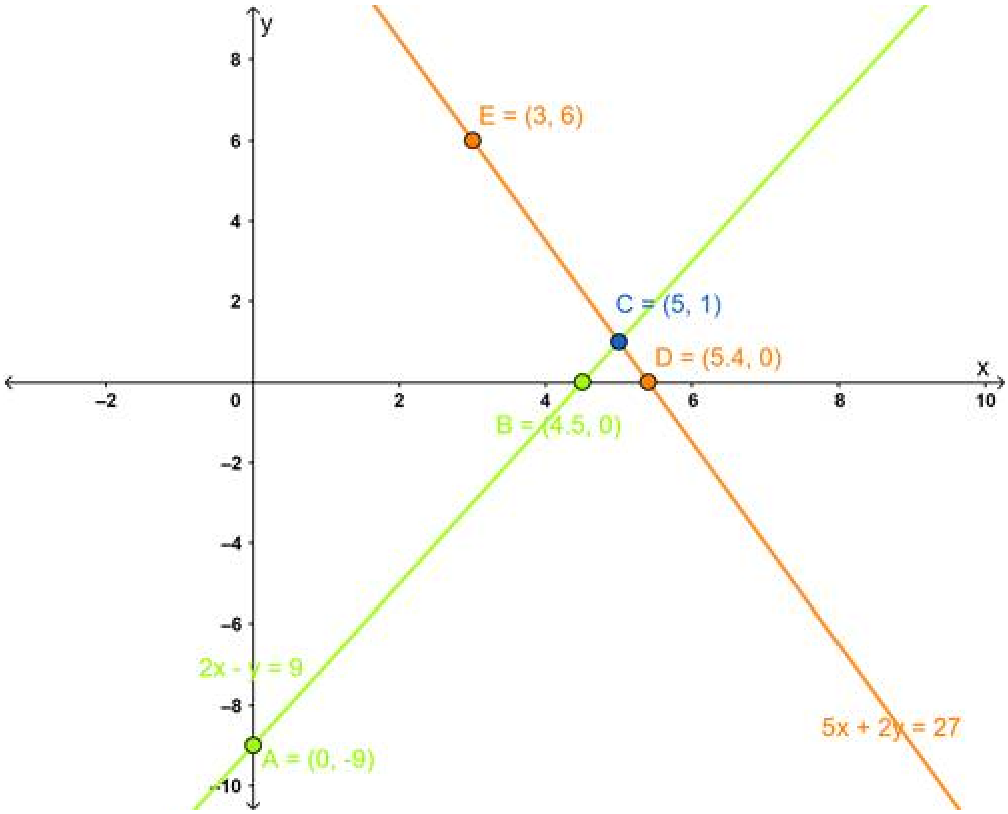
∴ The lines representing the given pair of linear equations will intersect at a point.

Now, table for $2x - y = 9$ or $y = 2x - 9$

x	0	4.5	5
$y = 2x - 9$	- 9	0	1

Now, table for $5x + 2y = 27$ or $y = \frac{27-5x}{2}$

x	5.4	5	3
$y = \frac{27-5x}{2}$	0	1	6



Here, the lines intersecting at point C i.e. (5,1)

Hence, the unique solution is x = 5 and y = 1.

9 F. Question

Solve graphically the following system of linear equations if it has unique solution:

3y = 5 — x

2x = y + 3

Answer

The given pair of linear equations is

x + 3y = 5 or x + 3y – 5 = 0

and 2x – y = 3 or 2x – y – 3 = 0

On comparing the given equations with standard form of pair of linear equations i.e. $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, we get

$a_1 = 1$, $b_1 = 3$ and $c_1 = -5$

and $a_2 = 2$, $b_2 = -1$ and $c_2 = -3$

$$\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{3}{-1} = -3 \text{ and } \frac{c_1}{c_2} = \frac{-5}{-3} = \frac{5}{3}$$

$$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

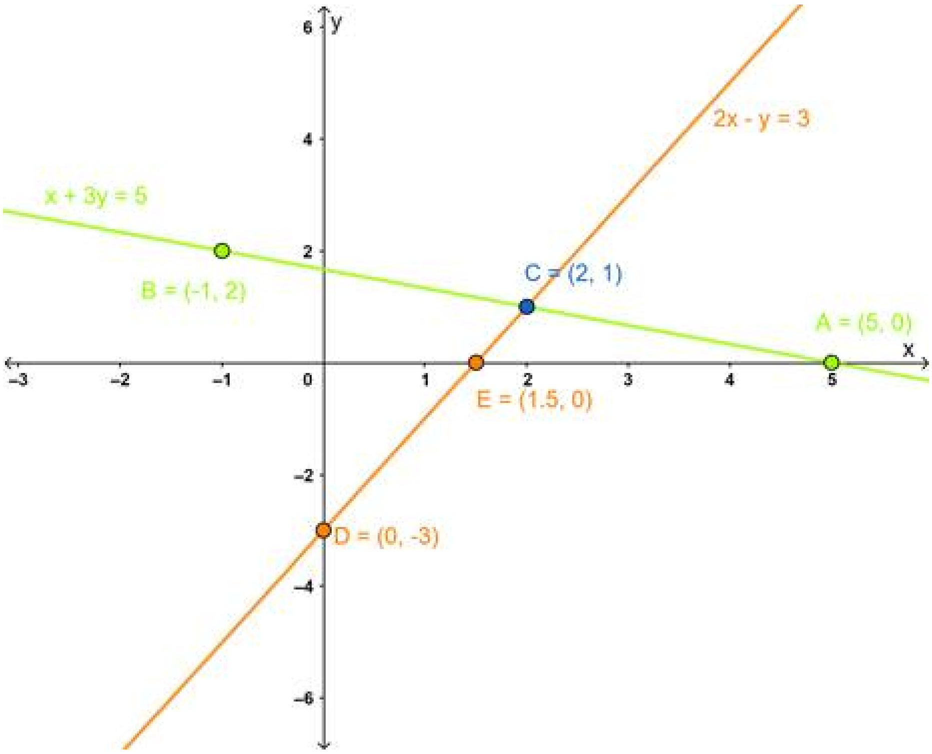
∴ The lines representing the given pair of linear equations will intersect at a point.

Now, table for $x + 3y = 5$ or $y = \frac{5-x}{3}$

x	5	- 1	2
$y = \frac{5-x}{3}$	0	2	1

Now, table for $2x - y = 3$ or $y = 2x - 3$

x	0	1.5	2
$y = 2x - 3$	- 3	0	1



Here, the lines intersecting at point C i.e. (2,1)

Hence, the unique solution is x = 2 and y = 1.

9 G. Question

Solve graphically the following system of linear equations if it has unique solution:

3x — 5y = —1

2x — y = —3

Answer

The given pair of linear equations is

3x – 5y = – 1 or 3x – 5y + 1 = 0

and 2x – y = – 3 or 2x – y + 3 = 0

On comparing the given equations with standard form of pair of linear equations i.e. $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, we get

$a_1 = 3$, $b_1 = -5$ and $c_1 = 1$

and $a_2 = 2$, $b_2 = -1$ and $c_2 = 3$

$$\frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{-5}{-1} = 5 \text{ and } \frac{c_1}{c_2} = \frac{1}{3}$$

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

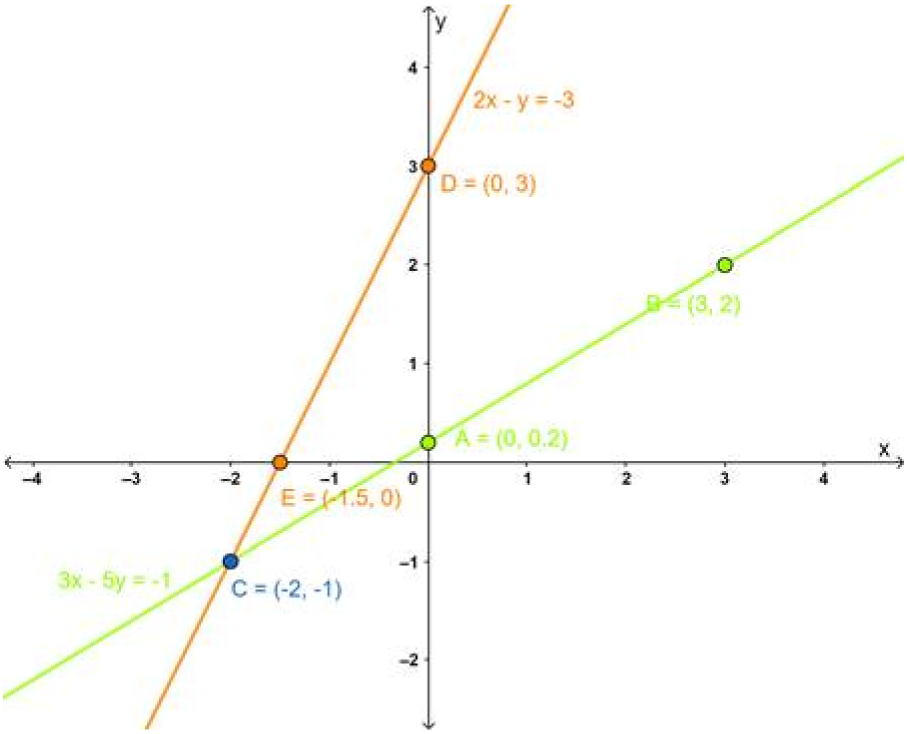
∴ The lines representing the given pair of linear equations will intersect at a point.

Now, table for $3x - 5y = -1$ or $y = \frac{3x+1}{5}$

x	0	3	- 2
$y = \frac{3x+1}{5}$	0.2	2	- 1

Now, table for $2x - y = -3$ or $y = 2x + 3$

x	0	- 1.5	- 2
$y = 2x + 3$	3	0	- 1



Here, the lines intersecting at point B i.e. (– 2, – 1)

Hence, the unique solution is x = – 2 and y = – 1.

9 H. Question

Solve graphically the following system of linear equations if it has unique solution:

2x — 6y + 10 = 0

3x — 9y + 15 = 0

Answer

The given pair of linear equations is

2x – 6y + 10 = 0

and 3x – 9y + 15 = 0

On comparing the given equations with standard form of pair of linear equations i.e. $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, we get

$a_1 = 2, b_1 = -6$ and $c_1 = 10$

and $a_2 = 3, b_2 = -9$ and $c_2 = 15$

$$\frac{a_1}{a_2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{-6}{-9} = \frac{2}{3} \text{ and } \frac{c_1}{c_2} = \frac{10}{15} = \frac{2}{3}$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

∴ The lines representing the given pair of linear equations will coincide.

9 I. Question

Solve graphically the following system of linear equations if it has unique solution:

3x + y — 11 = 0

x — y — 1 = 0

Answer

The given pair of linear equations is

3x + y – 11 = 0

and x – y – 1 = 0

On comparing the given equations with standard form of pair of linear equations i.e. $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, we get

$a_1 = 3, b_1 = 1$ and $c_1 = -11$

and $a_2 = 1, b_2 = -1$ and $c_2 = -1$

$$\frac{a_1}{a_2} = \frac{3}{1} = 3, \frac{b_1}{b_2} = \frac{1}{-1} = -1 \text{ and } \frac{c_1}{c_2} = \frac{-11}{-1} = 11$$

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

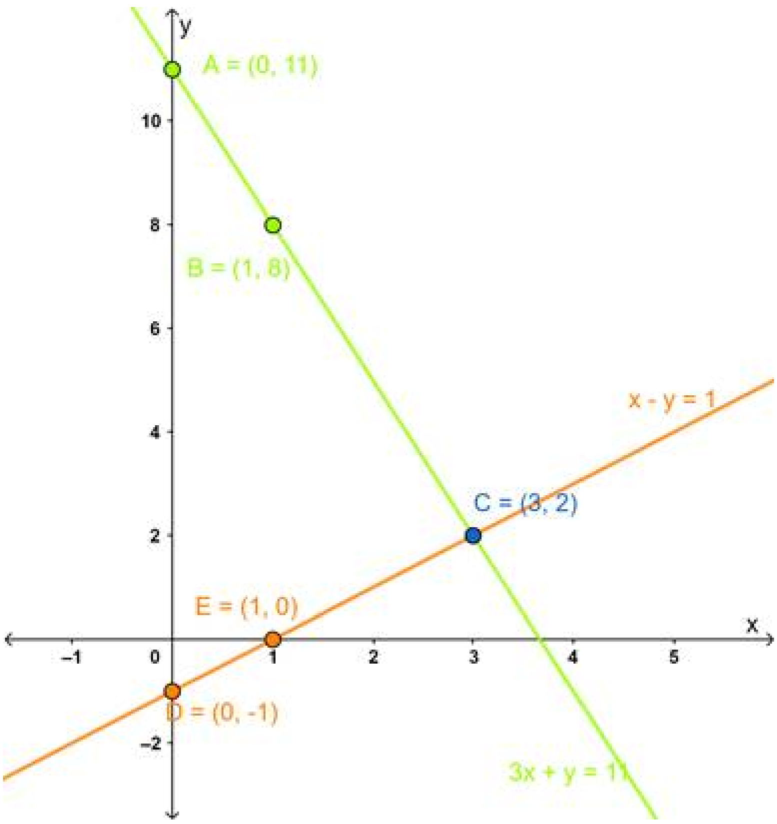
∴ The lines representing the given pair of linear equations will intersect at a point.

Now, table for 3x + y – 11 = 0 or y = 11 – 3x

x	0	1	3
y = 11 - 3x	11	8	2

Now, table for x – y – 1 = 0 or y = x – 1

x	0	1	3
y = x - 1	- 1	0	2



Here, the lines intersecting at point B i.e. (3,2)

Hence, the unique solution is x = 3 and y = 2.

10. Question

Solve the following system of linear equations graphically:

3x — 5y = 19, 3y — 7x + 1 = 0

Does the point (4, 9) lie on any of the lines? Write its equation.

Answer

The given equation is 3x – 5y = 19

and 3y – 7x + 1 = 0 or 7x – 3y = 1

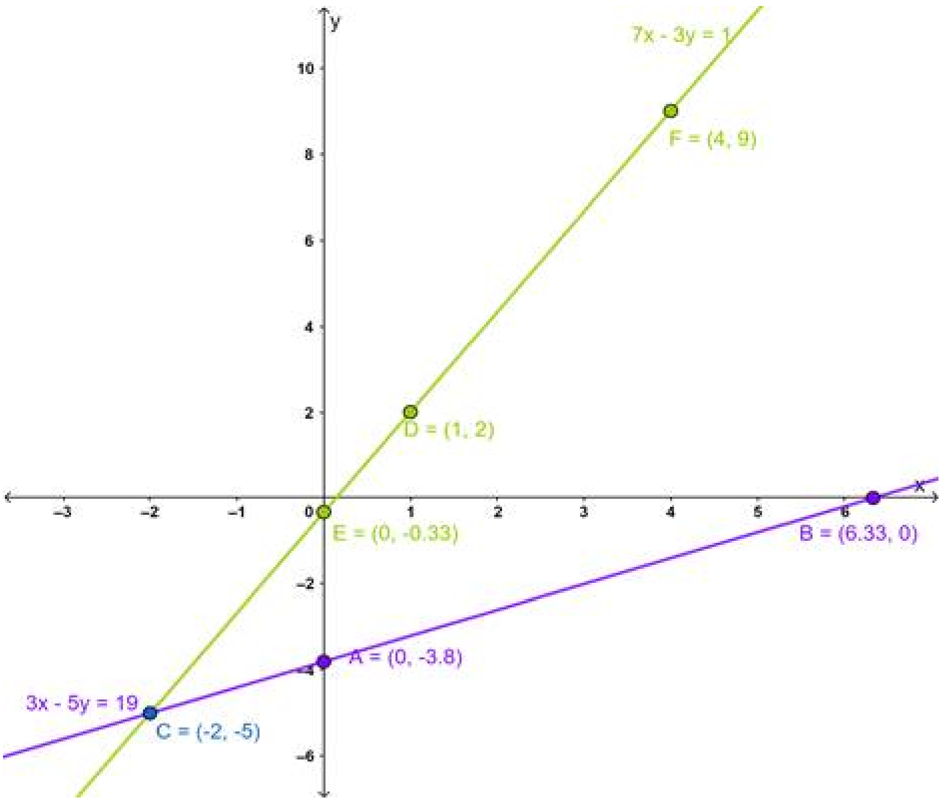
Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for $3x - 5y = 19$ or $y = \frac{3x-19}{5}$

x	0	6.33	- 2
$y = \frac{3x-19}{5}$	- 3.8	0	- 5

Now, table for $7x - 3y = 1$ or $y = \frac{7x-1}{3}$

x	1	0	4
$y = \frac{7x-1}{3}$	2	- 0.33	9



From the graph, it is clear that lines represented by the equations 3x – 5y = 19 and 7x – 3y – 1 = 0 are intersecting at a point C i.e. (– 2, – 5).

Yes, point (4,9) lie on 3y – 7x + 1 = 0.

11. Question

Solve the following system of linear equations graphically: 2x — 3y = 1, 3x — 4y = 1 Does the point (3, 2) lie on any of the lines? Write its equation.

Answer

The given equation is

$2x - 3y = 1$

and $3x - 4y = 1$

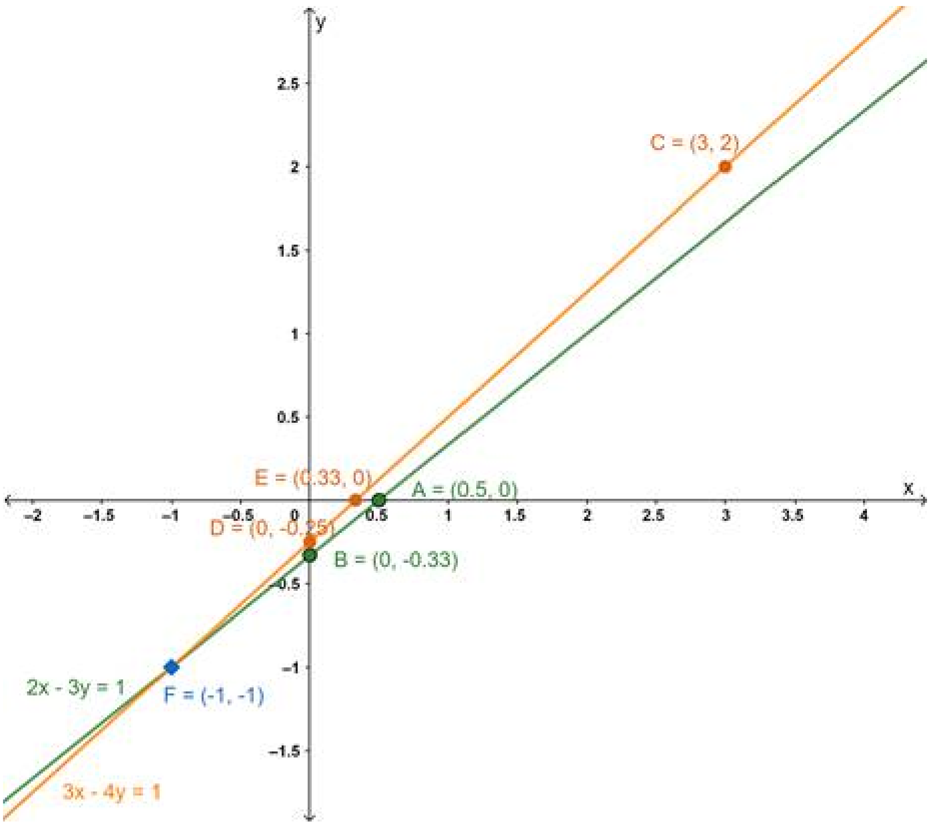
Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for $2x - 3y = 1$ or $y = \frac{2x-1}{3}$

x	0.5	0	3
$y = \frac{2x-1}{3}$	0	- 0.33	2

Now, table for $3x - 4y = 1$ or $y = \frac{3x-1}{4}$

x	0	0.33	- 1
$y = \frac{3x-1}{4}$	- 0.25	0	- 1



Here, the lines intersecting at point F i.e. (– 1, – 1)

Yes, point (3,2) lie on the line 3x – 4y = 1

12 A. Question

Solve the following system of equations graphically. Also find the points where the lines intersect x – axis.

x — 2y = – 3

2x + y = 4

Answer

The given equation is

x – 2y = – 3

and 2x + y = 4

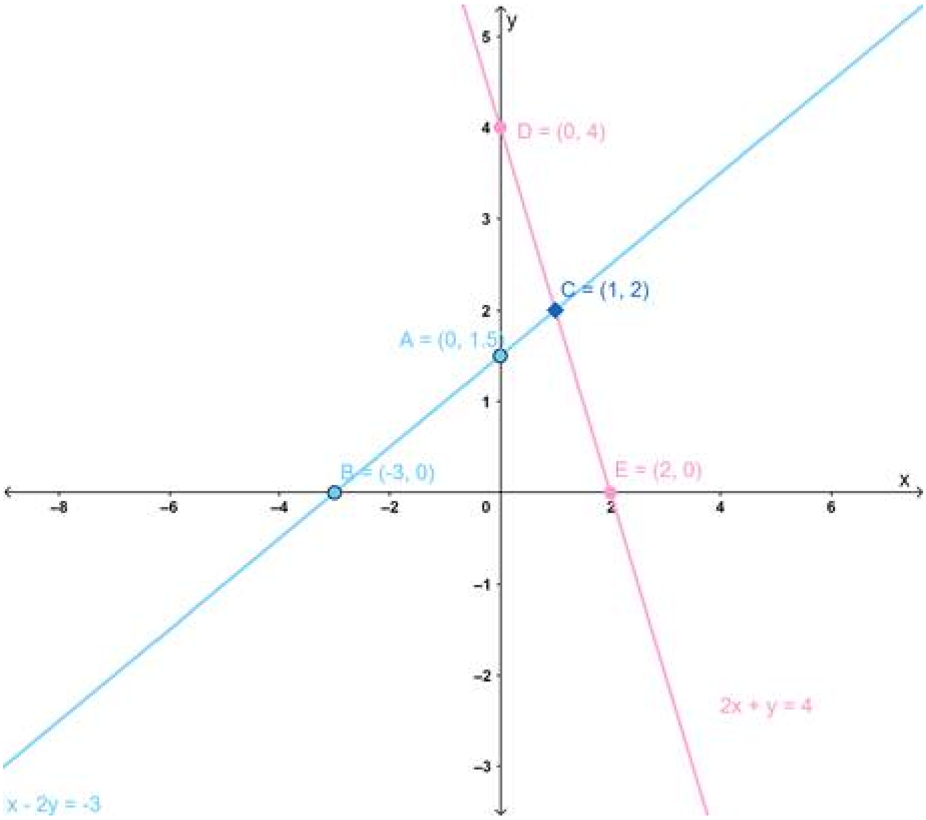
Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for $x - 2y = -3$ or $y = \frac{x+3}{2}$

x	0	- 3	1
$y = \frac{x+3}{2}$	1.5	0	2

Now, table for $2x + y = 4$ or $y = 4 - 2x$

x	2	0	1
$y = 4 - 2x$	0	4	2



Here, the lines intersecting at point C i.e. (1,2)

The points which intersect the x axis are B (– 3,0) and E (2,0)

12 B. Question

Solve the following system of equations graphically. Also find the points where the lines intersect x – axis.

2x + 3y = 8

x — 2y = — 3

Answer

The given equation is

2x + 3y = 8

and x – 2y = – 3

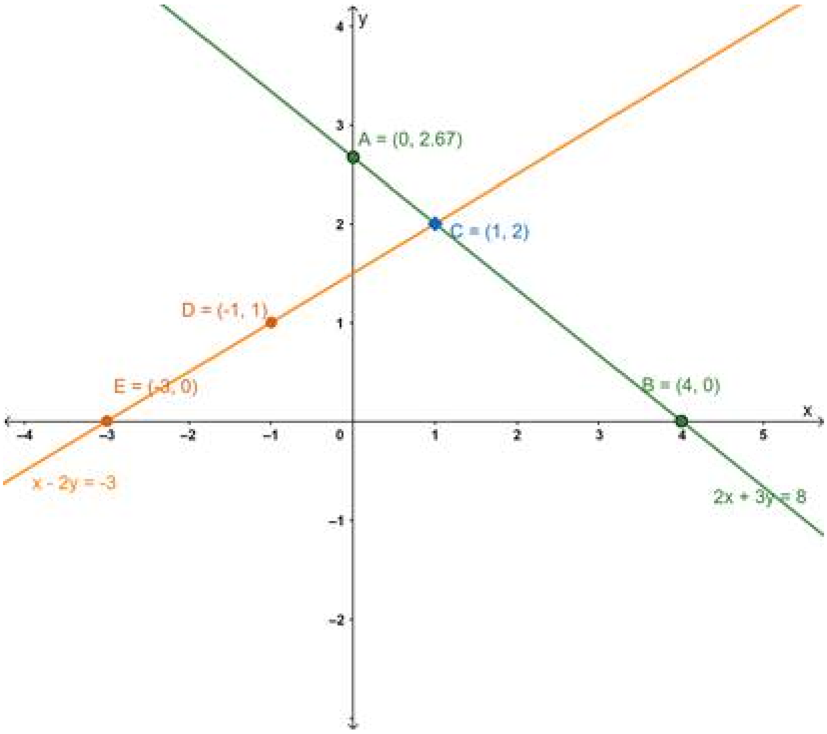
Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for $2x + 3y = 8$ or $y = \frac{8-2x}{3}$

x	0	4	1
$y = \frac{x+3}{2}$	$\frac{8}{3}$	0	2

Now, table for $x - 2y = -3$ or $y = \frac{x+3}{2}$

x	- 1	- 3	1
$y = 4 - 2x$	- 1	0	2



Here, the lines intersecting at point C i.e. (1,2)

The points which intersect at x axis are B (4,0) and E (– 3,0).

12 C. Question

Solve the following system of equations graphically. Also find the points where the lines intersect x – axis.

x + 2y = 5

2x — 3y = — 4

Answer

The given equation is

x + 2y = 5

and 2x – 3y = – 4

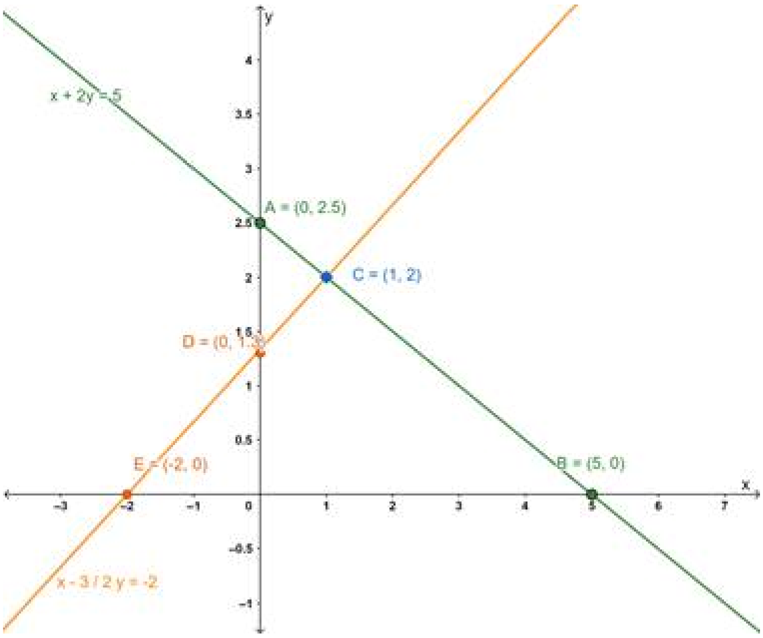
Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for $x + 2y = 5$ or $y = \frac{5-x}{2}$

x	0	5	1
$y = \frac{5-x}{2}$	$\frac{5}{2}$	0	2

Now, table for $2x - 3y = -4$ or $y = \frac{2x+4}{3}$

x	0	- 2	1
$y = \frac{2x + 4}{3}$	$\frac{4}{3}$	0	2



Here, the lines intersecting at point C i.e. (1,2)

The points which intersect the x axis are B (5,0) and E (– 2,0)

12 D. Question

Solve the following system of equations graphically. Also find the points where the lines intersect x – axis.

$x - y + 1 = 0$

$4x + 3y = 24$

Answer

The given equation is

$x - y + 1 = 0$

and $4x + 3y = 24$ or $x + \frac{3}{4}y = 6$

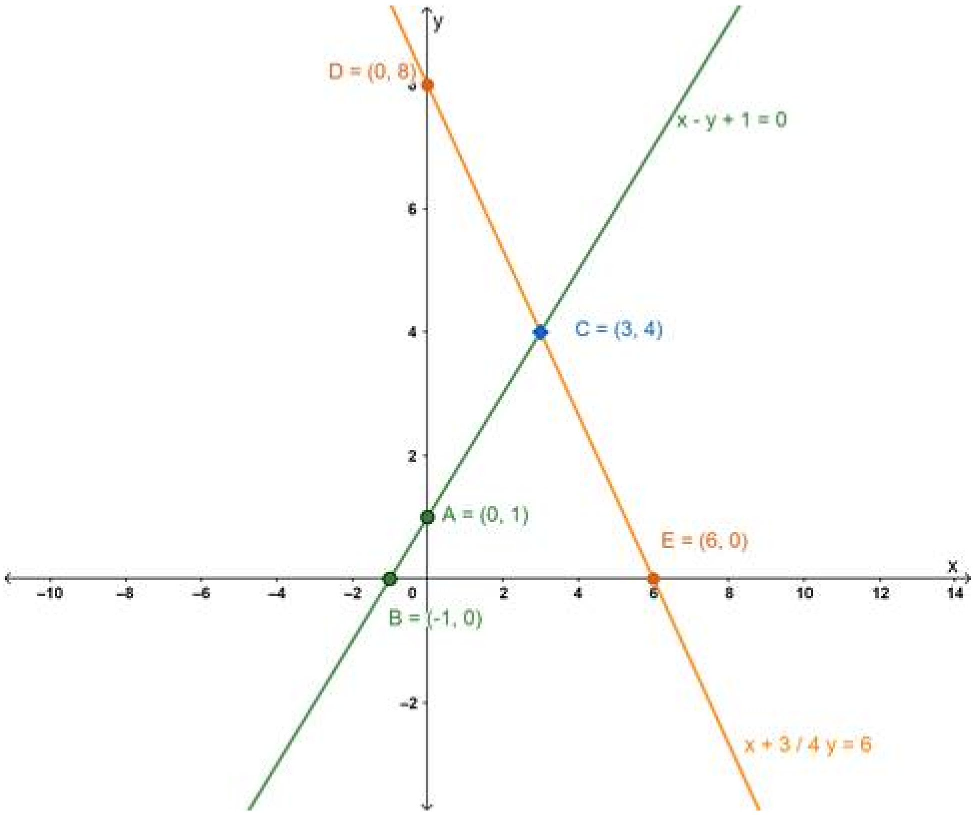
Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for $x - y + 1 = 0$ or $y = x + 1$

x	0	- 1	3
$y = x + 1$	1	0	4

Now, table for $4x + 3y = 24$ or $y = \frac{4x-24}{3}$

x	0	6	3
$y = \frac{4x-24}{3}$	8	0	4



Here, the lines intersecting at point C i.e. (3,4)

The points which intersect the x axis are B (– 1,0) and E (6,0)

12 E. Question

Solve the following system of equations graphically. Also find the points where the lines intersect x – axis.

x + 2y = 1

x— 2y = 7

Answer

The given equation is

x + 2y = 1

and x – 2y = 7

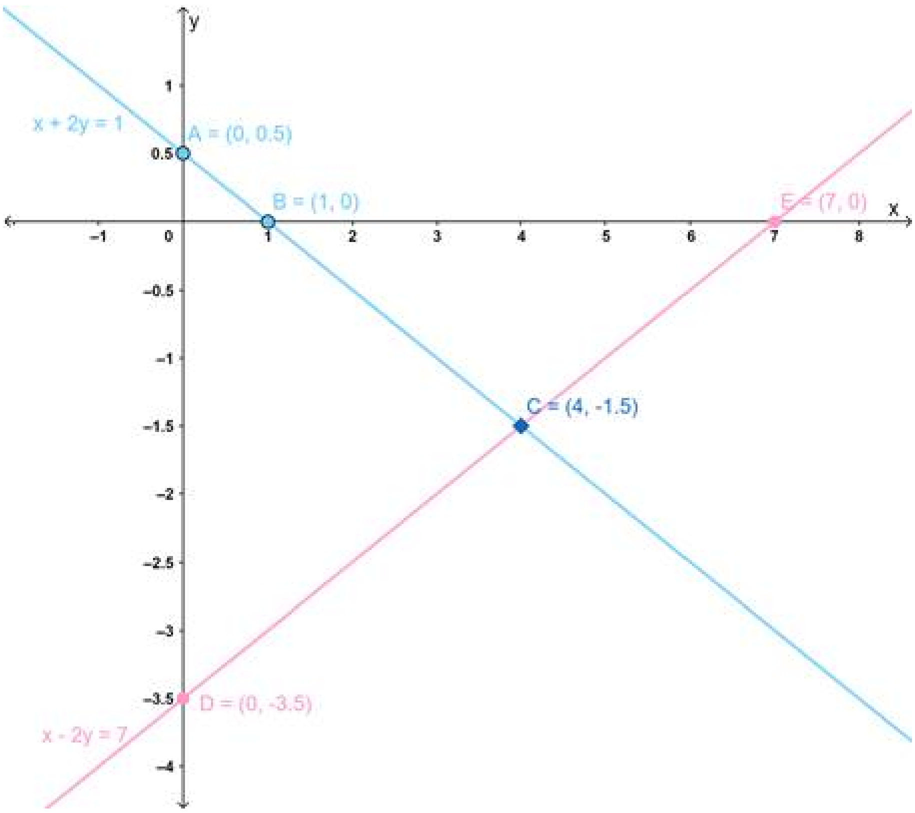
Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for $x + 2y = 1$ or $y = \frac{1-x}{2}$

x	0	1	4
$y = \frac{1-x}{2}$	$\frac{1}{2}$	0	$\frac{-3}{2}$

Now, table for $x - 2y = 7$ or $y = \frac{x-7}{2}$

x	0	7	4
$y = \frac{x-7}{2}$	$\frac{-7}{2}$	0	$\frac{-3}{2}$



Here, the lines intersecting at point B i.e. (4, – 1.5)

The points which intersect the x axis are B (1,0) and E (7,0)

12 F. Question

Solve the following system of equations graphically. Also find the points where the lines intersect x – axis.

x + 2y = 1

x — 2y = —7

Answer

The given equation is

x + 2y = 1

and x – 2y = – 7

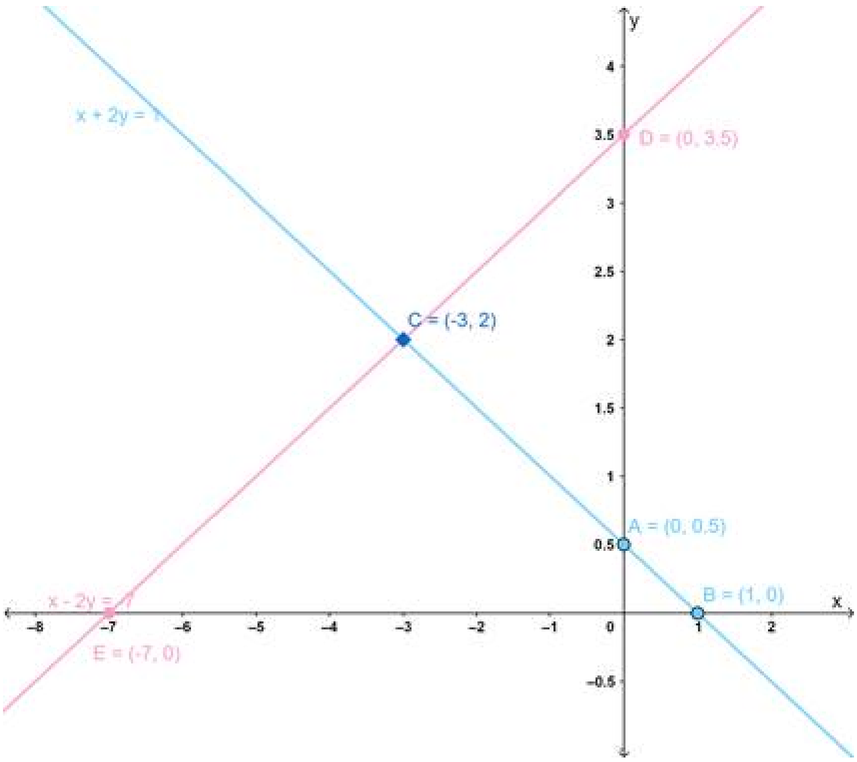
Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for $x + 2y = 1$ or $y = \frac{1-x}{2}$

x	0	1	- 3
$y = \frac{1-x}{2}$	$\frac{1}{2}$	0	2

Now, table for $x - 2y = -7$ or $y = \frac{x+7}{2}$

x	0	- 7	- 3
$y = \frac{x+7}{2}$	$\frac{7}{2}$	0	2



Here, the lines intersecting at point B i.e. (– 3,2)

The points which intersect the x axis are B (1,0) and E (– 7,0)

13 A. Question

Solve the following system of equations graphically. Also find the points where the lines meet the y – axis.

2x — y = 4

3y — x = 3

Answer

The given equation is

2x – y = 4

and 3y – x = 3

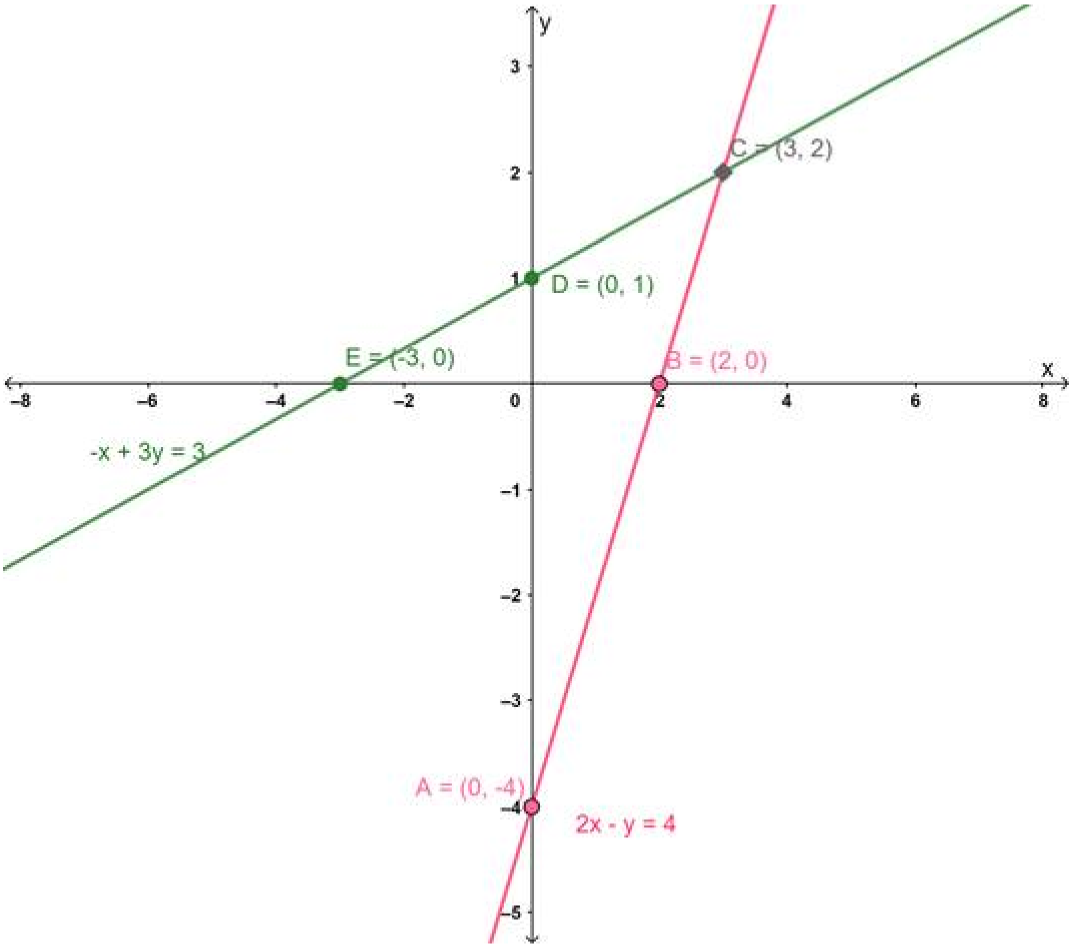
Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for 2x – y = 4 or y = 2x – 4

x	0	2	3
y = 2x - 4	- 4	0	2

Now, table for $3y - x = 3$ or $y = \frac{x+3}{3}$

x	0	- 3	3
$y = \frac{x+3}{3}$	1	0	2



Here, the lines intersecting at point C, i.e. (3,2)

The point which intersects at y axis are A (0, – 4) and D (0,1)

13 B. Question

Solve the following system of equations graphically. Also find the points where the lines meet the y – axis.

2x + 3y — 12 = 0

2x — y — 4 = 0

Answer

The given equation is

2x + 3y = 12

and 2x – y – 4 = 0

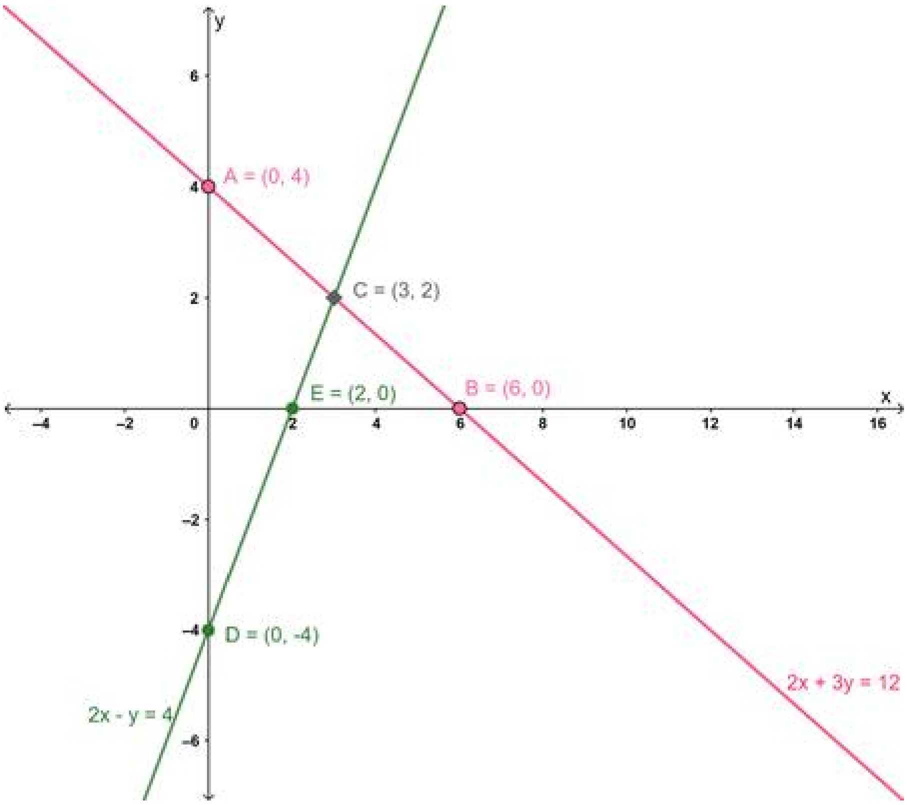
Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for $2x + 3y = 12$ or $y = \frac{12-x}{3}$

x	0	6	3
$y = \frac{12-x}{3}$	4	0	2

Now, table for $2x - y - 4 = 0$ or $y = 2x - 4$

x	0	2	3
$y = 2x - 4$	- 4	0	2



Here, the lines intersecting at point C, i.e. (3,2)

The points which intersects at y axis is A (0,4) and D (0, – 4)

13 C. Question

Solve the following system of equations graphically. Also find the points where the lines meet the y – axis.

2x — y — 5 = 0

x — y — 3 = 0

Answer

The given equation is

2x – y = 5

and x – y = 3

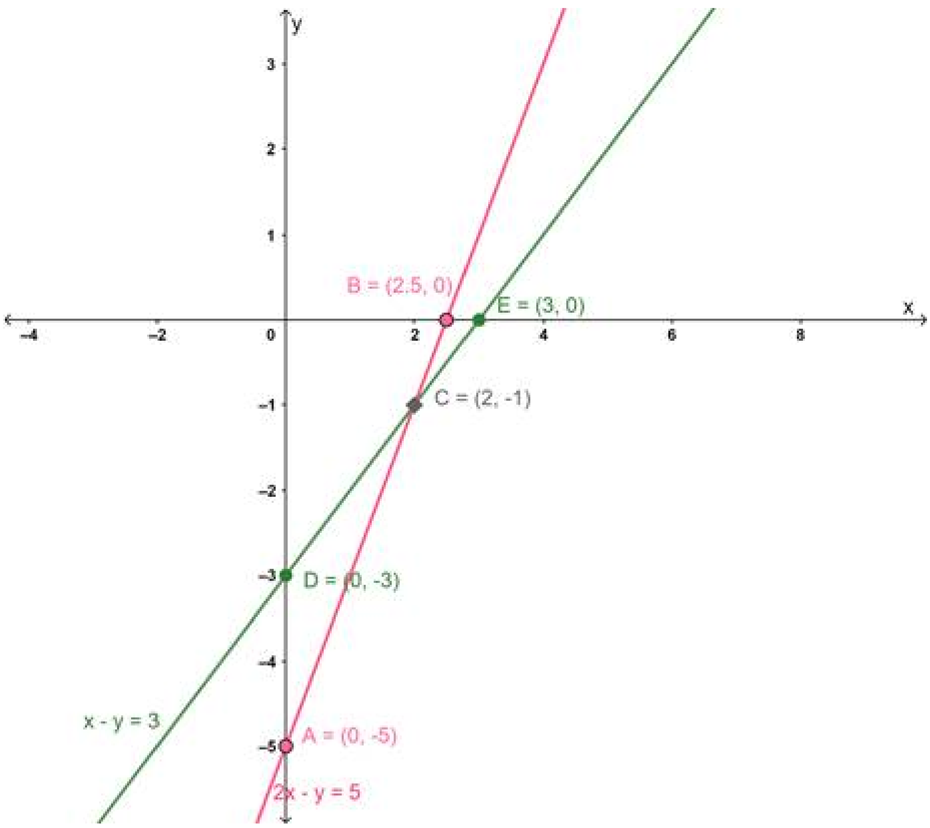
Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for 2x – y = 5 or y = 2x – 5

x	0	5/2	2
y = 2x - 5	- 5	0	- 1

Now, table for x – y = 3 or y = x – 3

x	0	3	2
y = x - 3	- 3	0	- 1



Here, the lines intersecting at point C, i.e. (2, – 1)

The point which intersects at y axis are A (0, – 5) and D (0, – 3)

13 D. Question

Solve the following system of equations graphically. Also find the points where the lines meet the y – axis.

2x — y — 4 = 0

x + y + 1 = 0

Answer

The given equation is

2x – y = 4

and x + y + 1 = 0

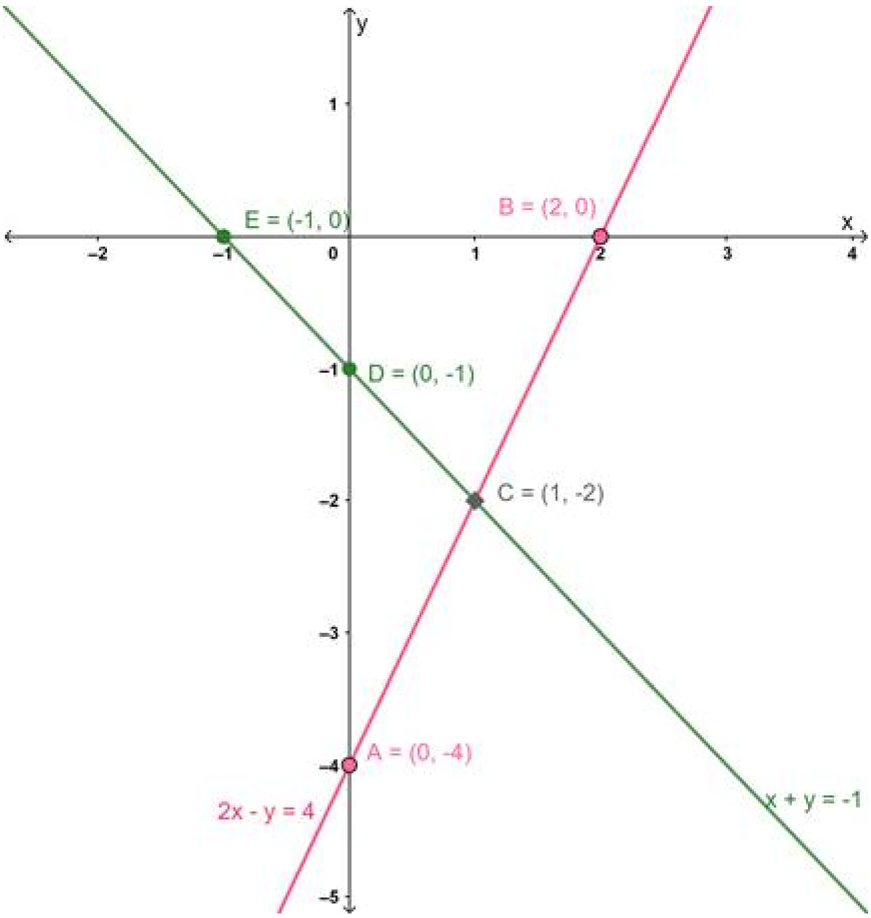
Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for 2x – y = 4 or y = 2x – 4

x	0	2	1
y = 2x - 4	- 4	0	- 2

Now, table for x + y + 1 = 0 or y = – (x + 1)

x	0	- 1	1
y = - (x + 1)	- 1	0	- 2



Here, the lines intersecting at point C, i.e. (1, – 2)

The point which intersects at y axis are A (0, – 4) and D (0, – 1)

13 E. Question

Solve the following system of equations graphically. Also find the points where the lines meet the y – axis.

3x + y — 5 = 0

2x — y — 5 = 0

Answer

The given equation is

3x + y = 5

and 2x – y = 5

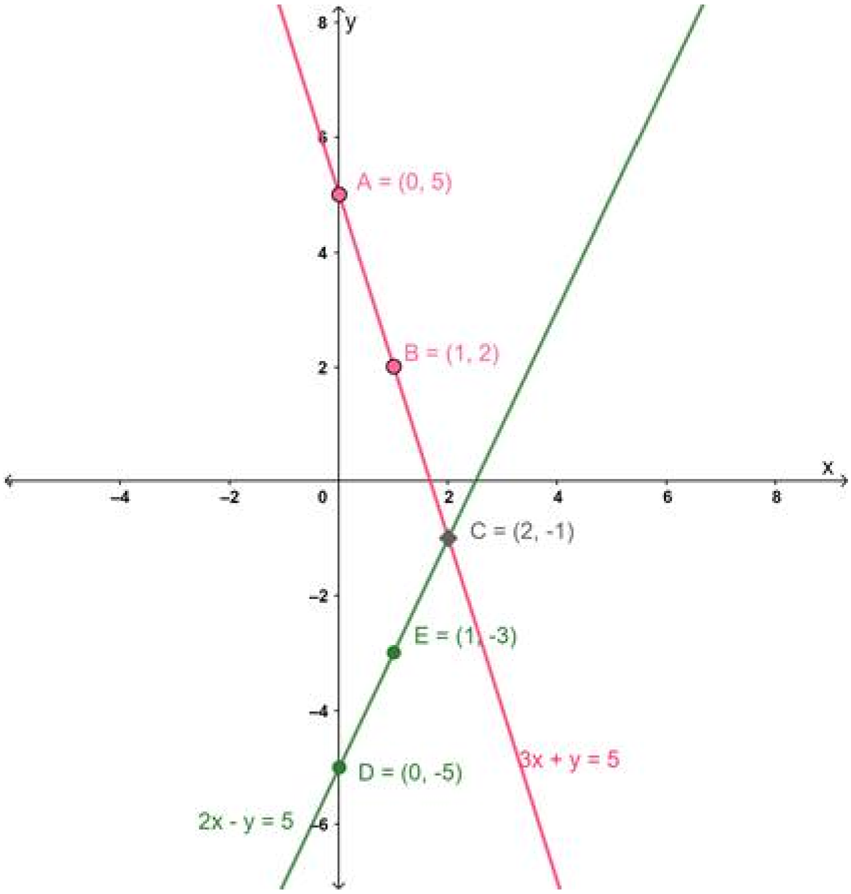
Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for 3x + y = 5 or y = 5 – 3x

x	0	1	2
y = 5 - 3x	5	2	- 1

Now, table for 2x – y = 5 or y = 2x + 5

x	0	1	2
y = 2x + 5	- 5	- 3	- 1



Here, the lines intersecting at point C i.e. (2, – 1)

The point which is intersect at y axis are A (0,5) and D (0, – 5)

14 A. Question

Solve the following system of linear equations graphically.

3x + 2y — 4 = 0

2x — 3y — 7 = 0

Shade the region bounded by the lines and the x – axis.

Answer

The given equation is

3x + 2y = 4

and 2x – 3y = 7

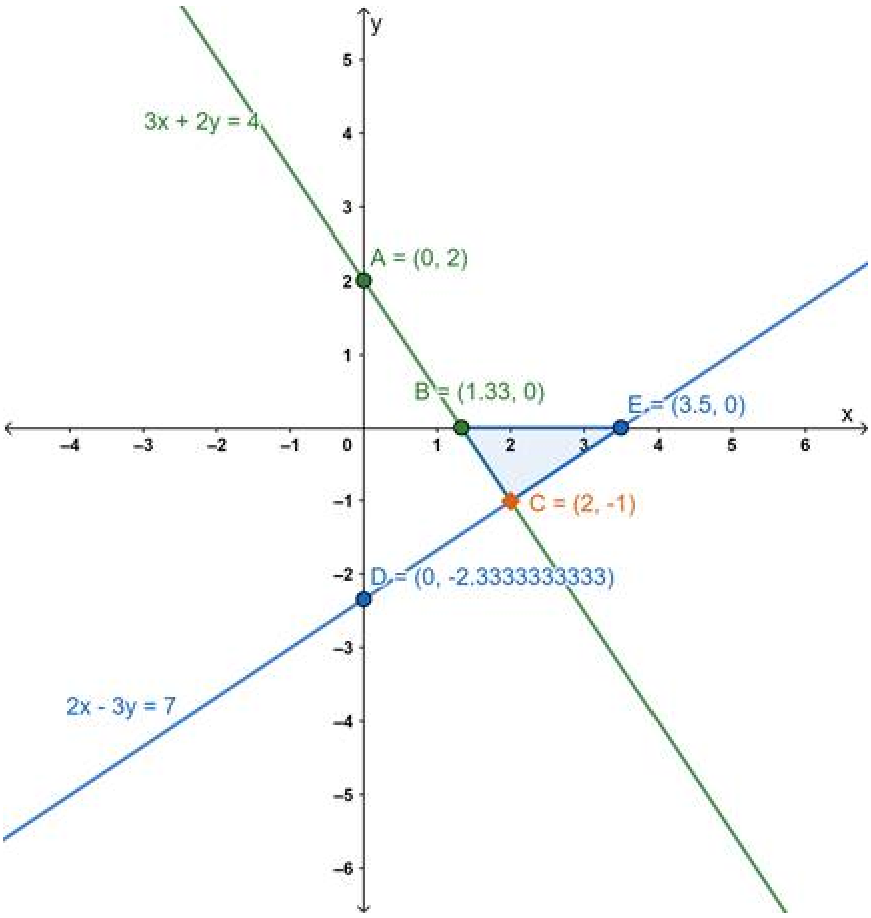
Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for $3x + 2y = 4$ or $y = \frac{4-3x}{2}$

x	0	$\frac{4}{3} = 1.33$	2
$y = \frac{4-3x}{2}$	2	0	- 1

Now, table for $2x - 3y = 7$ or $y = \frac{2x-7}{3}$

x	0	$\frac{7}{2}$	2
$y = \frac{2x-7}{3}$	$\frac{-7}{3}$	0	- 1



Here, the lines intersecting at a point C i.e. (2, – 1).

14 B. Question

Solve the following system of linear equations graphically.

3x — 2y – 1 = 0

2x — 3y + 6 = 0

Shade the region bounded by the lines and the x – axis.

Answer

The given equation is

3x – 2y = 1

and 2x – 3y = – 6

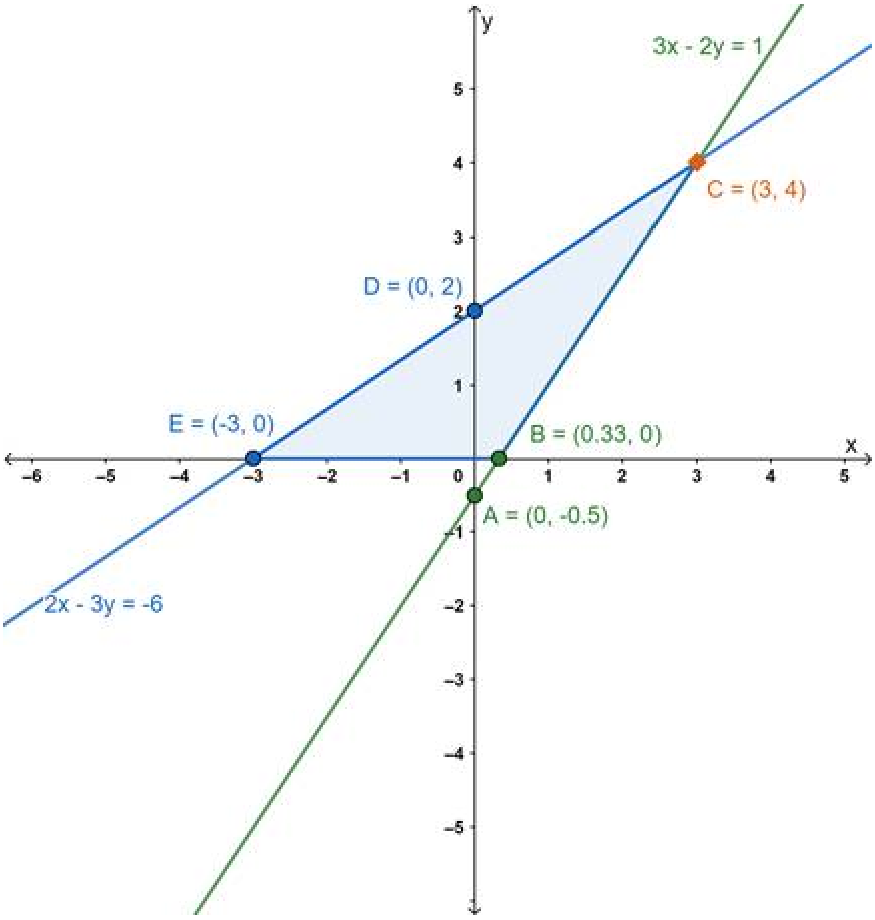
Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for $3x - 2y = 1$ or $y = \frac{3x-1}{2}$

x	0	$\frac{1}{3} = 0.33$	3
$y = \frac{3x-1}{2}$	- 0.5	0	4

Now, table for $2x - 3y = -6$ or $y = \frac{2x+6}{3}$

x	0	- 3	3
$y = \frac{2x+6}{3}$	2	0	4



Here, the lines intersecting at a point C, i.e. (3,4).

15 A1. Question

Solve the following pair of linear equations graphically and shade the region bounded by these lines and x – axis; also find the area of the shaded region.

2x + y = 6

2x — y = 0

Answer

The given equation is

2x + y = 6

and 2x – y = 0

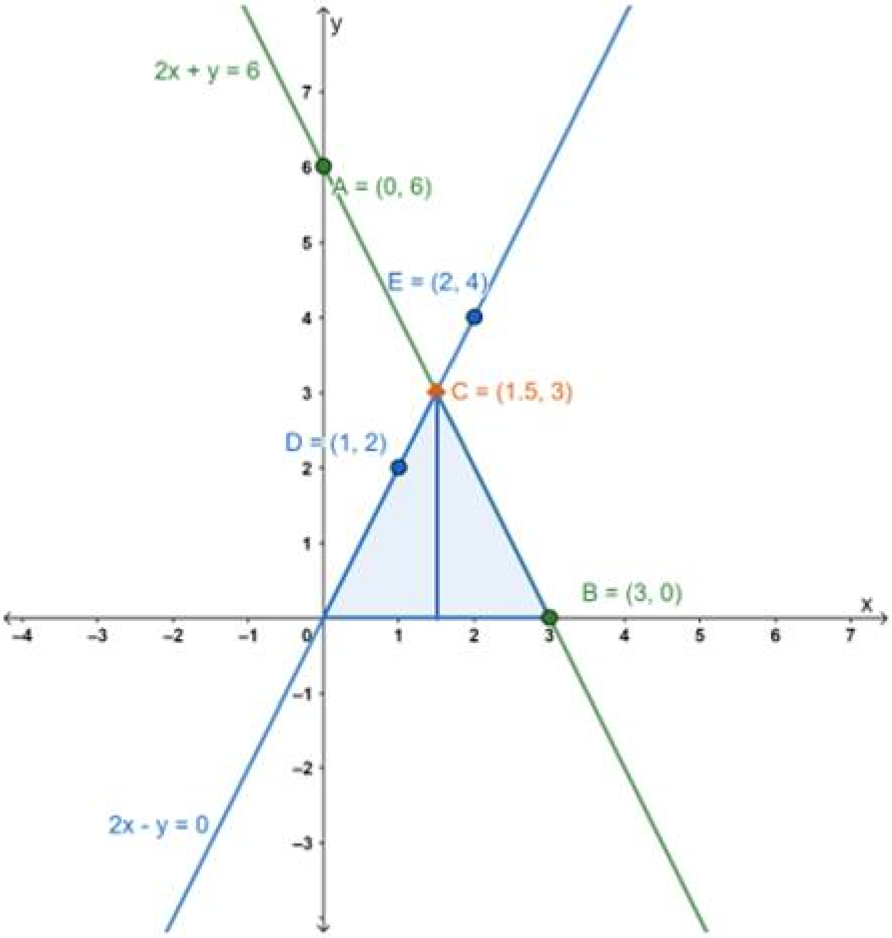
Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for 2x + y = 6 or y = 2x – 6

x	0	3	1.5
y = 2x - 6	6	0	3

Now, table for 2x – y = 0 or y = 2x

x	1	2	1.5
y = 2x	2	4	3



Here, the lines are intersecting at a point C $\left(\frac{3}{2}, 3\right)$.

The coordinates of the vertices of ΔCOB are C$\left(\frac{3}{2}, 3\right)$,O(0,0) and B(3,0).

Area = $A = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 3 \times 3 = \frac{9}{2} = 4.5\text{sq. units}$

15 A2. Question

Solve the following pair of linear equations graphically and shade the region bounded by these lines and x – axis; also find the area of the shaded region.

2x + 3y = —5

3x — 2y = 12

Answer

The given equation is

2x + 3y = – 5

and 3x – 2y = 12

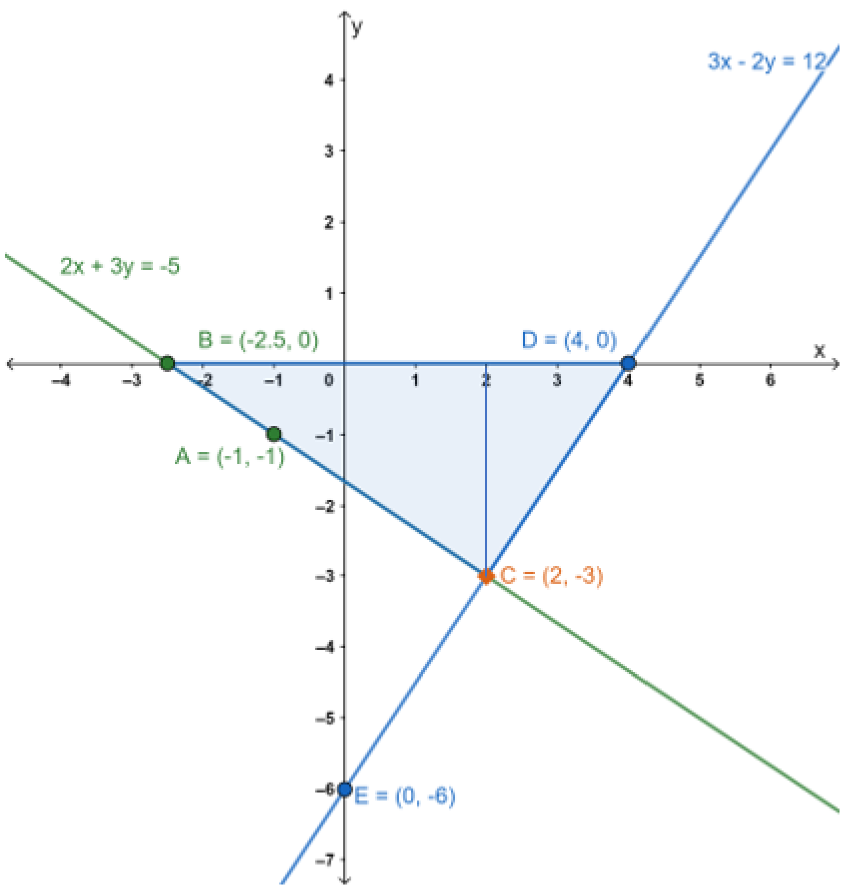
Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for $2x + 3y = -5$ or $y = \frac{-(2x+5)}{3}$

x	- 1	$-\frac{1}{2}$	2
$y = \frac{-(2x+5)}{3}$	- 1	0	- 3

Now, table for $3x - 2y = 12$ or $y = \frac{3x-12}{2}$

x	4	0	2
$y = \frac{3x-12}{2}$	0	- 6	- 3



Here, the lines are intersecting at a point C $(2, -3)$.

The coordinates of the vertices of ΔCBD are C$(2, -3)$,B$(-\frac{5}{2}, 0)$ and D(4,0)

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times \frac{13}{2} \times 3 \ (\because, \text{base} = (4 - (\frac{5}{2})) = \frac{13}{2})$$

$$= \frac{39}{4} \text{sq. units}$$

15 A3. Question

Solve the following pair of linear equations graphically and shade the region bounded by these lines and x – axis; also find the area of the shaded region.

4x — 3y + 4 = 0

4x + 3y — 20 = 0

Answer

The given equation is

4x – 3y = – 4

and 4x + 3y = 20

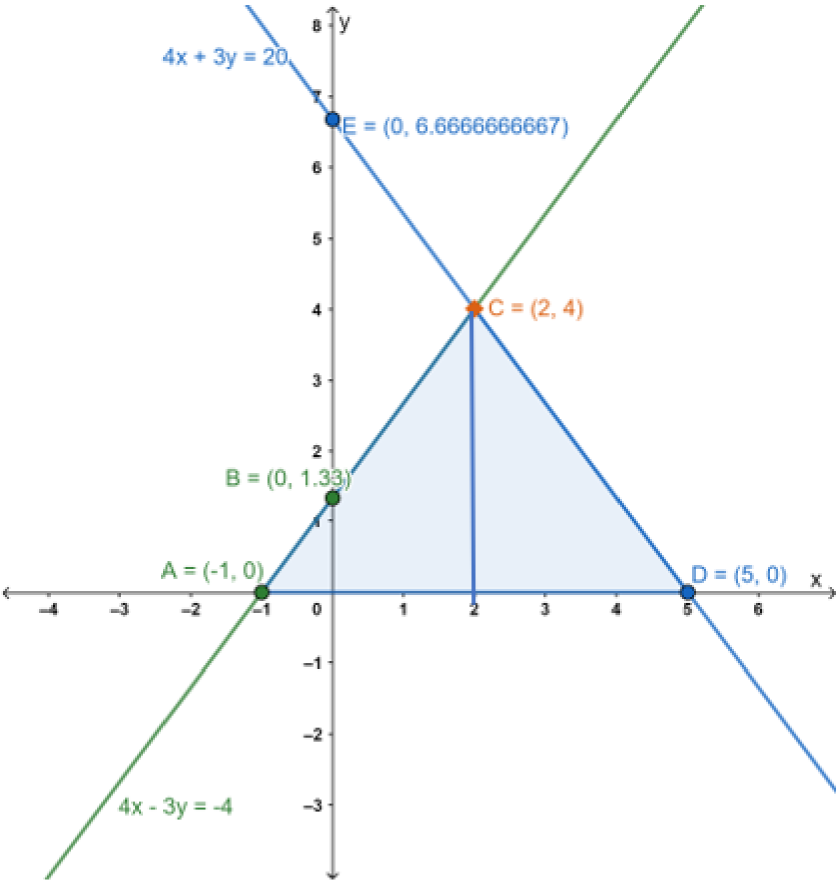
Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for $4x - 3y = -4 \text{ or } y = \frac{4x+4}{3}$

x	- 1	0	2
$y = \frac{4x + 4}{3}$	0	$\frac{4}{3}$	4

Now, table for $4x + 3y = 20 \text{ or } y = \frac{20-4x}{3}$

x	5	0	2
$y = \frac{20 - 4x}{3}$	0	$\frac{20}{3}$	4



Here, the lines are intersecting at a point C $(2, 4)$.

The coordinates of the vertices of ΔCAD are C$(2, 4)$, A$(-1, 0)$ and D(5,0)

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

$= \frac{1}{2} \times 6 \times 4$ (∵, base = (5–(–1)) = 6)

= 12 sq.units

15 A4. Question

Solve the following pair of linear equations graphically and shade the region bounded by these lines and x – axis; also find the area of the shaded region.

2x + y = 6

2x — y + 2 = 0

Answer

The given equation is

2x + y = 6

and 2x – y = – 2

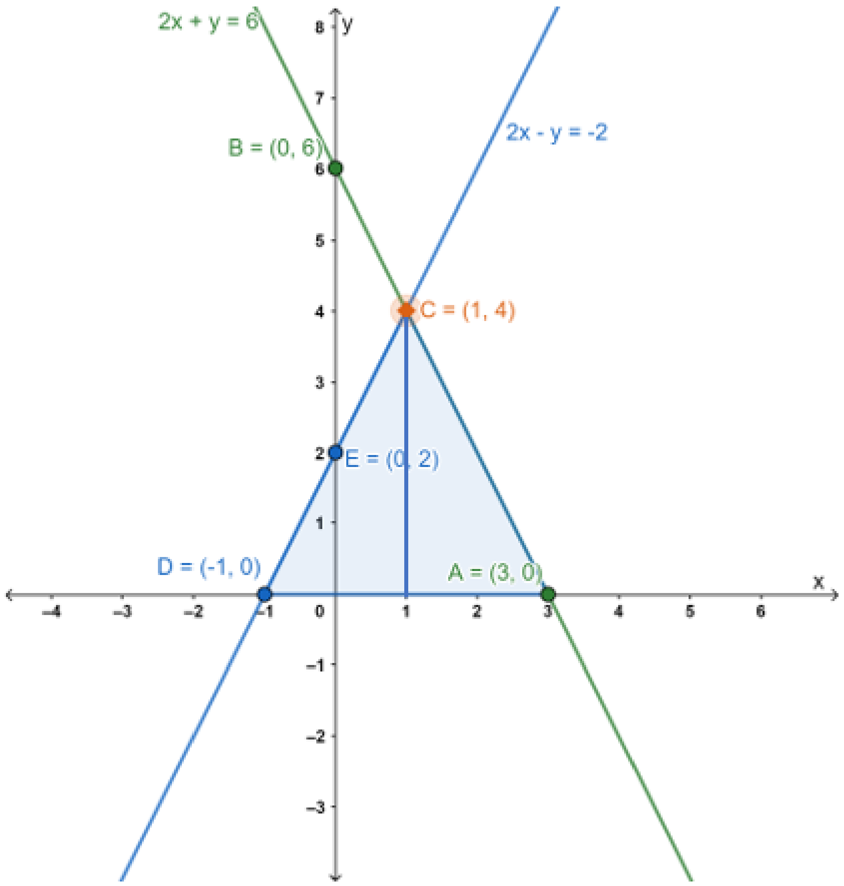
Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for $2x + y = 6$ or $y = 2x - 6$

x	3	0	4
y = 2x - 6	0	6	−1.5

Now, table for $2x - y = -2$ or $y = 2x + 2$

x	- 1	0	1
y = 2x + 2	0	2	4



Here, the lines are intersecting at a point C $(1, 4)$.

The coordinates of the vertices of ΔCAD are C$(1, 4)$, A(3,0) and D(– 1,0)

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

$= \frac{1}{2} \times 4 \times 4$ ($\because$, base $= (3 - (-1)) = 4$)

= 8 sq.units

15 B1. Question

Solve the following pair of linear equations graphically and shade the region bounded by these lines and y – axis. Also find the area of the shaded region.

x — y = 1

2x + y = 8

Answer

The given equation is

x – y = 1

and 2x + y = 8

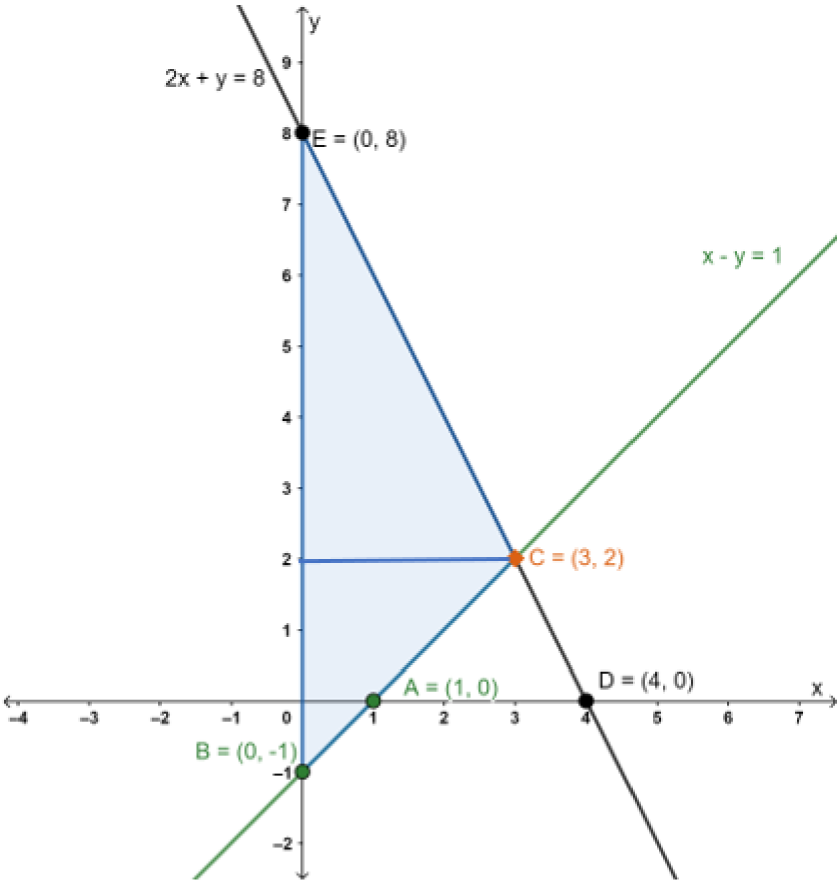
Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for x – y = 1 or y = x – 1

x	1	0	3
y = x - 1	0	- 1	2

Now, table for 2x + y = 8 or y = 2x – 8

x	4	0	3
y = 2x - 8	0	8	2



Here, the lines are intersecting at a point C $(3, 2)$.

The coordinates of the vertices of ΔCBE are C$(3, 2)$, B(0, – 1) and E(0,8)

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 9 \times 3 \ (\because, \text{base} = (8 - (-1)) = 9)$$

$$= \frac{27}{2} \text{sq. units}$$

15 B2. Question

Solve the following pair of linear equations graphically and shade the region bounded by these lines and y – axis. Also find the area of the shaded region.

3x + y — 11 = 0

x — y — 1 = 0

Answer

The given equation is

3x + y = 11

and x – y = 1

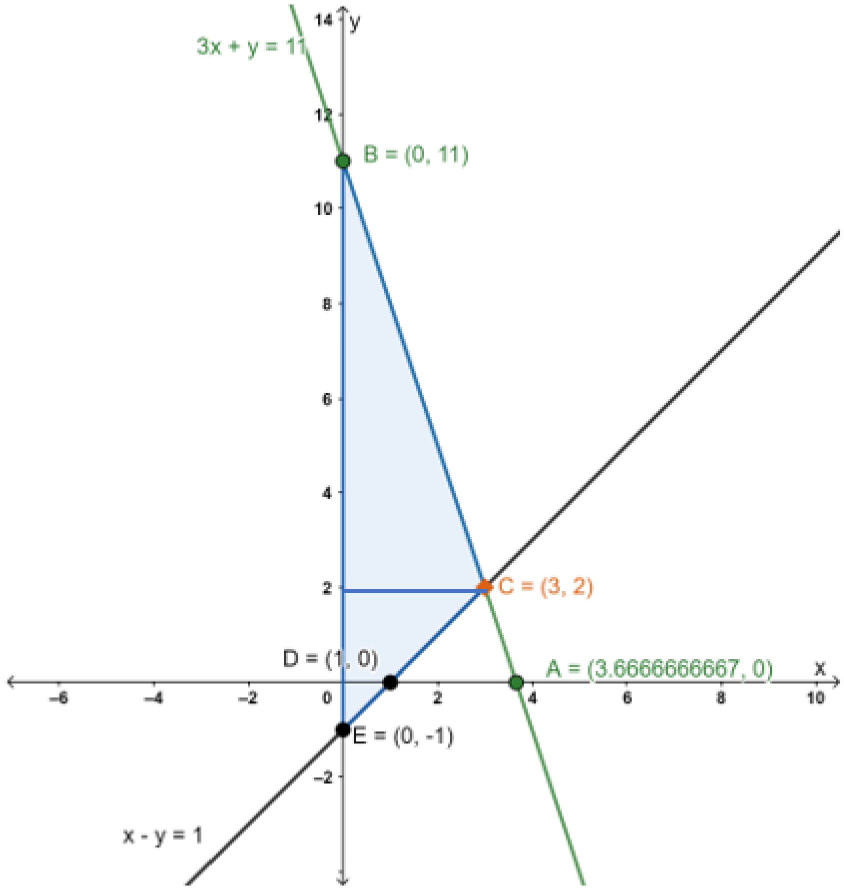
Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for 3x + y = 11 or y = 3x – 11

x	$\frac{11}{3}$	0	3
$y = \frac{1-x}{2}$	0	11	2

Now, table for x – y = 1or y = x – 1

x	1	0	3
y = x - 1	0	- 1	2



Here, the lines are intersecting at a point C $(3, 2)$.

The coordinates of the vertices of ΔCBE are C$(3, 2)$, B(0, 11) and E(0, – 1)

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

$= \frac{1}{2} \times 12 \times 3$ ($\because$, base $= (11 - (-1)) = 12$)

= 18 sq.units

15 B3. Question

Solve the following pair of linear equations graphically and shade the region bounded by these lines and y – axis. Also find the area of the shaded region.

x + 2y — 7 = 0

2x — y — 4 = 0

Answer

The given equation is

x + 2y = 7

and 2x – y = 4

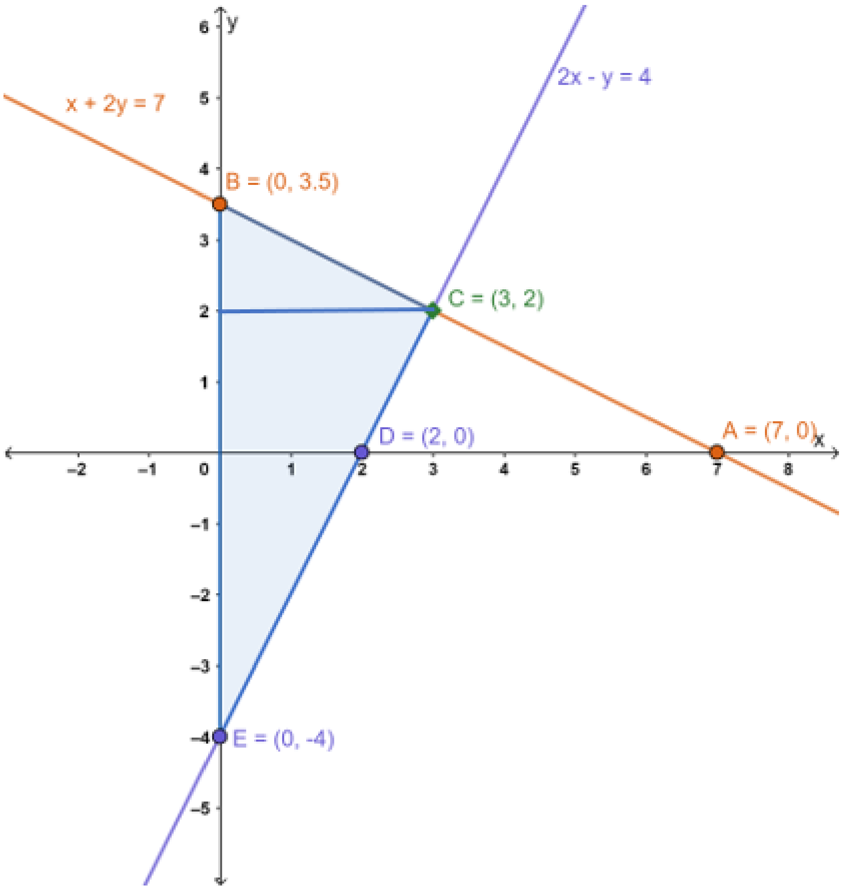
Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for $x + 2y = 7$ or $y = \frac{7-x}{2}$

x	7	0	4
$y = \frac{7-x}{2}$	0	$\frac{7}{2}$	$\frac{-3}{2}$

Now, table for $2x - y = 4$ or $y = 2x - 4$

x	2	0	3
$y = 2x - 4$	0	- 4	2



Here, the lines are intersecting at a point C (3, 2).

The coordinates of the vertices of ΔCBD are C(3, 2),B$(0, \frac{7}{2})$ and E(0, – 4)

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times \frac{15}{2} \times 3 \ (\because, \text{base} = \left(\left(\frac{7}{2}\right) - (-4)\right) = \frac{15}{2})$$

$$= \frac{45}{4} \text{sq. units}$$

16 A. Question

Solve the following system of linear equations graphically. Also shade the region bounded by the lines and y – axis.

4x — y = 4

3x + 2y = 14

Answer

The given equation is

4x – y = 4

and 3x + 2y = 14

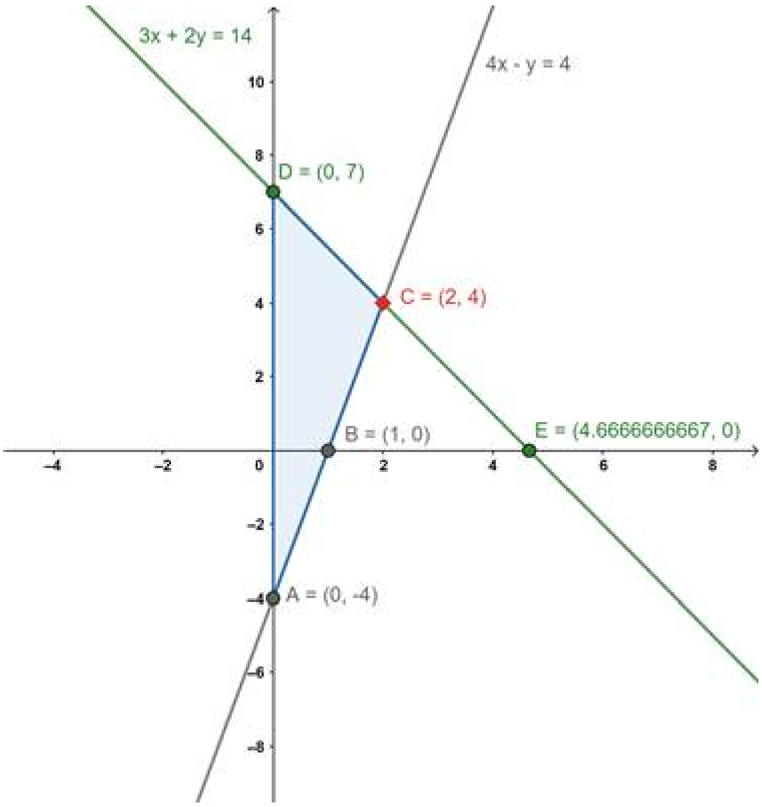
Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for 4x – y = 4 or y = 4x – 4

x	0	1	2
y = 4x - 4	- 4	0	4

Now, table for $3x + 2y = 14$ or $y = \frac{14-3x}{2}$

x	0	$\frac{14}{3}$	2
$y = \frac{14-3x}{2}$	7	0	4



Here, the lines are intersecting at point C(2,4).

16 B. Question

Solve the following system of linear equations graphically. Also shade the region bounded by the lines and y – axis.

x — y = 1

2x + y = 8

Answer

The given equation is

x – y = 1

and 2x + y = 8

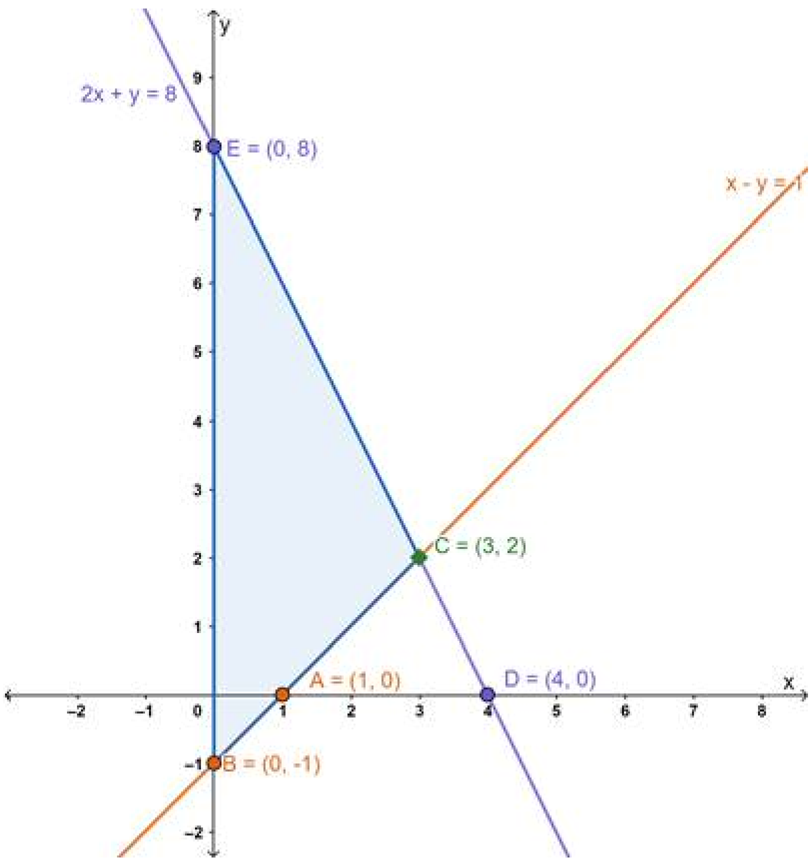
Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for x – y = 1 or y = x – 1

x	1	0	3
y = x - 1	0	- 1	2

Now, table for 2x + y = 8 or y = 8 – 2x

x	4	0	3
y = 8 - 2x	0	8	2



Here, the lines are intersecting at point C(3, 2).

17. Question

Solve the following system of linear equations graphically:

5x — 6y + 30 = 0; 5x + 4y — 20 = 0

Also find the vertices of the triangle formed by the two lines and x – axis.

Answer

The given equation is

5x – 6y = – 30

and 5x + 4y = 20

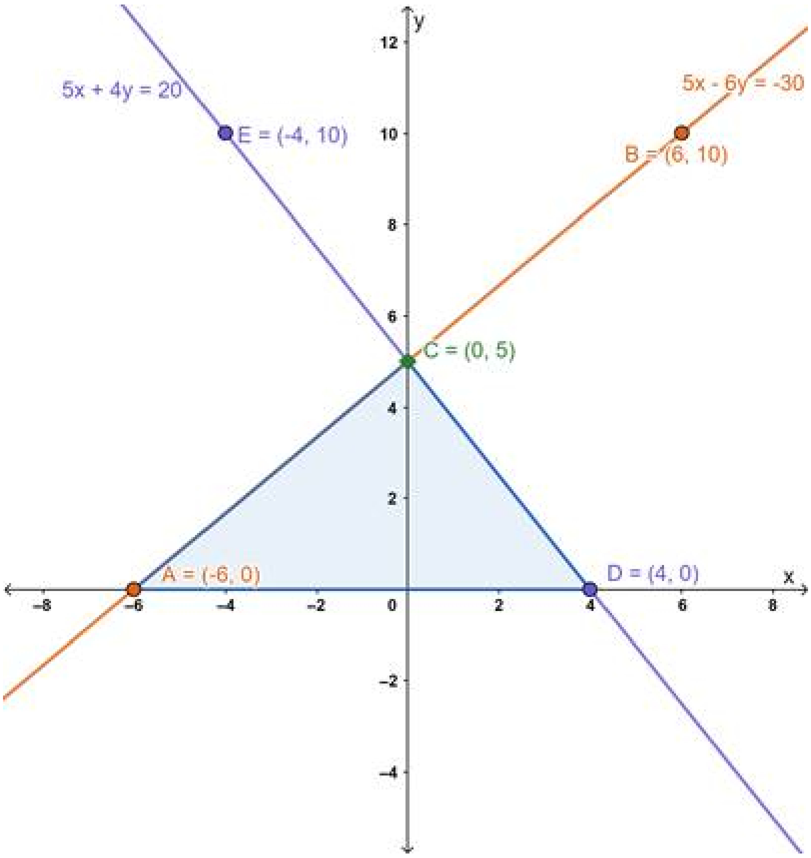
Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for $5x - 6y = -3$ or $y = \frac{5x+3}{6}$

x	- 6	6	0
$y = \frac{5x+3}{6}$	0	10	5

Now, table for $5x + 4y = 20$ or $y = \frac{20-5x}{4}$

x	4	- 4	0
$y = \frac{20-5x}{4}$	0	10	5



Here, the lines are intersecting at point C (0,5).

The coordinates of the vertices of ΔACD are A(– 6,0), C(0,5)and D(4,0)

18. Question

Draw the graphs of the equations 3x — y + 9 = 0 and 3x + 4y — 6 = 0.

Also determine the vertices of the triangle formed by the lines and the x – axis.

Answer

The given equation is

3x – y = – 9

and 3x + 4y = 6

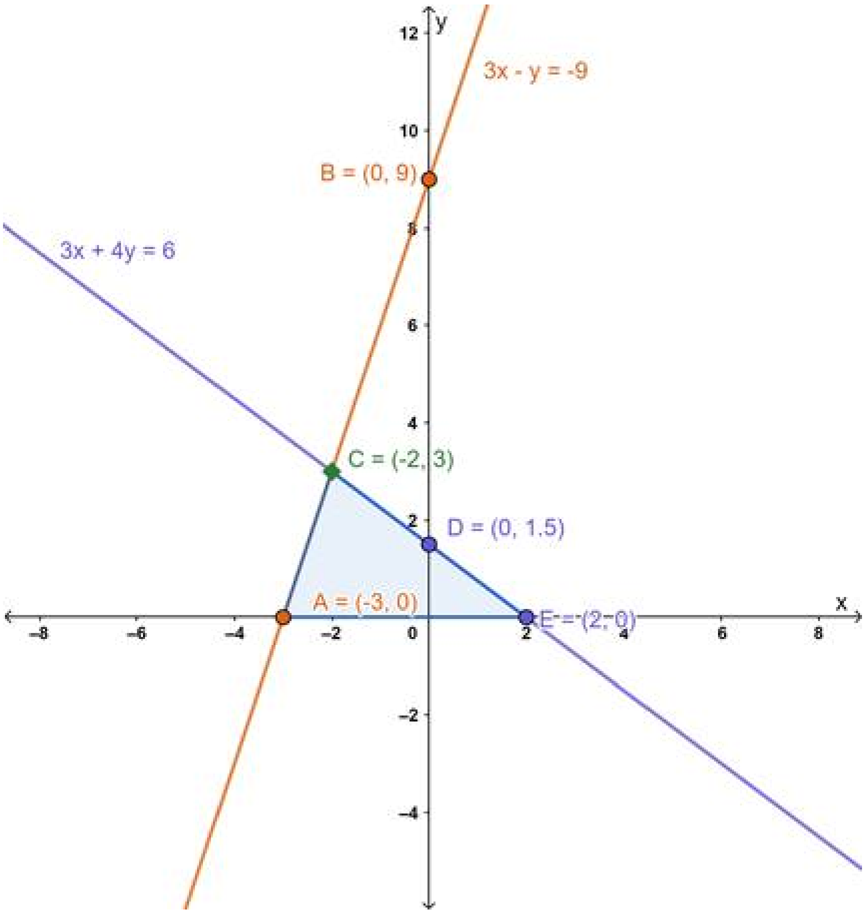
Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for 3x – y = – 9 or y = 3x + 9

x	- 3	0	- 2
y = 3x + 9	0	9	3

Now, table for $3x + 4y = 6$ or $y = \frac{6-3x}{4}$

x	0	2	- 2
$y = \frac{6-3x}{4}$	$\frac{3}{2}$	0	3



Here, the lines are intersecting at point C (– 2, 3).

The coordinates of the vertices of ΔACE are A(– 3,0), C(– 2,3)and E(2,0)

19. Question

Draw the graphs of the following equations 3x — 4y + 6 = 0; 3x + y — 9 = 0.

Also, determine the coordinates of the vertices of the triangle formed by these lines and the x – axis.

Answer

The given equation is

3x – 4y = – 9

and 3x + y = 9

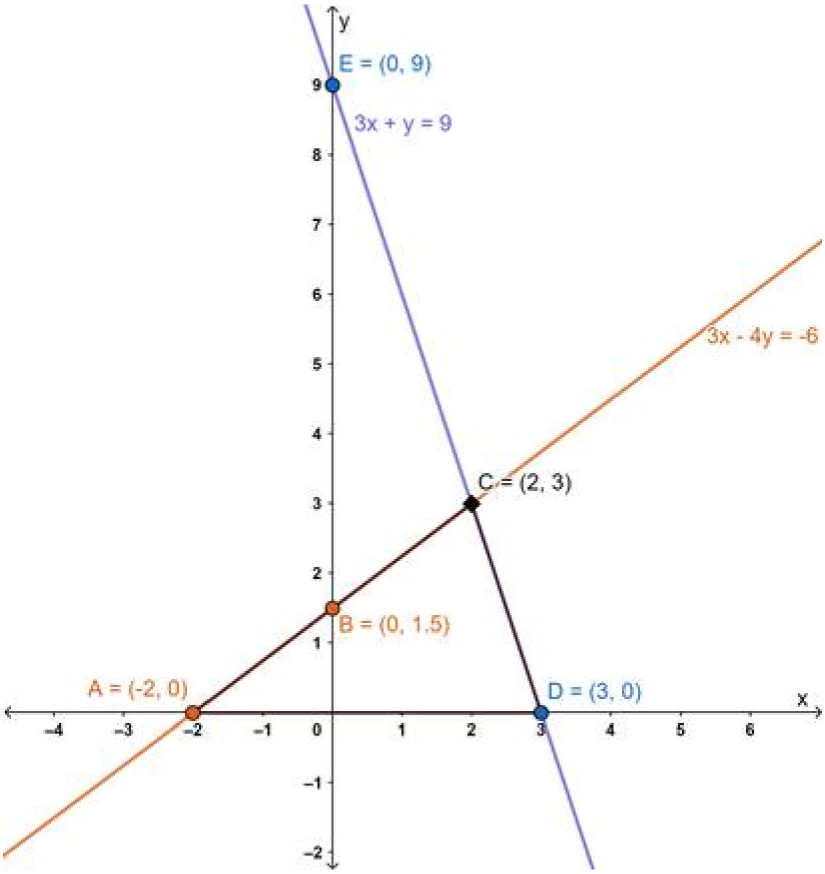
Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for $3x - 4y = -9$ or $y = \frac{3x+9}{4}$

x	- 2	0	4
$y = \frac{3x+9}{4}$	0	1.5	3

Now, table for $3x + y = 9$ or $y = 9 - 3x$

x	3	0	2
$y = 9 - 3x$	0	9	3



Here, the lines are intersecting at point C (2, 3).

The coordinates of the vertices of ΔACD are A(– 2,0), C(2,3)and D(3,0)

20 A. Question

Use a single graph paper and draw the graph of the following equations. Obtain the vertices of the triangle so formed:

2y – x = 8

5y – x = 14

y – 2x = 1

Answer

The given equation is

– x + 2y = 8

– x + 5y = 14

and – 2x + y = 1

Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for $-x + 2y = 8$ or $y = \frac{8 + x}{2}$

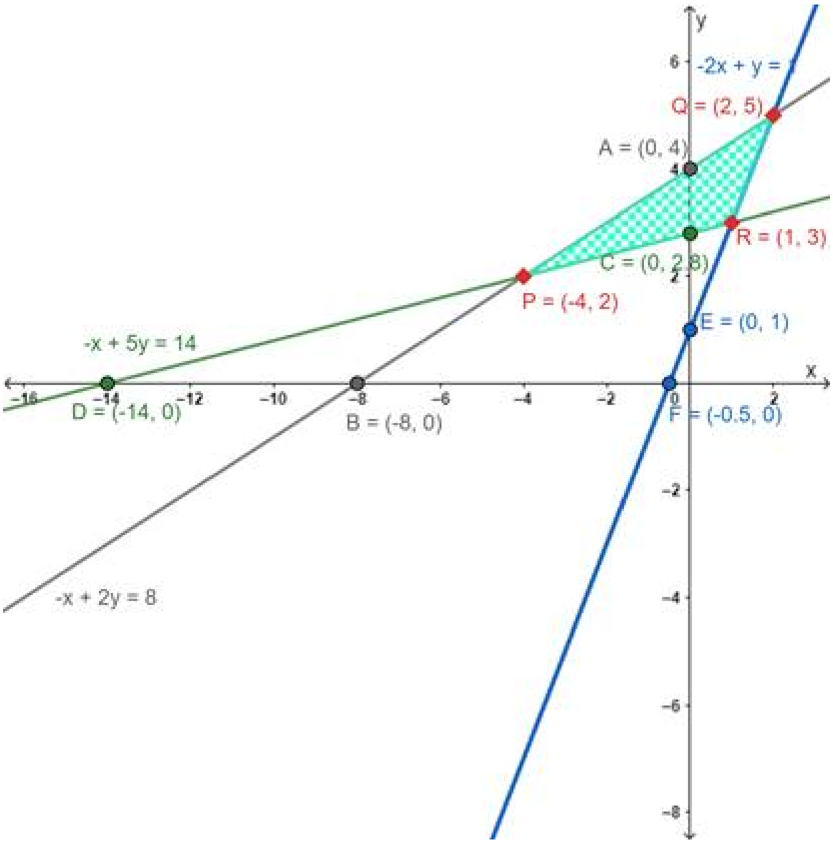
x	0	- 8
$y = \frac{3x + 9}{4}$	4	0

Now, table for $-x + 5y = 14$ or $y = \frac{14 + x}{5}$

x	0	- 14
$y = \frac{14 + x}{5}$	$\frac{14}{5}$	0

Now, table for $-2x + y = 1$ or $y = 1 + 2x$

x	0	$\frac{-1}{2}$
$y = 1 + 2x$	1	0



The coordinates of the vertices of ΔPQR are P(− 4, 2), Q(2, 5)and R(1, 3)

20 B. Question

Use a single graph paper and draw the graph of the following equations. Obtain the vertices of the triangle so formed:

y = x

Y = 2x

x + y = 6

Answer

The given equation is

y = x

y = 2x

and x + y = 6

Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for y = x

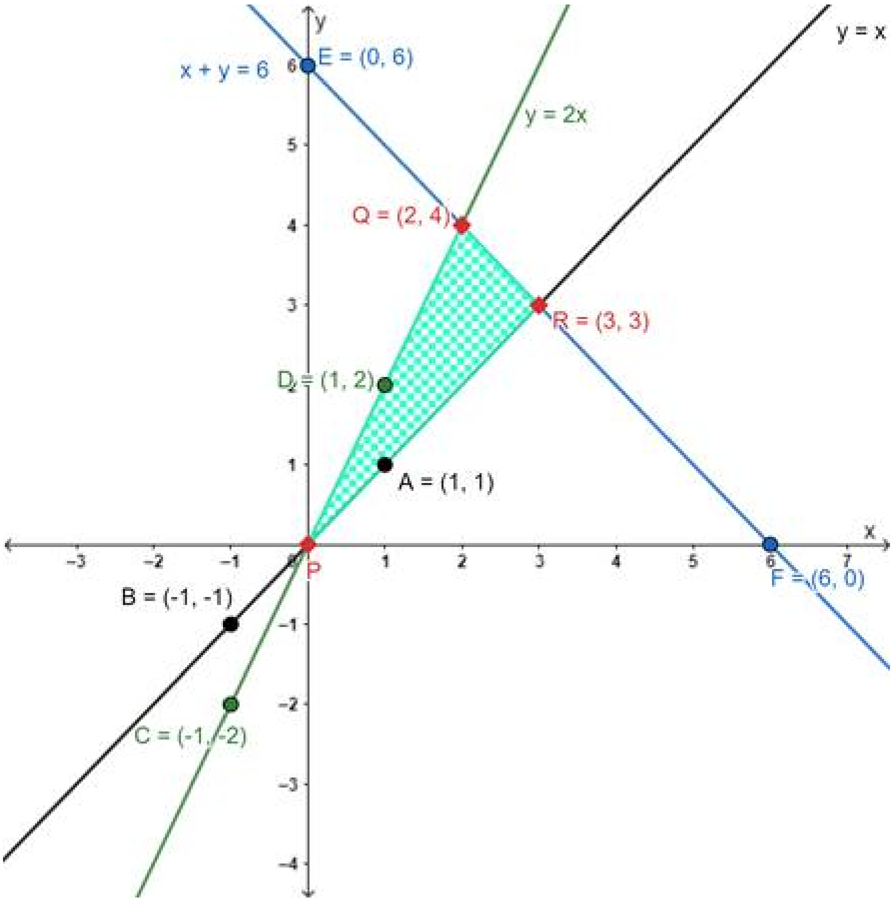
x	1	- 1
y = x	1	- 1

Now, table for y = 2x

x	1	- 1
y = 9 - 3x	2	- 2

Now, table for x + y = 6 or y = 6 – x

x	0	6
y = 6 - x	6	0



The coordinates of the vertices of ΔPQR are P(0, 0), Q(2, 4)and R(3, 3)

20 C. Question

Use a single graph paper and draw the graph of the following equations. Obtain the vertices of the triangle so formed:

y = x

3y = x

x + y = 8

Answer

The given equation is

y = x

3y = x

and x + y = 8

Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for y = x

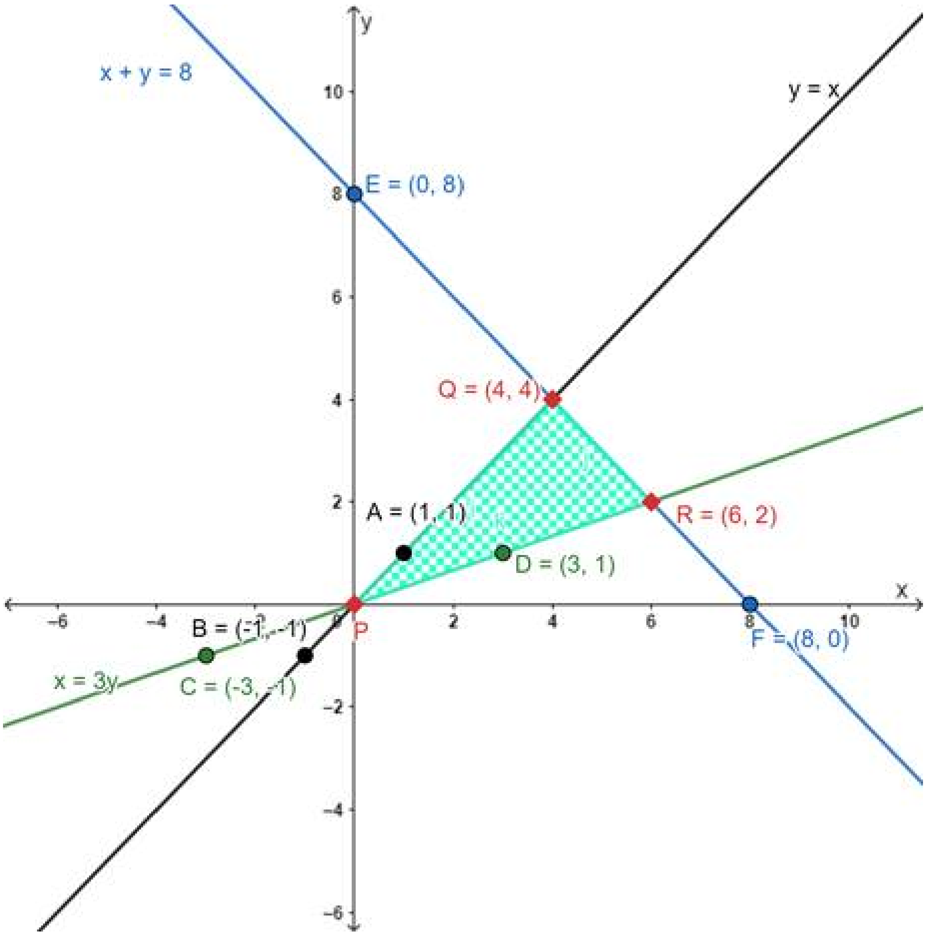
x	1	- 1
y = x	1	- 1

Now, table for 3y = x

x	3	- 3
$y = \frac{x}{3}$	1	- 1

Now, table for x + y = 8 or y = 8 – x

x	0	8
y = 8 - x	8	0



The coordinates of the vertices of ΔPQR are P(0, 0), Q(4, 4)and R(6, 2)

11 A. Question

Find the values of a and b for which the following system of linear equations has infinitely many solutions:

$2x + 3y = 7$, $(a + b) x + (2a - b) y = 3(a + b + 1)$

Answer

Given, pair of equations

$2x + 3y = 7$

and $(a + b)x + (2a - b)y = 3(a + b + 1)$

On comparing the given equation with standard form i.e. $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, we get

$a_1 = 2$, $b_1 = 3$ and $c_1 = -7$

and $a_2 = (a + b)$, $b_2 = (2a - b)$ and $c_2 = -(a + b + 1)$

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Here, $\frac{a_1}{a_2} = \frac{2}{a+b}, \frac{b_1}{b_2} = \frac{3}{2a-b}$ and $\frac{c_1}{c_2} = \frac{-7}{-3(a+b+1)}$

$$\therefore \underset{\text{I}}{\frac{2}{a+b}} = \underset{\text{II}}{\frac{3}{2a-b}} = \underset{\text{III}}{\frac{-7}{-3(a+b+1)}}$$

On taking I and II terms, we get

$$\frac{2}{a+b} = \frac{3}{2a-b}$$

$\Rightarrow 2(2a - b) = 3(a + b)$

$\Rightarrow 4a - 2b = 3a + 3b$

$\Rightarrow 4a - 3a - 3b - 2b = 0$

$\Rightarrow a - 5b = 0$ ...(1)

On taking I and III terms, we get

$$\Rightarrow \frac{2}{a+b} = \frac{7}{3(a+b+1)}$$

$\Rightarrow 6(a + b + 1) = 7(a + b)$

$\Rightarrow 6a + 6b + 6 = 7a + 7b$

$\Rightarrow 6a - 7a + 6b - 7b = -6$

$\Rightarrow -a - b = -6$

$\Rightarrow a + b = 6$ ...(2)

Solving eq^n (1) and (2), we get

$$\begin{array}{rcl} a - 5b &=& 0 \\ a + b &=& 6 \\ - \quad - & & - \\ \hline -6b &=& -6 \end{array}$$

$\Rightarrow$ **b = 1**

Now, substituting the value of b in eq^n (2), we get

$\Rightarrow a + b = 6$

$\Rightarrow a + 1 = 6$

$\Rightarrow$ **a = 5**

21 B. Question

Find the values of a and b for which the following system of linear equations has infinitely many solutions:

$(2a - 1)x - 3y = 5, 3x + (b - 2)y = 3$

Answer

Given, pair of equations

$(2a - 1)x - 3y = 5$

and $3x + (b - 2)y = 3$

On comparing the given equation with standard form i.e. $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, we get

$a_1 = (2a - 1)$, $b_1 = -3$ and $c_1 = -5$

and $a_2 = 3$, $b_2 = b - 2$ and $c_2 = 3$

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Here, $\frac{a_1}{a_2} = \frac{2a-1}{3}, \frac{b_1}{b_2} = \frac{-3}{b-2}$ and $\frac{c_1}{c_2} = \frac{-5}{-3} = \frac{5}{3}$

$$\therefore \underset{\text{I}}{\frac{2a-1}{3}} = \underset{\text{II}}{\frac{-3}{b-2}} = \underset{\text{III}}{\frac{5}{3}}$$

On taking I and III terms, we get

$$\frac{2a-1}{3} = \frac{5}{3}$$

$\Rightarrow 3(2a - 1) = 15$

$\Rightarrow 6a - 3 = 15$

$\Rightarrow 6a = 15 + 3$

$$\Rightarrow \mathbf{a} = \frac{18}{6} = \mathbf{3}$$

On taking II and III terms, we get

$$\Rightarrow \frac{-3}{b-2} = \frac{5}{3}$$

$\Rightarrow -9 = 5(b - 2)$

$\Rightarrow 5b - 10 = -9$

$\Rightarrow 5b = -9 + 10$

$$\Rightarrow \mathbf{b} = \frac{\mathbf{1}}{\mathbf{5}}$$

21 C. Question

Find the values of a and b for which the following system of linear equations has infinitely many solutions:

$kx + 3y - (k - 3) = 0, 12x + ky - k = 0$

Answer

Given, pair of equations

$kx + 3y - (k - 3) = 0$

and $12x + ky - k = 0$

On comparing the given equation with standard form i.e. $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, we get

$a_1 = k, b_1 = 3$ and $c_1 = -(k - 3)$

and $a_2 = 12, b_2 = k$ and $c_2 = -k$

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Here, $\frac{a_1}{a_2} = \frac{k}{12}, \frac{b_1}{b_2} = \frac{3}{k}$ and $\frac{c_1}{c_2} = \frac{-(k-3)}{-k} = \frac{(k-3)}{k}$

$\therefore \frac{k}{12} = \frac{3}{k} = \frac{(k-3)}{k}$ ...(1)

I II III

On taking I and II terms, we get

$$\frac{k}{12} = \frac{3}{k}$$

$\Rightarrow k^2 = 36$

$\Rightarrow k = \sqrt{36}$

$\Rightarrow k = \pm 6$

But k = – 6 not satisfies the last two terms of eq^n (1)

On taking II and III terms, we get

$$\frac{3}{k} = \frac{(k-3)}{k}$$

$\Rightarrow 3k = k(k - 3)$

$\Rightarrow 3k = k^{2} - 3k$

$\Rightarrow k^2 - 3k - 3k = 0$

$\Rightarrow k(k - 6) = 0$

$\Rightarrow k = 0$ and 6

Which satisfies the last two terms of eq^n (1)

Hence, the required value of k = 0, 6

21 D. Question

Find the values of a and b for which the following system of linear equations has infinitely many solutions:

$3x + 4y = 12, (a + b) x + 2 (a - b) y = 5a - 1$

Answer

Given, pair of equations

$3x + 4y = 12$

and $(a + b)x + 2(a - b)y = 5a - 1$

On comparing the given equation with standard form i.e. $a_1 x + b_1y + c_1 = 0$ and $a_2 x + b_2y + c_2 = 0$, we get

$a_1 = 3, b_1 = 4$ and $c_1 = -12$

and $a_2 = (a + b), b_2 = 2(a - b)$ and $c_2 = -(5a - 1) = 1 + 5a$

$a_1 = 3, \quad b_1 = 4, c_1 = -12$

and $a_2 = (a + b), \ b_2 = 2(a - b), c_2 = -(5a - 1) = 1 + 5a$

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Here, $\frac{a_1}{a_2} = \frac{3}{a+b}, \frac{b_1}{b_2} = \frac{4}{2(a-b)}$ and $\frac{c_1}{c_2} = \frac{-12}{-(5a-1)} = \frac{12}{(5a-1)}$

$$\therefore \frac{3}{a+b} = \frac{4}{2(a-b)} = \frac{-12}{-(5a-1)}$$

I II III

On taking I and II terms, we get

$$\frac{3}{a + b} = \frac{4}{2(a - b)}$$

$\Rightarrow 6(a - b) = 4(a + b)$

$\Rightarrow 6a - 6b = 4a + 4b$

$\Rightarrow 6a - 4a - 6b - 4b = 0$

$\Rightarrow 2a - 10b = 0$

$\Rightarrow a - 5b = 0$ ...(1)

On taking I and III terms, we get

$$\Rightarrow \frac{3}{a + b} = \frac{12}{5a - 1}$$

$\Rightarrow 3(5a - 1) = 12(a + b)$

$\Rightarrow 15a - 3 = 12a + 12b$

$\Rightarrow 15a - 12a - 12b = 3$

$\Rightarrow 3a - 12b = 3$

$\Rightarrow a - 4b = 1$ ...(2)

Solving eq^n (1) and (2), we get

$$\begin{array}{l} a - 5b = 0 \\ a - 4b = 1 \\ -\ \ +\ \ \ \ \ - \\ \hline -b = -1 \end{array}$$

$\Rightarrow$ **b = 1**

Now, substituting the value of b in eq^n (2), we get

$\Rightarrow a - 4b = 1$

$\Rightarrow a - 4 = 1$

$\Rightarrow$ **a = 1 + 4**

$\Rightarrow$ **a = 5**

21 E. Question

Find the values of a and b for which the following system of linear equations has infinitely many solutions:

$(a - 1)\,x + 3y = 2,\ 6x + (1 - 2b)\,y = 6$

Answer

Given, pair of equations

$(a - 1)x + 3y = 2$

and $6x + (1 - 2b)y = 6$

On comparing the given equation with standard form i.e. $a_1 x + b_1y + c_1 = 0$ and $a_2 x + b_2y + c_2 = 0$, we get

$a_1 = (a - 1),\ b_1 = 3$ and $c_1 = - 2$

and $a_2 = 6,\ b_2 = 1 - 2b$ and $c_2 = - 6$

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Here, $\frac{a_1}{a_2} = \frac{a-1}{6}, \frac{b_1}{b_2} = \frac{3}{1-2b}$ and $\frac{c_1}{c_2} = \frac{-2}{-6} = \frac{1}{3}$

$$\therefore \underset{\text{I}}{\frac{a-1}{6}} = \underset{\text{II}}{\frac{3}{1-2b}} = \underset{\text{III}}{\frac{1}{3}}$$

On taking I and III terms, we get

$$\frac{a-1}{6} = \frac{1}{3}$$

⇒ $3(a - 1) = 6$

⇒ $3a - 3 = 6$

⇒ $3a = 6 + 3$

$$\Rightarrow \mathbf{a} = \frac{9}{3} = \mathbf{3}$$

On taking II and III terms, we get

$$\Rightarrow \frac{3}{1-2b} = \frac{1}{3}$$

⇒ $9 = 1 - 2b$

⇒ $- 2b = 9 - 1$

⇒ $- 2b = 8$

⇒ $\mathbf{b = -4}$

21 F. Question

Find the values of a and b for which the following system of linear equations has infinitely many solutions:

$2x + 3y = 7$, $(a + b + 1)x + (a + 2b + 2)y = 4(a + b) + 1$

Answer

Given, pair of equations

$2x + 3y = 7$

and $(a + b + 1)x + (a + 2b + 2)y = 4(a + b) + 1$

On comparing the given equation with standard form i.e. $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

$a_1 = 2$, $b_1 = 3$ and $c_1 = -7$

and $a_2 = (a + b + 1)$, $b_2 = (a + 2b + 2)$ and $c_2 = -\{4(a + b) + 1\}$

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Here, $\frac{a_1}{a_2} = \frac{2}{a+b+1}, \frac{b_1}{b_2} = \frac{3}{a+2b+2}$ and $\frac{c_1}{c_2} = \frac{-7}{-\{4(a+b)+1\}}$

$$\therefore \underset{I}{\frac{2}{a+b+1}} = \underset{II}{\frac{3}{a+2b+2}} = \underset{III}{\frac{7}{\{4(a+b)+1\}}}$$

On taking I and II terms, we get

$$\frac{2}{a+b+1} = \frac{3}{a+2b+2}$$

$\Rightarrow 2(a + 2b + 2) = 3(a + b + 1)$

$\Rightarrow 2a + 4b + 4 = 3a + 3b + 3$

$\Rightarrow 2a - 3a - 3b + 4b = 3 - 4$

$\Rightarrow - a + b = - 1$

$\Rightarrow a - b = 1$ ...(1)

On taking I and III terms, we get

$$\Rightarrow \frac{2}{a+b+1} = \frac{7}{\{4(a+b)+1\}}$$

$\Rightarrow 2\{4(a + b) + 1)\} = 7(a + b + 1)$

$\Rightarrow 2(4a + 4b + 1) = 7a + 7b + 7$

$\Rightarrow 8a - 7a + 8b - 7b = - 2 + 7$

$\Rightarrow a + b = 5$ ...(2)

Solving eq^n (1) and (2), we get

$$\begin{array}{l} a - b = 1 \\ \underline{a + b\ = 5} \\ \quad 2a = 6 \end{array}$$

$\Rightarrow$ **a = 3**

Now, substituting the value of a in eq^n (1), we get

$\Rightarrow a - b = 1$

$\Rightarrow 3 - b = 1$

$\Rightarrow b = 2$

22 A. Question

For what value of a, the following system of linear equations has no solutions:

ax + 3y = a – 2, 12x + ay = a

Answer

Given, pair of equations

ax + 3y = a – 2

and 12x + ay = a

On comparing the given equation with standard form i.e. $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, we get

$a_1 = a$, $b_1 = 3$ and $c_1 = -(a-2)$

and $a_2 = 12$, $b_2 = a$ and $c_2 = -a$

For no solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\therefore \underset{\text{I}}{\frac{a}{12}} = \underset{\text{II}}{\frac{3}{a}} \neq \underset{\text{III}}{\frac{-(a-2)}{-a}}$$

On taking I and II terms, we get

$$\frac{a}{12} = \frac{3}{a}$$

$\Rightarrow a^2 = 36$

$\Rightarrow a = \sqrt{36}$

$\Rightarrow a = \pm 6$

22 B. Question

For what value of a, the following system of linear equations has no solutions:

x + 2y = 5, 3x + ay + 15 = 0

Answer

Given, pair of equations

x + 2y = 5

and $3x + ay + 15 = 0$

On comparing the given equation with standard form i.e. $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, we get

$a_1 = 1$, $b_1 = 2$ and $c_1 = -5$

and $a_2 = 3$, $b_2 = a$ and $c_2 = 15$

For no solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\therefore \quad \underset{\text{I}}{\frac{1}{3}} = \underset{\text{II}}{\frac{2}{a}} \neq \underset{\text{III}}{\frac{-5}{15}}$$

On taking I and II terms, we get

$$\frac{1}{3} = \frac{2}{a}$$

$$\Rightarrow \mathbf{a = 6}$$

22 C. Question

For what value of a, the following system of linear equations has no solutions:

$3x + y = 1$, $(2a - 1)x + (a - 1)y = 2a + 1$

Answer

Given, pair of equations

$3x + y = 1$

and $(2a - 1)x + (a - 1)y = 2a + 1$

On comparing the given equation with standard form i.e. $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, we get

$a_1 = 3$, $b_1 = 1$ and $c_1 = -1$

and $a_2 = (2a - 1)$, $b_2 = (a - 1)$ and $c_2 = -(2a + 1)$

For no solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$\therefore \frac{3}{2a-1} = \frac{1}{a-1} \neq \frac{-1}{-(2a+1)}$

I II III

On taking I and II terms, we get

$$\frac{3}{2a-1} = \frac{1}{a-1}$$

⇒ 3(a – 1) = 2a – 1

⇒ 3a – 3 = 2a – 1

⇒ 3a – 2a = – 1 + 3

⇒ **a = 2**

22 D. Question

For what value of a, the following system of linear equations has no solutions:

$(3a + 1)x + 3y - 2 = 0$, $(a^{2} + 1)x + (a - 2)y - 5 = 0$

Answer

Given, pair of equations

$(3a + 1)x + 3y - 2 = 0$

and $(a^2 + 1)x + (a - 2)y - 5 = 0$

On comparing the given equation with standard form i.e. $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, we get

$a_1 = 3a + 1$, $b_1 = 3$ and $c_1 = -2$

and $a_2 = a^2 + 1$, $b_2 = a - 2$ and $c_2 = -5$

For no solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$\therefore \frac{3a+1}{(a^2+1)} = \frac{3}{a-2} \neq \frac{-2}{-5}$

I II III

On taking I and II terms, we get

$$\frac{3a + 1}{(a^2 + 1)} = \frac{3}{a - 2}$$

$\Rightarrow (3a + 1)(a - 2) = 3(a^2 + 1)$

$\Rightarrow 3a^2 - 6a + a - 2 = 3a^2 + 3$

$\Rightarrow -5a = 2 + 3$

$\Rightarrow$ **$a = -1$**

23 A. Question

For what value of c, the following system of linear equations has infinite number of solutions:

$cx + 3y - (c - 3) = 0, 12x + cy - c = 0$

Answer

Given, pair of equations

$cx + 3y - (c - 3) = 0$

and $12x + cy - c = 0$

On comparing the given equation with standard form i.e. $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, we get

$a_1 = c$, $b_1 = 3$ and $c_1 = -(c - 3)$

and $a_2 = 12$, $b_2 = c$ and $c_2 = -c$

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Here, $\frac{a_1}{a_2} = \frac{c}{12}, \frac{b_1}{b_2} = \frac{3}{c}$ and $\frac{c_1}{c_2} = \frac{-(c-3)}{-c} = \frac{c-3}{c}$

$\therefore \frac{c}{12} = \frac{3}{c} = \frac{c-3}{c}$ ...(1)

I II III

On taking I and III terms, we get

$$\frac{c}{12} = \frac{3}{c}$$

$\Rightarrow c^2 = 36$

$\Rightarrow c = \sqrt{36}$

$\Rightarrow c = \pm 6$

But c = – 6 not satisfies the eq^n (1)

Hence, the required value of c = 6.

23 B. Question

For what value of c, the following system of linear equations has infinite number of solutions:

2x + 3y = 2, (c + 2) x + (2c + 1) y = 2 (c – 1)

Answer

Given, pair of equations

2x + 3y = 2

and (c + 2)x + (2c + 1)y = 2(c – 1)

On comparing the given equation with standard form i.e. $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, we get

$a_1 = 2$, $b_1 = 3$ and $c_1 = -2$

and $a_2 = c + 2$, $b_2 = 2c + 1$ and $c_2 = -2(c - 1)$

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Here, $\frac{a_1}{a_2} = \frac{2}{c+2}, \frac{b_1}{b_2} = \frac{3}{2c+1}$ and $\frac{c_1}{c_2} = \frac{-2}{-2(c-1)} = \frac{1}{c-1}$

$$\underset{\text{I}}{\frac{2}{c+2}} = \underset{\text{II}}{\frac{3}{2c+1}} = \underset{\text{III}}{\frac{1}{c-1}} \quad \ldots(1)$$

On taking I and III terms, we get

$$\frac{2}{c + 2} = \frac{3}{2c + 1}$$

$\Rightarrow 2(2c + 1) = 3(c + 2)$

$\Rightarrow 4c + 2 = 3c + 6$

$\Rightarrow 4c - 3c = 6 - 2$

$\Rightarrow$ **c = 4**

Hence, the required value of c = 4.

23 C. Question

For what value of c, the following system of linear equations has infinite number of solutions:

$x + (c + 1) y = 5, (c + 1) x + 9y = 8c - 1$

Answer

The pair of equations are:

$x + (c + 1) y = 5$

$(c + 1) x + 9y = 8c - 1$

These equations can be written as:

$x + (c + 1) y - 5 = 0$

$(c + 1) x + 9y - (8c - 1) = 0$

On comparing the given equation with standard form i.e.

$a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, we get

$a_1 = 1 , b_1 = c + 1 , c_1 = -5$

$a_2 = c + 1, b_2 = 9 , c_2 = -(8c - 1)$

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

So,

$$\frac{a_1}{a_2} = \frac{1}{c+1}, \qquad \frac{b_1}{b_2} = \frac{c+1}{9}, \qquad \frac{c_1}{c_2} = \frac{-5}{-8c+1}$$

So,

$$\frac{1}{c+1} = \frac{c+1}{9} = \frac{-5}{-8c+1}$$

(I) (II) (III)

$\Rightarrow$ From (I) and (II)

$$\frac{1}{c+1} = \frac{c+1}{9}$$

⇒ $9 = (c+1)^2$

⇒ $9 = c^2 + 1 + 2c$

⇒ $9 - 1 = c^2 + 2c$

⇒ $8 = c^2 + 2c$

⇒ $c^2 + 2c - 8 = 0$

Factorize by splitting the middle term,

$c^2 + 4c - 2c - 8 = 0$

⇒ $c (c + 4) - 2 (c + 4) = 0$

⇒ $(c+4) (c-2) = 0$

⇒ $c = -4, c = 2$

From (II) and (III)

$$\frac{c+1}{9} = \frac{-5}{-8c+1}$$

⇒ $(c+1)(-8c=1) = -5 \times 9$

⇒ $-8c^2 + c - 8c + 1 = -45$

⇒ $-8c^2 + c - 8c + 1 + 45 = 0$

⇒ $-8c^2 - 7c + 46 = 0$

⇒ $8c^2 + 7c - 46 = 0$

⇒ $8c^2 - 16c + 23c -46 = 0$

⇒ $8c (c-2) + 23 (c-2) = 0$

⇒ $(8c+23) (c-2) = 0$

⇒ $c = -23/8$ and $c = 2$

From (I) and (III)

$$\frac{1}{c+1} = \frac{-5}{-8c+1}$$

⇒ $-8c+1 = -5(c+1)$

$\Rightarrow -8c + 1 = -5c - 5$

$\Rightarrow -8c + 5c = -5 - 1$

$\Rightarrow -3c = -6$

$\Rightarrow c = 2$

So the value of c = 2.

23 D. Question

For what value of c, the following system of linear equations has infinite number of solutions:

$(c - 1)\,x - y = 5,\ (c + 1)\,x + (1 - c)\,y = 3c + 1$

Answer

The pair of equations are:

$(c - 1)\,x - y = 5$

$(c + 1)\,x + (1 - c)\,y = 3c + 1$

These equations can be written as:

$(c - 1)\,x - y - 5 = 0$

$(c + 1)\,x + (1 - c)\,y - (3c + 1)$

On comparing the given equation with standard form i.e.

$a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, we get

$a_1 = c - 1,\ b_1 = -1,\ c_1 = -5$

$a_2 = c + 1,\ b_2 = 1 - c,\ c_2 = -(3c + 1)$

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

So,

$$\frac{a_1}{a_2} = \frac{c-1}{c+1}, \qquad \frac{b_1}{b_2} = \frac{-1}{1-c}, \qquad \frac{c_1}{c_2} = \frac{-5}{-3c-1}$$

So,

$$\frac{c-1}{c+1} = \frac{-1}{1-c} = \frac{-5}{-3c-1}$$

(I) (II) (III)

From (I) and (II)

$$\frac{c-1}{c+1} = \frac{-1}{1-c}$$

⇒ $(c-1)(1-c) = -(c+1)$

⇒ $c - c^2 - 1 + c = -c - 1$

⇒ $c - c^2 - 1 + c + c + 1 = 0$

⇒ $3c - c^2 = 0$

⇒ $c(3 - c) = 0$

⇒ $c = 0, c = 3$

From (II) and (III)

$$\frac{-1}{1-c} = \frac{-5}{-3c-1}$$

⇒ $-(-3c-1) = -5(1-c)$

⇒ $3c + 1 = -5 + 5c$

⇒ $3c + 1 + 5 - 5c = 0$

⇒ $6 - 2c = 0$

⇒ $6 = 2c$

⇒ $c = 3$

From (I) and (III)

$$\frac{c-1}{c+1} = \frac{-5}{-3c-1}$$

⇒ $(c-1)(-3c-1) = -5(c+1)$

⇒ $-3c^2 - c + 3c + 1 = -5c - 5$

⇒ $-3c^2 - c + 3c + 1 + 5c + 5 = 0$

⇒ $-3c^2 + 7c + 6 = 0$

⇒ $3c^2 - 7c - 6 = 0$

$\Rightarrow 3c^2 - 9c + 2c - 6 = 0$

$\Rightarrow 3c\,(c-3) + 2\,(c-3) = 0$

$\Rightarrow (3c+2)\,(c-3) = 0$

$\Rightarrow c = -2/3$ and $c = 3$

Hence the value of c is 3.

24. Question

Solve the following system of equations graphically. Also determine the vertices of the triangle formed by the lines and y – axis.

$4x - 5y - 20 = 0, 3x + 5y - 15 = 0$

Answer

The given equation is

$4x - 5y = 20$

and $3x + 5y = 15$

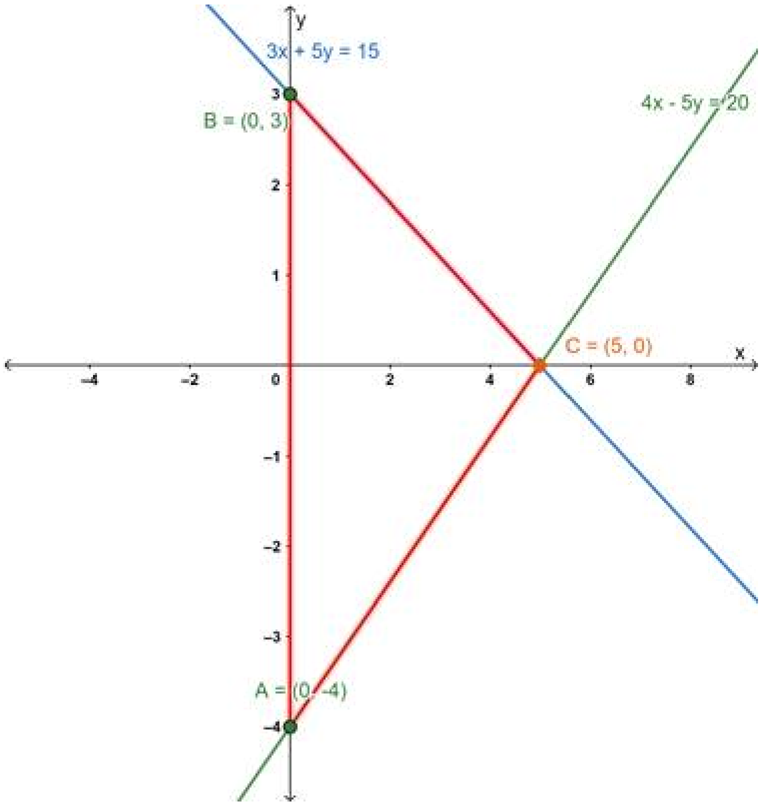
Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for $4x - 5y = 20$ or $y = \frac{4x-20}{5}$

x	0	5
$y = \frac{4x-20}{5}$	- 4	0

Now, table for $3x + 5y = 15$ or $y = \frac{15-3x}{5}$

x	0	5
$y = \frac{15-3x}{5}$	3	0



Here, the lines are intersecting at point C (5, 0).

The coordinates of the vertices of ΔABC are A(0, – 4), B(0, 3)and C(5,0)

25 A. Question

Find the value of a for which the following system of equations has unique solution:

ax + 2y = 5, 3x + y = 1

Answer

Given, pair of equations

ax + 2y = 5

and 3x + y = 1

On comparing the given equation with standard form i.e. $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, we get

$a_1 = a$, $b_1 = 2$ and $c_1 = -5$

and $a_2 = 3$, $b_2 = 1$ and $c_2 = -1$

For unique solutions,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Here, $\frac{a_1}{a_2} = \frac{a}{3}, \frac{b_1}{b_2} = \frac{2}{1}$

$\therefore \frac{a}{3} \neq \frac{2}{1}$ I II

On taking I and II terms, we get

$$\frac{a}{3} \neq \frac{2}{1}$$

$\Rightarrow a \neq 6$

Thus, given lines have a unique solution for all real values of **a,** except 6.

25 B. Question

Find the value of a for which the following system of equations has unique solution:

$9x + py - 1 = 0, 3x + 4y - 2 = 0$

Answer

Given, pair of equations

$9x + py - 1 = 0$

and $3x + 4y - 2 = 0$

On comparing the given equation with standard form i.e. $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, we get

$a_1 = 9, b_1 = p$ and $c_1 = - 1$

and $a_2 = 3, b_2 = 4$ and $c_2 = - 2$

For unique solutions,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Here, $\frac{a_1}{a_2} = \frac{9}{3} = 3, \frac{b_1}{b_2} = \frac{p}{4}$

$\therefore 3 \neq p/4$ I II

On taking I and II terms, we get

$p \neq 12$

Thus, given lines have a unique solution for all real values of **p,** except 12.

25 C. Question

Find the value of a for which the following system of equations has unique solution:

$3x + 2y = 4, ax - y = 3$

Answer

Given, pair of equations

$3x + 2y = 4$

and $ax - y = 3$

On comparing the given equation with standard form i.e. $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, we get

$a_1 = 3, b_1 = -2$ and $c_1 = -4$

and $a_2 = a, b_2 = -1$ and $c_2 = -3$

For unique solutions,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Here, $\frac{a_1}{a_2} = \frac{3}{a}, \frac{b_1}{b_2} = -\frac{2}{1}$

$$\therefore \underset{\text{I}}{\frac{3}{a}} \neq \underset{\text{II}}{\frac{2}{-1}}$$

On taking I and II terms, we get

$$\frac{3}{a} \neq \frac{2}{-1}$$

$$\Rightarrow a \neq -\frac{3}{2}$$

Thus, given lines have a unique solution for all real values of **a,** except $-\frac{3}{2}$.

25 D. Question

Find the value of a for which the following system of equations has unique solution:

$4x + py + 8 = 0, 2x + 2y + 2 = 0$

Answer

Given, pair of equations

4x + py + 8 = 0

and 2x + 2y + 2 = 0

On comparing the given equation with standard form i.e. $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, we get

$a_1 = 4$, $b_1 = p$ and $c_1 = 8$

and $a_2 = 2$, $b_2 = 2$ and $c_2 = 2$

For unique solutions,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Here, $\frac{a_1}{a_2} = \frac{4}{2} = 2, \frac{b_1}{b_2} = \frac{p}{2}$

$\therefore \underset{\text{I}}{2} \neq \underset{\text{II}}{\frac{p}{2}}$

On taking I and II terms, we get

$$2 \neq \frac{p}{2}$$

$\Rightarrow p \neq 4$

Thus, given lines have a unique solution for all real values of **p,** except 4.

26. Question

10 students of class X took part in mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

Answer

Let the number of boys = x

and the number of girls = y

Now, table for x + y – 10 = 0

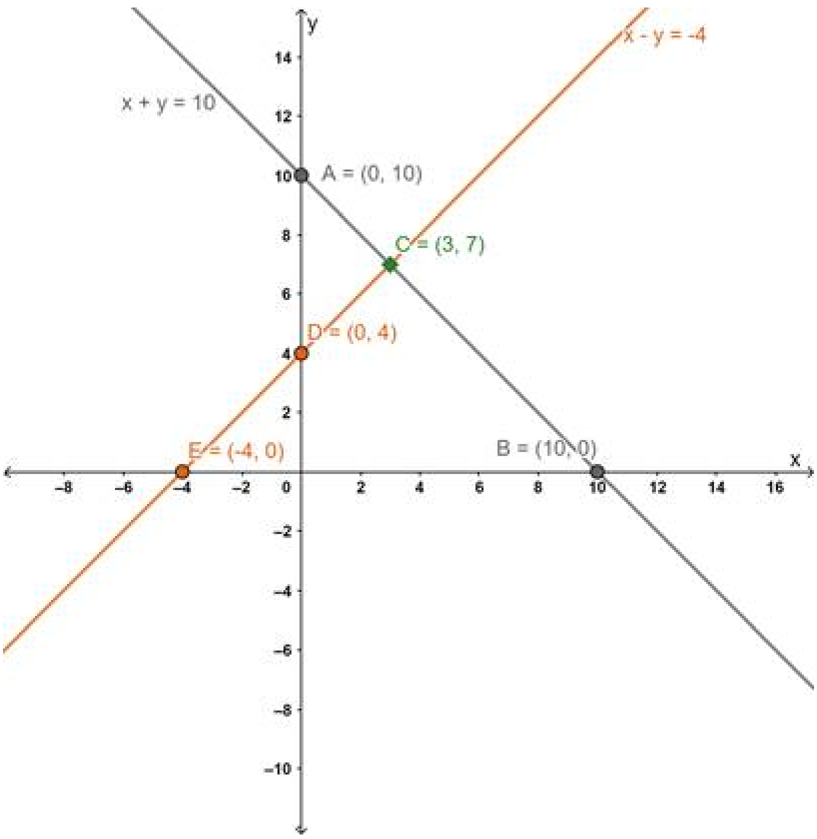
x	0	10
y = 10 - x	10	0

Now, table for x – y + 4 = 0

x	0	- 4
y	4	0

On plotting points on a graph paper and join them to get a straight line representing x + y – 10 = 0.

Similarly, on plotting the points on the same graph paper and join them to get a straight line representing x – y + 4 = 0.



∴ x = 3, y = 7 is the solution of the pair of linear equations.

Hence, the required number of boys is 3 and girls is 7.

27. Question

Form the pair of linear equations in the following problems and find their solutions graphically. Two years ago, a father was five times as old as his son. Two years later, his age will be 8 more than three times the age of the son.

Answer

Let the present age of father = x year

and the present age of his son = y year

Two years ago,

Father's age = (x – 2) year

His son's age = (y – 2) year

According to the question,

⇒ (x – 2) = 5(y – 2)

⇒ x – 2 = 5y – 10

⇒ x – 5y + 8 = 0 ...(1)

After two years,

Father's age = (x + 2) year

His son's age = (y + 2) year

According to the question,

⇒ (x + 2) = 3(y + 2) + 8

⇒ x + 2 = 3y + 6 + 8

⇒ x – 3y – 12 = 0 ...(2)

Now, table for x – 5y + 8 = 0

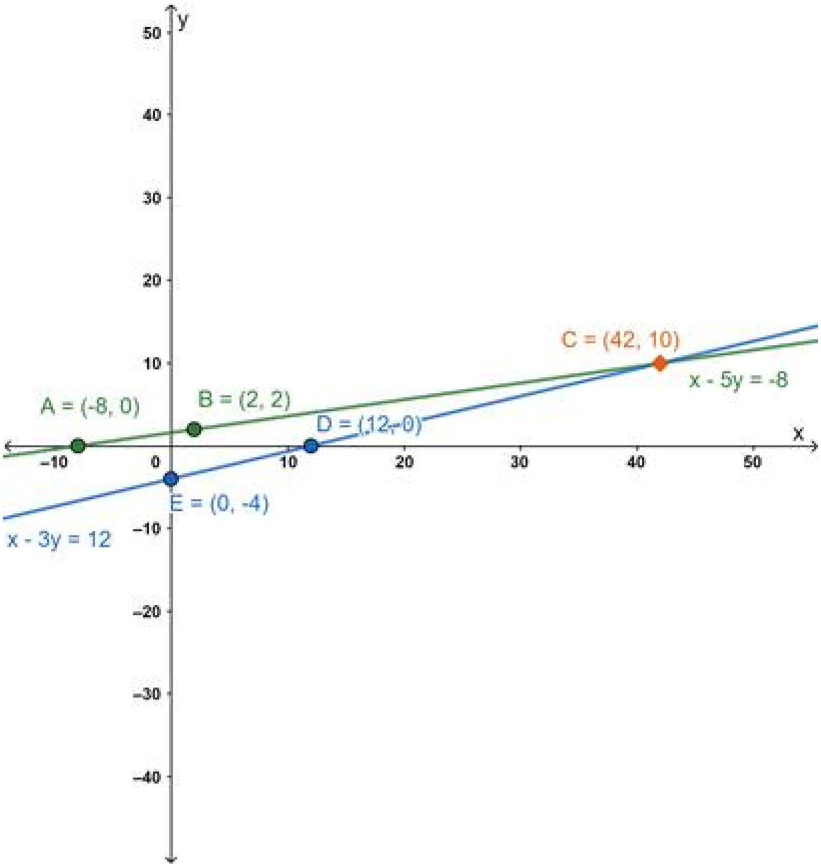
x	2	- 8
$y = \frac{x + 8}{5}$	2	0

Now, table for x – 3y – 12 = 0

x	0	12
$y = \frac{x - 12}{3}$	- 4	0

On plotting points on a graph paper and join them to get a straight line representing x – 5y + 8 = 0.

Similarly, on plotting the points on the same graph paper and join them to get a straight line representing x – 3y – 12 = 0.



∴ x = 42, y = 10 is the solution of the pair of linear equations.

Hence, the age of father is 42years and age of his son is 10 years.

28. Question

Champa went to a 'sale' to purchase some pants and skirts. When her friends asked her how many of each she had bought, she answered, "The number of skirts is two less than twice the number of pants purchased. Also the number of skirts is four less than tbur times the number of pants purchased". Find how many pants and skirts Champa bought?

Answer

Let the number of pants = x

and the number of skirts = y

According to the question

Number of skirts = 2(Number of pants) – 2

y = 2x – 2 …(i)

Also, Number of skirts = 4(Number of pants) – 4

y = 4x – 4 …(ii)

Substituting the value of y = 4x – 4 in eq^{n} (i),we get

4x – 4 = 2x – 2

⇒ 4x – 2x – 4 + 2 = 0

⇒ 2x = 2

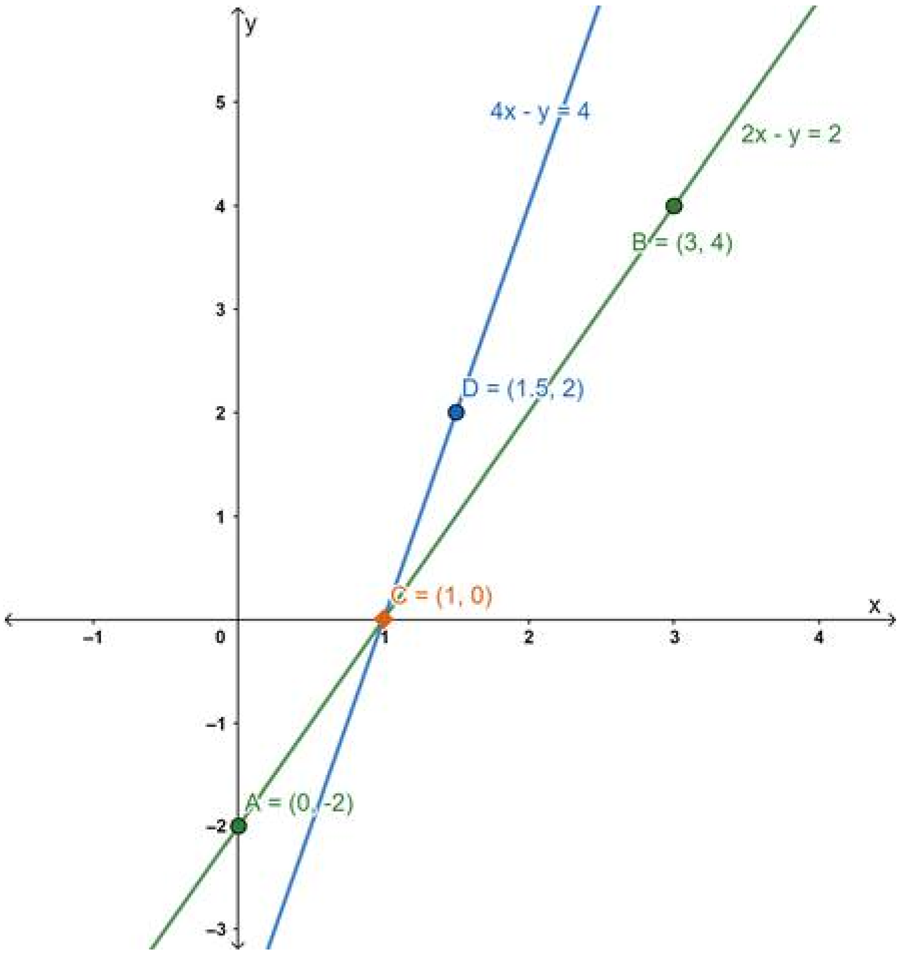
⇒ **x = 1**

Now, substitute the value of x in eq^n (ii), we get

y = 4(1) – 4 **= 0**

∴ x = 1, y = 0 is the solution of the pair of linear equations.

We can solve this problem through graphically also



Hence, the number of pants she purchased is 1 and the number of skirts, she purchased is zero i.e., she didn't buy any skirt.

29. Question

Priyanka purchased 2 pencils and 3 erasers for Rs. 9. Sayeeda purchased 1 pencil and two erasers for Rs. 5. Find the cost of one pencil and one eraser.

Answer

Let the cost of one pencil = Rs x

and the cost of one eraser = Rs y

According to the question

2x + 3y = 9 …(i)

x + 2y = 5 …(ii)

Now, table for 2x + 3y = 9

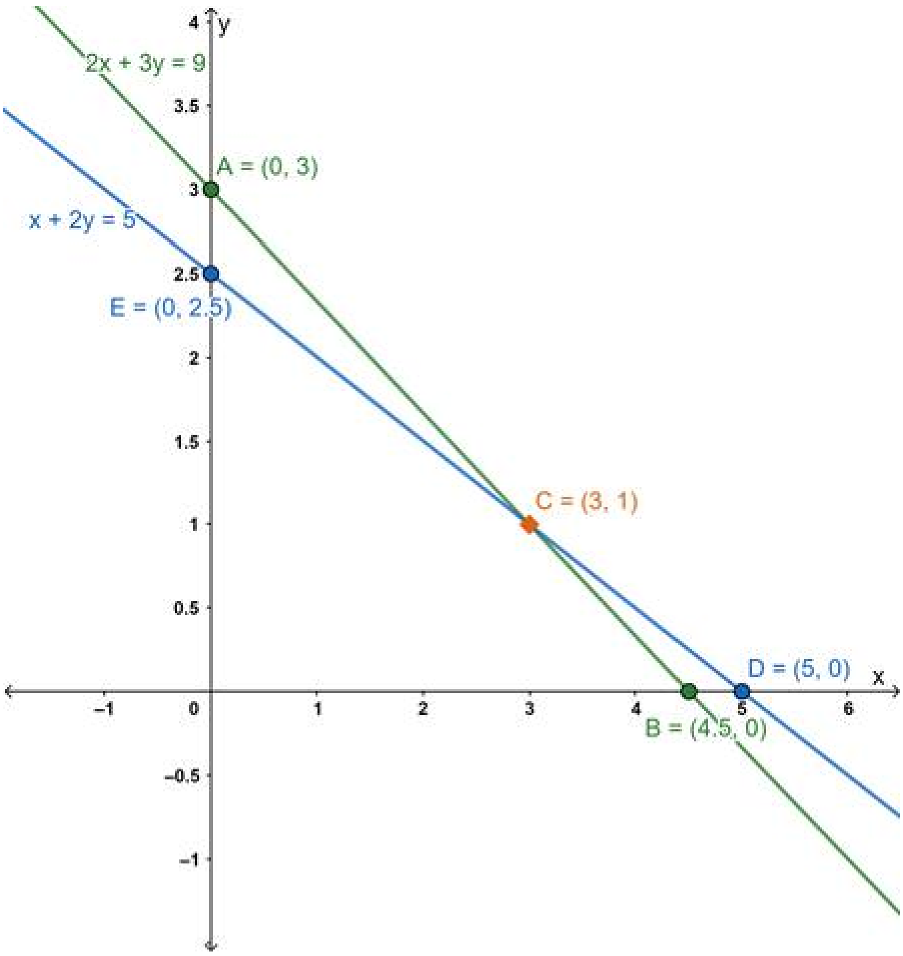
x	0	4.5
$y = \frac{9 - 2x}{3}$	3	0

Now, table for x + 2y = 5

x	0	5
$y = \frac{5 - x}{2}$	2.5	0

On plotting points on a graph paper and join them to get a straight line representing 2x + 3y = 9.

Similarly, on plotting the points on the same graph paper and join them to get a straight line representing x + 2y = 5.



∴ x = 3, y = 1 is the solution of the pair of linear equations.

Hence, the cost of one pencil is Rs 3 and cost of one eraser is Rs 1.

Exercise 3.2

1 A. Question

Solve the following pair of linear equations by substitution method:

7x— 15y = 2

x + 2y = 3

Answer

Given equations are

7x – 15y = 2 …(i)

x + 2y = 3 …(ii)

From eq^n (ii), x = 3 – 2y …(iii)

On substituting x = 3 – 2y in eq^n (i), we get

⇒ 7(3 – 2y) – 15y = 2

$\Rightarrow 21 - 14y - 15y = 2$

$\Rightarrow 21 - 29y = 2$

$\Rightarrow -29y = -19$

$$\Rightarrow y = \frac{19}{29}$$

Now, on putting $y = \frac{19}{29}$ in eq^n (iii), we get

$$\Rightarrow x = 3 - 2\left(\frac{19}{29}\right)$$

$$\Rightarrow x = 3 - \frac{38}{29}$$

$$\Rightarrow x = \frac{87 - 38}{29}$$

$$\Rightarrow x = \frac{49}{29}$$

Thus, $x = \frac{49}{29}$ and $y = \frac{19}{29}$ is the required solution.

1 B. Question

Solve the following pair of linear equations by substitution method:

$x + y = 14$

$x - y = 4$

Answer

Given equations are

$x + y = 14$ ...(i)

$x - y = 4$ ...(ii)

From eq^n (ii), $x = 4 + y$ ...(iii)

On substituting $x = 4 + y$ in eq^n (i), we get

$\Rightarrow 4 + y + y = 14$

$\Rightarrow 4 + 2y = 14$

$\Rightarrow 2y = 14 - 4$

$\Rightarrow 2y = 10$

$$\Rightarrow y = \frac{10}{2} = 5$$

Now, on putting $y = 5$ in eq^n (iii), we get

$\Rightarrow x = 4 + 5$

$\Rightarrow x = 9$

Thus, x = 9 and y = 5 is the required solution.

1. Question

Solve the following pair of linear equations by substitution method:

$3x - y = 3$

$9x — 3y = 9$

Answer

Given equations are

$3x - y = 3$ ...(i)

$9x - 3y = 9$ ...(ii)

From eq^n (i), $y = 3x - 3$ ...(iii)

On substituting $y = 3x - 3$ in eq^n (ii), we get

$\Rightarrow 9x - 3(3x - 3) = 9$

$\Rightarrow 9x - 9x + 9 = 9$

$\Rightarrow 9 = 9$

This equality is true for all values of x, therefore given pair of equations have infinitely many solutions.

1 C. Question

Solve the following pair of linear equations by substitution method:

$0.5x + 0.8y = 3.4$

$0.6x — 0.3y = 0.3$

Answer

Given equations are

$0.5x + 0.8y = 3.4$ ...(i)

$0.6x - 0.3y = 0.3$ ...(ii)

From eq^n (ii), $2x - y = 1$

$y = 2x - 1$ ...(iii)

On substituting $y = 0.2x - 1$ in eq^n (i), we get

$\Rightarrow 0.5x + 0.8(2x - 1) = 3.4$

$\Rightarrow 0.5x + 1.6x - 0.8 = 3.4$

$\Rightarrow 2.1x = 3.4 + 0.8$

$\Rightarrow 2.1x = 4.2$

$$\Rightarrow x = \frac{4.2}{2.1} = 2$$

Now, on putting $x = 2$ in eq^n (iii), we get

$\Rightarrow y = 2(2) - 1$

$\Rightarrow y = 4 - 1$

$\Rightarrow y = 3$

Thus, $x = 2$ and $y = 3$ is the required solution.

2 A. Question

Solve the following pair of linear equations by substitution method:

$x + y = a—b$

$ax —by = a^2 + b^2$

Answer

Given equations are

$x + y = a - b$ ...(i)

$ax - by = a^2 + b^2$ ...(ii)

From eq^n (i), $x = a - b - y$ ...(iii)

On substituting $x = a - b - y$ in eq^n (ii), we get

$\Rightarrow a(a - b - y) - by = a^2 + b^2$

$\Rightarrow a^2 - ab - ay - by = a^2 + b^2$

$\Rightarrow - ab - y(a + b) = b^2$

$\Rightarrow - y(a + b) = b^2 + ab$

$$\Rightarrow y = \frac{b^2 + ab}{-(\mathbf{a} + \mathbf{b})}$$

$$\Rightarrow \mathbf{y} = \frac{\mathbf{b}(\mathbf{b} + \mathbf{a})}{-(\mathbf{a} + \mathbf{b})} = -\mathbf{b}$$

Now, on putting **y = – b** in eq^n (iii), we get

$\Rightarrow x = a - b - (- b)$

$\Rightarrow$ **x = a**

Thus, **x = a** and **y = – b** is the required solution.

2 B. Question

Solve the following pair of linear equations by substitution method:

$x + y = 2m$

$mx — ny = m^2 + n^2$

Answer

Given equations are

$x + y = 2m$ ...(i)

$mx - ny = m^2 + n^2$ ...(ii)

From eq^n (i), $x = 2m - y$ ...(iii)

On substituting $x = 2m - y$ in eq^n (ii), we get

$\Rightarrow m(2m - y) - ny = m^2 + n^2$

$\Rightarrow 2m^2 - my - ny = m^2 + n^2$

$\Rightarrow - y(m + n) = m^2 - 2m^2 + n^2$

$\Rightarrow - y(m + n) = - m^2 + n^2$

$$\Rightarrow y = \frac{n^2 - m^2}{-(\mathbf{a} + \mathbf{b})} (\because, (a^2 - b^2) = (a - b)(a + b))$$

$$\Rightarrow y = \frac{(n-m)(n+m)}{-(n+m)}$$

$\Rightarrow y = -(n-m) = m - n$

Now, on putting **y = m – n** in eq^n (iii), we get

$\Rightarrow x = 2m - (m - n)$

$\Rightarrow x = 2m - m + n$

$\Rightarrow$ **$x = m + n$**

Thus, **$x = m + n$** and **$y = m - n$** is the required solution.

3 A. Question

Solve the following system of equations by substitution method:

$$\frac{x}{2} + y = 0.8$$

$$x + \frac{y}{2} = \frac{7}{10}$$

Answer

Given equations are

$\frac{x}{2} + y = 0.8$ ...(i)

$x + \frac{y}{2} = \frac{7}{10}$ ...(ii)

eq^n (i) can be re – written as,

$$\Rightarrow \frac{x}{2} + y = 0.8$$

$\Rightarrow \left(\frac{x+2y}{2}\right) = 0.8 \Rightarrow x + 2y = 0.8 \times 2$

$\Rightarrow x + 2y = 1.6$ ...(iii)

On substituting $x = 1.6 - 2y$ in eq^n (ii), we get

$$\Rightarrow 1.6 - 2y + \frac{y}{2} = \frac{7}{10}$$

$$\Rightarrow \frac{3.2 - 4y + y}{2} = \frac{7}{10}$$

$$\Rightarrow 3.2 - 3y = \frac{7}{10} \times 2$$

$$\Rightarrow -3y = \frac{7}{10} \times 2 - 3.2$$

$$\Rightarrow -3y = \frac{14}{10} - \frac{32}{10}$$

$$\Rightarrow -3y = -\frac{18}{10}$$

$$\Rightarrow y = \frac{18}{10} \times \frac{1}{3}$$

$$\Rightarrow \mathbf{y} = \frac{6}{10} = \mathbf{0.6}$$

Now, putting the y = 0.6 in eq^n (iii), we get

⇒ x + 2y = 1.6

⇒ x + 2(0.6) = 1.6

⇒ x + 1.2 = 1.6

⇒ x = 0.4

Thus, **x = 0.4** and **y = 0.6** is the required solution.

3 B. Question

Solve the following system of equations by substitution method:

s —t = 3

$$\frac{s}{3} + \frac{t}{2} = 6$$

Answer

Given equations are

s – t = 3 …(i)

$\frac{s}{3} + \frac{t}{2} = 6$ …(ii)

From eq^n (i), we get

⇒ s = 3 + t …(iii)

On substituting s = 3 + t in eq^n (ii), we get

$$\Rightarrow \frac{3 + t}{3} + \frac{t}{2} = 6$$

$$\Rightarrow \frac{2(3 + t) + 3t}{6} = 6$$

⇒ 6 + 2t + 3t = 6×6

⇒ 5t = 36 – 6

$$\Rightarrow \mathbf{t} = \frac{30}{5} = \mathbf{6}$$

Now, putting the t = 6 in eq^n (iii), we get

⇒ s = 3 + 6

⇒ s = 9

Thus, **s = 9** and **t = 6** is the required solution.

3 C. Question

Solve the following system of equations by substitution method:

$\frac{x}{a} + \frac{y}{b} = 2$, a ≠ 0, b ≠ 0

$ax - by = a^2 - b^2$

Answer

Given equations are

$\frac{x}{a} + \frac{y}{b} = 2$ ...(i)

$ax - by = a^2 - b^2$ ...(ii)

eq^n (i) can be re – written as,

$$\Rightarrow \frac{x}{a} + \frac{y}{b} = 2$$

$$\Rightarrow \left(\frac{bx + ay}{ab}\right) = 2$$

⇒ bx + ay = ab×2

⇒ bx + ay = 2ab

$\Rightarrow\ x = \frac{2ab-ay}{b}$ ...(iii)

On substituting $x = \frac{2ab-ay}{b}$ in eq^n (ii), we get

$$\Rightarrow\ a\left(\frac{2ab-ay}{b}\right) - by = a^2 - b^2$$

$$\Rightarrow\ \frac{2a^2b - a^2y}{b} - by = a^2 - b^2$$

$$\Rightarrow\ \frac{2a^2b - a^2y - b^2y}{b} = a^2 - b^2$$

$\Rightarrow 2a^2 b - a^2 y - b^2 y = b(a^2 - b^2)$

$\Rightarrow 2a^2 b - y(a^2 + b^2) = a^2b - b^3$

$\Rightarrow - y(a^2 + b^2) = a^2 b - b^3 - 2a^2b$

$\Rightarrow - y(a^2 + b^2) = - a^2b - b^3$

$\Rightarrow - y(a^2 + b^2) = - b(a^2 + b^2)$

$$\Rightarrow\ \mathbf{y} = \frac{-b(a^2 + b^2)}{-(a^2 + b^2)} = \mathbf{b}$$

Now, on putting **y = b** in eq^n (iii), we get

$$\Rightarrow\ x = \frac{2ab - ab}{b}$$

$\Rightarrow$ **x = a**

Thus, **x = a** and **y = b** is the required solution.

3 D. Question

Solve the following system of equations by substitution method:

$$\frac{bx}{a} + \frac{ay}{b} = a^2 + b^2$$

x + y = 2ab

Answer

Given equations are

$\frac{bx}{a} + \frac{ay}{b} = a^2 + b^2$ ...(i)

x + y = 2ab ...(ii)

eq^n (i) can be re - written as,

$$\Rightarrow \frac{bx}{a} + \frac{ay}{b} = a^2 + b^2$$

$$\Rightarrow \left(\frac{b^2x + a^2y}{ab}\right) = a^2 + b^2$$

$\Rightarrow b^2 x + a^2 y = ab \times (a^2 + b^2)$

$\Rightarrow x = \frac{ab\times(a^2 + b^2) - a^2y}{b^2}$ ...(iii)

Now, from eq^n (ii), y = 2ab – x ...(iv)

On substituting y = 2ab – x in eq^n (iii), we get

$$\Rightarrow x = \frac{ab \times (a^2 + b^2) - a^2(2ab - x)}{b^2}$$

$$\Rightarrow x = \frac{a^3b + b^3a - 2a^3b + a^2x}{b^2}$$

$\Rightarrow b^2 x = b^3 a - a^3b + a^2x$

$\Rightarrow b^2x - a^2x = b^3a - a^3b$

$\Rightarrow (b^2 - a^2) x = ab(b^2 - a^2)$

$\Rightarrow$ x = ab

Now, on putting **x = ab** in eq^n (iv), we get

$\Rightarrow$ y = 2ab – ab

$\Rightarrow$ **y = ab**

Thus, **x = ab** and **y = ab** is the required solution.

Exercise 3.3

1 A. Question

Solve the following system of equations by elimination method:

3x – 5y – 4 = 0

9x = 2y + 7

Answer

Given pair of linear equations is

3x – 5y – 4 = 0 ...(i)

And 9x = 2y + 7 ...(ii)

On multiplying Eq. (i) by 3 to make the coefficients of x equal, we get the equation as

9x – 15y – 12 = 0 ...(iii)

On subtracting Eq. (ii) from Eq. (iii), we get

9x – 15y – 12 – 9x = 0 – 2y – 7

⇒ – 15y + 2y = – 7 + 12

⇒ – 13y = 5

$$\Rightarrow y = -\frac{5}{13}$$

On putting $y = -\frac{5}{13}$ in Eq. (ii), we get

$$9x = 2\left(-\frac{5}{13}\right) + 7$$

$$\Rightarrow 9x = -\frac{10}{13} + 7$$

$$\Rightarrow 9x = \frac{-10 + 91}{13}$$

$$\Rightarrow 9x = \frac{81}{13}$$

$$\Rightarrow x = \frac{81}{13 \times 9}$$

$$\Rightarrow x = \frac{9}{13}$$

Hence, $x = \frac{9}{13}$ and $= -\frac{5}{13}$, which is the required solution.

1 B. Question

Solve the following system of equations by elimination method:

3x + 4y = 10

2x – 2y = 2

Answer

Given pair of linear equations is

3x + 4y = 10 …(i)

And 2x – 2y = 2 …(ii)

On multiplying Eq. (i) by 2 and Eq. (ii) by 3 to make the coefficients of x equal, we get the equation as

6x + 8y = 20 …(iii)

6x – 6y = 6 …(iv)

On subtracting Eq. (iii) from Eq. (iv), we get

6x – 6y – 6x – 8y = 6 – 20

⇒ – 14y = – 14

⇒ y = 1

On putting y = 1 in Eq. (ii), we get

2x – 2(1) = 2

⇒ 2x – 2 = 2

⇒ x = 2

Hence, x = 2 and y = 1 , which is the required solution.

1 C. Question

Solve the following system of equations by elimination method:

x + y = 5

2x – 3y = 4

Answer

Given pair of linear equations is

x + y = 5 …(i)

And 2x – 3y = 4 …(ii)

On multiplying Eq. (i) by 2 to make the coefficients of x equal, we get the equation as

2x + 2y = 10 ...(iii)

On subtracting Eq. (ii) from Eq. (iii), we get

2x + 2y – 2x + 3y = 10 – 4

⇒ 5y = 6

$$\Rightarrow y = \frac{6}{5}$$

On putting $y = \frac{6}{5}$ in Eq. (i), we get

x + y = 5

$$\Rightarrow x + \frac{6}{5} = 5$$

$$\Rightarrow x = \frac{25 - 6}{5}$$

$$\Rightarrow x = \frac{19}{5}$$

Hence, $x = \frac{19}{5}$ and $= \frac{6}{5}$, which is the required solution.

1 D. Question

Solve the following system of equations by elimination method:

2x + 3y = 8

4x + 6y = 7

Answer

Given pair of linear equations is

2x + 3y = 8 ...(i)

And 4x + 6y = 7 ...(ii)

On multiplying Eq. (i) by 2 to make the coefficients of x equal, we get the equation as

4x + 6y = 16 ...(iii)

On subtracting Eq. (ii) from Eq. (iii), we get

$4x + 6y - 4x - 6y = 16 - 7$

$\Rightarrow 0 = 9$

Which is a false equation involving no variable.

So, the given pair of linear equations has no solution i.e. this pair of linear equations is inconsistent.

1 E. Question

Solve the following system of equations by elimination method:

$8x + 5y = 9$

$3x + 2y = 4$

Answer

Given pair of linear equations is

$8x + 5y = 9$ ...(i)

And $3x + 2y = 4$ ...(ii)

On multiplying Eq. (i) by 2 and Eq. (ii) by 5 to make the coefficients of y equal, we get the equation as

$16x + 10y = 18$ ...(iii)

$15x + 10y = 20$ ...(iv)

On subtracting Eq. (iii) from Eq. (iv), we get

$15x + 10y - 16x - 10y = 20 - 18$

$\Rightarrow -x = 2$

$\Rightarrow x = -2$

On putting $x = -2$ in Eq. (ii), we get

$3x + 2y = 4$

$\Rightarrow 3(-2) + 2y = 4$

$\Rightarrow -6 + 2y = 4$

$\Rightarrow 2y = 4 + 6$

$\Rightarrow 2y = 10$

$\Rightarrow y = \frac{10}{2} = 5$

Hence, $x = -2$ and $= 5$, which is the required solution.

1 F. Question

Solve the following system of equations by elimination method:

2x + 3y = 46

3x + 5y = 74

Answer

Given pair of linear equations is

2x + 3y = 46 …(i)

And 3x + 5y = 74 …(ii)

On multiplying Eq. (i) by 3 and Eq. (ii) by 2 to make the coefficients of x equal, we get the equation as

6x + 9y = 138 …(iii)

6x + 10y = 148 …(iv)

On subtracting Eq. (iii) from Eq. (iv), we get

6x + 10y – 6x – 9y = 148 – 138

⇒ y = 10

On putting y = 10 in Eq. (ii), we get

3x + 5y = 74

⇒ 3x + 5(10) = 74

⇒ 3x + 50 = 74

⇒ 3x = 74 – 50

⇒ 3x = 24

⇒ x = 8

Hence, x = 8 and y = 10 , which is the required solution.

1 G. Question

Solve the following system of equations by elimination method:

0.4x – 1.5y = 6.5

0.3x + 0.2y = 0.9

Answer

Given pair of linear equations is

$0.4x - 1.5y = 6.5$ …(i)

And $0.3x + 0.2y = 0.9$ …(ii)

On multiplying Eq. (i) by 3 and Eq. (ii) by 4 to make the coefficients of x equal, we get the equation as

$1.2x - 4.5y = 19.5$ …(iii)

$1.2x + 0.8y = 3.6$ …(iv)

On subtracting Eq. (iii) from Eq. (iv), we get

$1.2x + 0.8y - 1.2x + 4.5y = 3.6 - 19.5$

$\Rightarrow 5.3y = -15.9$

$\Rightarrow y = -\frac{15.9}{5.3}$

$\Rightarrow y = -3$

On putting $y = -3$ in Eq. (ii), we get

$0.3x + 0.2y = 0.9$

$\Rightarrow 0.3x + 0.2(-3) = 0.9$

$\Rightarrow 0.3x - 0.6 = 0.9$

$\Rightarrow 0.3x = 1.5$

$\Rightarrow x = 1.5/0.3$

$\Rightarrow x = 5$

Hence, $x = 5$ and $y = -3$, which is the required solution.

1 H. Question

Solve the following system of equations by elimination method:

$\sqrt{2}x - \sqrt{3}y = 0$

$\sqrt{5}x + \sqrt{2}y = 0$

Answer

Given pair of linear equations is

$\sqrt{2}x - \sqrt{3}y = 0$ …(i)

And $\sqrt{5}x + \sqrt{2}y = 0$ …(ii)

On multiplying Eq. (i) by $\sqrt{2}$ and Eq. (ii) by $\sqrt{3}$ to make the coefficients of y equal, we get the equation as

$2x - \sqrt{6}y = 0$ …(iii)

$\sqrt{15}x + \sqrt{6}y = 0$ …(iv)

On adding Eq. (iii) and (iv), we get

$2x - \sqrt{6}y + \sqrt{15}x + \sqrt{6}y = 0$

$\Rightarrow 2x + \sqrt{15}x = 0$

$\Rightarrow x(2 + \sqrt{15}) = 0$

$\Rightarrow x = 0$

On putting x = 0 in Eq. (i), we get

$\sqrt{2}x - \sqrt{3}y = 0$

$\Rightarrow \sqrt{2}(0) - \sqrt{3}y = 0$

$\Rightarrow -\sqrt{3}y = 0$

$\Rightarrow y = 0$

Hence, x = 0 and y = 0 , which is the required solution.

1 I. Question

Solve the following system of equations by elimination method:

$2x + 5y = 1$

$2x + 3y = 3$

Answer

Given pair of linear equations is

$2x + 5y = 1$ …(i)

And $2x + 3y = 3$ …(ii)

On subtracting Eq. (ii) from Eq. (i), we get

$2x + 3y - 2x - 5y = 3 - 1$

$\Rightarrow -2y = 2$

$\Rightarrow$ y = – 1

On putting y = – 1 in Eq. (ii), we get

2x + 3(– 1) = 3

$\Rightarrow$ 2x – 3 = 3

$\Rightarrow$ 2x = 6

$\Rightarrow$ x = 6/2

$\Rightarrow$ x = 3

Hence, x = 3 and y = – 1 , which is the required solution.

2 A. Question

Solve the following system of equations by elimination method:

$$3x - \frac{8}{y} = 5$$

$$x - \frac{y}{3} = 3$$

Answer

Given pair of linear equations is

$\frac{x}{2} + \frac{2y}{3} = -1$ ...(i)

And $x - \frac{y}{3} = 3$ ...(ii)

On multiplying Eq. (ii) by 2 to make the coefficients of y equal, we get the equation as

$2x - \frac{2y}{3} = 6$ ...(iii)

On adding Eq. (i) and Eq. (ii), we get

$$\frac{x}{2} + \frac{2y}{3} + x - \frac{2y}{3} = -1 + 6$$

$$\Rightarrow \frac{x}{2} + 2x = 5$$

$$\Rightarrow \frac{x + 4x}{2} = 5$$

$$\Rightarrow \frac{5x}{2} = 5$$

$\Rightarrow x = 2$

On putting $x = 2$ in Eq. (ii), we get

$$x - \frac{y}{3} = 3$$

$$\Rightarrow 2 - \frac{y}{3} = 3$$

$$\Rightarrow -\frac{y}{3} = 3 - 2$$

$$\Rightarrow -\frac{y}{3} = 1$$

$\Rightarrow y = -3$

Hence, $x = 2$ and $y = -3$, which is the required solution.

2 B. Question

Solve the following system of equations by elimination method:

$$\frac{x}{6} + \frac{y}{15} = 4$$

$$\frac{x}{3} - \frac{y}{12} = \frac{19}{4}$$

Answer

Given pair of linear equations is

$\frac{x}{6} + \frac{y}{15} = 4$ ...(i)

And $\frac{x}{3} - \frac{y}{12} = \frac{19}{4}$ ...(ii)

On multiplying Eq. (ii) by $\frac{1}{2}$ to make the coefficients of x equal, we get the equation as

$\frac{x}{6} - \frac{y}{24} = \frac{19}{8}$ ...(iii)

On subtracting Eq. (ii) from Eq. (iii), we get

$$\frac{x}{6}-\frac{y}{24}-\frac{x}{6}-\frac{y}{15}=\frac{19}{8}-4$$

$$\Rightarrow -\frac{y}{24}-\frac{y}{15}=\frac{19-32}{8}$$

$$\Rightarrow \frac{-5y-8y}{120}=\frac{19-32}{8}$$

$$\Rightarrow \frac{-13y}{120}=\frac{-13}{8}$$

$$\Rightarrow y=\frac{13}{8}\times\frac{120}{13}$$

$\Rightarrow$ y = 15

On putting y = 15 in Eq. (ii), we get

$$\frac{x}{6}+\frac{y}{15}=4$$

$$\Rightarrow \frac{x}{6}+\frac{15}{15}=4$$

$$\Rightarrow \frac{x}{6}=4-1$$

$$\Rightarrow \frac{x}{6}=3$$

$\Rightarrow$ x = 18

Hence, x = 18 and y = 15 , which is the required solution.

2 C. Question

Solve the following system of equations by elimination method:

$$x+\frac{6}{y}=6$$

$$3x-\frac{8}{y}=5$$

Answer

Given pair of linear equations is

$x+\frac{6}{y}=6$ ...(i)

And $3x - \frac{8}{y} = 5$ ...(ii)

On multiplying Eq. (i) by 3 to make the coefficients of x equal, we get the equation as

$3x + \frac{18}{y} = 18$ ...(iii)

On subtracting Eq. (ii) from Eq. (iii), we get

$$3x + \frac{18}{y} - 3x + \frac{8}{y} = 18 - 5$$

$$\Rightarrow \frac{26}{y} = 13$$

$\Rightarrow y = 2$

On putting y = 2 in Eq. (i), we get

$$x + \frac{6}{y} = 6$$

$$\Rightarrow x + \frac{6}{2} = 6$$

$\Rightarrow x = 3$

Hence, x = 3 and y = 2 , which is the required solution.

3 A. Question

Solve the following equations by elimination method:

37x + 43y = 123

43x + 37y = 117

Answer

Given pair of linear equations is

37x + 43y = 123 ...(i)

And 43x + 37y = 117 ...(ii)

On multiplying Eq. (i) by 43 and Eq. (ii) by 37 to make the coefficients of x equal, we get the equation as

1591x + 1849y = 5289 ...(iii)

1591x + 1369y = 4329 ...(iv)

On subtracting Eq. (iii) from Eq. (iv), we get

⇒ 1591x + 1369y – 1591x – 1849y = 4329 – 5289

⇒ – 480y = – 960

$$\Rightarrow y = \frac{960}{480}$$

⇒ y = 2

On putting y = 2 in Eq. (ii), we get

⇒ 43x + 37(2) = 117 ⇒ 43x + 74 = 117

⇒ 43x = 117 – 74

⇒ 43x = 43

⇒ x = 1

Hence, x = 1 and y = 2 , which is the required solution.

3 B. Question

Solve the following equations by elimination method:

217x + 131y = 913

131x + 217y = 827

Answer

Given pair of linear equations is

217x + 131y = 913 ...(i)

And 131x + 217y = 827 ...(ii)

On multiplying Eq. (i) by 131 and Eq. (ii) by 217 to make the coefficients of x equal, we get the equation as

28427x + 17161y = 119603 ...(iii)

28427x + 47089y = 179459 ...(iv)

On subtracting Eq. (iii) from Eq. (iv), we get

⇒ 28427x + 47089y – 28427x – 17161y = 179459 – 119603

⇒ 47089y – 17161y = 179459 – 119603

$\Rightarrow 29928y = 59856$

$$\Rightarrow y = \frac{59856}{29928}$$

$\Rightarrow y = 2$

On putting y = 2 in Eq. (ii), we get

$\Rightarrow 131x + 217(2) = 827 \Rightarrow 131x + 434 = 827$

$\Rightarrow 131x = 393$

$$\Rightarrow x = \frac{393}{131}$$

$\Rightarrow x = 3$

Hence, x = 3 and y = 2 , which is the required solution.

3 C. Question

Solve the following equations by elimination method:

$99x + 101y = 499$

$101x + 99y = 501$

Answer

Given pair of linear equations is

$99x + 101y = 499$ …(i)

And $101x + 99y = 501$ …(ii)

On multiplying Eq. (i) by 101 and Eq. (ii) by 99 to make the coefficients of x equal, we get the equation as

$9999x + 10201y = 50399$ …(iii)

$9999x + 9801y = 49599$ …(iv)

On subtracting Eq. (iii) from Eq. (iv), we get

$\Rightarrow 9999x + 9801y - 9999x - 10201y = 49599 - 50399$

$\Rightarrow 9801y - 10201y = 49599 - 50399$

$\Rightarrow -400y = -800$

$$\Rightarrow y = \frac{800}{400}$$

⇒ $y = 2$

On putting $y = 2$ in Eq. (i), we get

⇒ $99x + 101(2) = 499$ ⇒ $99x + 202 = 499$

⇒ $99x = 297$

⇒ $x = 297/99$

⇒ $x = 3$

Hence, $x = 3$ and $y = 2$, which is the required solution.

3 D. Question

Solve the following equations by elimination method:

$29x - 23y = 110$

$23x - 29y = 98$

Answer

Given pair of linear equations is

$29x - 23y = 110$ ...(i)

And $23x - 29y = 98$ ...(ii)

On multiplying Eq. (i) by 23 and Eq. (ii) by 29 to make the coefficients of x equal, we get the equation as

$667x - 529y = 2530$ ...(iii)

$667x - 841y = 2842$ ...(iv)

On subtracting Eq. (iii) from Eq. (iv), we get

⇒ $667x - 841y - 667x + 529y = 2842 - 2530$

⇒ $-312y = 312$

⇒ $y = -1$

On putting $y = 2$ in Eq. (ii), we get

⇒ $29x - 23(-1) = 110$ ⇒ $29x + 23 = 110$

⇒ $29x = 110 - 23$

⇒ $29x = 87$

⇒ $x = 3$

Hence, x = 3 and y = – 1 , which is the required solution.

4 A. Question

Solve the following system of equations by elimination method:

$$\frac{1}{x}-\frac{1}{y}=1$$

$$\frac{1}{x}+\frac{1}{y}=7, x \neq 0, y \neq 0$$

Answer

Given pair of linear equations is

$\frac{1}{x}-\frac{1}{y} = 1$ ...(i)

And $\frac{1}{x} + \frac{1}{y} = 7$ ...(ii)

Adding Eq. (i) and Eq. (ii), we get

$$\frac{1}{x}-\frac{1}{y}+\frac{1}{x}+\frac{1}{y} = 1 + 7$$

$$\Rightarrow \frac{1}{x}+\frac{1}{x} = 8$$

$$\Rightarrow \frac{2}{x} = 8$$

$$\Rightarrow \frac{2}{8} = x$$

$$\Rightarrow x = \frac{1}{4}$$

On putting $x = \frac{1}{4}$ in Eq. (ii), we get

$$\frac{1}{x}+\frac{1}{y} = 7$$

$$\Rightarrow \frac{1}{\frac{1}{4}}+\frac{1}{y} = 7$$

$$\Rightarrow 4 + \frac{1}{y} = 7 \Rightarrow \frac{1}{y} = 7 - 4 = 3$$

$$\Rightarrow y = \frac{1}{3}$$

Hence, $x = \frac{1}{4}$ and $= \frac{1}{3}$, which is the required solution.

4 B. Question

Solve the following system of equations by elimination method:

$$\frac{2}{x} + \frac{3}{y} = 13$$

$$\frac{5}{x} - \frac{4}{y} = -2, x \neq 0, y \neq 0$$

Answer

Given pair of linear equations is

$\frac{2}{x} + \frac{3}{y} = 13$ ...(i)

And $\frac{5}{x} - \frac{4}{y} = -2$ ...(ii)

On multiplying Eq. (i) by 5 and Eq. (ii) by 2 to make the coefficients of x equal, we get the equation as

$\frac{10}{x} + \frac{15}{y} = 65$ ...(iii)

$\frac{10}{x} - \frac{8}{y} = -4$ ...(iv)

On subtracting Eq. (iii) from Eq. (iv), we get

$$\frac{10}{x} - \frac{8}{y} - \frac{10}{x} - \frac{15}{y} = -4 - 65$$

$$\Rightarrow -\frac{8}{y} - \frac{15}{y} = -69$$

$$\Rightarrow -\frac{23}{y} = -69$$

$\Rightarrow \frac{23}{69} = y$

$\Rightarrow y = \frac{1}{2}$

On putting $y = \frac{1}{2}$ in Eq. (ii), we get

$\frac{5}{x} - \frac{4}{y} = -2$

$\Rightarrow \frac{5}{x} - \frac{4}{\frac{1}{2}} = 7$

$\Rightarrow \frac{5}{x} - 8 = 7 \Rightarrow \frac{5}{x} = 15$

$\Rightarrow x = \frac{1}{3}$

Hence, $x = \frac{1}{3}$ and $= \frac{1}{2}$, which is the required solution.

4 C. Question

Solve the following system of equations by elimination method:

$$\frac{3a}{x} - \frac{2b}{y} + 5 = 0, \frac{a}{x} + \frac{3b}{y} - 2 = 0, (x \neq 0, y \neq 0)$$

Answer

Given pair of linear equations is

$\frac{4}{x} + \frac{7}{y} = 11$ ...(i)

And $\frac{3}{x} - \frac{5}{y} = -2$ ...(ii)

On multiplying Eq. (i) by 3 and Eq. (ii) by 4 to make the coefficients of x equal, we get the equation as

$\frac{12}{x} + \frac{21}{y} = 33$ ...(iii)

$\frac{12}{x} - \frac{20}{y} = -8$ ...(iv)

On subtracting Eq. (iii) from Eq. (iv), we get

$$\frac{12}{x} - \frac{20}{y} - \frac{12}{x} - \frac{21}{y} = -8 - 33$$

$$\Rightarrow -\frac{20}{y} - \frac{21}{y} = -41$$

$$\Rightarrow -\frac{41}{y} = -41$$

$$\Rightarrow \frac{41}{41} = y$$

$$\Rightarrow y = 1$$

On putting $y = 1$ in Eq. (ii), we get

$$\frac{3}{x} - \frac{5}{y} = -2$$

$$\Rightarrow \frac{3}{x} - 5 = -2$$

$$\Rightarrow \frac{3}{x} = -2 + 5 \Rightarrow \frac{3}{x} = 3$$

$$\Rightarrow x = 1$$

Hence, $x = \frac{1}{3}$ and $= \frac{1}{2}$, which is the required solution.

4 D. Question

Solve the following system of equations by elimination method:

$$\frac{3a}{x} - \frac{2b}{y} + 5 = 0, \frac{a}{x} + \frac{3b}{y} - 2 = 0, (x \neq 0, y \neq 0)$$

Answer

Given pair of linear equations is

$$\frac{3a}{x} - \frac{2b}{y} = -5 \text{ ...(i)}$$

And $\frac{a}{x} + \frac{3b}{y} = 2$ ...(ii)

On multiplying Eq. (ii) by 3 to make the coefficients of x equal, we get the equation as

$\frac{3a}{x} + \frac{9b}{y} = 6$ ...(iii)

On subtracting Eq. (i) from Eq. (iii), we get

$$\frac{3a}{x} + \frac{9b}{y} - \frac{3a}{x} + \frac{2b}{y} = 6 - (-5)$$

$$\Rightarrow \frac{9b}{y} + \frac{2b}{y} = 11$$

$$\Rightarrow \frac{11b}{y} = 11$$

$\Rightarrow$ y = b

On putting y = b in Eq. (ii), we get

$$\frac{a}{x} + \frac{3b}{y} = 2$$

$$\Rightarrow \frac{a}{x} + \frac{3b}{b} = 2$$

$$\Rightarrow \frac{a}{x} = 2 - 3 \Rightarrow \frac{a}{x} = -1$$

$\Rightarrow$ x = – a

Hence, x = – a and y = b , which is the required solution.

5 A. Question

Solve the following system of equations by elimination method:

$\frac{2x + 5y}{xy} = 6, \ \frac{4x - 5y}{xy} = -3$, where $x \neq 0$ and $y \neq 0$

Answer

Given pair of linear equations is

$$\frac{2x + 5y}{xy} = 6$$

Or 2x + 5y = 6xy ...(i)

And $\frac{4x-5y}{xy} = -3$

Or 4x – 5y = – 3xy ...(ii)

On adding Eq. (i) and Eq. (ii), we get

2x + 5y + 4x – 5y = 6xy – 3xy

⇒ 6x = 3xy

$$\Rightarrow \frac{6x}{3x} = y$$

⇒ y = 2 and x = 0

On putting y = 2 in Eq. (ii), we get

2x + 5(2) = 6xy

⇒ 2x + 10 = 6x(2)

⇒ 2x + 10 = 12x ⇒ 2x – 12x = – 10

⇒ – 10x = – 10

⇒ x = 1

On putting x = 0 , we get y = 0

Hence, x = 0,1 and y = 0,2 , which is the required solution.

5 B. Question

Solve the following system of equations by elimination method:

x + y = 2xy

x – y = 6xy

Answer

Given pair of linear equations is

x + y = 2xy ...(i)

And x – y = 6xy ...(ii)

On adding Eq. (i) and Eq. (ii), we get

x + y + x – y = 2xy + 6xy

⇒ 2x = 8xy

$$\Rightarrow \frac{2x}{8x} = y$$

$\Rightarrow y = \frac{1}{4}$ and $x = 0$

On putting $y = \frac{1}{4}$ in Eq. (ii), we get

$x - \frac{1}{4} = 6xy$

$\Rightarrow \frac{4x - 1}{4} = 6x\left(\frac{1}{4}\right)$

$\Rightarrow 4x - 1 = 6x \Rightarrow - 1 = 6x - 4x$

$\Rightarrow - 1 = 2x$

$\Rightarrow x = -\frac{1}{2}$

On putting x = 0 , we get y = 0

Hence, $x = -\frac{1}{2}, 0$ and $= \frac{1}{4}, 0$, which is the required solution.

5 C. Question

Solve the following system of equations by elimination method:

5x + 3y = 19xy

7x – 2y = 8xy

Answer

Given pair of linear equations is

5x + 3y = 19xy ...(i)

And 7x – 2y = 8xy ...(ii)

On multiplying Eq. (i) by 2 and Eq. (ii) by 3 to make the coefficients of y equal, we get the equation as

10x + 6y = 38xy ...(iii)

And 21x – 6y = 24xy ...(iv)

On adding Eq. (i) and Eq. (ii), we get

10x + 6y + 21x – 6y = 38xy + 24xy

$\Rightarrow$ 31x = 62xy

$\Rightarrow \frac{31x}{62x} = y$

$\Rightarrow y = \frac{1}{2}$ and $x = 0$

On putting $y = \frac{1}{2}$ in Eq. (ii), we get

7x – 2y = 8xy

$\Rightarrow 7x - 2\left(\frac{1}{2}\right) = 8x\left(\frac{1}{2}\right)$

⇒ 7x – 1 = 4x ⇒ – 1 = 4x – 7x

⇒ – 1 = – 3x

$\Rightarrow x = \frac{1}{3}$

On putting x = 0 , we get y = 0

Hence, $x = \frac{1}{3}, 0$ and $= \frac{1}{2}, 0$, which is the required solution.

5 D. Question

Solve the following system of equations by elimination method:

x + y = 7xy

2x – 3y = – xy

Answer

Given pair of linear equations is

x + y = 7xy ...(i)

And 2x – 3y = – xy ...(ii)

On multiplying Eq. (i) by 2 to make the coefficients of x equal, we get the equation as

2x + 2y = 14xy ...(iii)

On subtracting Eq. (ii) and Eq. (iii), we get

2x + 2y – 2x + 3y = 14xy + xy

⇒ 2y + 3y = 15xy

⇒ 5y = 15xy

$$\Rightarrow \frac{5y}{15y} = x$$

$$\Rightarrow x = \frac{1}{3} \text{ and } y = 0$$

On putting $x = \frac{1}{3}$ in Eq. (ii), we get

2x – 3y = – xy

$$\Rightarrow 2\left(\frac{1}{3}\right) - 3y = 6\left(\frac{1}{3}\right)y$$

$$\Rightarrow \left(\frac{2}{3}\right) - 3y = 2y \Rightarrow \left(\frac{2}{3}\right) = 5y$$

$$\Rightarrow y = \frac{2}{15}$$

On putting x = 0 , we get x = 0

Hence, $x = \frac{1}{3}, 0$ and $= \frac{2}{15}, 0$, which is the required solution.

6 A. Question

Solve for x and y the following system of equations:

$$\frac{1}{2(2x+3y)} + \frac{12}{7(3x-2y)} = \frac{1}{2}, \frac{7}{(2x+3y)} + \frac{4}{(3x-2y)} = 2$$

Where (2x + 3y) ≠ 0 and (3x – 2y) ≠ 0

Answer

Given pair of linear equations is

$$\frac{1}{2(2x+3y)} + \frac{12}{7(3x-2y)} = \frac{1}{2} \text{ ...(i)}$$

And $\frac{7}{(2x+3y)} + \frac{4}{(3x-2y)} = 2$ ...(ii)

On multiplying Eq. (i) by 7 and Eq. (ii) by $\frac{1}{2}$ to make the coefficients equal of first term, we get the equation as

$$\frac{7}{2(2x + 3y)} + \frac{7 \times 12}{7(3x - 2y)} = \frac{7}{2}$$

$$\Rightarrow \frac{7}{2(2x+3y)} + \frac{12}{(3x-2y)} = \frac{7}{2} \text{ ...(iii)}$$

$$\Rightarrow \frac{7}{2(2x + 3y)} + \frac{4}{2(3x - 2y)} = \frac{2}{2}$$

$$\Rightarrow \frac{7}{2(2x+3y)} + \frac{2}{(3x-2y)} = 1 \text{ ...(iv)}$$

On substracting Eq. (iii) from Eq. (iv), we get

$$\frac{7}{2(2x + 3y)} + \frac{2}{(3x - 2y)} - \frac{7}{2(2x + 3y)} - \frac{12}{(3x - 2y)} = 1 - \frac{7}{2}$$

$$\Rightarrow \frac{2}{(3x - 2y)} - \frac{12}{(3x - 2y)} = 1 - \frac{7}{2}$$

$$\Rightarrow \frac{-10}{(3x - 2y)} = \frac{2 - 7}{2}$$

$$\Rightarrow \frac{-10}{(3x - 2y)} = \frac{-5}{2}$$

$$\Rightarrow \frac{1}{(3x - 2y)} = \frac{5}{2 \times 10}$$

$$\Rightarrow 3x - 2y = 4$$

$$\Rightarrow 3x = 4 + 2y$$

$$\Rightarrow x = \frac{4 + 2y}{3} \text{ ...(a)}$$

On multiplying Eq. (ii) by $\frac{3}{7}$ to make the coefficients equal of second term, we get the equation as

$$\frac{7 \times 3}{7(2x + 3y)} + \frac{4 \times 3}{7(3x - 2y)} = \frac{2 \times 3}{7}$$

$$\Rightarrow \frac{3}{(2x+3y)} + \frac{12}{7(3x-2y)} = \frac{6}{7} \text{ ...(v)}$$

On substracting Eq. (i) from Eq. (iv), we get

$$\frac{3}{(2x + 3y)} + \frac{12}{7(3x - 2y)} - \frac{1}{2(2x + 3y)} - \frac{12}{7(3x - 2y)} = \frac{6}{7} - \frac{1}{2}$$

$$\Rightarrow \frac{3}{(2x + 3y)} - \frac{1}{2(2x + 3y)} = \frac{6}{7} - \frac{1}{2}$$

$$\Rightarrow \frac{6 - 1}{2(2x + 3y)} = \frac{12 - 7}{14}$$

$$\Rightarrow \frac{5}{2(2x + 3y)} = \frac{5}{14}$$

$$\Rightarrow \frac{1}{(2x + 3y)} = \frac{2}{14}$$

$$\Rightarrow 2x + 3y = 7$$

$$\Rightarrow x = \frac{7-3y}{2} \text{ ...(b)}$$

From Eq. (a) and (b), we get

$$\frac{4 + 2y}{3} = \frac{7 - 3y}{2}$$

$\Rightarrow 2(4 + 2y) = 3(7 - 3y)$

$\Rightarrow 8 + 4y = 21 - 9y$

$\Rightarrow 4y + 9y = 21 - 8$

$\Rightarrow 13y = 13$

$\Rightarrow y = 1$

On putting the value of y = 1 in Eq. (b), we get

$$\Rightarrow x = \frac{7 - 3(1)}{2}$$

$$\Rightarrow x = \frac{4}{2} = 2$$

Hence, x = 2 and y = 1 , which is the required solution.

6 B. Question

Solve for x and y the following system of equations:

$$\frac{6}{x-1} - \frac{3}{y-2} = 1, x \neq 1, y \neq 1$$

$$\frac{3}{x-1} + \frac{2}{y+1} = \frac{13}{6}, x \neq 1, y \neq 1$$

Answer

Given pair of linear equations is

$\frac{2}{x-1} + \frac{3}{y+1} = 2$ ...(i)

And $\frac{3}{x-1} + \frac{2}{y+1} = \frac{13}{6}$ ...(ii)

On multiplying Eq. (i) by 3 and Eq. (ii) by 2 to make the coefficients equal of first term, we get the equation as

$\frac{6}{x-1} + \frac{9}{y+1} = 6$ ...(iii)

$\frac{6}{x-1} + \frac{4}{y+1} = \frac{13}{3}$ ...(iv)

On substracting Eq. (iii) from Eq. (iv), we get

$$\frac{6}{x-1} + \frac{4}{y+1} - \frac{6}{x-1} - \frac{9}{y+1} = \frac{13}{3} - 6$$

$$\Rightarrow \frac{4}{y+1} - \frac{9}{y+1} = \frac{13}{3} - 6$$

$$\Rightarrow \frac{-5}{y+1} = \frac{13-18}{3}$$

$$\Rightarrow \frac{-5}{y+1} = \frac{-5}{3}$$

$$\Rightarrow \frac{1}{y+1} = \frac{1}{3}$$

$\Rightarrow y + 1 = 3$

$\Rightarrow y = 3 - 1$

$\Rightarrow y = 2$

On putting the value of y = 2 in Eq. (ii), we get

$$\frac{6}{x-1} + \frac{4}{2+1} = \frac{13}{3}$$

$$\Rightarrow \frac{6}{x-1} + \frac{4}{3} = \frac{13}{3}$$

$$\Rightarrow \frac{6}{x-1} = \frac{13}{3} - \frac{4}{3}$$

$$\Rightarrow \frac{6}{x-1} = \frac{9}{3}$$

$$\Rightarrow \frac{1}{x-1} = \frac{9}{3 \times 6}$$

$\Rightarrow x - 1 = 2$

$\Rightarrow x = 3$

Hence, x = 3 and y = 2 , which is the required solution.

6 C. Question

Solve for x and y the following system of equations:

$$\frac{44}{x+y} + \frac{30}{x-y} = 10$$

$$\frac{55}{x+y} + \frac{40}{x-y} = 13, x + y \neq 0, x - y \neq 0$$

Answer

Given pair of linear equations is

$\frac{44}{x+y} + \frac{30}{x-y} = 10$ ...(i)

And $\frac{55}{x+y} + \frac{40}{x-y} = 13$ ...(ii)

On multiplying Eq. (i) by 4 and Eq. (ii) by 3 to make the coefficients equal of second term, we get the equation as

$\frac{176}{x+y} + \frac{120}{x-y} = 40$ ...(iii)

$\frac{165}{x+y} + \frac{120}{x-y} = 39$ ...(iv)

On substracting Eq. (iii) from Eq. (iv), we get

$$\frac{176}{x+y} + \frac{120}{x-y} - \frac{165}{x+y} - \frac{120}{x-y} = 40 - 39$$

$$\Rightarrow \frac{176 - 165}{x+y} = 1$$

$$\Rightarrow \frac{11}{x+y} = 1$$

$\Rightarrow x + y = 11$ ...(a)

On putting the value of x + y = 11 in Eq. (1), we get

$$\Rightarrow \frac{44}{11} + \frac{30}{x-y} = 10$$

$$\Rightarrow \frac{30}{x-y} = 10 - 4$$

⇒ 6(x – y) = 30

⇒ x – y = 5 ...(b)

Adding Eq. (a) and (b), we get

⇒ 2x = 16

⇒ x = 8

On putting value of x = 8 in eq. (a), we get

8 + y = 11

⇒ y = 3

Hence, x = 8 and y = 3 , which is the required solution.

6 D. Question

Solve for x and y the following system of equations:

$$\frac{5}{x-1} + \frac{1}{y-2} = 2$$

$$\frac{6}{x-1} - \frac{3}{y-2} = 1,$$

Answer

Given pair of linear equations is

$\frac{5}{x-1} + \frac{1}{y-2} = 2$ ...(i)

And $\frac{6}{x-1} - \frac{3}{y-2} = 1$ ...(ii)

On multiplying Eq. (i) by 3 to make the coefficients equal of second term, we get the equation as

$\frac{15}{x-1} + \frac{3}{y-2} = 6$ ...(iii)

On adding Eq. (ii) and Eq. (iii), we get

$$\frac{6}{x-1}-\frac{3}{y-2}+\frac{15}{x-1}+\frac{3}{y-2}=6+1$$

$$\Rightarrow \frac{6}{x-1}-\frac{15}{x-1}=7$$

$$\Rightarrow \frac{21}{x-1}=7$$

$\Rightarrow x - 1 = 3$

$\Rightarrow x = 3 + 1$

$\Rightarrow x = 4$

On putting the value of x = 4 in Eq. (ii), we get

$$\frac{6}{4-1}-\frac{3}{y-2}=1$$

$$\Rightarrow 2-\frac{3}{y-2}=1$$

$$\Rightarrow -\frac{3}{y-2}=1-2$$

$$\Rightarrow -\frac{3}{y-2}=-1$$

$\Rightarrow (y - 2) = 3$

$\Rightarrow y = 3 + 2$

$\Rightarrow y = 5$

Hence, x = 4 and y = 5 , which is the required solution.

7 A. Question

Form the pair of linear equations for the following problems and find their solution by elimination method:

Aftab tells his daughter, "seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." Find their present ages.

Answer

Let the present age of father i.e. Aftab = x yr

And the present age of his daughter = y yr

Seven years ago,

Aftab's age = $(x - 7)$yr

Daughter's age = $(y - 7)$yr

According to the question,

$(x - 7) = 7(y - 7)$

$\Rightarrow x - 7 = 7y - 49$

$\Rightarrow x - 7y = - 42$ ...(i)

After three years,

Aftab's age = $(x + 3)$yr

Daughter's age = $(y + 3)$yr

According to the question,

$(x + 3) = 3(y + 3)$

$\Rightarrow x + 3 = 3y + 9$

$\Rightarrow x - 3y = 6$ ...(ii)

Now, we can solve this by an elimination method

On subtracting Eq. (ii) from (i) we get

$x - 3y - x + 7y = 6 - (- 42)$

$\Rightarrow - 3y + 7y = 6 + 42$

$\Rightarrow 4y = 48$

$\Rightarrow y = 12$

On putting y = 12 in Eq. (ii) we get

$x - 3(12) = 6$

$\Rightarrow x - 36 = 6$

$\Rightarrow x = 42$

Hence, the age of Aftab is 42years and age of his daughter is 12years.

7 B. Question

Form the pair of linear equations for the following problems and find their solution by elimination method:

Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

Answer

Let the present age of Nuri = x yr

And the present age of Sonu = y yr

Five years ago,

Nuri's age = (x – 5)yr

Sonu's age = (y – 5)yr

According to the question,

$(x - 5) = 3(y - 5)$

$\Rightarrow x - 5 = 3y - 15$

$\Rightarrow x - 3y = -10$ …(i)

After ten years,

Aftab's age = (x + 10)yr

Daughter's age = (y + 10)yr

According to the question,

$(x + 10) = 2(y + 10)$

$\Rightarrow x + 10 = 2y + 20$

$\Rightarrow x - 2y = 10$ …(ii)

Now, we can solve this by an elimination method

On subtracting Eq. (ii) from (i) we get

$x - 2y - x + 3y = 10 - (-10)$

$\Rightarrow -2y + 3y = 10 + 10$

$\Rightarrow y = 20$

On putting y = 20 in Eq. (i) we get

$x - 3(20) = -10$

⇒ x – 60 = – 10

⇒ x = 50

Hence, the age of Nuri is 50 years and age of Sonu is 20 years.

7 C. Question

Form the pair of linear equations for the following problems and find their solution by elimination method:

The difference between two numbers is 26 and one number is three times the other. Find them.

Answer

Let the one number = x

And the other number = y

According to the question,

x – y = 26 …(i)

and x = 3y

or x – 3y = 0 …(ii)

Now, we can solve this by an elimination method

On subtracting Eq. (ii) from (i) we get

x – 3y – x + y = 0 – 26

⇒ – 3y + y = – 26

⇒ – 2y = – 26

⇒ y = 13

On putting y = 13 in Eq. (ii) we get

x – 3(13) = 0

⇒ x – 39 = 0

⇒ x = 39

Hence, the two numbers are 39 and 13.

Exercise 3.4

1 A. Question

Solve the following pair of linear equation by cross - multiplication method:

8x + 5y = 9

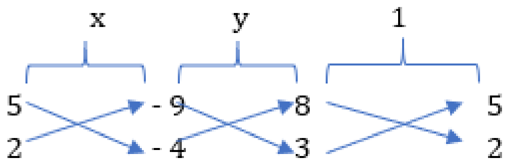
3x + 2y = 4

Answer

Given, pair of equations is

8x + 5y – 9 = 0 and 3x + 2y – 4 = 0

By cross - multiplication method, we have



$$\Rightarrow \frac{x}{-20 + 18} = \frac{y}{-27 + 32} = \frac{1}{16 - 15}$$

$$\Rightarrow \underset{\text{I}}{\frac{x}{-2}} = \underset{\text{II}}{\frac{y}{5}} = \underset{\text{III}}{\frac{1}{1}}$$

On taking I and III ratio, we get

$$\Rightarrow \frac{x}{-2} = \frac{1}{1}$$

⇒ x = – 2

On taking II and III ratio, we get

$$\Rightarrow \frac{y}{5} = \frac{1}{1}$$

⇒ y = 5

1 B. Question

Solve the following pair of linear equation by cross - multiplication method:

2x + 3y = 46

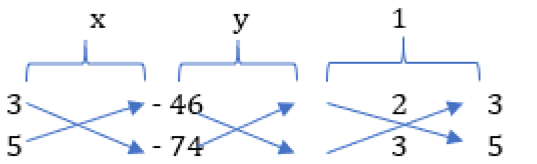
3x + 5y = 74

Answer

Given, pair of equations is

2x + 3y – 46 = 0 and 3x + 5y – 74 = 0

By cross - multiplication method, we have



$$\Rightarrow \frac{x}{-222 + 230} = \frac{y}{-138 + 148} = \frac{1}{10 - 9}$$

$$\Rightarrow \frac{x}{8} = \frac{y}{10} = \frac{1}{1}$$

I II III

On taking I and III ratio, we get

$$\Rightarrow \frac{x}{8} = \frac{1}{1}$$

⇒ x = 8

On taking II and III ratio, we get

$$\Rightarrow \frac{y}{10} = \frac{1}{1}$$

⇒ y = 10

1 C. Question

Solve the following pair of linear equation by cross - multiplication method:

x + 4y + 9 = 0

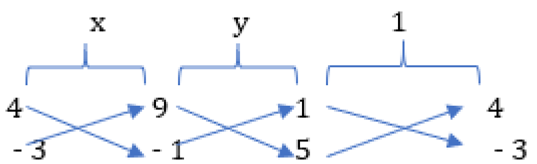
5x – 1 = 3y

Answer

Given, pair of equations is

x + 4y + 9 = 0 and 5x – 3y – 1 = 0

By cross - multiplication method, we have



$$\Rightarrow \frac{x}{-4 + 27} = \frac{y}{45 + 1} = \frac{1}{-3 - 20}$$

$$\Rightarrow \frac{x}{23} = \frac{y}{46} = \frac{1}{-23}$$

I II III

On taking I and III ratio, we get

$$\Rightarrow \frac{x}{23} = \frac{1}{-23}$$

⇒ x = – 1

On taking II and III ratio, we get

$$\Rightarrow \frac{y}{46} = \frac{1}{-23}$$

⇒ y = – 2

1 D. Question

Solve the following pair of linear equation by cross - multiplication method:

2x + 3y – 7 = 0

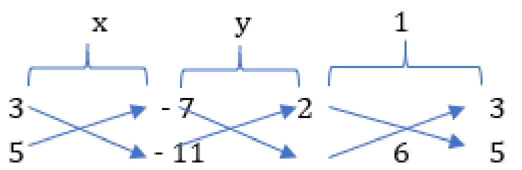
6x + 5y – 11 = 0

Answer

Given, pair of equations is

2x + 3y – 7 = 0 and 6x + 5y – 11 = 0

By cross - multiplication method, we have



$$\Rightarrow \frac{x}{-33 + 35} = \frac{y}{-42 + 22} = \frac{1}{10 - 18}$$

$$\Rightarrow \frac{x}{2} = \frac{y}{-20} = \frac{1}{-8}$$

I II III

On taking I and III ratio, we get

$$\Rightarrow \frac{x}{2} = \frac{1}{-8}$$

$$\Rightarrow x = \frac{1}{-4}$$

On taking II and III ratio, we get

$$\Rightarrow \frac{y}{-20} = \frac{1}{-8}$$

$$\Rightarrow y = \frac{5}{2}$$

1 E. Question

Solve the following pair of linear equation by cross - multiplication method:

$$\frac{2}{x} + \frac{3}{y} = 13$$

$$\frac{5}{x} - \frac{4}{y} = -2$$

Answer

Given, pair of equations is

$$\frac{2}{x} + \frac{3}{y} = 13$$

$$\frac{5}{x} - \frac{4}{y} = -2$$

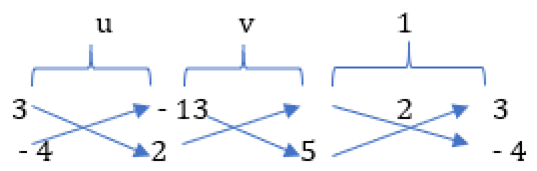
Let $u = \frac{1}{x}$ and $v = \frac{1}{y}$

So, Eq. (1) and (2) reduces to

2u + 3v – 13 = 0

5u – 4v + 2 = 0

By cross - multiplication method, we have



$$\Rightarrow \frac{u}{6 - 52} = \frac{v}{-65 - 4} = \frac{1}{-8 - 15}$$

$$\Rightarrow \frac{u}{-46} = \frac{v}{-69} = \frac{1}{-23}$$

I II III

On taking I and III ratio, we get

$$\Rightarrow \frac{u}{-46} = \frac{1}{-23}$$

$\Rightarrow$ u = 2

On taking II and III ratio, we get

$$\Rightarrow \frac{v}{-69} = \frac{1}{-23}$$

$\Rightarrow$ v = 3

So, u = $\frac{1}{x} = 2 \Rightarrow x = \frac{1}{2}$

and v = $\frac{1}{y} = 3 \Rightarrow y = \frac{1}{3}$

1 F. Question

Solve the following pair of linear equation by cross - multiplication method:

$$\frac{x}{3} - \frac{y}{12} = \frac{19}{4}$$

$$\frac{x}{3} - \frac{y}{12} = \frac{19}{4}$$

Answer

Given, pair of equations is

$$\frac{x}{6} + \frac{y}{15} = 4$$

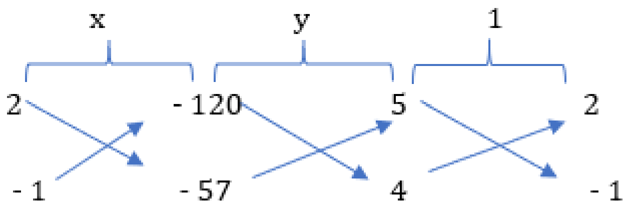
$$\Rightarrow 5x + 2y = 4 \times 30$$

$$\Rightarrow 5x + 2y - 120 = 0$$

And $\frac{x}{3} - \frac{y}{12} = \frac{19}{4}$

$$\Rightarrow 4x - y = 57$$

By cross - multiplication method, we have



$$\Rightarrow \frac{x}{-114-120} = \frac{y}{-480 + 285} = \frac{1}{-5-8}$$

$$\Rightarrow \underset{\text{I}}{\frac{x}{-234}} = \underset{\text{II}}{\frac{y}{-195}} = \underset{\text{III}}{\frac{1}{-13}}$$

On taking I and III ratio, we get

$$\Rightarrow \frac{x}{-234} = \frac{1}{-13}$$

$\Rightarrow x = 18$

On taking II and III ratio, we get

$$\Rightarrow \frac{y}{-195} = \frac{1}{-13}$$

$\Rightarrow y = 15$

2 A. Question

Solve the following pair of equations by cross - multiplication method.

$ax + by = a - b$

$bx - ay = a + b$

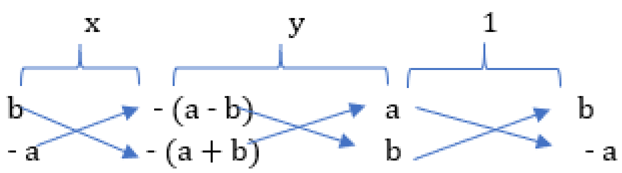
Answer

Given, pair of equations is

$ax + by = a - b \Rightarrow ax + by - (a - b) = 0$

$bx - ay = a + b \Rightarrow bx - ay - (a + b) = 0$

By cross - multiplication method, we have



$$\Rightarrow \frac{x}{-b(a+b)-a(a-b)} = \frac{y}{-b(a-b)+a(a+b)} = \frac{1}{-a^2-b^2}$$

$$\Rightarrow \frac{x}{-ba-b^2-a^2+ab} = \frac{y}{-ba+b^2+a^2+ab} = \frac{1}{-a^2-b^2}$$

$$\Rightarrow \underset{\text{I}}{\frac{x}{-b^2-a^2}} = \underset{\text{II}}{\frac{y}{b^2+a^2}} = \underset{\text{III}}{\frac{1}{-a^2-b^2}}$$

On taking I and III ratio, we get

$$\Rightarrow \frac{x}{-b^2-a^2} = \frac{1}{-a^2-b^2}$$

$\Rightarrow x = 1$

On taking II and III ratio, we get

$$\Rightarrow \frac{y}{b^2+a^2} = \frac{1}{-a^2-b^2}$$

$\Rightarrow y = -1$

2 B. Question

Solve the following pair of equations by cross - multiplication method.

$$a^2 - b^2$$

$$\frac{x}{a^2} + \frac{y}{b^2} = 2$$

$$a \neq 0, b \neq 0$$

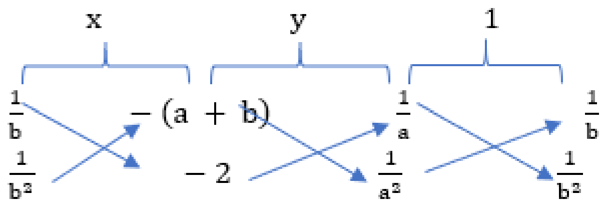
Answer

Given, pair of equations is

$$\frac{x}{a} + \frac{y}{b} = a + b \Rightarrow \frac{x}{a} + \frac{y}{b} - (a+b) = 0$$

And $\frac{x}{a^2} + \frac{y}{b^2} = 2 \Rightarrow \frac{x}{a^2} + \frac{y}{b^2} - 2 = 0$

By cross - multiplication method, we have



$$\Rightarrow \frac{x}{\frac{-2}{b} + \frac{a+b}{b^2}} = \frac{y}{-\frac{a+b}{a^2} + \frac{2}{a}} = \frac{1}{\frac{1}{ab^2} - \frac{1}{a^2b}}$$

$$\Rightarrow \frac{b^2x}{-2b + a + b} = \frac{a^2y}{-a - b + 2a} = \frac{a^2b^2}{a - b}$$

$$\Rightarrow \underset{\text{I}}{\frac{b^2x}{a-b}} = \underset{\text{II}}{\frac{a^2y}{a-b}} = \underset{\text{III}}{\frac{a^2b^2}{a-b}}$$

On taking I and III ratio, we get

$$\Rightarrow \frac{b^2x}{a-b} = \frac{a^2b^2}{a-b}$$

$$\Rightarrow \frac{x}{1} = \frac{a^2b^2}{b^2}$$

$\Rightarrow x = a^2$

On taking II and III ratio, we get

$$\Rightarrow \frac{a^2y}{a-b} = \frac{a^2b^2}{a-b}$$

$\Rightarrow y = b^2$

2 C. Question

Solve the following pair of equations by cross - multiplication method.

$x - y = a + b$

$ax + by = a^2 - b^2$

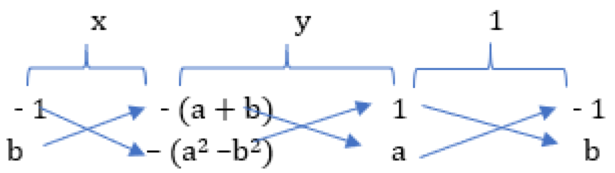
Answer

Given, pair of equations is

$x - y = a + b \Rightarrow x - y - (a + b) = 0$

$ax + by = a^2 - b^2 \Rightarrow ax + by - (a^2 - b^2) = 0$

By cross – multiplication method, we have



$$\Rightarrow \frac{x}{a^2 - b^2 + b(a + b)} = \frac{y}{-a(a + b) + a^2 - b^2} = \frac{1}{b + a}$$

$$\Rightarrow \frac{x}{a^2 + ab} = \frac{y}{-ba - b^2} = \frac{1}{b + a}$$

$$\Rightarrow \underset{\text{I}}{\frac{x}{a(a + b)}} = \underset{\text{II}}{\frac{y}{-b(a + b)}} = \underset{\text{III}}{\frac{1}{a + b}}$$

On taking I and III ratio, we get

$$\Rightarrow \frac{x}{a(a + b)} = \frac{1}{a + b}$$

$\Rightarrow x = a$

On taking II and III ratio, we get

$$\Rightarrow \frac{y}{-b(a + b)} = \frac{1}{a + b}$$

$\Rightarrow y = - b$

2 D. Question

Solve the following pair of equations by cross - multiplication method.

$$\frac{2x}{a} + \frac{y}{b} = 2$$

$$\frac{x}{a} - \frac{y}{b} = 4$$

$a \neq 0, b \neq 0$

Answer

$$\frac{2x}{a} + \frac{y}{b} = 2$$

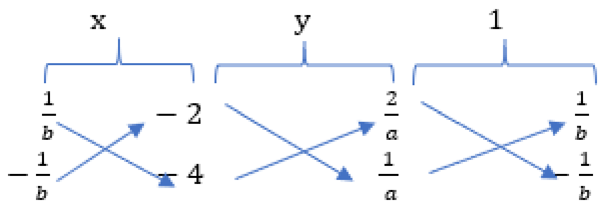
$$\frac{x}{a} - \frac{y}{b} = 4$$

Given, pair of equations is

$$\frac{2x}{a} + \frac{y}{b} = 2 \Rightarrow \frac{2x}{a} + \frac{y}{b} - 2 = 0$$

And $\frac{x}{a} - \frac{y}{b} = 4 \Rightarrow \frac{x}{a} - \frac{y}{b} - 4 = 0$

By cross - multiplication method, we have



$$\Rightarrow \frac{x}{\frac{-4}{b} - \frac{2}{b}} = \frac{y}{\frac{-2}{a} + \frac{8}{a}} = \frac{1}{-\frac{2}{ab} - \frac{1}{ab}}$$

$$\Rightarrow \frac{bx}{-6} = \frac{ay}{6} = \frac{ab}{-3}$$

I II III

On taking I and III ratio, we get

$$\Rightarrow \frac{bx}{-6} = \frac{ab}{-3}$$

$\Rightarrow x = 2a$

On taking II and III ratio, we get

$$\Rightarrow \frac{ay}{6} = \frac{ab}{-3}$$

$\Rightarrow y = -2b$

2 E. Question

Solve the following pair of equations by cross - multiplication method.

$2ax + 3by = a + 2b$

$3ax + 2by = 2a + b$

Answer

$2ax + 3by = a + 2b$

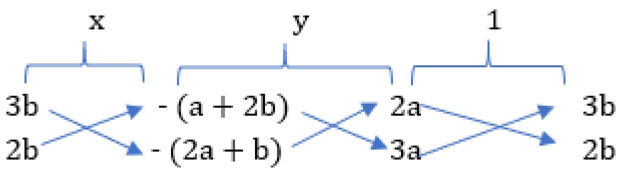
$3ax + 2by = 2a + b$

Given, pair of equations is

$2ax + 3by = a + 2b \Rightarrow 2ax + 3by - (a + 2b) = 0$

$3ax + 2by = 2a + b \Rightarrow 3ax + 2by - (2a + b) = 0$

By cross - multiplication method, we have



$$\Rightarrow \frac{x}{-3b(2a + b) + 2b(a + 2b)} = \frac{y}{-3a(a + 2b) + 2a(2a + b)} = \frac{1}{4ab - 9ab}$$

$$\Rightarrow \frac{x}{-4ba - 3b^2 + 4ab + 4b^2} = \frac{y}{-3a^2 - 6ba + 4a^2 + 2ab} = \frac{1}{-5ab}$$

$$\Rightarrow \underset{\text{I}}{\frac{x}{b^2 - 4ab}} = \underset{\text{II}}{\frac{y}{-4ab + a^2}} = \underset{\text{III}}{\frac{1}{-5ab}}$$

On taking I and III ratio, we get

$$\Rightarrow \frac{x}{b^2 - 4ab} = \frac{1}{-5ab}$$

$$\Rightarrow x = \frac{b(b - 4a)}{-5ab}$$

$$\Rightarrow x = \frac{4a - b}{5a}$$

On taking II and III ratio, we get

$$\Rightarrow \frac{y}{-4ab + a^2} = \frac{1}{-5ab}$$

$$\Rightarrow y = \frac{a(a - 4b)}{-5ab}$$

$$\Rightarrow y = \frac{4b - a}{5b}$$

2 F. Question

Solve the following pair of equations by cross - multiplication method.

$$\frac{x}{a} + \frac{y}{b} = 2$$

$ax + by = a^2 - b^2$

Answer

$$\frac{x}{a} + \frac{y}{b} = 2$$

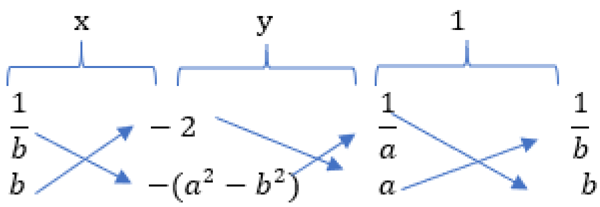
$ax + by = a^2 - b^2$

Given, pair of equations is

$$\frac{x}{a} + \frac{y}{b} = 2 \Rightarrow \frac{x}{a} + \frac{y}{b} - 2 = 0$$

$ax + by = a^2 - b^2 \Rightarrow ax + by - (a^2 - b^2) = 0$

By cross - multiplication method, we have



$$\Rightarrow \frac{x}{\frac{-(a^2 + b^2)}{b} + 2b} = \frac{y}{-2a + \frac{(a^2 + b^2)}{a}} = \frac{1}{\frac{b}{a} - \frac{a}{b}}$$

$$\Rightarrow \frac{bx}{-a^2 - b^2 + 2b^2} = \frac{ay}{-2a^2 + a^2 + b^2} = \frac{ab}{b^2 - a^2}$$

$$\Rightarrow \frac{bx}{-a^2 + b^2} = \frac{ay}{-a^2 + b^2} = \frac{ab}{b^2 - a^2}$$

$$\Rightarrow \underset{\text{I}}{\frac{bx}{b^2 - a^2}} = \underset{\text{II}}{\frac{ay}{b^2 - a^2}} = \underset{\text{III}}{\frac{ab}{b^2 - a^2}}$$

On taking I and III ratio, we get

$$\Rightarrow \frac{bx}{b^2 - a^2} = \frac{ab}{b^2 - a^2}$$

$\Rightarrow x = a$

On taking II and III ratio, we get

$$\Rightarrow \frac{ay}{b^2 - a^2} = \frac{ab}{b^2 - a^2}$$

$\Rightarrow y = b$

3. Question

Solve the following system of equations by cross - multiplication method.

$a(x + y) + b(x - y) = a^2 - ab + b^2$

$a(x + y) - b(x - y) = a^2 + ab + b^2$

Answer

The given system of equations can be re - written as

$ax + ay + bx - by - a^2 + ab - b^2 = 0$

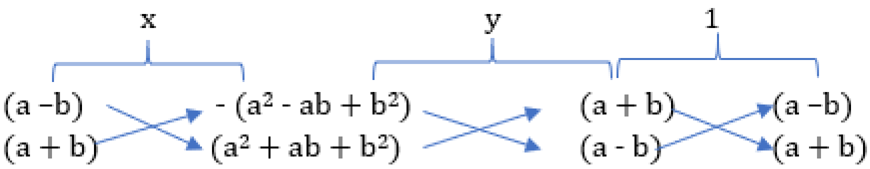
$\Rightarrow (a + b)x + (a - b)y - (a^2 - ab + b^2) = 0$ ...(1)

and $ax + ay - bx + by - a^2 - ab - b^2 = 0$

$\Rightarrow (a - b)x + (a + b)y - (a^2 + ab + b^2) = 0$ ...(2)

Now, by cross – multiplication method, we have

By cross - multiplication method, we have



$$\Rightarrow \frac{x}{-(a-b)(a^2 + ab + b^2) + (a + b)(a^2 - ab + b^2)}$$

$$= \frac{y}{-(a^2 - ab + b^2)(a - b) + (a^2 + ab + b^2)(a + b)}$$

$$= \frac{1}{(a + b)(a + b) - (a - b)(a - b)}$$

$$\Rightarrow \frac{x}{-(a^3 - b^3) + (a^3 + b^3)}$$

$$= \frac{y}{-a^3 + a^2b - b^2a + ba^2 - ab^2 + b^3 + a^3 + a^2b + b^2a + ba^2 + ab^2 + b^3}$$

$$= \frac{1}{a^2 + 2ab + b^2 - a^2 + 2ab - b^2}$$

$$\Rightarrow \underset{\text{I}}{\frac{x}{2b^3}} = \underset{\text{II}}{\frac{y}{4a^2b + 2b^3}} = \underset{\text{III}}{\frac{1}{4ab}}$$

On taking I and III ratio, we get

$$\Rightarrow \frac{x}{2b^3} = \frac{1}{4ab}$$

$$\Rightarrow x = \frac{b^2}{2a}$$

On taking II and III ratio, we get

$$\Rightarrow \frac{y}{4a^2b + 2b^3} = \frac{1}{4ab}$$

$$\Rightarrow y = \frac{2a^2 + b^2}{2a}$$

4 A. Question

Which of the following pair of linear equations has unique solution, no solution or infinitely many solutions. In case there is a unique solution, find it by using cross - multiplication method:

$x - 3y - 7 = 0$

$3x - 3y - 15 = 0$

Answer

Given pair of linear equations

$x - 3y - 7 = 0$

$3x - 3y - 15 = 0$

$\Rightarrow x - y - 5 = 0$ ...(ii)

As we can see that $a_1 = 1$, $b_1 = -3$ and $c_1 = -7$

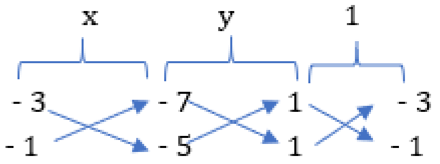
and $a_2 = 1$, $b_2 = -1$ and $c_2 = -5$

$$\frac{a_1}{a_2} = \frac{1}{1} = 1, \frac{b_1}{b_2} = \frac{-3}{-1} = 3 \text{ and } \frac{c_1}{c_2} = \frac{-7}{-5} = \frac{7}{5}$$

$$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

∴ Given pair of equations has unique solution

By cross - multiplication method, we have



$$\Rightarrow \frac{x}{15-7} = \frac{y}{-7+5} = \frac{1}{-1+3}$$

$$\Rightarrow \frac{x}{8} = \frac{y}{-2} = \frac{1}{2}$$

I II III

On taking I and III ratio, we get

$$\Rightarrow \frac{x}{8} = \frac{1}{2}$$

$\Rightarrow$ x = 4

On taking II and III ratio, we get

$$\Rightarrow \frac{y}{-2} = \frac{1}{2}$$

$\Rightarrow$ y = – 1

4 B. Question

Which of the following pair of linear equations has unique solution, no solution or infinitely many solutions. In case there is a unique solution, find it by using cross - multiplication method:

2x + y = 5

3x + 2y = 8

Answer

Given pair of linear equations

2x + y = 5

3x + 2y = 8

As we can see that $a_1 = 2$, $b_1 = 1$ and $c_1 = -5$

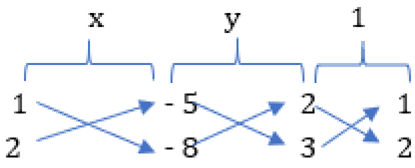
and $a_2 = 3$, $b_2 = 2$ and $c_2 = -8$

$$\frac{a_1}{a_2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{1}{2} \text{ and } \frac{c_1}{c_2} = \frac{-5}{-8} = \frac{5}{8}$$

$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

∴ Given pair of equations has unique solution

By cross - multiplication method, we have



$\Rightarrow \frac{x}{-8 + 10} = \frac{y}{-15 + 16} = \frac{1}{4 - 3}$

$\Rightarrow \underset{\text{I}}{\frac{x}{2}} = \underset{\text{II}}{\frac{y}{1}} = \underset{\text{III}}{\frac{1}{1}}$

On taking I and III ratio, we get

$\Rightarrow \frac{x}{2} = 1$

⇒ x = 2

On taking II and III ratio, we get

$\Rightarrow \frac{y}{1} = 1$

⇒ y = 1

4 C. Question

Which of the following pair of linear equations has unique solution, no solution or infinitely many solutions. In case there is a unique solution, find it by using cross - multiplication method:

3x – 5y = 20

6x – 10y = 40

Answer

Given pair of linear equations

3x – 5y = 20

6x – 10y = 40

⇒ 3x – 5y = 20 …(ii)

As we can see that $a_1 = 3$, $b_1 = -5$ and $c_1 = -20$

and $a_2 = 3$, $b_2 = -5$ and $c_2 = -20$

$$\frac{a_1}{a_2} = \frac{3}{3} = 1, \frac{b_1}{b_2} = \frac{-5}{-5} = 1 \text{ and } \frac{c_1}{c_2} = \frac{-20}{-20} = 1$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

∴ Given pair of equations has infinitely many solutions.

4 D. Question

Which of the following pair of linear equations has unique solution, no solution or infinitely many solutions. In case there is a unique solution, find it by using cross - multiplication method:

$x - 3y - 3 = 0$

$3x - 9y - 2 = 0$

Answer

Given pair of linear equations

$x - 3y - 3 = 0$

$3x - 9y - 2 = 0$

As we can see that $a_1 = 1$, $b_1 = -3$ and $c_1 = -3$

and $a_2 = 3$, $b_2 = -9$ and $c_2 = -2$

$$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3} \text{ and } \frac{c_1}{c_2} = \frac{-3}{-2} = \frac{3}{2}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

∴ Given pair of equations has no solution

4 E. Question

Which of the following pair of linear equations has unique solution, no solution or infinitely many solutions. In case there is a unique solution, find it by using cross - multiplication method:

$x + y = 2$

$2x + 2y = 4$

Answer

Given pair of linear equations

x + y = 2

2x + 2y = 4

⇒ x + y – 2 = 0

As we can see that a_1 = 1, b_1 = 1 and c_1 = – 2

and a_2 = 1, b_2 = 1 and c_2 = – 2

$$\frac{a_1}{a_2} = \frac{1}{1} = 1, \frac{b_1}{b_2} = \frac{1}{1} = 3 \text{ and } \frac{c_1}{c_2} = \frac{-2}{-2} = 1$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

∴ Given pair of equations has infinitely many solution

4 F. Question

Which of the following pair of linear equations has unique solution, no solution or infinitely many solutions. In case there is a unique solution, find it by using cross - multiplication method:

x + y = 2

2x + 2y = 6

Answer

Given pair of linear equations

x + y = 2

2x + 2y = 6

⇒ x + y – 3 = 0

As we can see that a_1 = 1, b_1 = 1 and c_1 = – 2

and a_2 = 1, b_2 = 1 and c_2 = – 3

$$\frac{a_1}{a_2} = \frac{1}{1} = 1, \frac{b_1}{b_2} = \frac{1}{1} = 1 \text{ and } \frac{c_1}{c_2} = \frac{-2}{-3} = \frac{2}{3}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

∴ Given pair of equations has no solution

5 A. Question

Solve the following system of linear equations by cross - multiplication method.

$$\frac{15}{x+y}+\frac{7}{x-y}-10=0$$

$$\frac{15}{x+y}+\frac{7}{x-y}-10=0$$

[Hint: Let u = $\frac{1}{x-y}$ and v = $\frac{1}{x-y}$]

Answer

Given, pair of equations is

$\frac{5}{x+y} + \frac{2}{x-y} + 1 = 0$ ...(1)

$\frac{15}{x+y} + \frac{7}{x-y} - 10 = 0$ ...(2)

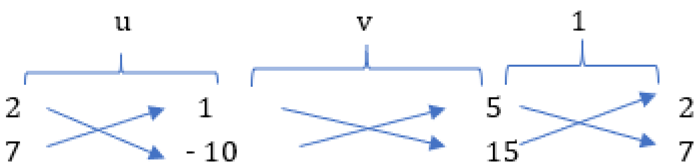
Let $u = \frac{1}{x+y}$ and $v = \frac{1}{x-y}$

Now, the Eq. (1) and (2) reduces to

5u + 2v + 1 = 0

15u + 7v – 10 = 0

By cross - multiplication method, we have



$$\Rightarrow \frac{u}{-20-7} = \frac{v}{15+50} = \frac{1}{35-30}$$

$$\Rightarrow \underset{\text{I}}{\frac{u}{-27}} = \underset{\text{II}}{\frac{v}{65}} = \underset{\text{III}}{\frac{1}{5}}$$

On taking I and III ratio, we get

$$\Rightarrow \frac{u}{-27} = \frac{1}{5}$$

$$\Rightarrow u = \frac{-27}{5}$$

On taking II and III ratio, we get

$$\Rightarrow \frac{v}{65} = \frac{1}{5}$$

$\Rightarrow$ v = 13

So, u = $\frac{1}{x+y} = \frac{-27}{5} \Rightarrow x + y = -\frac{5}{27}$ ...(a)

and v = $\frac{1}{x-y} = 13 \Rightarrow x - y = \frac{1}{13}$ ...(b)

On adding Eq. (a) and (b), we get

$$2x = -\frac{5}{27} + \frac{1}{13}$$

$$\Rightarrow 2x = \frac{-65 + 27}{27 \times 13}$$

$$\Rightarrow 2x = \frac{-38}{351}$$

$$\Rightarrow x = \frac{-19}{351}$$

On putting the value of $x = \frac{-19}{351}$ in Eq. (a), we get

$$\frac{-19}{351} + y = -\frac{5}{27}$$

$$\Rightarrow y = -\frac{5}{27} + \frac{19}{351}$$

$$\Rightarrow y = \frac{-65 + 19}{351}$$

$$\Rightarrow y = -\frac{46}{351}$$

5 B. Question

Solve the following system of linear equations by cross - multiplication method.

ax – ay = 2

(a – 1)x + (a + 1)y = $2(a^2 + 1)$

[Hint: Let u = $\frac{1}{x-y}$ and v = $\frac{1}{x-y}$]

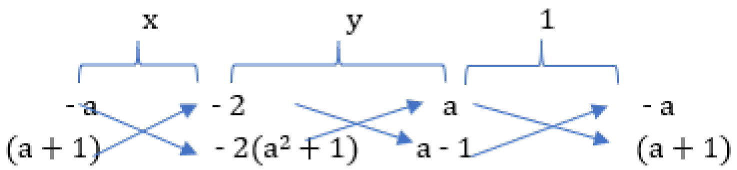
Answer

Given, pair of equations is

ax – ay = 2

(a – 1)x + (a + 1)y = $2(a^2 + 1)$

By cross - multiplication method, we have



$$\Rightarrow \frac{x}{2a(a^2+1)+2(a+1)} = \frac{y}{-2(a-1)+2a(a^2+1)} = \frac{1}{a(a+1)+a(a-1)}$$

$$\Rightarrow \frac{x}{2a^3+2a+2a+2} = \frac{y}{-2a+2+2a^3+2a} = \frac{1}{a^2+a+a^2-a}$$

$$\Rightarrow \underset{\text{I}}{\frac{x}{2a^3+4a+2}} = \underset{\text{II}}{\frac{y}{2a^3+2}} = \underset{\text{III}}{\frac{1}{2a^2}}$$

On taking I and III ratio, we get

$$\Rightarrow \frac{x}{2a^3+4a+2} = \frac{1}{2a^2}$$

$$\Rightarrow x = \frac{a^3+2a+1}{a^2}$$

On taking II and III ratio, we get

$$\Rightarrow \frac{y}{2a^3+2} = \frac{1}{2a^2}$$

$$\Rightarrow y = \frac{a^3+1}{a^2}$$

6. Question

If the cost of 2 pencils and 3 erasers is Rs. 9 and the cost of 4 pencils and 6 erasers is Rs. 18. Find the cost of each pencil and each eraser.

Answer

Let the cost of one pencil = Rs x

and cost of one eraser = Rs y

According to the question

2x + 3y = 9 ...(1)

4x + 6y = 18

⇒2(2x + 3y) = 18

⇒2x + 3y = 9 ...(2)

As we can see From Eq. (1) and (2)

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

∴Given pair of linear equations has infinitely many solutions.

7. Question

The paths traced by the wheels of two trains are given by equations x + 2y – 4 = 0 and 2x + 4y – 12 = 0. Will the paths cross each other?

Answer

Given paths traced by the wheel of two trains are

x + 2y – 4 = 0 ...(i)

2x + 4y – 12 = 0

⇒ x + 2y – 6 = 0 ...(ii)

As we can see that $a_1 = 1$, $b_1 = 2$ and $c_1 = -4$

and $a_2 = 1$, $b_2 = 2$ and $c_2 = -6$

$$\frac{a_1}{a_2} = \frac{1}{1} = 1, \frac{b_1}{b_2} = \frac{2}{2} = 1 \text{ and } \frac{c_1}{c_2} = \frac{-4}{-6} = \frac{2}{3}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

∴ Given pair of equations has no solution

Hence, two paths will not cross each other.

8. Question

The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditure is 4 : 3. If each of them manages to save Rs. 2000 per month, find their monthly incomes.

Answer

Given ratio of incomes = 9:7

And the ratio of their expenditures = 4:3

Saving of each person = Rs. 2000

Let incomes of two persons = 9x and 7x

And their expenditures = 4y and 3y

According to the question,

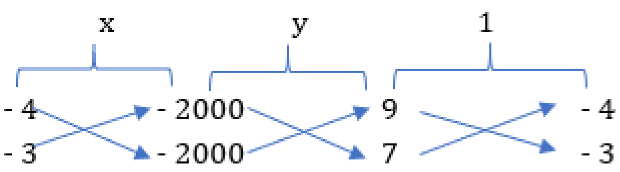
9x – 4y = 2000

⇒9x – 4y – 2000 = 0 ...(i)

7x – 3y = 2000

⇒7x – 3y – 2000 = 0 ...(ii)

By cross - multiplication method, we have



$$\Rightarrow \frac{x}{8000 - 6000} = \frac{y}{-14000 + 18000} = \frac{1}{-27 + 28}$$

$$\Rightarrow \underset{\text{I}}{\frac{x}{2000}} = \underset{\text{II}}{\frac{y}{4000}} = \underset{\text{III}}{\frac{1}{1}}$$

On taking I and III ratios, we get

$$\Rightarrow \frac{x}{2000} = \frac{1}{1}$$

⇒ x = 2000

On taking II and III ratios, we get

$$\Rightarrow \frac{y}{4000} = \frac{1}{1}$$

⇒ y = 4000

Hence, the monthly incomes of two persons are 9(2000) = Rs18000 and 7(2000) = Rs14000

9. Question

The sum of two - digits number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there?

Answer

Let unit's digit = y

and the ten's digit = x

So, the original number = 10x + y

The sum of the number = 10x + y

The sum of the digit = x + y

reversing number = x + 10y

According to the question,

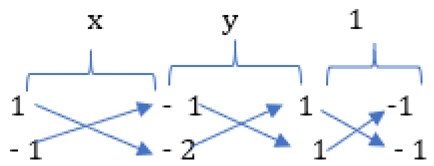
10x + y + x + 10y = 66

⇒11x + 11y = 66

⇒ x + y = 6 ...(i)

x – y = 2 ...(ii)

By cross - multiplication method, we have



$$\Rightarrow \frac{x}{-2-6} = \frac{y}{-6+2} = \frac{1}{-1-1}$$

$$\Rightarrow \frac{x}{-8} = \frac{y}{-4} = \frac{1}{-2}$$

I II III

On taking I and III ratio, we get

$$\Rightarrow \frac{x}{-8} = \frac{1}{-1}$$

$\Rightarrow x = 4$

On taking II and III ratio, we get

$$\Rightarrow \frac{y}{-4} = \frac{1}{-2}$$

$\Rightarrow y = 2$

So, the original number = 10x + y

= 10(4) + 2

= 42

Reversing the number = x + 10y

= 24

Hence, the two digit number is 42 and 24. These are two such numbers.

10. Question

If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes 1/2 if we add 1 to the denominator. What is the fraction?

Answer

Let the numerator = x

and the denominator = y

So, the fraction $= \frac{x}{y}$

According to the question,

Condition I:

$$\frac{x + 1}{y - 1} = 1$$

⇒ x + 1 = y – 1

⇒ x – y = – 2

⇒ x – y + 2 = 0 ...(i)

Condition II:

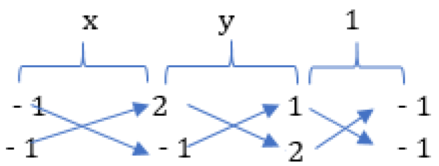
$$\frac{x}{y + 1} = \frac{1}{2}$$

⇒ 2x = y + 1

⇒ 2x – y = 1

⇒ 2x – y – 1 = 0 ...(ii)

By cross - multiplication method, we have



$$\Rightarrow \frac{x}{1 + 2} = \frac{y}{4 + 1} = \frac{1}{-1 + 2}$$

$$\Rightarrow \underset{I}{\frac{x}{3}} = \underset{II}{\frac{y}{5}} = \underset{III}{\frac{1}{1}}$$

On taking I and III ratio, we get

$$\Rightarrow \frac{x}{3} = \frac{1}{1}$$

⇒ x = 3

On taking II and III ratio, we get

$$\Rightarrow \frac{y}{5} = \frac{1}{1}$$

⇒ y = 5

So, the numerator is 3 and the denominator is 5

Hence, the fraction is $\frac{3}{5}$

11. Question

The cost of 5 oranges and 3 apples is Rs. 35 and the cost of 2 oranges and 4 apples is Rs. 28. Find the cost of an orange and an apple.

Answer

Let the cost of an orange = Rs x

And the cost of an apple = Rs y

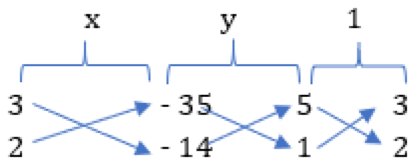
According to the question,

$5x + 3y = 35$

And $2x + 4y = 28$

$\Rightarrow x + 2y = 14$

By cross - multiplication method, we have



$$\Rightarrow \frac{x}{-42 + 70} = \frac{y}{-35 + 70} = \frac{1}{10 - 3}$$

$$\Rightarrow \underset{\text{I}}{\frac{x}{28}} = \underset{\text{II}}{\frac{y}{35}} = \underset{\text{III}}{\frac{1}{7}}$$

On taking I and III ratio, we get

$$\Rightarrow \frac{x}{28} = \frac{1}{7}$$

$\Rightarrow x = 4$

On taking II and III ratio, we get

$$\Rightarrow \frac{y}{35} = \frac{1}{7}$$

$\Rightarrow y = 5$

Hence, the cost of an orange is Rs. 4 and cost of an apple is Rs. 5

12. Question

A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days, she has to pay Rs. 1000 as hostel charges, whereas a student B, who takes food for 26 days, pays Rs. 1180 as hostel charges. Find the fixed charges and cost of food per day.

Answer

Let fixed hostel charge (monthly) = Rs x

and cost of food for one day = Rs y

In case of student A,

x + 20y = 1000

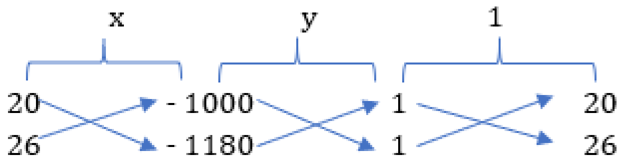
x + 20y – 1000 = 0 ...(i)

In case of student B,

x + 26y = 1180

x + 26y – 1180 = 0 ...(ii)

By cross - multiplication method, we have



$$\Rightarrow \frac{x}{-23600 + 26000} = \frac{y}{-1000 + 1180} = \frac{1}{26 - 20}$$

$$\Rightarrow \underset{\text{I}}{\frac{x}{2400}} = \underset{\text{II}}{\frac{y}{180}} = \underset{\text{III}}{\frac{1}{6}}$$

On taking I and III ratio, we get

$$\Rightarrow \frac{x}{2400} = \frac{1}{6}$$

⇒ x = 400

On taking II and III ratio, we get

$$\Rightarrow \frac{y}{180} = \frac{1}{6}$$

⇒ y = 30

Hence, monthly fixed charges is Rs. 400 and cost of food per day is Rs. 30

13. Question

A fraction becomes 1/3 when 1 is subtracted from the numerator and it becomes 1/4 when 8 is added to its denominator. Find the fraction.

Answer

Let the numerator = x

and the denominator = y

So, the fraction $= \frac{x}{y}$

According to the question,

Condition I:

$$\frac{x-1}{y} = \frac{1}{3}$$

⇒ 3(x – 1) = y

⇒ 3x – 3 = y

⇒ 3x – y – 3 = 0 ...(i)

Condition II:

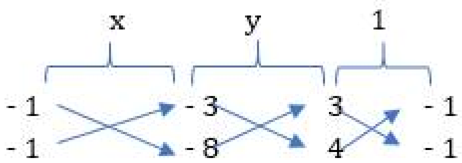
$$\frac{x}{y+8} = \frac{1}{4}$$

⇒ 4x = y + 8

⇒ 4x – y = 8

⇒ 4x – y – 8 = 0 ...(ii)

By cross - multiplication method, we have



$$\Rightarrow \frac{x}{8-3} = \frac{y}{-12+24} = \frac{1}{-3+4}$$

$$\Rightarrow \underset{\text{I}}{\frac{x}{5}} = \underset{\text{II}}{\frac{y}{12}} = \underset{\text{III}}{\frac{1}{1}}$$

On taking I and III ratio, we get

$$\Rightarrow \frac{x}{5} = \frac{1}{1}$$

$\Rightarrow x = 5$

On taking II and III ratio, we get

$$\Rightarrow \frac{y}{12} = \frac{1}{1}$$

$\Rightarrow y = 12$

So, the numerator is 5 and the denominator is 12

Hence, the fraction is $\frac{3}{5}$

Exercise 3.5

1. Question

The sum of the two numbers is 18. The sum of their reciprocals is 1/4. Find the numbers.

Answer

Let the two numbers be x and y.

According to the question,

$x + y = 18$ ...(i)

$\frac{1}{x} + \frac{1}{y} = \frac{1}{4}$ ...(ii)

Eq. (ii) can be re - written as

$\frac{y + x}{xy} = \frac{1}{4}$ ...(iii)

On putting the value of $x + y = 18$ in Eq. (iii), we get

$$\frac{18}{xy} = \frac{1}{4}$$

$\Rightarrow xy = 72$

$$\Rightarrow x = \frac{72}{y}$$

On putting the value of $x = \frac{72}{y}$ in Eq. (i), we get

$$\frac{72}{y} + y = 18$$

$\Rightarrow 72 + y^2 = 18y$

$\Rightarrow y^2 - 18y + 72 = 0$

$\Rightarrow y^2 - 12y - 6y + 72 = 0$

$\Rightarrow y(y - 12) - 6(y - 12) = 0$

$\Rightarrow (y - 6)(y - 12) = 0$

$\Rightarrow y = 6$ and 12

$$\text{If } y = 6, \text{then } x = \frac{72}{6} = 12$$

$$\text{If } y = 12, \text{then } x = \frac{72}{12} = 6$$

Hence, the two numbers are 6 and 12.

2. Question

The sum of two numbers is 15 and sum of their reciprocals is $\frac{3}{10}$. Find the numbers.

Answer

Let the two numbers be x and y.

According to the question,

$x + y = 15$ ...(i)

$\frac{1}{x} + \frac{1}{y} = \frac{3}{10}$ ...(ii)

Eq. (ii) can be re - written as

$\frac{y + x}{xy} = \frac{3}{10}$ ...(iii)

On putting the value of $x + y = 18$ in Eq. (iii), we get

$$\frac{15}{xy} = \frac{3}{10}$$

$\Rightarrow xy = 50$

$\Rightarrow x = \frac{50}{y}$

On putting the value of $x = \frac{50}{y}$ in Eq. (i), we get

$\frac{50}{y} + y = 18$

$\Rightarrow 50 + y^2 = 15y$

$\Rightarrow y^2 - 15y + 50 = 0$

$\Rightarrow y^2 - 10y - 5y + 50 = 0$

$\Rightarrow y(y - 10) - 5(y - 10) = 0$

$\Rightarrow (y - 5)(y - 10) = 0$

$\Rightarrow y = 5 \text{ and } 10$

If $y = 5$, then $x = \frac{50}{5} = 10$

If $y = 10$, then $x = \frac{50}{10} = 5$

Hence, the two numbers are 5 and 10.

3. Question

Two numbers are in the ratio of 5 : 6. If 8 is subtracted from each of the numb, they become in the ratio of 4 : 5. Find the numbers.

Answer

Let the two numbers be x and y.

According to the question,

$\frac{x}{y} = \frac{5}{6}$

$\Rightarrow y = \frac{6x}{5}$ ...(i)

Also, $\frac{x-8}{y-8} = \frac{4}{5}$

$\Rightarrow 5(x - 8) = 4(y - 8)$

$\Rightarrow 5x - 40 = 4y - 32$

$\Rightarrow 5x - 4y = 8$ ...(ii)

On putting the value of $y = \frac{6x}{5}$ in Eq. (ii), we get

$$5x - 4\left(\frac{6x}{5}\right) = 8$$

$$\Rightarrow \frac{25x - 24x}{5} = 8$$

$\Rightarrow x = 40$

On putting the value of x = 40 in Eq. (i), we get

$$y = \frac{6 \times 40}{5} = 48$$

Hence, the two numbers are 40 and 48.

4. Question

The sum of two numbers is 16 and the sum of their reciprocals is 1/3. Find the numbers.

Answer

Let the two numbers be x and y.

According to the question,

$x + y = 16$ ...(i)

$\frac{1}{x} + \frac{1}{y} = \frac{1}{3}$ ...(ii)

Eq. (ii) can be re - written as

$\frac{y+x}{xy} = \frac{1}{3}$ ...(iii)

On putting the value of x + y = 16 in Eq. (iii), we get

$$\frac{16}{xy} = \frac{1}{3}$$

$\Rightarrow xy = 48$

$$\Rightarrow x = \frac{48}{y}$$

On putting the value of $x = \frac{48}{y}$ in Eq. (i), we get

$$\frac{48}{y} + y = 16$$

$\Rightarrow 48 + y^2 = 16y$

$\Rightarrow y^2 - 16y + 48 = 0$

$\Rightarrow y^2 - 12y - 4y + 48 = 0$

$\Rightarrow y(y - 12) - 4(y - 12) = 0$

$\Rightarrow (y - 4)(y - 12) = 0$

$\Rightarrow y = 4$ and 12

$$\text{If } y = 4, \text{then } x = \frac{48}{4} = 12$$

$$\text{If } y = 12, \text{then } x = \frac{48}{12} = 4$$

Hence, the two numbers are 4 and 12.

5. Question

Two positive numbers differ by 3 and their product is 54. Find the numbers.

Answer

Let the two numbers be x and y.

According to the question,

$x - y = 3$ ...(i)

Also, $x \times y = 54$

$\Rightarrow x = \frac{54}{y}$ ...(ii)

On putting the value of $x = \frac{54}{y}$ in Eq. (i), we get

$$\frac{54}{y} - y = 3$$

$\Rightarrow 54 - y^2 = 3y$

$\Rightarrow y^2 + 3y - 54 = 0$

$\Rightarrow y^2 + 9y - 6y - 54 = 0$

$\Rightarrow y(y + 9) - 6(y + 9) = 0$

$\Rightarrow (y - 6)(y + 9) = 0$

$\Rightarrow y = -9$ and 6

But $y = -9$ can't be the one number as it is given that the numbers are positive.

$$\Rightarrow y = 6, \text{then } x = \frac{54}{6} = 9$$

Hence, the two numbers are 9 and 6.

6. Question

Two numbers are in the ratio of 3 : 5. If 5 is subtracted from each of the number they become in the ratio of 1 : 2. Find the numbers.

Answer

Let the two numbers be x and y.

According to the question,

$$\frac{x}{y} = \frac{3}{5}$$

$\Rightarrow y = \frac{5x}{3}$ ...(i)

Also, $\frac{x-5}{y-5} = \frac{1}{2}$

$\Rightarrow 2(x - 5) = (y - 5)$

$\Rightarrow 2x - 10 = y - 5$

$\Rightarrow 2x - y = 5$ ...(ii)

On putting the value of $y = \frac{5x}{3}$ in Eq. (ii), we get

$$2x - \left(\frac{5x}{3}\right) = 5$$

$$\Rightarrow \frac{6x - 5x}{3} = 5$$

$\Rightarrow x = 15$

On putting the value of x = 15 in Eq. (i), we get

$$y = \frac{5 \times 15}{3} = 25$$

Hence, the two numbers are 15 and 25.

7. Question

Two numbers are in the ratio of 3 : 4. If 8 is added to each number, they become in the ratio of 4 : 5. Find the numbers.

Answer

Let the two numbers be x and y.

According to the question,

$$\frac{x}{y} = \frac{3}{4}$$

$\Rightarrow y = \frac{4x}{3}$ ...(i)

Also, $\frac{x+8}{y+8} = \frac{4}{5}$

$\Rightarrow 5(x + 8) = 4(y + 8)$

$\Rightarrow 5x + 40 = 4y + 32$

$\Rightarrow 5x - 4y = -8$ ...(ii)

On putting the value of $y = \frac{6x}{5}$ in Eq. (ii), we get

$$5x - 4\left(\frac{4x}{3}\right) = -8$$

$$\Rightarrow \frac{15x - 16x}{3} = -8$$

$\Rightarrow x = 24$

On putting the value of x = 24 in Eq. (i), we get

$$y = \frac{4 \times 24}{3} = 32$$

Hence, the two numbers are 24 and 32.

8. Question

Two numbers differ by 2 and their product is 360. Find the numbers.

Answer

Let the two numbers be x and y.

According to the question,

$x - y = 2$ ...(i)

Also, $x \times y = 360$

$\Rightarrow x = \frac{360}{y}$ ...(ii)

On putting the value of $x = \frac{360}{y}$ in Eq. (i), we get

$$\frac{360}{y} - y = 2$$

$\Rightarrow 360 - y^2 = 2y$

$\Rightarrow y^2 + 2y - 360 = 0$

$\Rightarrow y^2 + 20y - 18y - 360 = 0$

$\Rightarrow y(y + 20) - 18(y + 20) = 0$

$\Rightarrow (y - 18)(y + 20) = 0$

$\Rightarrow y = -20$ and 18

But $y = -20$ can't be the one number as it is given that the numbers are positive.

$\Rightarrow y = 18, \text{then } x = \frac{360}{18} = 20$

Hence, the two numbers are 20 and 18.

9. Question

Two numbers differ by 4 and their product is 192. Find the numbers.

Answer

Let the two numbers be x and y.

According to the question,

$x - y = 4$ ...(i)

Also, x×y = 192

$\Rightarrow x = \frac{192}{y}$ ...(ii)

On putting the value of $x = \frac{192}{y}$ in Eq. (i), we get

$$\frac{192}{y} - y = 4$$

$\Rightarrow 192 - y^2 = 4y$

$\Rightarrow y^2 + 4y - 192 = 0$

$\Rightarrow y^2 + 16y - 12y - 192 = 0$

$\Rightarrow y(y + 16) - 12(y + 16) = 0$

$\Rightarrow (y - 12)(y + 16) = 0$

$\Rightarrow y = -16$ and 12

But y = – 16 can't be the one number as it is given that the numbers are positive.

$\Rightarrow y = 12$, then $x = \frac{192}{12} = 16$

Hence, the two numbers are 16 and 12.

10. Question

Two numbers differ by 4 and their product is 96. Find the numbers.

Answer

Let the two numbers be x and y.

According to the question,

x – y = 4 ...(i)

Also, x×y = 96

$\Rightarrow x = \frac{96}{y}$ ...(ii)

On putting the value of $x = \frac{96}{y}$ in Eq. (i), we get

$$\frac{96}{y} - y = 4$$

$\Rightarrow 96 - y^2 = 4y$

$\Rightarrow y^2 + 4y - 96 = 0$

$\Rightarrow y^2 + 12y - 8y - 96 = 0$

$\Rightarrow y(y + 12) - 8(y + 12) = 0$

$\Rightarrow (y - 8)(y + 12) = 0$

$\Rightarrow y = -8$ and 12

But $y = -8$ can't be the one number as it is given that the numbers are positive.

$$\Rightarrow y = 12, \text{then } x = \frac{96}{12} = 8$$

Hence, the two numbers are 8 and 12.

11. Question

The monthly incomes of A and B are in the ratio of 5 : 4 and their monthly expenditures are in the ratio of 7 : 5. If each saves Rs. 3000 per month, find the monthly income of each.

Answer

Given ratio of incomes = 5:4

And the ratio of their expenditures = 7:5

Saving of each person = Rs. 3000

Let incomes of two persons = 5x and 4x

And their expenditures = 7y and 5y

According to the question,

$5x - 7y = 3000$ ...(i)

$4x - 5y = 3000$ ...(ii)

On multiplying Eq. (i) by 4 and Eq. (ii) by 5 to make the coefficients of x equal, we get

$20x - 28y = 12000$ ...(iii)

$20x - 25y = 15000$ ...(iv)

On subtracting Eq. (iii) from (iv), we get

$20x - 25y - 20x + 28y = 15000 - 12000$

$\Rightarrow 3y = 3000$

$\Rightarrow y = 1000$

On putting the y = 1000 in Eq. (i), we get

$5x - 7y = 3000$

$\Rightarrow 5x - 7(1000) = 3000$

$\Rightarrow 5x = 10000$

$\Rightarrow x = 2000$

Thus, monthly income of both the persons are 5(2000) and 4(2000), i.e. Rs. 10000 and Rs. 8000

12. Question

Scooter charges consist of fixed charges and the remaining depending upon the distance travelled in kilometres. If a person travels 12 km, he pays Rs. 45 and for travelling 20 km, he pays Rs. 73. Express the above statements in the form of simultaneous equations and hence, find the fixed charges and the rate per km.

Answer

Let fixed charge = Rs x

and charge per kilometer = Rs y

According to the question,

$x + 12y = 45$ ...(i)

and $x + 20y = 73$ ...(ii)

On subtracting Eq. (i) from Eq. (ii), we get

$x + 20y - x - 12y = 73 - 45$

$\Rightarrow 8y = 28$

$$\Rightarrow y = \frac{28}{8} = 3.5$$

On putting the value of y = 3.5 in Eq. (i), we get

$x + 12(3.5) = 45$

$\Rightarrow x + 42 = 45$

$\Rightarrow x = 45 - 42 = 3$

Hence, monthly fixed charges is Rs. 3 and charge per kilometer is Rs. 3.5

13. Question

A part of monthly hostel charges in a college is fixed and the remaining depend on the number of days one has taken food in the mess. When a student A, takes food for 22 days, he has to pay Rs. 1380 as hostel charges, whereas a student B, who takes food for 28 days, pays Rs. 1680 as hostel charges. Find the fixed charge and the cost of food per day.

Answer

Let fixed hostel charge (monthly) = Rs x

and cost of food for one day = Rs y

In case of student A,

$x + 22y = 1380$ …(i)

In case of student B,

$x + 28y = 1680$ …(ii)

On subtracting Eq. (i) from Eq. (ii), we get

$x + 28y - x - 22y = 1680 - 1380$

$\Rightarrow 6y = 300$

$\Rightarrow y = 50$

On putting the value of $y = 50$ in Eq. (i), we get

$x + 22(50) = 1380$

$\Rightarrow x + 1100 = 1380$

$\Rightarrow x = 1380 - 1100 = 280$

Hence, monthly fixed charges is Rs. 280 and cost of food per day is Rs. 50

14. Question

Taxi charges in a city consist of fixed charges per day and the remaining depending upon the distance travelled in kilometers. If a person travels 110 km, he pays Rs. 690, and for travelling 200 km, he pays Rs. 1050. Find the fixed charges per day and the rate per km.

Answer

Let fixed charge = Rs. x

and charge per kilometer = Rs. y

According to the question,

x + 110y = 690 …(i)

and x + 200y = 1050 …(ii)

On subtracting Eq. (i) from Eq. (ii), we get

x + 200y – x – 110y = 1050 – 690

⇒ 90y = 360

⇒ y = 40

On putting the value of y = 40 in Eq. (i), we get

x + 110(40) = 690

⇒ x + 440 = 690

⇒ x = 690 – 440 = 250

Hence, monthly fixed charges is Rs. 250 and charge per kilometer is Rs. 40

15. Question

A part of monthly hostel charges in a college are fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 25 days, he has to pay Rs. 1750 as hostel charges whereas a student a d who takes food for 28 days, pays Rs. 1900 as hostel charges. Find the fixed charges and the cost of the food per day.

Answer

Let fixed hostel charge (monthly) = Rs x

and cost of food for one day = Rs y

In case of student A,

x + 25y = 1750 …(i)

In case of student B,

x + 28y = 1900 …(ii)

On subtracting Eq. (i) from Eq. (ii), we get

x + 28y – x – 25y = 1900 – 1750

⇒ 3y = 150

⇒ y = 50

On putting the value of y = 50 in Eq. (i), we get

x + 25(50) = 1750

⇒ x + 1250 = 1750

⇒ x = 1750 – 1250 = 500

Hence, monthly fixed charges is Rs. 500 and cost of food per day is Rs. 50

16. Question

The total expenditure per month of a household consists of a fixed rent of the house and the mess charges, depending upon the number of people sharing the house. The total monthly expenditure is Rs. 3,900 for 2 people and Rs. 7,500 for 5 people. Find the rent of the house and the mess charges per head per month.

Answer

Let fixed rent of the house = Rs. x

And the mess charges per hesd per month = Rs. y

According to the question,

x + 2y = 3900 …(i)

and x + 5y = 7500 …(ii)

On subtracting Eq. (i) from Eq. (ii), we get

x + 5y – x – 2y = 7500 – 3900

⇒ 3y = 3600

⇒ y = 1200

On putting the value of y = 1200 in Eq. (i), we get

x + 2 (1200) = 3900

⇒ x + 2400 = 3900

⇒ x = 1500

Hence, fixed rent of the house is Rs. 1500 and the mess charges per head per month is Rs. 1200.

17. Question

The car rental charges in a city comprise a fixed charge together with the charge for the distance covered. For a journey of 13 km, the charge paid is Rs. 96 and for a journey of 18 km, the charge paid is Rs. 131. What will a person have to pay for travelling a distance of 25 km?

Answer

Let fixed charge = Rs. x

and charge per kilometer = Rs. y

According to the question,

$x + 13y = 96$ ...(i)

and $x + 18y = 131$ ...(ii)

On subtracting Eq. (i) from Eq. (ii), we get

$x + 18y - x - 13y = 131 - 96$

$\Rightarrow 5y = 35$

$\Rightarrow y = 7$

On putting the value of $y = 7$ in Eq. (i), we get

$x + 13(7) = 96$

$\Rightarrow x + 91 = 96$

$\Rightarrow x = 5$

Hence, monthly fixed charges is Rs. 5 and charge per kilometer is Rs. 7

Now, amount to be paid for travelling 25 km

= Fixed charge + Rs 7 ×25

= 5 + 175

= Rs. 180

Hence, the amount paid by a person for travelling 25km is Rs. 180

18. Question

The sum of a two - digit number and the number formed by interchanging the digits is 132. If 12 is added to the number, the new number becomes 5 times the sum of the digits. Find the number.

Answer

Let unit's digit = y

and the ten's digit = x

So, the original number = 10x + y

After interchanging the digits, New number = x + 10y

The sum of the number = 10x + y

The sum of the digit = x + y

According to the question,

$(10x + y) + (x + 10y) = 132$

$\Rightarrow 11x + 11y = 132$

$\Rightarrow 11(x + y) = 132$

$\Rightarrow x + y = 12$ ...(i)

and $10x + y + 12 = 5(x + y)$

$\Rightarrow 10x + y + 12 = 5x + 5y$

$\Rightarrow 10x - 5x + y - 5y = - 12$

$\Rightarrow 5x - 4y = - 12$ ...(ii)

From Eq. (i), we get

$x = 12 - y$ ...(iii)

On substituting the value of $x = 12 - y$ in Eq. (ii), we get

$5(12 - y) - 4y = - 12$

$\Rightarrow 60 - 5y - 4y = - 12$

$\Rightarrow - 9y = - 12 - 60$

$\Rightarrow - 9y = - 72$

$\Rightarrow y = 8$

On putting the value of $y = 8$ in Eq. (iii), we get

$x = 12 - 8 = 4$

So, the Original number = 10x + y

$= 10 \times 4 + 8$

$= 48$

Hence, the two digit number is 48.

19. Question

A two - digit number is 4 times the sum of its digits. If 18 is added to the number, the digits are reversed. Find the number.

Answer

Let unit's digit = y

and the ten's digit = x

So, the original number = 10x + y

The sum of the two digit number = 10x + y

The sum of the digit = x + y

According to the question,

$(10x + y) = 4(x + y)$

$\Rightarrow 10x + y = 4x + 4y$

$\Rightarrow 10x - 4x + y - 4y = 0$

$\Rightarrow 6x - 3y = 0$

$\Rightarrow 2x - y = 0$

$\Rightarrow y = 2x$ ...(i)

After interchanging the digits, New number = x + 10y

and 10x + y + 18 = x + 10y

$\Rightarrow 10x + y + 18 = x + 10y$

$\Rightarrow 10x - x + y - 10y = -18$

$\Rightarrow 9x - 9y = -18$

$\Rightarrow x - y = -2$ ...(ii)

On substituting the value of y = 2x in Eq. (ii), we get

$x - y = -18$

$\Rightarrow x - 2x = -2$

$\Rightarrow - x = - 2$

$\Rightarrow x = 2$

On putting the value of x = 2 in Eq. (i), we get

$y = 2 \times 2 = 4$

So, the Original number = 10x + y

$= 10 \times 2 + 4$

$= 20 + 4$

$= 24$

Hence, the two digit number is 24.

20. Question

A number consists of two digits. When it is divided by the sum of its digits, the quotient is 6 with no remainder. When the number is diminished by 9, the digits are reversed. Find the number.

Answer

Let unit's digit = y

and the ten's digit = x

So, the original number = 10x + y

The sum of the number = 10x + y

The sum of the digit = x + y

According to the question,

$$\frac{(10x + y)}{x + y} = 6$$

$\Rightarrow 10x + y = 6(x + y)$

$\Rightarrow 10x + y = 6x + 6y$

$\Rightarrow 10x + y - 6x - 6y$

$\Rightarrow 4x - 5y = 0$ ...(i)

The reverse of the number = x + 10y

and 10x + y – 9 = x + 10y

$\Rightarrow 10x + y - 9 = x + 10y$

$\Rightarrow 10x - x + y - 10y = 9$

$\Rightarrow 9x - 9y = 9$

$\Rightarrow x - y = 1$

$\Rightarrow x = y + 1$ …(ii)

On substituting the value of $x = y + 1$ in Eq. (i), we get

$4x - 5y = 0$

$\Rightarrow 4(y + 1) - 5y = 0$

$\Rightarrow 4y + 4 - 5y = 0$

$\Rightarrow 4 - y = 0$

$\Rightarrow y = 4$

On substituting the value of $y = 4$ in Eq. (ii), we get

$x = y + 1$

$\Rightarrow x = 4 + 1$

$\Rightarrow x = 5$

So, the Original number $= 10x + y$

$= 10\times5 + 4$

$= 50 + 4$

$= 54$

Hence, the two digit number is 54.

21. Question

The sum of the digits of a two - digit number is 12. The number obtained by interchanging its digits exceeds the given number by 18. Find the number.

Answer

Let unit's digit $= y$

and the ten's digit $= x$

So, the original number $= 10x + y$

The sum of the number $= 10x + y$

The sum of the digit = $x + y$

According to the question,

$x + y = 12$ ...(i)

After interchanging the digits, the number = $x + 10y$

and $10x + y + 18 = x + 10y$

$\Rightarrow 10x + y + 18 = x + 10y$

$\Rightarrow 10x - x + y - 10y = -18$

$\Rightarrow 9x - 9y = -18$

$\Rightarrow x - y = -2$ ...(ii)

On adding Eq. (i) and (ii) , we get

$x + y + x - y = 12 - 2$

$\Rightarrow 2x = 10$

$\Rightarrow x = 5$

On substituting the value of $x = 5$ in Eq. (i), we get

$x + y = 12$

$\Rightarrow 5 + y = 12$

$\Rightarrow y = 7$

So, the Original number = $10x + y$

$= 10\times5 + 7$

$= 50 + 7$

$= 57$

Hence, the two digit number is 57.

22. Question

A two - digit number is 3 more than 4 times the sum of its digits. If 18 is added to the number, the digits are reversed. Find the number.

Answer

Let unit's digit = y

and the ten's digit = x

So, the original number = $10x + y$

The sum of the number = $10x + y$

The sum of the digit = $x + y$

According to the question,

$10x + y = 3 + 4(x + y)$

$\Rightarrow 10x + y = 3 + 4x + 4y$

$\Rightarrow 10x + y - 4x - 4y = 3$

$\Rightarrow 6x - 3y = 3$

$\Rightarrow 2x - y = 1$ ...(i)

The reverse number = $x + 10y$

and $10x + y + 18 = x + 10y$

$\Rightarrow 10x + y + 18 = x + 10y$

$\Rightarrow 10x - x + y - 10y = -18$

$\Rightarrow 9x - 9y = -18$

$\Rightarrow x - y = -2$ ...(ii)

On subtracting Eq. (i) from Eq. (ii) , we get

$x - y - 2x + y = -2 - 1$

$\Rightarrow -x = -3$

$\Rightarrow x = 3$

On substituting the value of $x = 3$ in Eq. (i), we get

$2(3) - y = 1$

$\Rightarrow 6 - y = 1$

$\Rightarrow -y = 1 - 6$

$\Rightarrow -y = -5$

$\Rightarrow y = 5$

So, the Original number = $10x + y$

$= 10\times3 + 5$

= 30 + 5

= 35

Hence, the two digit number is 35.

23. Question

A number consisting of two digits is seven times the sum of its digits. When 27 is subtracted from the number, the digits are reversed. Find the number.

Answer

Let unit's digit = y

and the ten's digit = x

So, the original number = 10x + y

The sum of the number = 10x + y

The sum of the digit = x + y

According to the question,

$10x + y = 7(x + y)$

$\Rightarrow 10x + y = 7x + 7y$

$\Rightarrow 10x + y - 7x - 7y = 0$

$\Rightarrow 3x - 6y = 0$

$\Rightarrow x - 2y = 0$

$\Rightarrow x = 2y$ ...(i)

The reverse number = x + 10y

and $10x + y - 27 = x + 10y$

$\Rightarrow 10x + y - 27 = x + 10y$

$\Rightarrow 10x - x + y - 10y = 27$

$\Rightarrow 9x - 9y = 27$

$\Rightarrow x - y = 3$ ...(ii)

On substituting the value of x = 2y in Eq. (ii), we get

$x - y = 3$

$\Rightarrow 2y - y = 3$

⇒ y = 3

On putting the value of y = 3 in Eq. (i), we get

x = 2(3) = 6

So, the Original number = 10x + y

= 10×6 + 3

= 60 + 3

= 63

Hence, the two digit number is 63.

24. Question

The sum of the digits of a two - digit number is 15. The number obtained by interchanging the digits exceeds the given number by 9. Find the number.

Answer

Let unit's digit = y

and the ten's digit = x

So, the original number = 10x + y

The sum of the number = 10x + y

The sum of the digit = x + y

According to the question,

x + y = 15 …(i)

After interchanging the digits, the number = x + 10y

and 10x + y + 9 = x + 10y

⇒ 10x + y + 9 = x + 10y

⇒ 10x – x + y – 10y = – 9

⇒ 9x – 9y = – 9

⇒ x – y = – 1 …(ii)

On adding Eq. (i) and (ii) , we get

x + y + x – y = 15 – 1

⇒ 2x = 14

⇒ x = 7

On substituting the value of x = 5 in Eq. (i), we get

x + y = 15

⇒ 7 + y = 15

⇒ y = 8

So, the Original number = 10x + y

= 10×7 + 8

= 70 + 8

= 78

Hence, the two digit number is 78.

25. Question

The sum of the numerator and denominator of a fraction is 3 less than twice the denominator. If the numerator and denominator are decreased by 1, the numerator becomes half the denominator. Determine the fraction.

Answer

Let the numerator = x

and the denominator = y

So, the fraction $= \frac{x}{y}$

According to the question,

Condition I:

x + y = 2y – 3

⇒ x + y – 2y = – 3

⇒ x – y = – 3 ...(i)

Condition II:

$$\frac{x-1}{y-1} = \frac{1}{2}$$

⇒ 2(x – 1) = y – 1

⇒ 2x – 2 = y – 1

$\Rightarrow 2x - y = 1$ ...(ii)

On subtracting Eq. (i) from Eq. (ii), we get

$2x - y - x + y = 1 + 3$

$\Rightarrow x = 4$

On putting the value of x in Eq. (i), we get

$4 - y = -3$

$\Rightarrow y = 7$

So, the numerator is 4 and the denominator is 7

Hence, the fraction is $\frac{4}{7}$

26. Question

The sum of the numerator and denominator of a fraction is 4 more than twice the numerator. If the numerator and denominator are increased by 3, then are in the ratio 2 : 3. Determine the fraction.

Answer

Let the numerator = x

and the denominator = y

So, the fraction $= \frac{x}{y}$

According to the question,

Condition I:

$x + y = 2x + 4$

$\Rightarrow x + y - 2x = 4$

$\Rightarrow -x + y = 4$

$\Rightarrow y = 4 + x$ ...(i)

Condition II:

$$\frac{x + 3}{y + 3} = \frac{2}{3}$$

$\Rightarrow 3(x + 3) = 2(y + 3)$

$\Rightarrow 3x + 9 = 2y + 6$

$\Rightarrow 3x - 2y = -3$ ...(ii)

On putting the value of y in Eq.(ii) , we get

$3x - 2(4 + x) = -3$

$\Rightarrow 3x - 8 - 2x = -3$

$\Rightarrow x = 5$

On putting the value of x in Eq. (i), we get

$y = 4 + 5$

$\Rightarrow y = 9$

So, the numerator is 5 and the denominator is 9

Hence, the fraction is $\frac{5}{9}$

27. Question

The sum of the numerator and denominator of a fraction is 8. If 3 is added to both 3 the numerator and the denominator, the fraction becomes 3/4. Find the fraction.

Answer

Let the numerator = x

and the denominator = y

So, the fraction $= \frac{x}{y}$

According to the question,

Condition I:

$x + y = 8$

$\Rightarrow y = 8 - x$ ...(i)

Condition II:

$$\frac{x + 3}{y + 3} = \frac{3}{4}$$

$\Rightarrow 4(x + 3) = 3(y + 3)$

$\Rightarrow 4x + 12 = 3y + 9$

$\Rightarrow 4x - 3y = -3$ ...(ii)

On putting the value of y in Eq.(ii) , we get

4x – 3(8 – x) = – 3

⇒ 4x – 24 + 3x = – 3

⇒ 7x = 21

⇒ x = 3

On putting the value of x in Eq. (i), we get

y = 8 – 3

⇒ y = 5

So, the numerator is 3 and the denominator is 5

Hence, the fraction is $\frac{3}{5}$

28. Question

The numerator of a fraction is one less than its denominator. If 3 is added to each of the numerator and denominator, the fraction is increased by 3/28, find the fraction. 28

Answer

Let the denominator = x

Given that numerator is one less than the denominator

⇒ numerator = x – 1

So, the fraction = $\frac{x-1}{x}$

According to the question,

$$\frac{x-1+3}{x+3} = \frac{x-1}{x} + \frac{3}{28}$$

$$\Rightarrow \frac{x+2}{x+3} = \frac{x-1}{x} + \frac{3}{28}$$

$$\Rightarrow \frac{x+2}{x+3} - \frac{x-1}{x} = \frac{3}{28}$$

$$\Rightarrow \frac{(x+2)x-(x+3)(x-1)}{(x+3)(x)} = \frac{3}{28}$$

⇒ $28\{(x^2 + 2x) - (x^2 - x + 3x - 3)\} = 3\,(x^2 + 3x)$

$\Rightarrow 28x^2 + 56x - 28x^2 - 56x + 84 = 3x^2 + 9x$

$\Rightarrow 3x^2 + 9x - 84 = 0$

$\Rightarrow x^2 + 3x - 28 = 0$

$\Rightarrow x^2 + 7x - 4x - 28 = 0$

$\Rightarrow x(x + 7) - 4(x + 7) = 0$

$\Rightarrow (x - 4)(x + 7) = 0$

$\Rightarrow x = 4$ and -7

But x is a natural number

Hence, $x = 4$

So, the fraction is $\frac{x-1}{x} = \frac{4-1}{4} = \frac{3}{4}$

29. Question

The age of the father is 3 years more than 3 times the son's age. 3 years here, the age of the father will be 10 years more than twice the age of the son. Find their present ages.

Answer

Let the age of father = x years

And the age of his son = y years

According to the question,

$x = 3 + 3y$ ...(i)

Three year here,

Father's age = $(x + 3)$ years

Son's age = $(y + 3)$ years

According to the question,

$(x + 3) = 10 + 2(y + 3)$

$\Rightarrow x + 3 = 10 + 2y + 6$

$\Rightarrow x = 2y + 13$ ...(ii)

From Eq. (i) and (ii), we get

$3 + 3y = 13 + 2y$

$\Rightarrow 3y - 2y = 13 - 3$

$\Rightarrow y = 10$

On putting the value of $y = 7$ in Eq. (i), we get

$x = 3 + 3(10)$

$\Rightarrow x = 3 + 30$

$\Rightarrow x = 33$

Hence, the age of father is 33 years and the age of his son is 10 years.

30. Question

Two years ago, a man was five times as old as his son. Two years later his age will be 8 more than three times the age of the son. Find the present ages of man and his son.

Answer

Let the age of a man = x years

And the age of his son = y years

Two years ago,

Man's age = (x – 2) years

Son's age = (y – 2) years

According to the question,

$(x - 2) = 5(y - 2)$

$\Rightarrow x - 2 = 5y - 10$

$\Rightarrow x = 5y - 10 + 2$

$\Rightarrow x = 5y - 8$ ...(i)

Two years later,

Father's age = (x + 2) years

Son's age = (y + 2) years

According to the question,

$(x + 2) = 8 + 3(y + 2)$

$\Rightarrow x + 2 = 8 + 3y + 6$

$\Rightarrow x = 3y + 12$ …(ii)

From Eq. (i) and (ii), we get

$5y - 8 = 3y + 12$

$\Rightarrow 5y - 3y = 12 + 8$

$\Rightarrow 2y = 20$

$\Rightarrow y = 10$

On putting the value of $y = 11$ in Eq. (i), we get

$x = 5(10) - 8$

$\Rightarrow x = 50 - 8$

$\Rightarrow x = 42$

Hence, the age of man is 42 years and the age of his son is 10 years.

31. Question

Father's age is three times the sum of ages of his two children. After 5 years, his age will be twice the sum of ages of two children. Find the age of father.

Answer

Let the age of two children be x and y

So, the father's present age = $3(x + y)$

After five years,

Age of two children = $(x + 5) + (y + 5)$ years

= $(x + y + 10)$ years

So, the age of father after five years = $3(x + y) + 5$

$= 3x + 3y + 5$

According to the question,

$3x + 3y + 5 = 2(x + y + 10)$

$\Rightarrow 3x + 3y + 5 = 2x + 2y + 20$

$\Rightarrow 3x - 2x + 3y - 2y = 20 - 5$

$\Rightarrow x + y = 15$

So, the age of two children = 15 years

And the age of father = 3(15) = 45years

Hence, the age of father is 45 years and the age of his two children is 15 years.

32. Question

Five years ago, A was thrice as old as B and ten years later, A shall be twice as old as B. What are the present ages of A and B?

Answer

Let the age of A = x years

And the age of B = y years

Five years ago,

A’s age = (x – 5) years

B’s age = (y – 5) years

According to the question,

$(x - 5) = 3(y - 5)$

$\Rightarrow x - 5 = 3y - 15$

$\Rightarrow x = 3y - 10$ …(i)

Ten years later,

A’s age = (x + 10)

B’s age = (y + 10)

According to the question,

$(x + 10) = 2(y + 10)$

$\Rightarrow x + 10 = 2y + 20$

$\Rightarrow x = 2y + 10$ …(ii)

From Eq. (i) and (ii), we get

$3y - 10 = 2y + 10$

$\Rightarrow 3y - 2y = 10 + 10$

$\Rightarrow y = 20$

On putting the value of y = 20 in Eq. (i), we get

$x = 3(20) - 10$

$\Rightarrow x = 50$

Hence, the age of person A is 50years and Age of B is 20years.

33. Question

Ten years hence, a man's age will be twice the age of his son. Ten years ago, the man was four times as old as his son. Find their present ages.

Answer

Let the age of a man = x years

And the age of his son = y years

Ten years hence,

Man's age = (x + 10) years

Son's age = (y + 10) years

According to the question,

$(x + 10) = 2(y + 10)$

$\Rightarrow x + 10 = 2y + 20$

$\Rightarrow x = 2y + 20 - 10$

$\Rightarrow x = 2y + 10$ ...(i)

Ten years ago,

Father's age = (x – 10) years

Son's age = (y – 10) years

According to the question,

$(x - 10) = 4(y - 10)$

$\Rightarrow x - 10 = 4y - 40$

$\Rightarrow x = 4y - 30$ ...(ii)

From Eq. (i) and (ii), we get

$2y + 10 = 4y - 30$

$\Rightarrow 2y - 4y = -30 - 10$

$\Rightarrow -2y = -40$

⇒ y = 20

On putting the value of y = 20 in Eq. (i), we get

x = 2y + 10

⇒ x = 2(20) + 10

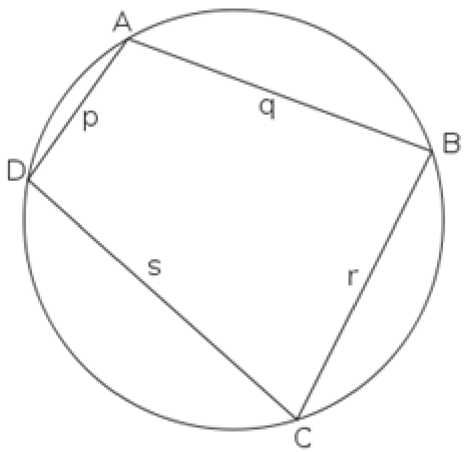
⇒ x = 50

Hence, the age of man is 50 years and the age of his son is 20 years.

34. Question

Find a cyclic quadrilateral ABCD, ∠A = (2x + 4)°, ∠B = (y + 3)°, ∠C = (2y + 10) and ∠D = (4x — 5)°. Find the four angles.

Answer



We know that, in a cyclic quadrilateral, the sum of two opposite angles is 180°

∴ ∠A + ∠C = 180° and ∠B + ∠D = 180°

⇒ 2x + 4 + 2y + 10 = 180 and y + 3 + 4x – 5 = 180

⇒ 2x + 2y = 180 – 14 and 4x + y – 2 = 180

⇒ x + y = 83 and 4x + y = 182

So, we get pair of linear equation i.e.

x + y = 83 ...(i)

4x + y = 182 ...(ii)

On subtracting Eq.(i) from (ii), we get

4x + y – x – y = 182 – 83

$\Rightarrow 3x = 99$

$\Rightarrow x = 33$

On putting the value of x = 33 in Eq. (i) we get,

$33 + y = 83$

$\Rightarrow y = 83 - 33 = 50$

On putting the values of x and y, we calculate the angles as

$\angle A = (2x + 43)° = 2(33) + 4 = 66 + 4 = 70°$

$\angle B = (y + 3)° = 50 + 3 = 53°$

$\angle C = (2y + 10)° = 2(50) + 10 = 100 + 10 = 110°$

and $\angle D = (4x — 5)° = 4(33) - 5 = 132 - 5 = 127°$

Hence, the angles are $\angle A = 63°$, $\angle B = 57°$, $\angle C = 117°$, $\angle D = 123°$

35. Question

Find the four angles of a cyclic quadrilateral ABCD in which $\angle A = (2x — 3)°$,

$\angle B = (y + 7)°$, $\angle C = (2y + 17)°$ and $\angle D = (4x — 9)°$.

$\angle A = 63°$, $\angle B = 57°$, $\angle C = 117°$, $\angle D = 123°$.

Answer

We know that, in a cyclic quadrilateral, the sum of two opposite angles is 180°

$\therefore \angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$

$\Rightarrow 2x - 3 + 2y + 17 = 180$ and $y + 7 + 4x - 9 = 180$

$\Rightarrow 2x + 2y + 14 = 180$ and $4x + y - 2 = 180$

$\Rightarrow 2x + 2y = 180 - 14$ and $4x + y = 182$

$\Rightarrow x + y = 83$ and $4x + y = 182$

So, we get pair of linear equation i.e.

$x + y = 83$ …(i)

$4x + y = 182$ …(ii)

On subtracting Eq.(i) from (ii), we get

$4x + y - x - y = 182 - 83$

$\Rightarrow 3x = 99$

$\Rightarrow x = 33$

On putting the value of x = 33 in Eq. (i) we get,

$33 + y = 83$

$\Rightarrow y = 83 - 33 = 50$

On putting the values of x and y, we calculate the angles as

$\angle A = (2x - 3)^\circ = 2(33) - 3 = 66 - 3 = 63^\circ$

$\angle B = (y + 7)^\circ = 50 + 7 = 57^\circ$

$\angle C = (2y + 17)^\circ = 2(50) + 17 = 100 + 17 = 117^\circ$

and $\angle D = (4x - 9)^\circ = 4(33) - 9 = 132 - 9 = 123^\circ$

Hence, the angles are $\angle A = 63^\circ$, $\angle B = 57^\circ$, $\angle C = 117^\circ$, $\angle D = 123^\circ$

36. Question

In a ΔABC, $\angle C = 3\angle B = 2(\angle A + \angle B)$. Find the three angles.

$\angle A = 20^\circ, \angle B = 40^\circ, \angle C = 120^\circ$.

Answer

We know that, in a triangle , the sum of three angles is 180°

$\therefore \angle A + \angle B + \angle C = 180^\circ$ ...(a)

According to the question,

$\underset{\text{I}}{\angle C} = \underset{\text{II}}{3\angle B} = \underset{\text{III}}{2(\angle A + \angle B)}$

On taking II and III, we get

$\Rightarrow 3\angle B = 2(\angle A + \angle B)$

$\Rightarrow 3\angle B = 2\angle A + 2\angle B$

$\Rightarrow \angle B = 2\angle A$ ...(i)

Now, on taking I and II, we get

$\angle C = 3\angle B$

$\Rightarrow \angle C = 3(2\angle A)$ (from eq. (i))

$\Rightarrow \angle C = 6\angle A$ ...(ii)

On substituting the value of $\angle B$ and $\angle C$ in Eq. (a), we get

$\angle A + 2\angle A + 6\angle A = 180°$

$\Rightarrow 9\angle A = 180°$

$\Rightarrow \angle A = 20°$

On puuting the value of $\angle A = 20°$ in Eq. (i) and (ii), we get

$\angle B = 2\angle A = 2(20) = 40°$

$\angle C = 6\angle A = 6(20) = 120°$

Hence, the angles are $\angle A = 20°$, $\angle B = 40°$, $\angle C = 120°$

37. Question

In a ΔABC, $\angle A = x°$, $\angle B = (3x)°$ and $\angle C = y°$.

If $3y — 5x = 30$, show that the triangle is right – angled.

Answer

We know that, in a triangle , the sum of three angles is 180°

$\therefore \angle A + \angle B + \angle C = 180°$

According to the question,

$x + 3x + y = 180$

$\Rightarrow 4x + y = 180$

$\Rightarrow y = 180 – 4x$ ...(i)

Given that $3y — 5x = 30$ ...(ii)

On substituting the value of y in Eq. (ii), we get

$3(180 – 4x) – 5x = 30$

$\Rightarrow 540 – 12x – 5x = 30$

$\Rightarrow 540 – 17x = 30$

$\Rightarrow – 17x = 30 – 540$

$\Rightarrow – 17x = – 510$

$\Rightarrow x = 30$

Now, we substitute the value of x in Eq.(i), we get

$\Rightarrow y = 180 - 4(30)$

$\Rightarrow y = 60$

On putting the value of x and y, we calculate the angles

$\angle A = x° = 30°$

$\angle B = (3x)° = 3(30) = 90°$

and $\angle C = y° = 60°$

Here, we can see that $\angle B = 90°$, so triangle is a right angled.

38. Question

The area of a rectangle gets reduced by 8 m^2, when its length is reduced by 5m and its breadth is increased by 3 m. If we increase the length by 3 m and breadth by 2 m, the area is increased by 74 m^2. Find the length and the breadth of the rectangle.

Answer

Let the length of a rectangle = x m

and the breadth of a rectangle = y m

Then, Area of rectangle = xy m^2

Condition I :

Area is reduced by 8m^2, when length = (x – 5) m and breadth = (y + 3) m

Then, area of rectangle = $(x - 5)\times(y + 3)$ m^2

According to the question,

$xy - (x - 5)\times(y + 3) = 8$

$\Rightarrow xy - (xy + 3x - 5y - 15) = 8$

$\Rightarrow xy - xy - 3x + 5y + 15 = 8$

$\Rightarrow - 3x + 5y = 8 - 15$

$\Rightarrow 3x - 5y = 7$ ...(i)

Condition II:

Area is increased by $74m^2$, when length = (x + 3) m and breadth = (y + 2) m

Then, area of rectangle = $(x + 3)\times(y + 2)\ m^2$

According to the question,

$(x + 3)\times(y + 2) - xy = 74$

$\Rightarrow (xy + 3y + 2x + 6) - xy = 74$

$\Rightarrow xy + 2x + 3y + 6 - xy = 74$

$\Rightarrow 2x + 3y = 74 - 6$

$\Rightarrow 2x + 3y = 68$ …(ii)

On multiplying Eq. (i) by 2 and Eq. (ii) by 3, we get

$6x - 10y = 14$ …(iii)

$6x + 9y = 204$ …(iv)

On subtracting Eq. (i) from Eq. (ii), we get

$6x + 9y - 6x + 10y = 204 - 14$

$\Rightarrow 19y = 190$

$\Rightarrow y = 10$

On putting the value of y = 10 in Eq. (i), we get

$3x - 5\ (10) = 7$

$\Rightarrow 3x - 50 = 7$

$\Rightarrow 3x = 57$

$\Rightarrow x = 19$

Hence, the length of the rectangle is 19m and the breadth of a rectangle is 10m

39. Question

The length of a room exceeds its breadth by 3 metres. If the length is increased by 3 metres and the breadth is decreased by 2 metres, the area remains the same. Find the length and the breadth of the room.

Answer

Let the breadth of a room = x m

According to the question,

Length of the room = x + 3

Then, Area of room = (x + 3)× (x) m^2

= $x^2 + 3x$

Condition II:

Area remains same,

when length = (x + 3 + 3) m = (x + 6) m

and breadth = (x – 2) m

According to the question,

$x^2 + 3x = (x + 6)(x - 2)$

$\Rightarrow x^2 + 3x = x^2 - 2x + 6x - 12$

$\Rightarrow 3x = 4x - 12$

$\Rightarrow 3x - 4x = -12$

$\Rightarrow x = 12$

So, length of the room = (x + 3) = 12 + 3 = 15m

Hence, the length of the room is 15m and the breadth of a room is 12m

40. Question

Two places A and B are 120 km apart from each other on a highway. A car starts from A and another from B at the same time. If they move in the same direction, they meet in 6 hours, and if they move in opposite directions, they meet in 1 hour 12 minutes. Find the speed of each car.

Answer

Let the speed of car I = x km/hr

And the speed of car II = y km/hr

Car I starts from point A and Car II starts from point B.

Let two cars meet at C after 6h.

Distance travelled by car I in 6h = 6x km

Distance travelled by car II in 6h = 6y km

Since, they are travelling in same direction, sign should be negative

6x – 6y = 120

⇒ x –y = 20 ...(i)

Now, Let two cars meet after 1hr 12min

1hr 12min = $1 + \frac{12}{60} = \frac{6}{5}$ hr

Since they are travelling in opposite directions, sign should be positive.

$$\frac{6}{5}x + \frac{6}{5}y = 120$$

⇒ 6x + 6y = 120 × 5

⇒ x + y = 100 ...(ii)

On adding (i) and (ii) , we get

x – y + x + y = 20 + 100

⇒ 2x = 120

⇒ x = 60

Putting the value of x = 60 in Eq. (i), we get

60 – y = 20

⇒ y = 40

So, the speed of the two cars are 60km/h and 40 km/hr respectively.

41. Question

A train travels a distance of 300 km at a constant speed. If the speed of SE the 20 train is increased by 5 km an hour, the journey would have taken 2 hours less. Find the original speed of the train.

Answer

Total distance travelled = 300km

Let the speed of train = x km/hr

We know that,

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

Hence, time taken by train = $\frac{300}{x}$

According to the question,

Speed of the train is increased by 5km an hour

∴ the new speed of the train = (x + 5)km/hr

Time taken to cover 300km = $\frac{300}{x+5}$

Given that time taken is 2hrs less from the previous time

$$\Rightarrow \frac{300}{x} - \frac{300}{x+5} = 2$$

$$\Rightarrow \frac{300(x+5) - 300(x)}{x(x+5)} = 2$$

⇒ 300x + 1500 – 300x = 2x (x + 5)

⇒ 1500 = $2x^2$ + 10x

⇒ 750 = x^2 + 5x

⇒ x^2 + 5x – 750 = 0

⇒ x^2 + 30x – 25x – 750 = 0

⇒ x (x + 30) – 25 (x + 30) = 0

⇒ (x – 25) (x + 30) = 0

⇒ (x – 25) = 0 or (x + 30) = 0

∴ x = 25 or x = – 30

Since, speed can't be negative.

Hence, the speed of the train is 25km/hr

42. Question

A plane left 30 minutes later than the scheduled time and in order to reach the destination 1500 km away in time, it has to increase the speed by 250 km/hr from the usual speed. Find its usual speed.

Answer

Let the usual time taken by the aeroplane = x km/hr

Distance to the destination = 1500km

We know that,

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

Hence, speed $= \frac{1500}{x}$ hrs

According to the question,

Plane left 30min later than the scheduled time

30min $= \frac{30}{60} = \frac{1}{2}$ hr

Time taken by the aeroplane $= x - \frac{1}{2}$ hrs

∴ the speed of the plane $= \frac{1500}{x - \frac{1}{2}}$

Given that speed has to increase by 250 km/hr

$$\Rightarrow \frac{1500}{x - \frac{1}{2}} - \frac{1500}{x} = 250$$

$$\Rightarrow \frac{1500}{\frac{2x-1}{2}} - \frac{1500}{x} = 250$$

$$\Rightarrow \frac{2}{2x-1} - \frac{1}{x} = \frac{250}{1500}$$

$$\Rightarrow \frac{2x - (2x-1)}{(2x-1)x} = \frac{1}{6}$$

$\Rightarrow 6(2x - 2x + 1) = 2x^2 - x$

$\Rightarrow 6 = 2x^2 - x$

$\Rightarrow 2x^2 - x - 6 = 0$

$\Rightarrow 2x^2 - 4x + 3x - 6 = 0$

$\Rightarrow 2x(x - 2) + 3(x - 2) = 0$

$\Rightarrow (2x + 3)(x - 2) = 0$

$\Rightarrow (2x + 3) = 0$ or $(x - 2) = 0$

$\therefore x = \frac{3}{2}$ or $x = 2$

Since, time can't be negative.

Hence, the time taken by the aeroplane is 2hrs and the speed is 750km/hr

43. Question

A man travels 600 km partly by train and partly by car. If he covers 400 km by train and the rest by car, it takes him 6 hours and 30 minutes. But, if he travels 200 km by train and the rest by car, he takes half an hour longer. Find the speed of the train and that of the car.

Answer

Let the speed of a train = x km/hr

And the speed of a car = y km/hr

Total distance travelled = 600km

According to the question,

If he covers 400km by train and rest by car i.e. (600 – 400) = 200km

Time take = 6hrs 30min $= 6 + \frac{30}{60} = 6.5\text{hrs}$

If he travels 200km by train and rest by car i.e. (600 – 200) = 400km

He takes half hour longer i.e. 7 hours

So, total time = train time + car time

We know that,

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

$\Rightarrow \frac{400}{x} + \frac{200}{y} = 6.5$ ...(i)

$\Rightarrow \frac{200}{x} + \frac{400}{y} = 7$ ...(ii)

Let take $\frac{1}{x} = u$ and $\frac{1}{y} = v$

400u + 200v = 6.5 ...(iii)

and 200u + 400v = 7 ...(iv)

On multiplying Eq. (iii) by 2 and Eq. (iv) by 4, we get

800u + 400v = 13 ...(a)

800u + 1600v = 28 ...(b)

On subtracting Eq. (a) from Eq. (b), we get

800u + 1600v – 800u – 400v = 28 – 13

⇒ 1200v = 15

$$\Rightarrow v = \frac{15}{1200}$$

$$\Rightarrow v = \frac{1}{80}$$

On putting the value of v in Eq. (iv), we get

$$200u + 400\left(\frac{1}{80}\right) = 7$$

⇒ 200u + 5 = 7

⇒ 200u = 2

$$\Rightarrow u = \frac{1}{100}$$

So, we get $u = \frac{1}{100}$ and $v = \frac{1}{80}$

⇒ x = 100 and y = 80

Hence, the speed of the train is 100km/hr and the speed of the car is 80km/hr.

44. Question

Places A and B are 80 km apart from each other on a highway. One car starts from A and another from B at the same time. If they move in the same direction, they meet in 8 hours and if they move in opposite directions, they meet in 1 hour and 20 minutes. Find speed of the cars.

Answer

Let the speed of car I = x km/hr

And the speed of car II = y km/hr

Car I starts from point A and Car II starts from point B.

Let two cars meet at C after 8h.

Distance travelled by car I in 8h = 8x km

Distance travelled by car II in 8h = 8y km

Since, they are travelling in same direction, sign should be negative

8x – 8y = 80

⇒ x –y = 10 ...(i)

Now, Let two cars meet after 1hr 20 min

1hr 20min = $1 + \frac{20}{60} = \frac{4}{3}$ hr

Since they are travelling in opposite directions, sign should be positive.

$$\frac{4}{3}x + \frac{4}{3}y = 80$$

⇒ 4x + 4y = 240

⇒ x + y = 60 ...(ii)

On adding (i) and (ii) , we get

x – y + x + y = 10 + 60

⇒ 2x = 70

⇒ x = 35

Putting the value of x = 25 in Eq. (i), we get

35 – y = 10

⇒ y = 25

So, the speed of the two cars are 35km/h and 25 km/hr respectively.

45. Question

A boat goes 16 km upstream and 24 km downstream in 6 hours. Also, it covers 12 km upstream and 36 km downstream in the same time. Find the speed of the boat in still water and that of the stream.

Answer

Let speed of the boat in still water = x km/hr

and speed of the stream = y km/hr

Then, the speed of the boat downstream = (x + y)km/hr

And speed of the boat upstream = (x – y)km/hr

According to the question

Condition I: When boat goes 16 km upstream, let the time taken be t_1.

Then,

$$t_1 = \frac{16}{x - y} h \left[\because \text{time} = \frac{\text{distance}}{\text{speed}}\right]$$

When boat goes 24 km downstream, let the time taken be t_2.

Then,

$$t_2 = \frac{24}{x + y} h$$

But total time taken $(t_1 + t_2) = 6$ hours

$$\therefore \frac{16}{x-y} + \frac{24}{x+y} = 6 \text{ ...(a)}$$

Condition II: When boat goes 12 km upstream, let the time taken be T_1.

Then,

$$T_1 = \frac{12}{x - y} h \left[\because \text{time} = \frac{\text{distance}}{\text{speed}}\right]$$

When boat goes 36 km downstream, let the time taken be T_2.

Then,

$$T_2 = \frac{36}{x + y} h$$

But total time taken $(T_1 + T_2) = 6$ hours

$$\therefore \frac{12}{x-y} + \frac{36}{x+y} = 6 \text{ ...(b)}$$

Now, we solve tis pair of linear equations by elimination method

$$\frac{16}{x+y} + \frac{24}{x-y} = 6 \text{ ...(i)}$$

And $\frac{12}{x+y} + \frac{36}{x-y} = 6$ ...(ii)

On multiplying Eq. (i) by 3 and Eq. (ii) by 4 to make the coefficients equal of first term, we get the equation as

$$\frac{48}{x+y} + \frac{72}{x-y} = 18 \ldots(iii)$$

$$\frac{48}{x+y} + \frac{144}{x-y} = 24 \ldots(iv)$$

On substracting Eq. (iii) from Eq. (iv), we get

$$\frac{48}{x+y} + \frac{144}{x-y} - \frac{48}{x+y} - \frac{72}{x-y} = 24 - 18$$

$$\Rightarrow \frac{144-72}{x-y} = 6$$

$$\Rightarrow \frac{72}{x-y} = 6$$

$\Rightarrow$ x – y = 12 ...(a)

On putting the value of x – y = 12 in Eq. (i), we get

$$\Rightarrow \frac{16}{x+y} + \frac{24}{12} = 6$$

$$\Rightarrow \frac{16}{x+y} = 6 - 2$$

$$\Rightarrow \frac{16}{x+y} = 4$$

$\Rightarrow$ x + y = 4 ...(b)

Adding Eq. (a) and (b), we get

$\Rightarrow$ 2x = 16

$\Rightarrow$ x = 8

On putting value of x = 8 in eq. (a), we get

8 – y = 12

$\Rightarrow$ y = – 4 but speed can't be negative

$\Rightarrow$ y = 4

Hence, x = 8 and y = 4 , which is the required solution.

Hence, the speed of the boat in still water is 8km/hr and speed of the stream is 4km/hr

46. Question

A man travels 370 km, partly by train and partly by car. If he covers 250 km by train and the rest by car, it takes him 4 hours. But, if he travels 130 km by train and the rest by car, he takes 18 minutes longer. Find the speed of the train and that of the car.

Answer

Let the speed of a train = x km/hr

And the speed of a car = y km/hr

Total distance travelled = 370km

According to the question,

If he covers 250km by train and rest by car i.e. (370 – 250) = 120km

Time take = 4hrs

If he travels 130km by train and rest by car i.e. (370 – 130) = 240km

He takes 18min longer i.e. $4 + \frac{18}{60} = 4.3\text{hrs}$

So, total time = train time + car time

We know that,

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

$\Rightarrow \frac{250}{x} + \frac{120}{y} = 4$ ...(i)

$\Rightarrow \frac{130}{x} + \frac{240}{y} = 4.3$ ...(ii)

Let take $\frac{1}{x} = u$ and $\frac{1}{y} = v$

250u + 120v = 4 ...(iii)

and 130u + 240v = 4.3 ...(iv)

On multiplying Eq. (iii) by 2

500u + 240v = 8 ...(v)

On subtracting Eq. (iv) from Eq. (v), we get

500u + 240v – 130u – 240v = 8 – 4.3

$\Rightarrow 370u = 3.7$

$$\Rightarrow u = \frac{3.7}{370}$$

$$\Rightarrow u = \frac{1}{100}$$

On putting the value of v in Eq. (iv), we get

$$130\left(\frac{1}{100}\right) + 240v = 4.3$$

$\Rightarrow 1.3 + 240v = 4.3$

$\Rightarrow 240v = 3$

$$\Rightarrow v = \frac{1}{80}$$

So, we get $u = \frac{1}{100}$ and $v = \frac{1}{80}$

$\Rightarrow x = 100$ and $y = 80$

Hence, the speed of the train is 100km/hr and the speed of the car is 80km/hr.