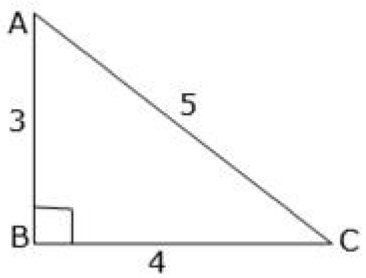
4. Trigonometric Ratios and Identities

Exercise 4.1

1. Question

From the given figure, find the value of the following:



(i) sin C

(ii) sin A

(iii) cos C

(iv) cos A

(v) tan C

(vi) tan A

Answer

(i) Sin C

We know that,

$$\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}}$$

So, here θ = C

Side opposite to ∠C = AB = 3

Hypotenuse = AC = 5

So, $\text{Sin C} = \frac{AB}{AC} = \frac{3}{5}$

(ii) Sin A

So, here θ = A

The side opposite to ∠A = BC = 4

Hypotenuse = AC = 5

So, Sin A $= \frac{BC}{AC} = \frac{4}{5}$

(iii) Cos C

We know that,

$$\cos\theta = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}}$$

So, here θ = C

Side adjacent to ∠C = BC = 4

Hypotenuse = AC = 5

So, Cos C $= \frac{BC}{AC} = \frac{4}{5}$

(iv) Cos A

Here, θ = A

Side adjacent to ∠A = AB = 3

Hypotenuse = AC = 5

So, Cos A $= \frac{AB}{AC} = \frac{3}{5}$

(v) tan C

We know that,

$$\tan\theta = \frac{\text{side opposite to angle }\theta}{\text{side adjacent to angle }\theta}$$

So, here θ = C

Side opposite to ∠C = AB = 3

Side adjacent to ∠C = BC = 4

So, tan C $= \frac{AB}{BC} = \frac{3}{4}$

(vi) tan A

here θ = A

Side opposite to ∠A = BC = 4

Side adjacent to ∠A = AB = 3

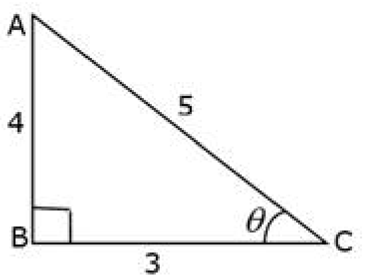
So, $\tan A = \frac{AB}{BC} = \frac{4}{3}$

2. Question

From the given figure, find the value of :

(i) tan θ

(ii) cos θ



Answer

(i) tan θ

We know that,

$$\tan\theta = \frac{\text{side opposite to angle }\theta}{\text{side adjacent to angle }\theta}$$

Side opposite to θ = AB = 4

Side adjacent to θ = BC = 3

So, $\tan\theta = \frac{AB}{BC} = \frac{4}{3}$

(ii) cos θ

We know that,

$$\cos\theta = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}}$$

Side adjacent to θ = BC = 3

Hypotenuse = AC = 5

So, $\text{Cos}\ \theta = \frac{BC}{AC} = \frac{3}{5}$

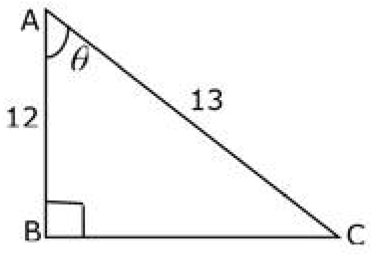
3. Question

From the given figure, find the value of

(i) sin θ

(ii) tan θ

(iii) tan A – cot C



Answer

(i) sin θ

We know that,

$$\sin\theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Side opposite to θ = BC = ?

Hypotenuse = AC = 13

Firstly we have to find the value of BC.

So, we can find the value of BC with the help of Pythagoras theorem.

According to Pythagoras theorem,

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (12)^2 + (BC)^2 = (13)^2$

$\Rightarrow 144 + (BC)^2 = 169$

$\Rightarrow (BC)^2 = 169 - 144$

$\Rightarrow (BC)^2 = 25$

$\Rightarrow BC = \sqrt{25}$

$\Rightarrow$ BC =±5

But side BC can't be negative. So, BC = 5

Now, BC = 5 and AC = 13

So, $\text{Sin } \theta = \frac{BC}{AC} = \frac{5}{13}$

(ii) tan θ

We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

Side opposite to θ = BC = 5

Side adjacent to θ = AB = 12

So, $\tan \theta = \frac{BC}{AB} = \frac{5}{12}$

(iii) tan A – cot C

We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

and

$$\cot \theta = \frac{\text{side adjacent to angle } \theta}{\text{side opposite to angle } \theta}$$

tan A

Here, θ = A

Side opposite to ∠A = BC = 5

Side adjacent to ∠A = AB = 12

So, $\tan A = \frac{BC}{AB} = \frac{5}{12}$

Cot C

Here, θ = C

Side adjacent to ∠C = BC = 5

Side opposite to ∠C = AB = 12

So, $\cot C = \frac{BC}{AB} = \frac{5}{12}$

So, $\tan A - \cot C = \frac{5}{12} - \frac{5}{12} = 0$

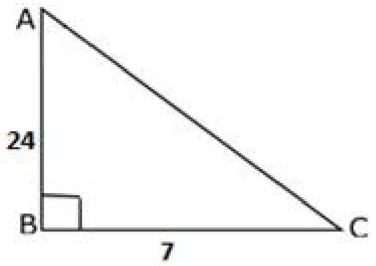
4 A. Question

In ΔABC, right angled at B, AB = 24 cm, BC = 7 cm. Determine

a. sin A, cos A

b. sin C, cos C

Answer



(i)

(a) sin A

We know that,

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

So, here θ = A

Side opposite to ∠A = BC = 7

Hypotenuse = AC = ?

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem.

According to Pythagoras theorem,

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (24)^2 + (7)^2 = (AC)^2$

$\Rightarrow 576 + 49 = (AC)^2$

⇒ $(AC)^2 = 625$

⇒ $AC = \sqrt{625}$

⇒ $AC = \pm 25$

But side AC can't be negative. So, AC = 25cm

Now, BC = 7 and AC = 25

So, $\text{Sin A} = \frac{BC}{AC} = \frac{7}{25}$

Cos A

We know that,

$$\cos\theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

So, here θ = A

Side adjacent to ∠A = AB = 24

Hypotenuse = AC = 25

So, $\text{Cos A} = \frac{AB}{AC} = \frac{24}{25}$

(b) sin C

We know that,

$$\sin\theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

So, here θ = C

The side opposite to ∠C = AB = 24

Hypotenuse = AC = 25

So, $\text{Sin C} = \frac{AB}{AC} = \frac{24}{25}$

Cos C

We know that,

$$\cos\theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

So, here θ = C

Side adjacent to ∠C = BC = 7

Hypotenuse = AC = 25

So, $\text{Cos } C = \frac{BC}{AC} = \frac{7}{25}$

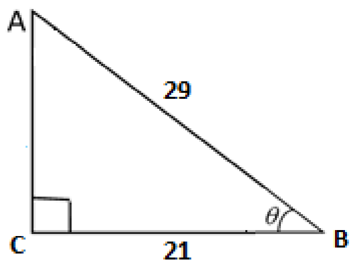
4 B. Question

Consider ΔACB, right angled at C, in which AB = 29 units, BC = 21 units and ∠ABC=θ. Determine the values of

a. $\cos^2 \theta + \sin^2 \theta$

b. $\cos^2 \theta - \sin^2 \theta$

Answer



(a) $\text{Cos}^2\theta + \sin^2 \theta$

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem.

According to Pythagoras theorem,

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

$\Rightarrow (AC)^2 + (BC)^2 = (AB)^2$

$\Rightarrow (AC)^2 + (21)^2 = (29)^2$

$\Rightarrow (AC)^2 = (29)^2 - (21)^2$

Using the identity $a^2 - b^2 = (a+b)(a - b)$

$\Rightarrow (AC)^2 = (29-21)(29+21)$

$\Rightarrow (AC)^2 = (8)(50)$

$\Rightarrow (AC)^2 = 400$

$\Rightarrow AC = \sqrt{400}$

⇒ AC =±20

But side AC can't be negative. So, AC = 20units

Now, we will find the sin θ and cos θ

$$\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}}$$

In ΔACB, Side opposite to angle θ = AC = 20

and Hypotenuse = AB = 29

So, $\text{Sin}\ \theta = \frac{AC}{AB} = \frac{20}{29}$

Now, We know that

$$\cos\theta = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}}$$

In ΔACB, Side adjacent to angle θ = BC = 21

and Hypotenuse = AB = 29

So, $\cos\theta = \frac{BC}{AB} = \frac{21}{29}$

So, $\cos^2\theta + \sin^2\theta = \left(\frac{21}{29}\right)^2 + \left(\frac{20}{29}\right)^2$

$$= \frac{441 + 400}{29 \times 29}$$

$$= \frac{841}{841}$$

=1

$\text{Cos}^2\theta + \sin^2\theta = 1$

(b) $\text{Cos}^2\theta - \sin^2\theta$

Putting values, we get

$$\cos^2\theta - \sin^2\theta = \left(\frac{21}{29}\right)^2 - \left(\frac{20}{29}\right)^2$$

$$= \frac{441 - 400}{29 \times 29}$$

$= \frac{41}{841}$

4 C. Question

In ΔABC, ∠A is a right angle, then find the values of sin B, cos C and tan B in each of the following :

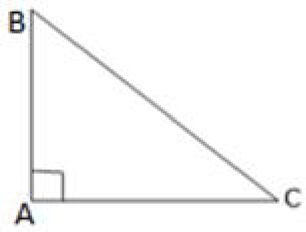
a. AB = 12, AC = 5, BC = 13

b. AB = 20, AC = 21, BC = 29

c. BC = √2, AB = AC = 1

Answer

Given that ∠A is a right angle.



(a) AB = 12, AC = 5, BC = 13

To Find : sin B, cos C and tan B

We know that,

$$\sin\theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Here, θ = B

Side opposite to angle B = AC = 5

Hypotenuse = BC =13

So, $\text{Sin B} = \frac{AC}{BC} = \frac{5}{13}$

Now, Cos C

We know that,

$$\cos\theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Here, θ = C

Side adjacent to angle C = AC = 5

Hypotenuse = BC =13

So, $\text{Cos C} = \frac{AC}{BC} = \frac{5}{13}$

Now, tan B

We know that,

$$\tan\theta = \frac{\text{side opposite to angle }\theta}{\text{side adjacent to angle }\theta}$$

Here, θ = B

The side opposite to angle B = AC = 5

The side adjacent to angle B = AB = 12

So, $\tan B = \frac{AC}{AB} = \frac{5}{12}$

(b) AB = 20, AC = 21, BC = 29

To Find: sin B, cos C and tan B

We know that,

$$\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}}$$

Here, θ = B

The side opposite to angle B = AC =21

Hypotenuse = BC =29

So, $\text{Sin B} = \frac{AC}{BC} = \frac{21}{29}$

Now, Cos C

We know that,

$$\cos\theta = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}}$$

Here, θ = C

Side adjacent to angle C = AC = 21

Hypotenuse = BC = 29

So, $\text{Cos C} = \frac{AC}{BC} = \frac{21}{29}$

Now, tan B

We know that,

$$\tan\theta = \frac{\text{side opposite to angle }\theta}{\text{side adjacent to angle }\theta}$$

Here, θ = B

The side opposite to angle B = AC = 21

The side adjacent to angle B = AB = 20

So, $\tan B = \frac{AC}{AB} = \frac{21}{20}$

(c) BC =√2, AB = AC = 1

To Find: sin B, cos C and tan B

We know that,

$$\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}}$$

Here, θ = B

The side opposite to angle B = AC =1

Hypotenuse = BC =√2

So, $\text{Sin B} = \frac{AC}{BC} = \frac{1}{\sqrt{2}}$

Now, Cos C

We know that,

$$\cos\theta = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}}$$

Here, θ = C

Side adjacent to angle C = AC = 1

Hypotenuse = BC = √2

So, $\text{Cos C} = \frac{AC}{BC} = \frac{1}{\sqrt{2}}$

Now, tan B

We know that,

$$\tan\theta = \frac{\text{side opposite to angle }\theta}{\text{side adjacent to angle }\theta}$$

Here, θ = B

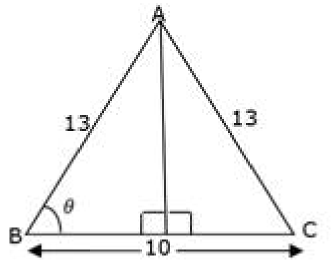
The side opposite to angle B = AC = 1

The side adjacent to angle B = AB = 1

So, $\tan B = \frac{AC}{AB} = \frac{1}{1} = 1$

5 A. Question

Find the value of the following : (a) sin θ (b) cos θ (c) tan θ from the figures given below :



Answer

Firstly, we give the name to the midpoint of BC i.e. M

BC = BM + MC = 2BM or 2MC

⇒ BM = 5 and MC = 5

Now, we have to find the value of AM, and we can find out with the help of Pythagoras theorem.

So, In ΔAMB

⇒ $(AM)^2 + (BM)^2 = (AB)^2$

⇒ $(AM)^2 + (5)^2 = (13)^2$

⇒ $(AM)^2 = (13)^2 – (5)^2$

Using the identity $a^2 –b^2 = (a+b) (a – b)$

⇒ $(AM)^2 = (13–5)(13+5)$

⇒ $(AM)^2 = (8)(18)$

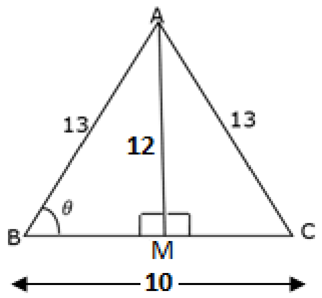
⇒ $(AM)^2 = 144$

⇒ AM =√144

⇒ AM =±12

But side AM can't be negative. So, AM = 12

a. sin θ



We know that,

$$\sin\theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

In ΔAMB

Side opposite to θ = AM = 12

Hypotenuse = AB=13

So, $\sin\theta = \frac{AM}{AB} = \frac{12}{13}$

So, $\text{Sin}\ \theta = \frac{12}{13}$

b. cos θ

We know that, $\cos\theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$

In ΔAMB

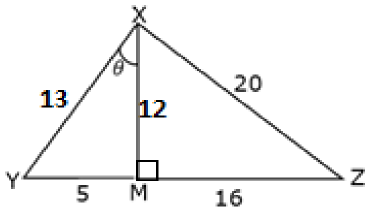
The side adjacent to θ = BM = 5

Hypotenuse = AB = 13

So, $\cos\theta = \frac{BM}{AB} = \frac{5}{13}$

So, $\cos\theta = \frac{5}{13}$

c. tan θ



We know that,

$$\tan\theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

In ΔAMB

Side opposite to θ = AM = 12

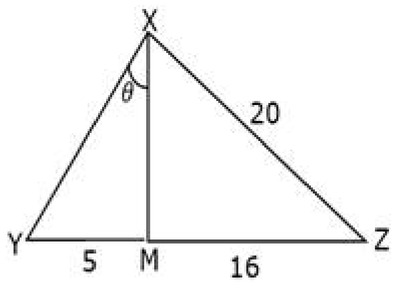
The side adjacent to θ = BM = 5

So, $\tan\theta = \frac{AM}{BM} = \frac{12}{5}$

So, $\tan\theta = \frac{12}{5}$

5 B. Question

Find the value of the following : (a) sin θ (b) cos θ (c) tan θ from the figures given below :



Answer

Firstly, we have to find the value of XM and we can find out with the help of Pythagoras theorem

So, In ΔXMZ

⇒ $(XM)^2 + (MZ)^2 = (XZ)^2$

⇒ $(XM)^2 + (16)^2 = (20)^2$

⇒ $(XM)^2 = (20)^2 – (16)^2$

Using the identity $a^2 – b^2 = (a+b)(a – b)$

⇒ $(XM)^2 = (20–16)(20+16)$

$\Rightarrow (XM)^2 = (4)(36)$

$\Rightarrow (XM)^2 = 144$

$\Rightarrow XM = \sqrt{144}$

$\Rightarrow XM = \pm 12$

But side XM can't be negative. So, XM = 12

Now, In ΔXMY we have the value of XM and MY but we don't have the value of XY.

So, again we apply the Pythagoras theorem in ΔXMY

$\Rightarrow (XM)^2 + (MY)^2 = (XY)^2$

$\Rightarrow (12)^2 + (5)^2 = (XY)^2$

$\Rightarrow 144 + 25 = (XY)^2$

$\Rightarrow (XY)^2 = 169$

$\Rightarrow XY = \sqrt{169}$

$\Rightarrow XY = \pm 13$

But side XY can't be negative. So, XY = 13

a. sin θ

We know that,

$$\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}}$$

In ΔXMY

Side opposite to θ = MY = 5

Hypotenuse = XY = 13

So, $\sin\theta = \frac{MY}{XY} = \frac{5}{13}$

b. cos θ

We know that,

$$\cos\theta = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}}$$

In ΔXMY

Side adjacent to θ = XM = 12

Hypotenuse = XY = 13

So, $\cos\theta = \frac{XM}{XY} = \frac{12}{13}$

c. tan θ

We know that,

$$\tan\theta = \frac{\text{side opposite to angle }\theta}{\text{side adjacent to angle }\theta}$$

In ΔXMY

The side opposite to θ = MY = 5

Side adjacent to θ = XM = 12

So, $\tan\theta = \frac{MY}{XM} = \frac{5}{12}$

6. Question

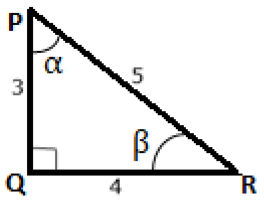
In ΔPQR, ∠Q is a right angle PQ = 3, QR = 4. If ∠P=α and ∠R=β, then find the values of

(i) sin α (ii) cos α

(iii) tan α (iv) sin β

(v) cos β (vi) tan β

Answer



Given : PQ = 3, QR = 4

⇒ $(PQ)^2 + (QR)^2 = (PR)^2$

⇒ $(3)^2 + (4)^2 = (PR)^2$

⇒ $9 + 16 = (PR)^2$

⇒ $(PR)^2 = 25$

⇒ PR =$\sqrt{25}$

$\Rightarrow$ PR $= \pm 5$

But side PR can't be negative. So, PR = 5

(i) sin α

We know that,

$$\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}}$$

Here, θ = α

The side opposite to angle α = QR =4

Hypotenuse = PR =5

So, $\sin\alpha = \frac{4}{5}$

(ii) cos α

We know that,

$$\cos\theta = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}}$$

Here, θ = α

The side adjacent to angle α = PQ =3

Hypotenuse = PR =5

So, $\cos\alpha = \frac{3}{5}$

(iii) tan α

We know that,

$$\tan\theta = \frac{\text{side opposite to angle }\theta}{\text{side adjacent to angle }\theta}$$

Here, θ = α

Side opposite to angle α = QR =4

Side adjacent to angle α = PQ =3

So, $\tan\alpha = \frac{4}{3}$

(iv) sin β

We know that,

$$\sin\theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Here, θ = β

The side opposite to angle β = PQ =3

Hypotenuse = PR =5

So, $\sin\beta = \frac{3}{5}$

(v) cos β

We know that,

$$\cos\theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Here, θ = β

Side adjacent to angle β = QR =4

Hypotenuse = PR =5

So, $\cos\beta = \frac{4}{5}$

(vi) tan β

We know that,

$$\tan\theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

Here, θ = β

Side opposite to angle β = PQ =3

Side adjacent to angle β = QR =4

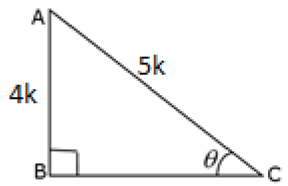
So, $\tan\beta = \frac{3}{4}$

7 A. Question

If $\sin\theta = \frac{4}{5}$, then find the values of cos θ and tan θ.

Answer

Given: Sin θ $= \frac{4}{5}$



We know that,

$$\sin\theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Or $\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

$$\text{Sin } \theta = \frac{4}{5} \Rightarrow \frac{P}{H} = \frac{4}{5} \Rightarrow \frac{AB}{AC} = \frac{4}{5}$$

Let,

Perpendicular =AB =4k

and Hypotenuse =AC =5k

where, k is any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

In right angled Δ ABC, we have

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (4k)^2 + (BC)^2 = (5k)^2$

$\Rightarrow 16k^2 + (BC)^2 = 25k^2$

$\Rightarrow (BC)^2 = 25\ k^2 - 16\ k^2$

$\Rightarrow (BC)^2 = 9\ k^2$

$\Rightarrow BC = \sqrt{9}\ k^2$

$\Rightarrow BC = \pm 3k$

But side BC can’t be negative. So, BC = 3k

Now, we have to find the value of cos θ and tan θ

We know that,

$\cos\theta = \frac{\text{base}}{\text{hypotenuse}}$

The side adjacent to angle θ or base = BC =3k

Hypotenuse = AC =5k

So, $\cos\theta = \frac{3k}{5k} = \frac{3}{5}$

Now,

We know that, $\tan\theta = \frac{\text{perpendicular}}{\text{base}}$

Perpendicular = AB =4k

Base = BC =3k

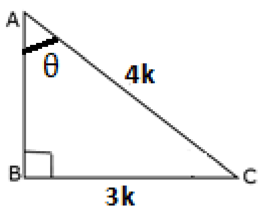
So, $\tan\theta = \frac{4k}{3k} = \frac{4}{3}$

7 B. Question

If $\sin A = \frac{3}{4}$, calculate cos A and tan A.

Answer

Given: Sin A $= \frac{3}{4}$



We know that,

$\sin\theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$

Or $\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

$\text{Sin } \theta = \frac{3}{4} \Rightarrow \frac{P}{H} = \frac{3}{4} \Rightarrow \frac{BC}{AC} = \frac{3}{4}$

Let,

Side opposite to angle θ = BC =3k

and Hypotenuse = AC =4k

where, k is any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (AB)^2 + (3k)^2 = (4k)^2$

$\Rightarrow (AB)^2 + 9k^2 = 16k^2$

$\Rightarrow (AB)^2 = 16\ k^2 - 9\ k^2$

$\Rightarrow (AB)^2 = 7\ k^2$

$\Rightarrow AB = k\sqrt{7}$

So, $AB = k\sqrt{7}$

Now, we have to find the value of cos A and tan A

We know that,

$$\cos\theta = \frac{\text{Side adjacent to angle } \theta}{\text{hypotenuse}}$$

Here, θ = A

The side adjacent to angle A = AB $=k\sqrt{7}$

Hypotenuse = AC =4k

So, $\cos A = \frac{k\sqrt{7}}{4k} = \frac{\sqrt{7}}{4}$

Now,

We know that,

$$\tan\theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

The side opposite to angle A = BC =3k

The side adjacent to angle A = AB $=k\sqrt{7}$

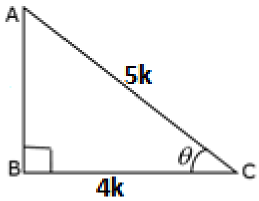
So, $\tan A = \frac{3k}{k\sqrt{7}} = \frac{3}{\sqrt{7}}$

8. Question

If $\sin\theta = \frac{3}{5}$, then find the values cos θ and tan θ.

Answer

Given: $\text{Sin}\ \theta = \frac{3}{5}$



We know that,

$$\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}}$$

Or $\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

$$\text{Sin}\ \theta = \frac{3}{5} \Rightarrow \frac{P}{H} = \frac{3}{5} \Rightarrow \frac{AB}{AC} = \frac{3}{5}$$

Let,

Perpendicular =AB =3k

and Hypotenuse =AC =5k

where, k is any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (3k)^2 + (BC)^2 = (5k)^2$

$\Rightarrow 9k^2 + (BC)^2 = 25k^2$

$\Rightarrow (BC)^2 = 25\ k^2 - 9\ k^2$

$\Rightarrow (BC)^2 = 16\ k^2$

$\Rightarrow BC = \sqrt{16\ k^2}$

$\Rightarrow BC = \pm 4k$

But side BC can't be negative. So, BC = 4k

Now, we have to find the value of cos θ and tan θ

We know that,

$$\cos\theta = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}}$$

The side adjacent to angle θ = BC =4k

Hypotenuse = AC =5k

So, $\cos\theta = \frac{4k}{5k} = \frac{4}{5}$

Now, tan θ

We know that, $\tan\theta = \frac{\text{perpendicular}}{\text{base}}$

Perpendicular = AB =3k

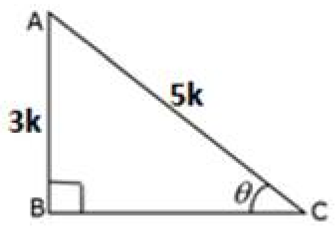
Base = BC =4k

So, $\tan\theta = \frac{3k}{4k} = \frac{3}{4}$

9. Question

If $\cos\theta = \frac{4}{5}$, then find the value of tan θ.

Answer



We know that,

$$\cos\theta = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}}$$

Or $\cos\theta = \frac{\text{base}}{\text{Hypotenuse}}$

$$\text{Cos }\theta = \frac{4}{5} \Rightarrow \frac{B}{H} = \frac{4}{5} \Rightarrow \frac{BC}{AC} = \frac{4}{5}$$

Let,

Base =BC = 4k

Hypotenuse =AC = 5k

Where, k ia any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (AB)^2 + (4k)^2 = (5k)^2$

$\Rightarrow (AB)^2 + 16k^2 = 25k^2$

$\Rightarrow (AB)^2 = 25\ k^2 - 16\ k^2$

$\Rightarrow (AB)^2 = 9\ k^2$

$\Rightarrow AB = \sqrt{9}\ k^2$

$\Rightarrow AB = \pm 3k$

But side AB can't be negative. So, AB = 3k

Now, we have to find tan θ

We know that,

$$\tan\theta = \frac{\text{side opposite to angle }\theta}{\text{side adjacent to angle }\theta}$$

Side opposite to angle θ = BC =4k

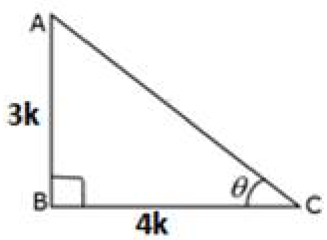
Side adjacent to angle θ = AB =3k

So, $\tan\theta = \frac{4k}{3k} = \frac{4}{3}$

10 A. Question

If $\tan\theta = \frac{3}{4}$, then find the values of cos θ and sin θ.

Answer



We know that,

$$\tan\theta = \frac{\text{side opposite to angle }\theta}{\text{side adjacent to angle }\theta}$$

Or $\tan\theta = \frac{\text{perpendicular}}{\text{base}}$

$$\tan\theta = \frac{3}{4} \Rightarrow \frac{P}{B} = \frac{3}{4} \Rightarrow \frac{AB}{BC} = \frac{3}{4}$$

Let,

The side opposite to angle θ =AB = 3k

The side adjacent to angle θ =BC = 4k

where k is any positive integer

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (3k)^2 + (4k)^2 = (AC)^2$

$\Rightarrow (AC)^2 = 9\,k^2 + 16\,k^2$

$\Rightarrow (AC)^2 = 25\,k^2$

$\Rightarrow AC = \sqrt{25\,k^2}$

$\Rightarrow AC = \pm 5k$

But side AC can't be negative. So, AC = 5k

Now, we will find the sin θ and cos θ

$$\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}}$$

Side opposite to angle θ = AB = 3k

and Hypotenuse = AC = 5k

So, $\text{Sin}\,\theta = \frac{AB}{AC} = \frac{3k}{5k} = \frac{3}{5}$

Now, We know that

$$\cos\theta = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}}$$

Side adjacent to angle θ = BC = 4k

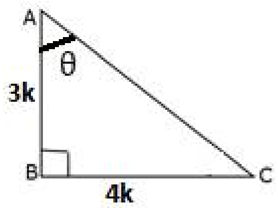
and Hypotenuse = AC = 5k

So, $\cos\theta = \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5}$

10 B. Question

If tan A= 4/3. Find the other trigonometric ratios of the angle A.

Answer



We know that,

$$\tan\theta = \frac{\text{side opposite to angle }\theta}{\text{side adjacent to angle }\theta}$$

Or $\tan\theta = \frac{\text{perpendicular}}{\text{base}}$

Here, θ = A

$$\tan A = \frac{4}{3} \Rightarrow \frac{P}{B} = \frac{4}{3} \Rightarrow \frac{BC}{AB} = \frac{4}{3}$$

Let,

The side opposite to angle A =BC = 4k

The side adjacent to angle A =AB = 3k

where k is any positive integer

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (3k)^2 + (4k)^2 = (AC)^2$

$\Rightarrow (AC)^2 = 9k^2 + 16k^2$

$\Rightarrow (AC)^2 = 25\ k^2$

$\Rightarrow AC = \sqrt{25\ k^2}$

$\Rightarrow AC = \pm 5k$

But side AC can't be negative. So, AC = 5k

Now, we will find the sin A and cos A

$$\sin\theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Side opposite to angle A = BC = 4k

and Hypotenuse = AC = 5k

So, $\text{Sin A} = \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5}$

Now, We know that

$$\cos\theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Side adjacent to angle A = AB = 3k

and Hypotenuse = AC = 5k

So, $\cos A = \frac{AB}{AC} = \frac{3k}{5k} = \frac{3}{5}$

Now, we find other trigonometric ratios

$$\text{Cosec A} = \frac{1}{\sin A}$$

$$= \frac{1}{\frac{4}{5}}$$

$$= \frac{5}{4}$$

$$\sec A = \frac{1}{\cos A}$$

$$= \frac{1}{\frac{3}{5}}$$

$= \frac{5}{3}$

$\cot A = \frac{1}{\tan A}$

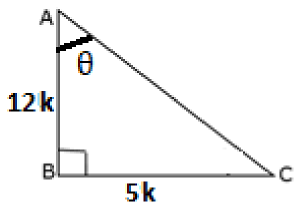
$= \frac{1}{\frac{4}{3}}$

$= \frac{3}{4}$

11. Question

If $\cot\theta = \frac{12}{5}$, then find the value of sin θ.

Answer



We know that,

$\cot\theta = \frac{\text{side adjacent to angle } \theta}{\text{side opposite to angle } \theta}$

Or $\cot\theta = \frac{\text{base}}{\text{perpendicular}}$

$\cot\theta = \frac{12}{5} \Rightarrow \frac{B}{P} = \frac{12}{5} \Rightarrow \frac{AB}{BC} = \frac{12}{5}$

Let,

Side adjacent to angle θ =AB = 12k

The side opposite to angle θ =BC = 5k

where k is any positive integer

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (12k)^2 + (5k)^2 = (AC)^2$

$\Rightarrow (AC)^2 = 144\ k^2 + 25\ k^2$

$\Rightarrow (AC)^2 = 169\ k^2$

$\Rightarrow AC = \sqrt{169\ k^2}$

$\Rightarrow AC = \pm 13k$

But side AC can't be negative. So, AC = 13k

Now, we will find the sin θ

$$\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}}$$

Side opposite to angle θ = BC = 5k

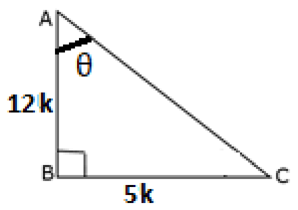
and Hypotenuse = AC = 13k

So, $\text{Sin}\ \theta = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$

12. Question

If $\tan\theta = \frac{5}{12}$, then find the value of cos θ.

Answer



We know that,

$$\tan\theta = \frac{\text{side opposite to angle }\theta}{\text{side adjacent to angle }\theta}$$

Or $\tan\theta = \frac{\text{perpendicular}}{\text{base}}$

$$\tan\theta = \frac{5}{12} \Rightarrow \frac{P}{B} = \frac{5}{12} \Rightarrow \frac{BC}{AB} = \frac{5}{12}$$

Let,

The side opposite to angle θ =BC = 5k

The side adjacent to angle θ =AB = 12k

where k is any positive integer

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (12k)^2 + (5k)^2 = (AC)^2$

$\Rightarrow (AC)^2 = 144\ k^2 + 25\ k^2$

$\Rightarrow (AC)^2 = 169\ k^2$

$\Rightarrow AC = \sqrt{169\ k^2}$

$\Rightarrow AC = \pm 13k$

But side AC can't be negative. So, AC = 13k

Now, We know that

$$\cos\theta = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}}$$

Side adjacent to angle θ = AB = 12k

and Hypotenuse = AC = 13k

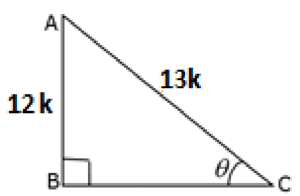
So, $\cos\theta = \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13}$

13. Question

If $\sin\theta = \frac{12}{13}$, then find the value of cos θ and tan θ.

Answer

Given: $\text{Sin }\theta = \frac{12}{13}$



We know that,

$$\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}}$$

Or $\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

$$\text{Sin }\theta = \frac{12}{13} \Rightarrow \frac{P}{H} = \frac{12}{13} \Rightarrow \frac{AB}{AC} = \frac{12}{13}$$

Let,

Side opposite to angle θ = 12k

and Hypotenuse = 13k

where, k is any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (12k)^2 + (BCk)^2 = (13)^2$

$\Rightarrow 144\ k^2 + (BC)^2 = 169\ k^2$

$\Rightarrow (BC)^2 = 169\ k^2 - 144\ k^2$

$\Rightarrow (BC)^2 = 25\ k^2$

$\Rightarrow BC = \sqrt{25}\ k^2$

$\Rightarrow BC = \pm 5k$

But side BC can’t be negative. So, BC = 5k

Now, we have to find the value of cos θ and tan θ

We know that,

$$\cos\theta = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}}$$

Side adjacent to angle θ = BC =5k

Hypotenuse = AC =13k

So, $\cos\theta = \frac{5k}{13k} = \frac{5}{13}$

Now, tan θ

We know that,

$$\tan\theta = \frac{\text{side opposite to angle }\theta}{\text{side adjacent to angle }\theta}$$

side opposite to angle θ = AB =12k

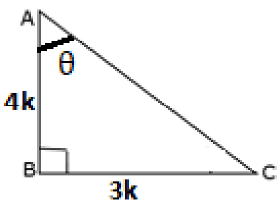
Side adjacent to angle θ = BC =5k

So, $\tan\theta = \frac{12k}{5k} = \frac{12}{5}$

14. Question

If tan θ =0.75, then find the value of sin θ.

Answer



We know that,

$$\tan\theta = \frac{\text{side opposite to angle }\theta}{\text{side adjacent to angle }\theta}$$

Or $\tan\theta = \frac{\text{perpendicular}}{\text{base}}$

Given: tan θ =0.75

$$\Rightarrow \tan\theta = \frac{75}{100} = \frac{3}{4}$$

$$\tan\theta = \frac{3}{4} \Rightarrow \frac{P}{B} = \frac{3}{4} \Rightarrow \frac{BC}{AB} = \frac{3}{4}$$

Let,

The side opposite to angle θ =BC = 3k

The side adjacent to angle θ =AB = 4k

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (4k)^2 + (3k)^2 = (AC)^2$

$\Rightarrow (AC)^2 = 16\ k^2 + 9\ k^2$

$\Rightarrow (AC)^2 = 25\ k^2$

$\Rightarrow AC = \sqrt{25\ k^2}$

$\Rightarrow AC = \pm 5k$

But side AC can't be negative. So, AC = 5k

Now, we will find the sin θ

$$\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}}$$

Side opposite to angle θ = BC = 3k

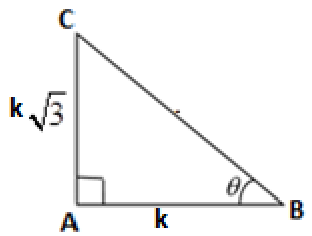
and Hypotenuse = AC = 5k

So, $\text{Sin}\ \theta = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$

15. Question

If tan B= √3, then find the values of sin B and cos B.

Answer



We know that,

$$\tan\theta = \frac{\text{side opposite to angle }\theta}{\text{side adjacent to angle }\theta}$$

Or $\tan\theta = \frac{\text{perpendicular}}{\text{base}}$

Given: tan B = √3

$$\Rightarrow \tan B = \frac{\sqrt{3}}{1}$$

$\tan B = \frac{\sqrt{3}}{1} \Rightarrow \frac{P}{B} = \frac{\sqrt{3}}{1} \Rightarrow \frac{AC}{AB} = \frac{\sqrt{3}}{1}$

Let,

Side opposite to angle B =AC = √3k

The side adjacent to angle B =AB = 1k

where k is any positive integer

Firstly we have to find the value of BC.

So, we can find the value of AC with the help of Pythagoras theorem

$\Rightarrow (AB)^2 + (AC)^2 = (BC)^2$

$\Rightarrow (1k)^2 + (\sqrt{3}k)^2 = (BC)^2$

$\Rightarrow (BC)^2 = 1\,k^2 + 3\,k^2$

$\Rightarrow (BC)^2 = 4\,k^2$

$\Rightarrow BC = \sqrt{2}\,k^2$

$\Rightarrow BC = \pm 2k$

But side BC can't be negative. So, BC = 2k

Now, we will find the sin B and cos B

$$\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}}$$

Side opposite to angle B = AC = k√3

and Hypotenuse = BC = 2k

So, $\text{Sin B} = \frac{AC}{BC} = \frac{k\sqrt{3}}{2k} = \frac{\sqrt{3}}{2}$

Now, we know that,

$$\cos\theta = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}}$$

The side adjacent to angle B = AB =1k

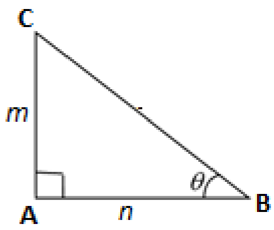
Hypotenuse = BC =2k

So, $\cos B = \frac{AB}{BC} = \frac{1k}{2k} = \frac{1}{2}$

16. Question

If $\tan\theta = \frac{m}{n}$, then find the values of cos θ and sin θ.

Answer



We know that,

$$\tan\theta = \frac{\text{side opposite to angle }\theta}{\text{side adjacent to angle }\theta}$$

Or $\tan\theta = \frac{\text{perpendicular}}{\text{base}}$

Here, $\tan\theta = \frac{m}{n}$

So, Side opposite to angle θ =AC = m

The side adjacent to angle θ =AB = n

Firstly we have to find the value of BC.

So, we can find the value of AC with the help of Pythagoras theorem

⇒ $(AB)^2 + (AC)^2 = (BC)^2$

⇒ $(n)^2 + (m)^2 = (BC)^2$

⇒ $(BC)^2 = m^2 + n^2$

⇒ BC $=\sqrt{m^2 + n^2}$

So, BC $=\sqrt{(m^2 + n^2)}$

Now, we will find the sin B and cos B

$$\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}}$$

Side opposite to angle θ = AC = m

and Hypotenuse = BC $=\sqrt{(m^2 + n^2)}$

So, Sin θ $= \frac{AC}{BC} = \frac{m}{\sqrt{m^2+n^2}}$

Now, we know that,

$$\cos\theta = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}}$$

Side adjacent to angle θ = AB =n

Hypotenuse = BC =$\sqrt{(m^2 + n^2)}$

So, cos θ $= \frac{AB}{BC} = \frac{n}{\sqrt{m^2+n^2}}$

17. Question

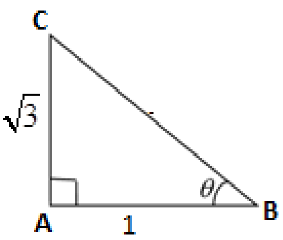
If sin θ = √3 cos θ, then find the values of cos θ and sin θ.

Answer

Given : sin θ =√3cos θ

$$\Rightarrow \frac{\sin\theta}{\cos\theta} = \sqrt{3}$$

⇒ tan θ =√3



We know that,

$$\tan\theta = \frac{\text{side opposite to angle }\theta}{\text{side adjacent to angle }\theta}$$

Or tan θ $= \frac{\text{perpendicular}}{\text{base}}$

and tan θ = √3

$$\Rightarrow \tan\theta = \frac{\sqrt{3}}{1}$$

$$\tan\theta = \frac{\sqrt{3}}{1} \Rightarrow \frac{P}{B} = \frac{\sqrt{3}}{1} \Rightarrow \frac{AC}{AB} = \frac{\sqrt{3}}{1}$$

Let,

The side opposite to angle θ =AC = k√3

The side adjacent to angle θ =AB = 1k

where k is any positive integer

Firstly we have to find the value of BC.

So, we can find the value of AC with the help of Pythagoras theorem

$\Rightarrow (AB)^2 + (AC)^2 = (BC)^2$

$\Rightarrow (1k)^2 + (k\sqrt{3})^2 = (BC)^2$

$\Rightarrow (BC)^2 = 1\ k^2 + 3\ k^2$

$\Rightarrow (BC)^2 = 4\ k^2$

$\Rightarrow BC = \sqrt{2}\ k^2$

$\Rightarrow BC = \pm 2k$

But side BC can’t be negative. So, BC = 2k

Now, we will find the sin θ and cos θ

$$\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}}$$

Side opposite to angle θ = AC = k√3

and Hypotenuse = BC = 2k

So, $\text{Sin}\,\theta = \frac{AC}{BC} = \frac{k\sqrt{3}}{2k} = \frac{\sqrt{3}}{2}$

Now, we know that,

$$\cos\theta = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}}$$

The side adjacent to angle θ = AB =1k

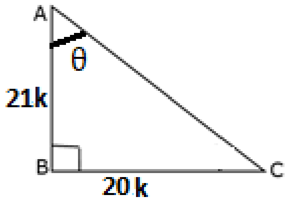
Hypotenuse = BC =2k

So, $\cos\theta = \frac{AB}{BC} = \frac{1k}{2k} = \frac{1}{2}$

18 A. Question

If $\cot\theta = \dfrac{21}{20}$, then find the values of cos θ and sin θ.

Answer



We know that,

$$\cot\theta = \frac{\text{side adjacent to angle } \theta}{\text{side opposite to angle } \theta}$$

Or $\cot\theta = \dfrac{\text{base}}{\text{perpendicular}}$

$$\cot\theta = \frac{21}{20} \Rightarrow \frac{B}{P} = \frac{21}{20} \Rightarrow \frac{AB}{BC} = \frac{21}{20}$$

Let,

Side adjacent to angle θ =AB = 21k

The side opposite to angle θ =BC = 20k

where k is any positive integer

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (21k)^2 + (20k)^2 = (AC)^2$

$\Rightarrow (AC)^2 = 441\ k^2 + 400\ k^2$

$\Rightarrow (AC)^2 = 841\ k^2$

$\Rightarrow AC = \sqrt{841\ k^2}$

$\Rightarrow AC = \pm 29k$

But side AC can’t be negative. So, AC = 29k

Now, we will find the sin θ

$$\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}}$$

Side opposite to angle θ = BC = 20k

and Hypotenuse = AC = 29k

So, Sin θ $= \frac{BC}{AC} = \frac{20k}{29k} = \frac{20}{29}$

Now, we know that,

$$\cos\theta = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}}$$

Side adjacent to angle θ = AB =21k

Hypotenuse = AC =29k

So, cos θ $= \frac{AB}{AC} = \frac{21k}{29k} = \frac{21}{29}$

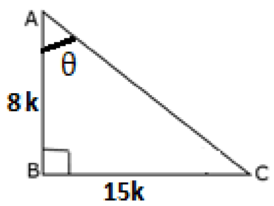
18 B. Question

If 15 cot A=18, find sin A and sec A.

Answer

Given: 15 cot A = 8

$$\Rightarrow \cot A = \frac{8}{15}$$



And we know that,

$$\cot\theta = \frac{\text{side adjacent to angle }\theta}{\text{side opposite to angle }\theta}$$

Or cot θ $= \frac{\text{base}}{\text{perpendicular}}$

$$\cot A = \frac{8}{15} \Rightarrow \frac{B}{P} = \frac{8}{15} \Rightarrow \frac{BC}{AC} = \frac{8}{15}$$

Let,

Side adjacent to angle A =AB = 8k

The side opposite to angle A =BC = 15k

where k is any positive integer

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (8k)^2 + (15k)^2 = (AC)^2$

$\Rightarrow (AC)^2 = 64\ k^2 + 225\ k^2$

$\Rightarrow (AC)^2 = 289\ k^2$

$\Rightarrow AC = \sqrt{289\ k^2}$

$\Rightarrow AC = \pm 17k$

But side AC can't be negative. So, AC = 17k

Now, we will find the sin θ

$$\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}}$$

Side opposite to angle θ = BC = 15k

and Hypotenuse = AC = 17k

So, $\text{Sin}\ \theta = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$

Now, we know that,

$$\cos\theta = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}}$$

The side adjacent to angle θ = AB =8

Hypotenuse = AC =17

So, $\cos\theta = \frac{AB}{AC} = \frac{8k}{17k} = \frac{8}{17}$

$$\therefore \sec\theta = \frac{1}{\cos\theta}$$

$= \frac{1}{\frac{8}{17}}$

$= \frac{17}{8}$

19. Question

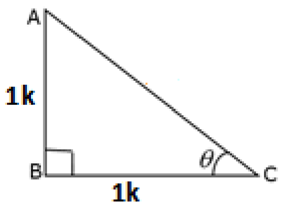
If sin θ = cos θ and 0° < θ <90°, then find the values of sin θ and cos θ.

Answer

Given: sinθ = cosθ

$\Rightarrow \frac{\sin\theta}{\cos\theta} = 1$

⇒ tan θ = 1



$\tan\theta = \frac{1}{1} \Rightarrow \frac{P}{B} = \frac{1}{1} \Rightarrow \frac{AB}{BC} = \frac{1}{1}$

Let,

Side opposite to angle θ = AB =1k

The side adjacent to angle θ = BC =1k

where k is any positive integer

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (1k)^2 + (1k)^2 = (AC)^2$

$\Rightarrow (AC)^2 = 1k^2 + 1k^2$

$\Rightarrow (AC)^2 = 2k^2$

$\Rightarrow AC = \sqrt{2k^2}$

$\Rightarrow AC = k\sqrt{2}$

So, $AC = k\sqrt{2}$

Now, we will find the $\sin\theta$

$$\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}}$$

Side opposite to angle θ = AB= 1k

and Hypotenuse = AC = $k\sqrt{2}$

So, $\text{Sin}\,\theta = \frac{AB}{AC} = \frac{1k}{k\sqrt{2}} = \frac{1}{\sqrt{2}}$

Now, we know that,

$$\cos\theta = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}}$$

The side adjacent to angle θ = BC =1k

Hypotenuse = AC =$k\sqrt{2}$

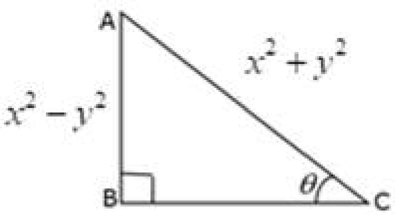
So, $\cos\theta = \frac{BC}{AC} = \frac{1k}{k\sqrt{2}} = \frac{1}{\sqrt{2}}$

20. Question

If $\sin\theta = \frac{x^2 - y^2}{x^2 + y^2}$, then find the values of $\cos\theta$ and $\frac{1}{\tan\theta}$.

Answer

$\sin\theta = \frac{x^2 - y^2}{x^2 + y^2}$



We know that,

$$\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}}$$

Or, $\sin\theta = \frac{\text{Perpendicular}}{\text{hypotenuse}}$

$$\sin\theta = \frac{x^2 - y^2}{x^2 + y^2} \Rightarrow \frac{P}{H} = \frac{x^2 - y^2}{x^2 + y^2} \Rightarrow \frac{AB}{AC} = \frac{x^2 - y^2}{x^2 + y^2}$$

Let,

Side opposite to angle θ = AB = $x^2 - y^2$

and Hypotenuse = AC = $x^2 + y^2$

In right angled ΔABC, we have

$(AB)^2 + (BC)^2 = (AC)^2$ [by using Pythagoras theorem]

$\Rightarrow (x^2 - y^2)^2 + (BC)^2 = (x^2 + y^2)^2$

$\Rightarrow (BC)^2 = (x^2 + y^2)^2 - (x^2 - y^2)^2$

Using the identity, $a^2 - b^2 = (a+b)(a - b)$

$\Rightarrow (BC)^2 = [(x^2 + y^2 + x^2 - y^2)][x^2 + y^2 - (x^2 - y^2)]$

$\Rightarrow (BC)^2 = (2x^2)(2y^2)$

$\Rightarrow (BC)^2 = (4x^2y^2)$

$\Rightarrow BC = \sqrt{4x^2y^2}$

$\Rightarrow BC = \pm 2xy$

$\Rightarrow BC = 2xy$ [taking positive square root since, side cannot be negative]

$$\therefore \cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{2xy}{x^2 + y^2}$$

and $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{AB}{BC} = \frac{x^2 - y^2}{2xy}$

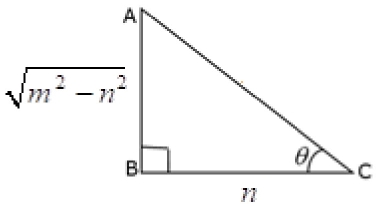
So, $\frac{1}{\tan\theta} = \frac{1}{\frac{x^2 - y^2}{2xy}} = \frac{2xy}{x^2 - y^2}$

21. Question

If $\tan\theta = \frac{\sqrt{m^2 - n^2}}{n}$, then find the values of sin θ and cos θ.

Answer

Given: $\tan\theta = \frac{\sqrt{m^2 - n^2}}{n}$



We know that,

$$\tan\theta = \frac{\sqrt{m^2-n^2}}{n} \Rightarrow \frac{P}{B} = \frac{\sqrt{m^2-n^2}}{n} \Rightarrow \frac{AB}{BC} = \frac{\sqrt{m^2-n^2}}{n}$$

Let,

AB = $\sqrt{(m^2 - n^2)}$ and BC = n

In right angled ΔABC, we have

$(AB)^2 + (BC)^2 = (AC)^2$ [by using Pythagoras theorem]

$\Rightarrow (\sqrt{(m^2 - n^2)})^2 + (n)^2 = (AC)^2$

$\Rightarrow m^2 - n^2 + n^2 = (AC)^2$

$\Rightarrow (AC)^2 = (m^2)$

$\Rightarrow AC = \sqrt{m^2}$

$\Rightarrow AC = \pm m$

$\Rightarrow$ AC = m [taking positive square root since, side cannot be negative]

Now, we have to find the value of cos θ and sin θ

We, know that

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$= \frac{BC}{AC} = \frac{n}{m}$$

and

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

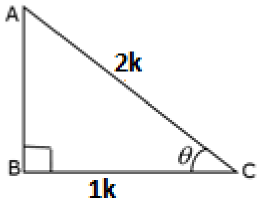
$$= \frac{AB}{AC} = \frac{\sqrt{m^2-n^2}}{m}$$

22 A. Question

If sec θ = 2, then find the values of other t–ratios of angle θ.

Answer

Given: sec θ = 2



We know that,

$$\sec\theta = \frac{\text{hypotenuse}}{\text{base}}$$

$$\text{Sec}\ \theta = \frac{2}{1} \Rightarrow \frac{H}{B} = \frac{2}{1} \Rightarrow \frac{AC}{BC} = \frac{2}{1}$$

Let,

BC = 1k and AC = 2k

where, k is any positive integer.

In right angled ΔABC, we have

$(AB)^2 + (BC)^2 = (AC)^2$ [by using Pythagoras theorem]

$\Rightarrow (AB)^2 + (1k)^2 = (2k)^2$

$\Rightarrow (AB)^2 + k^2 = 4k^2$

$\Rightarrow (AB)^2 = 4k^2 - k^2$

$\Rightarrow (AB)^2 = 3k^2$

$\Rightarrow AB = k\sqrt{3}$

Now, we have to find the value of other trigonometric ratios.

We, know that

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$= \frac{AB}{AC} = \frac{k\sqrt{3}}{2k} = \frac{\sqrt{3}}{2}$$

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$= \frac{BC}{AC} = \frac{1k}{2k} = \frac{1}{2}$$

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$= \frac{AB}{BC} = \frac{k\sqrt{3}}{1k} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

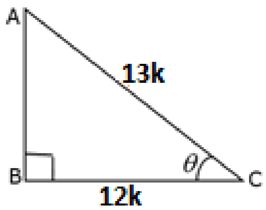
$$\text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\sqrt{3}}$$

22 B. Question

Given $\sec\theta = \frac{13}{12}$, calculate all other trigonometric ratios.

Answer



Given: $\sec\theta = \frac{13}{12}$

We know that,

$$\sec\theta = \frac{\text{hypotenuse}}{\text{base}}$$

$$\sec\theta = \frac{13}{12} \Rightarrow \frac{H}{B} = \frac{13}{12} \Rightarrow \frac{AC}{BC} = \frac{13}{12}$$

Let,

BC = 12k and AC = 13k

where, k is any positive integer.

In right angled ΔABC, we have

$(AB)^2 + (BC)^2 = (AC)^2$ [by using Pythagoras theorem]

$\Rightarrow (AB)^2 + (12k)^2 = (13k\ )^2$

$\Rightarrow (AB)^2 + 144k^2 = 169k^2$

$\Rightarrow (AB)^2 = 169k^2 - 144k^2$

$\Rightarrow (AB)^2 = 25k^2$

$\Rightarrow AB = \sqrt{25k^2}$

$\Rightarrow AB = \pm 5k$ [taking positive square root since, side cannot be negative]

Now, we have to find the value of other trigonometric ratios.

We, know that

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$= \frac{AB}{AC} = \frac{5k}{13k} = \frac{5}{13}$$

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$= \frac{BC}{AC} = \frac{12k}{13k} = \frac{12}{13}$$

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$= \frac{AB}{BC} = \frac{5k}{12k} = \frac{5}{12}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{5}{13}} = \frac{13}{5}$$

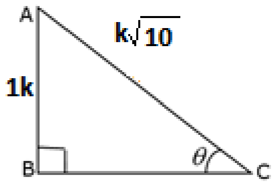
$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{5}{12}} = \frac{12}{5}$$

23. Question

If $\operatorname{cosec}\theta = \sqrt{10}$, then find the values of other t–ratios of angle θ.

Answer

Given: cosec θ = $\sqrt{10}$



We know that,

$$\text{cosec}\,\theta = \frac{\text{hypotenuse}}{\text{perpendicular}}$$

$$\text{cosec}\,\theta = \frac{\sqrt{10}}{1} \Rightarrow \frac{H}{P} = \frac{\sqrt{10}}{1} \Rightarrow \frac{AC}{AB} = \frac{\sqrt{10}}{1}$$

Let,

AB = 1k and AC = $k\sqrt{10}$

where, k is any positive integer.

In right angled ΔABC, we have

$(AB)^2 + (BC)^2 = (AC)^2$ [by using Pythagoras theorem]

$\Rightarrow (1k)^2 + (BC)^2 = (k\sqrt{10})^2$

$\Rightarrow (BC)^2 = 10k^2 - k^2$

$\Rightarrow (BC)^2 = 9k^2$

$\Rightarrow BC = \sqrt{9k^2}$

$\Rightarrow BC = \pm 3k$ [taking positive square root since, side cannot be negative]

Now, we have to find the value of other trigonometric ratios.

We, know that

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$= \frac{AB}{AC} = \frac{1k}{k\sqrt{10}} = \frac{1}{\sqrt{10}}$$

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$= \frac{BC}{AC} = \frac{3k}{k\sqrt{10}} = \frac{3}{\sqrt{10}}$$

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$= \frac{AB}{BC} = \frac{1k}{3k} = \frac{1}{3}$$

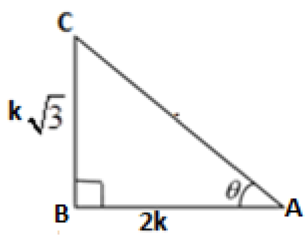
$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{3}{\sqrt{10}}} = \frac{\sqrt{10}}{3}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{1}{3}} = 3$$

24 A. Question

If $\tan A = \frac{\sqrt{3}}{2}$, then find the values of sin A + cos A.

Answer



We know that,

$$\tan\theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

Or $\tan\theta = \frac{\text{perpendicular}}{\text{base}}$

Given: $\tan A = \frac{\sqrt{3}}{2}$

$$\Rightarrow \tan A = \frac{\sqrt{3}}{2}$$

$$\tan A = \frac{\sqrt{3}}{2} \Rightarrow \frac{P}{B} = \frac{\sqrt{3}}{2} \Rightarrow \frac{BC}{AB} = \frac{\sqrt{3}}{2}$$

Let,

Side opposite to angle A =BC = k√3

Side adjacent to angle A =AB = 2k

where, k is any positive integer

Firstly we have to find the value of BC.

So, we can find the value of AC with the help of Pythagoras theorem

⇒ $(AB)^2 + (BC)^2 = (AC)^2$

⇒ $(2k)^2 + (\sqrt{3}k)^2 = (AC)^2$

⇒ $(AC)^2 = 4\,k^2 + 3\,k^2$

⇒ $(AC)^2 = 7\,k^2$

⇒ $AC = \sqrt{7}\,k^2$

⇒ AC =k√7

So, AC = k√7

Now, we will find the sin A and cos A

$$\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}}$$

Side opposite to angle A = BC = k√3

and Hypotenuse = AC = k√7

So, $\text{Sin A} = \frac{BC}{AC} = \frac{k\sqrt{3}}{k\sqrt{7}} = \frac{\sqrt{3}}{\sqrt{7}}$

Now, we know that,

$$\cos\theta = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}}$$

Side adjacent to angle A = AB =2k

Hypotenuse = AC = k√7

So, $\cos A = \frac{AB}{AC} = \frac{2k}{k\sqrt{7}} = \frac{2}{\sqrt{7}}$

Now, we have to find sin A +cos A

Putting values of sin A and cos A, we get

$$\text{Sin A} + \text{Cos A} = \frac{\sqrt{3}}{\sqrt{7}} + \frac{2}{\sqrt{7}} = \frac{\sqrt{3}+2}{\sqrt{7}}$$

24 B. Question

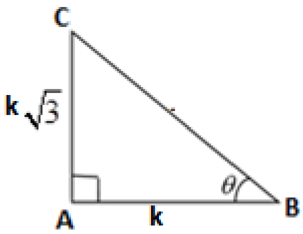
If $\sin\theta = \sqrt{3}\cos\theta$, find the value of cos θ – sin θ.

Answer

Given: sin θ =√3cos θ

$$\Rightarrow \frac{\sin\theta}{\cos\theta} = \sqrt{3}$$

⇒ tan θ = √3



We know that,

$$\tan\theta = \frac{\text{side opposite to angle }\theta}{\text{side adjacent to angle }\theta}$$

$$\text{Or } \tan\theta = \frac{\text{perpendicular}}{\text{base}}$$

Given: tan θ = √3

$$\Rightarrow \tan\theta = \frac{\sqrt{3}}{1}$$

$$\tan\theta = \frac{\sqrt{3}}{1} \Rightarrow \frac{P}{B} = \frac{\sqrt{3}}{1} \Rightarrow \frac{AC}{AB} = \frac{\sqrt{3}}{1}$$

Let,

Side opposite to angle θ =AC = √3k

Side adjacent to angle θ =AB = 1k

where k is any positive integer

Firstly we have to find the value of BC.

So, we can find the value of AC with the help of Pythagoras theorem

$\Rightarrow (AB)^2 + (AC)^2 = (BC)^2$

$\Rightarrow (1k)^2 + (\sqrt{3}k)^2 = (BC)^2$

$\Rightarrow (BC)^2 = 1\ k^2 + 3\ k^2$

$\Rightarrow (BC)^2 = 4\ k^2$

$\Rightarrow BC = \sqrt{2}\ k^2$

$\Rightarrow BC = \pm 2k$

But side BC can't be negative. So, BC = 2k

Now, we will find the sin B and cos B

$$\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}}$$

Side opposite to angle θ = AC = $k\sqrt{3}$

and Hypotenuse = BC = 2k

So, $\text{Sin}\ \theta = \frac{AC}{BC} = \frac{k\sqrt{3}}{2k} = \frac{\sqrt{3}}{2}$

Now, we know that,

$$\cos\theta = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}}$$

The side adjacent to angle θ = AB =1k

Hypotenuse = BC =2k

So, $\cos\theta = \frac{AB}{BC} = \frac{1k}{2k} = \frac{1}{2}$

Now, we have to find the value of cos θ – sin θ

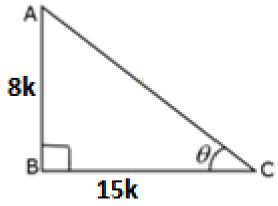
Putting the values of sin θ and cos θ, we get

$$\text{Cos}\ \theta - \sin\theta = \frac{1}{2} - \frac{\sqrt{3}}{2} = \frac{1-\sqrt{3}}{2}$$

24 C. Question

If $\tan\theta = \frac{8}{15}$, find the value of $1 + \cos^2\theta$.

Answer



We know that,

$$\tan\theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

Or $\tan\theta = \frac{\text{perpendicular}}{\text{base}}$

$$\tan\theta = \frac{8}{15} \Rightarrow \frac{P}{B} = \frac{8}{15} \Rightarrow \frac{AB}{BC} = \frac{8}{15}$$

Let,

Side opposite to angle θ =AB = 8k

Side adjacent to angle θ =BC = 15k

where, k is any positive integer

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (8k)^2 + (15k)^2 = (AC)^2$

$\Rightarrow (AC)^2 = 64k^2+225k^2$

$\Rightarrow (AC)^2 = 289\ k^2$

$\Rightarrow AC = \sqrt{289\ k^2}$

$\Rightarrow AC = \pm 17k$

But side AC can’t be negative. So, AC = 17k

Now, we will find the cos θ

We know that

$$\cos\theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Side adjacent to angle θ = BC = 15k

and Hypotenuse = AC = 17k

So, $\cos\theta = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$

Now, we have to find the value of $1 + \cos^2\theta$

Putting the value of cos θ, we get

$$1 + \cos^2\theta = 1 + \left(\frac{15}{17}\right)^2$$

$$= 1 + \frac{225}{289}$$

$$= \frac{289 + 225}{289}$$

$$= \frac{514}{289}$$

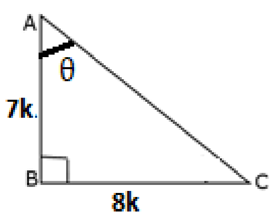
25. Question

If $\cot\theta = \frac{7}{8}$, evaluate

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

(ii) $\cot^2\theta$

Answer



Given: $\cot\theta = \frac{7}{8}$

We know that,

$$\cot\theta = \frac{\text{side adjacent to angle }\theta}{\text{side opposite to angle }\theta}$$

Or $\cot\theta = \frac{\text{base}}{\text{perpendicular}}$

$\cot\theta = \frac{7}{8} \Rightarrow \frac{B}{P} = \frac{7}{8} \Rightarrow \frac{AB}{BC} = \frac{7}{8}$

Let,

Side adjacent to angle θ =AB = 7k

Side opposite to angle θ =BC = 8k

where, k is any positive integer

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (7k)^2 + (8k)^2 = (AC)^2$

$\Rightarrow (AC)^2 = 49\ k^2 + 69\ k^2$

$\Rightarrow (AC)^2 = 113\ k^2$

$\Rightarrow AC = \sqrt{113}\ k^2$

$\Rightarrow AC = k\sqrt{113}$

$\therefore\ \text{Sin}\,\theta = \frac{P}{H} = \frac{BC}{AC} = \frac{8k}{k\sqrt{113}} = \frac{8}{\sqrt{113}}$

and $\cos\theta = \frac{B}{H} = \frac{AB}{AC} = \frac{7k}{k\sqrt{113}} = \frac{7}{\sqrt{113}}$

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

We know that,

$(a+b)(a - b) = (a^2 - b^2)$

So, using this identity, we get

$$= \frac{(1)^2 - (\sin\theta)^2}{(1)^2 - (\cos\theta)^2}$$

$$= \frac{1 - \sin^2\theta}{1 - \cos^2\theta}$$

$$= \frac{1 - \left(\frac{8}{\sqrt{113}}\right)^2}{1 - \left(\frac{7}{\sqrt{113}}\right)^2}$$

$$= \frac{1 - \frac{64}{113}}{1 - \frac{49}{113}}$$

$$= \frac{\frac{113 - 64}{113}}{\frac{113 - 49}{113}}$$

$$= \frac{49}{64}$$

(ii) $\cot^2 \theta$

Given $\cot \theta = \frac{7}{8}$

$$= \left(\frac{7}{8}\right)^2$$

$$= \frac{49}{64}$$

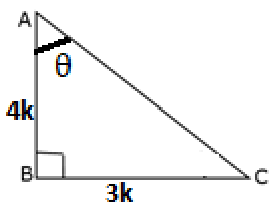
26 A. Question

If 3 cot A = 4, check whether $\frac{1 - \tan^2 A}{1 + \tan^2 A}$ $= \cos^2 A - \sin^2 A$ or not.

Answer

Given: 3cot A = 4

$$\Rightarrow \cot A = \frac{4}{3}$$



We know that,

$$\cot\theta = \frac{\text{side adjacent to angle }\theta}{\text{side opposite to angle }\theta}$$

Or $\cot\theta = \frac{\text{base}}{\text{perpendicular}}$

$$\cot A = \frac{4}{3} \Rightarrow \frac{B}{P} = \frac{4}{3} \Rightarrow \frac{AB}{BC} = \frac{4}{3}$$

Let,

Side adjacent to angle A =AB = 4k

The side opposite to angle A =BC = 3k

where k is any positive integer

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (4k)^2 + (3k)^2 = (AC)^2$

$\Rightarrow (AC)^2 = 16\,k^2 + 9\,k^2$

$\Rightarrow (AC)^2 = 25\,k^2$

$\Rightarrow AC = \sqrt{25k^2}$

$\Rightarrow AC = \pm 5k$ [taking positive square root since, side cannot be negative]

$$\therefore\ \tan A = \frac{1}{\cot A} = \frac{1}{\frac{4}{3}} = \frac{3}{4}$$

$$\text{Sin } A = \frac{P}{H} = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$$

and $\cos A = \frac{B}{H} = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$

Now, LHS $= \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2}$

$$= \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}}$$

$$= \frac{\frac{16-9}{16}}{\frac{16+9}{16}}$$

$= \frac{7}{25}$ ...(i)

And RHS = $\cos^2 A - \sin^2 A$

$$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$= \frac{16}{25} - \frac{9}{25}$$

$= \frac{7}{25}$ ...(ii)

From Eqs. (i) and (ii) LHS =RHS

Hence Proved

26 B. Question

In a right triangle ABC, right angled at B, if tan A = 1, then verify that 2 sin A cos A = 1.

Answer

tan A = 1

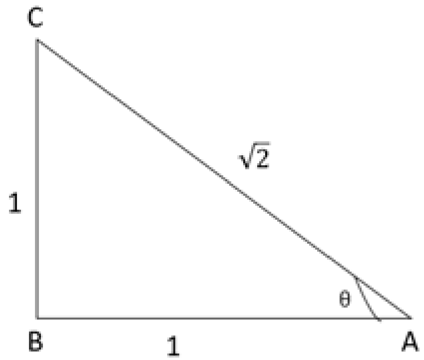
As we know

$$\tan\theta = \frac{\text{perpedicular}}{\text{base}}$$

Now construct a right angle triangle right angled at B such that

$\angle$ BAC = θ

Hence perpendicular = BC = 1 and base = AB = 1



By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = (1)^2 + (1)^2$

$\Rightarrow AC^2 = 2$

$\Rightarrow AC = \sqrt{2}$

As,

$$\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}} \text{ and } \cos\theta = \frac{\text{base}}{\text{hypotenuse}}$$

$\Rightarrow \sin\theta = \frac{1}{\sqrt{2}}$ and $\cos\theta = \frac{1}{\sqrt{2}}$

Hence,

$2 \sin A \cos A = 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$

$\Rightarrow 2 \sin A \cos A = 2 \times \frac{1}{2}$

$\Rightarrow 2 \sin A \cos A = 1$

= R.H.S

Hence proved.

27. Question

If $4\sin^2 \theta = 3$ and $0^o < \theta < 90^o$, find the value of $1 + \cos \theta$.

Answer

$4\sin^2 \theta = 3$

$$\Rightarrow \sin^2\theta = \frac{3}{4}$$

$$\Rightarrow \sin\theta = \pm\frac{\sqrt{3}}{2}$$

But it is given $0^o < \theta < 90^o$

So, $\sin\theta = \frac{\sqrt{3}}{2}$

$\sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \frac{P}{H} = \frac{\sqrt{3}}{2}$

Let, P =k√3 and H =2k

In right angled ΔABC, we have

$B^2 + P^2 = H^2$

$\Rightarrow B^2 + (k\sqrt{3})^2 = (2k)^2$

$\Rightarrow B^2 + 3k^2 = 4k^2$

$\Rightarrow B^2 = 4k^2 - 3k^2$

$\Rightarrow B^2 = k^2$

$\Rightarrow B = \pm k$

⇒ B = k [taking positive square root since, side cannot be negative]

$\therefore \cos\theta = \frac{B}{H} = \frac{k}{2k} = \frac{1}{2}$

So, $1 + \cos\theta = 1 + \frac{1}{2} = \frac{2+1}{2} = \frac{3}{2}$

28. Question

If $\tan\theta = \frac{p}{q}$, find the value of $\frac{p\sin\theta - q\cos\theta}{p\sin\theta + q\cos\theta}$.

Answer

Given: $\tan\theta = \frac{p}{q}$

Now, $\frac{p\sin\theta - q\cos\theta}{p\sin\theta + q\cos\theta}$

$\Rightarrow \frac{\cos\theta\left(\frac{p\sin\theta}{\cos\theta} - q\right)}{\cos\theta\left(\frac{p\sin\theta}{\cos\theta} + q\right)}$

$\Rightarrow \frac{p\tan\theta - q}{p\tan\theta + q}$ $[\because \tan\theta = \frac{\sin\theta}{\cos\theta}]$

$\Rightarrow \frac{p\left(\frac{p}{q}\right) - q}{p\left(\frac{p}{q}\right) + q}$

$$\Rightarrow \frac{\frac{p^2 - q^2}{q}}{\frac{p^2 + q^2}{q}}$$

$$\Rightarrow \frac{p^2 - q^2}{p^2 + q^2}$$

$$\therefore \frac{p\sin\theta - q\cos\theta}{p\sin\theta + q\cos\theta} = \frac{p^2 - q^2}{p^2 + q^2}$$

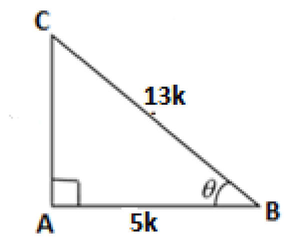
29. Question

If 13 cos θ = 5, $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$.

Answer

Given: 13 cosθ = 5

$$\Rightarrow \cos\theta = \frac{5}{13}$$



We know that,

$$\cos\theta = \frac{\text{Base}}{\text{hypotenuse}}$$

$$\text{Cos } \theta = \frac{5}{13} \Rightarrow \frac{B}{H} = \frac{5}{13}$$

Let AB =5k and BC = 13k

In right angled ΔABC, we have

$B^2 + P^2 = H^2$

$\Rightarrow (5k)^2 + P^2 = (13k)^2$

$\Rightarrow P^2 + 25k^2 = 169k^2$

$\Rightarrow P^2 = 169k^2 – 25k^2$

$\Rightarrow P^2 = 144k^2$

$\Rightarrow P = \sqrt{144k^2}$

$\Rightarrow P = \pm 12k$

$\Rightarrow P = 12k$ [taking positive square root since, side cannot be negative]

$\therefore \sin\theta = \frac{P}{H} = \frac{12}{13}$

Now, $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$

$= \frac{\frac{12}{13} + \frac{5}{13}}{\frac{12}{13} - \frac{5}{13}}$

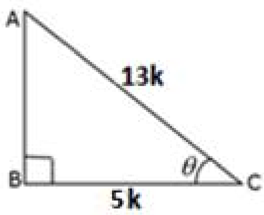
$= \frac{17}{7}$

30. Question

If $\sec\theta = \frac{13}{5}$, show that $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = 3$.

Answer

Given: $\sec\theta = \frac{13}{5}$



We know that,

$\sec\theta = \frac{\text{hypotenuse}}{\text{base}}$

$\text{Sec}\,\theta = \frac{13}{5} \Rightarrow \frac{H}{B} = \frac{13}{5} \Rightarrow \frac{AC}{BC} = \frac{13}{5}$

Let,

BC = 5k and AC = 13k

where, k is any positive integer.

In right angled ΔABC, we have

$(AB)^2 + (BC)^2 = (AC)^2$ [by using Pythagoras theorem]

$\Rightarrow (AB)^2 + (5k)^2 = (13k)^2$

$\Rightarrow (AB)^2 + 25k^2 = 169k^2$

$\Rightarrow (AB)^2 = 169k^2 - 25k^2$

$\Rightarrow (AB)^2 = 144k^2$

$\Rightarrow AB = \sqrt{144k^2}$

$\Rightarrow AB = \pm 12k$ [taking positive square root since, side cannot be negative]

Now, we have to find the value of other trigonometric ratios.

We, know that

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$= \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13}$$

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$= \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$$

Now, LHS = $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$

$$= \frac{2\left(\frac{12}{13}\right) - 3\left(\frac{5}{13}\right)}{4\left(\frac{12}{13}\right) - 9\left(\frac{5}{13}\right)}$$

$$= \frac{24 - 15}{48 - 45}$$

$$= \frac{9}{3}$$

=3 =RHS

Hence Proved

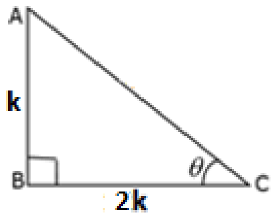
31. Question

If 2 tan θ = 1, find the value of $\frac{3\cos\theta + \sin\theta}{2\cos\theta - \sin\theta}$.

Answer

Given: 2 tan θ = 1

$\Rightarrow \tan\theta = \frac{1}{2}$



We know that,

$\tan\theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$

Or $\tan\theta = \frac{\text{perpendicular}}{\text{base}}$

$\tan\theta = \frac{1}{2} \Rightarrow \frac{P}{B} = \frac{1}{2} \Rightarrow \frac{AB}{BC} = \frac{1}{2}$

Let,

Side opposite to angle θ =AB = 1k

Side adjacent to angle θ =BC = 2k

where, k is any positive integer

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (k)^2 + (2k)^2 = (AC)^2$

$\Rightarrow (AC)^2 = k^2+4k^2$

$\Rightarrow (AC)^2 = 5k^2$

$\Rightarrow AC = \sqrt{5k^2}$

$\Rightarrow AC = \pm k\sqrt{5}$

But side AC can't be negative. So, AC = k√5

Now, we will find the sin θ and cos θ

We know that

$$\cos\theta = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}}$$

Side adjacent to angle θ = BC = 2k

and Hypotenuse = AC = k√5

So, $\cos\theta = \frac{BC}{AC} = \frac{2k}{k\sqrt{5}} = \frac{2}{\sqrt{5}}$

And $\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}}$

Side adjacent to angle θ =AB = 1k

And Hypotenuse =AC = k√5

So, $\sin\theta = \frac{AB}{AC} = \frac{1k}{k\sqrt{5}} = \frac{1}{\sqrt{5}}$

Now, $\frac{3\cos\theta+\sin\theta}{2\cos\theta-\sin\theta}$

$$= \frac{3\left(\frac{2}{\sqrt{5}}\right) + \frac{1}{\sqrt{5}}}{2\left(\frac{2}{\sqrt{5}}\right) - \frac{1}{\sqrt{5}}}$$

$$= \frac{6+1}{4-1}$$

$$= \frac{7}{3}$$

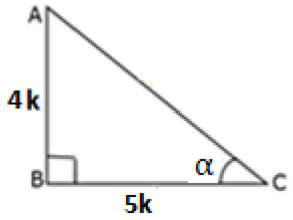
32. Question

If 5 tan α = 4, show that $\frac{5\sin\alpha - 3\cos\alpha}{5\sin\alpha + 2\cos\alpha} = \frac{1}{6}$.

Answer

Given: 5 tan = 4

$\Rightarrow \tan\alpha = \frac{4}{5}$



We know that,

$$\tan\theta = \frac{\text{side opposite to angle }\theta}{\text{side adjacent to angle }\theta}$$

Or $\tan\theta = \frac{\text{perpendicular}}{\text{base}}$

$$\tan\alpha = \frac{4}{5} \Rightarrow \frac{P}{B} = \frac{4}{5} \Rightarrow \frac{AB}{BC} = \frac{4}{5}$$

Let,

The side opposite to angle α =AB = 4k

The side adjacent to angle α =BC = 5k

where k is any positive integer

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (4k)^2 + (5k)^2 = (AC)^2$

$\Rightarrow (AC)^2 = 16k^2+25k^2$

$\Rightarrow (AC)^2 = 41k^2$

$\Rightarrow AC = \sqrt{41k^2}$

$\Rightarrow AC = \pm k\sqrt{41}$

But side AC can't be negative. So, AC = $k\sqrt{41}$

Now, we will find the sin α and cos α

We know that

$$\cos\theta = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}}$$

Side adjacent to angle α = BC = 5k

and Hypotenuse = AC = k√41

So, $\cos \alpha = \frac{BC}{AC} = \frac{5k}{k\sqrt{41}} = \frac{5}{\sqrt{41}}$

And $\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$

Side adjacent to angle α =AB = 4k

And Hypotenuse =AC = k√5

So, $\sin \alpha = \frac{AB}{AC} = \frac{4k}{k\sqrt{41}} = \frac{4}{\sqrt{41}}$

Now, LHS $= \frac{5\sin\alpha - 3\cos\alpha}{5\sin\alpha + 2\cos\alpha}$

$$= \frac{5\left(\frac{4}{\sqrt{41}}\right) - 3\left(\frac{5}{\sqrt{41}}\right)}{5\left(\frac{4}{\sqrt{41}}\right) + 2\left(\frac{5}{\sqrt{41}}\right)}$$

$$= \frac{20 - 15}{20 + 10}$$

$$= \frac{5}{30}$$

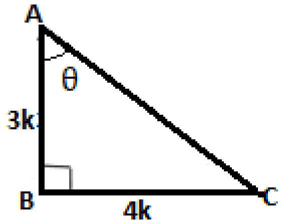
$= \frac{1}{6}$ = RHS

Hence Proved

33. Question

If $\cot\theta = \frac{3}{4}$, prove that $\sqrt{\frac{\sec\theta + \operatorname{cosec}\theta}{\sec\theta - \operatorname{cosec}\theta}} = \sqrt{7}$.

Answer



We know that,

$$\cot\theta = \frac{\text{side adjacent to angle }\theta}{\text{side opposite to angle }\theta}$$

Or $\cot\theta = \frac{\text{base}}{\text{perpendicular}}$

$$\text{Cot}\,\theta = \frac{3}{4} \Rightarrow \frac{B}{P} = \frac{3}{4} \Rightarrow \frac{AB}{BC} = \frac{3}{4}$$

Let,

Side adjacent to angle θ =AB = 3k

The side opposite to angle θ =BC = 4k

where k is any positive integer

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (3k)^2 + (4k)^2 = (AC)^2$

$\Rightarrow (AC)^2 = 9k^2 + 16k^2$

$\Rightarrow (AC)^2 = 25k^2$

$\Rightarrow AC = \sqrt{25k^2}$

$\Rightarrow AC = \pm 5k$

But side AC can’t be negative. So, AC = 5k

Now, we will find the sin θ

$$\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}}$$

Side opposite to angle θ = BC = 4k

and Hypotenuse = AC = 5k

So, $\text{Sin}\,\theta = \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5}$

Now, we know that,

$$\cos\theta = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}}$$

The side adjacent to angle θ = AB =3k

Hypotenuse = AC =5k

So, $\cos\theta = \frac{AB}{AC} = \frac{3k}{5k} = \frac{3}{5}$

$\therefore \sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{3}{5}} = \frac{5}{3}$

And

$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{4}{5}} = \frac{5}{4}$

Now, LHS $= \sqrt{\frac{\sec\theta+\operatorname{cosec}\theta}{\sec\theta-\operatorname{cosec}\theta}}$

$= \sqrt{\frac{\left(\frac{5}{3}\right)+\left(\frac{5}{4}\right)}{\left(\frac{5}{3}\right)-\left(\frac{5}{4}\right)}}$

$= \sqrt{\frac{\left(\frac{20+15}{12}\right)}{\left(\frac{20-15}{12}\right)}}$

$= \sqrt{\frac{35}{5}}$

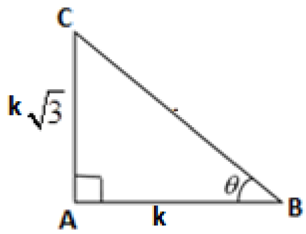
$= \sqrt{7}$ = RHS

Hence Proved

34. Question

If $\cot\theta = \frac{1}{\sqrt{3}}$, verify that: $\frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{3}{5}$.

Answer



We know that,

$$\cot\theta = \frac{\text{side adjacent to angle }\theta}{\text{side opposite to angle }\theta}$$

Or $\cot\theta = \frac{\text{base}}{\text{perpendicular}}$

$$\text{Cot}\,\theta = \frac{1}{\sqrt{3}} \Rightarrow \frac{B}{P} = \frac{1}{\sqrt{3}} \Rightarrow \frac{AB}{AC} = \frac{1}{\sqrt{3}}$$

Let,

Side adjacent to angle θ =AB = 1k

Side opposite to angle θ =AC = k√3

where, k is any positive integer

Firstly we have to find the value of BC.

So, we can find the value of AC with the help of Pythagoras theorem

$\Rightarrow (AB)^2 + (AC)^2 = (BC)^2$

$\Rightarrow (1k)^2 + (\sqrt{3}k)^2 = (BC)^2$

$\Rightarrow (BC)^2 = 1\,k^2 + 3\,k^2$

$\Rightarrow (BC)^2 = 4\,k^2$

$\Rightarrow BC = \sqrt{2}\,k^2$

$\Rightarrow BC = \pm 2k$

But side BC can't be negative. So, BC = 2k

Now, we will find the sin θ and cos θ

$$\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}}$$

Side opposite to angle θ = AC = k√3

and Hypotenuse = BC = 2k

So, $\text{Sin}\ \theta = \frac{AC}{BC} = \frac{k\sqrt{3}}{2k} = \frac{\sqrt{3}}{2}$

Now, we know that,

$$\cos\theta = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}}$$

Side adjacent to angle θ = AB =1k

Hypotenuse = BC =2k

So, $\cos\theta = \frac{AB}{BC} = \frac{1k}{2k} = \frac{1}{2}$

Now, LHS $= \frac{1-\cos^2\theta}{2-\sin^2\theta}$

$$= \frac{1-\left(\frac{1}{2}\right)^2}{2-\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{1-\frac{1}{4}}{2-\frac{3}{4}}$$

$$= \frac{\frac{4-1}{4}}{\frac{8-3}{4}}$$

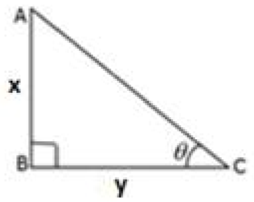
$= \frac{3}{5}$ = RHS

Hence Proved

35. Question

If $\tan\theta = \frac{x}{y}$, find the value of x sin θ + y cos θ.

Answer



We know that,

$$\tan\theta = \frac{\text{side opposite to angle }\theta}{\text{side adjacent to angle }\theta}$$

Or $\tan\theta = \frac{\text{perpendicular}}{\text{base}}$

$$\tan\theta = \frac{x}{y} \Rightarrow \frac{P}{B} = \frac{x}{y} \Rightarrow \frac{AB}{BC} = \frac{x}{y}$$

Let,

Side opposite to angle θ =AB = x

Side adjacent to angle θ =BC = y

where, k is any positive integer

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (x)^2 + (y)^2 = (AC)^2$

$\Rightarrow (AC)^2 = x^2+y^2$

$\Rightarrow AC = \sqrt{(x^2+y^2)}$

Now, we will find the sin θ and cos θ

We know that

$$\cos\theta = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}}$$

Side adjacent to angle θ = BC = y

and Hypotenuse = AC = $\sqrt{(x^2+y^2)}$

So, $\cos\theta = \frac{BC}{AC} = \frac{y}{\sqrt{x^2+y^2}}$

And $\sin\theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$

Side adjacent to angle θ =AB = x

And Hypotenuse =AC = $\sqrt{(x^2+y^2)}$

So, $\sin\theta = \frac{AB}{AC} = \frac{x}{\sqrt{x^2+y^2}}$

Now, x sin θ +y cos θ

$$= x\left(\frac{x}{\sqrt{x^2+y^2}}\right) + y\left(\frac{y}{\sqrt{x^2+y^2}}\right)$$

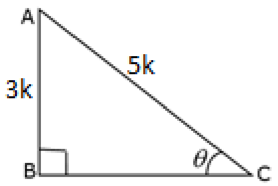
$$= \frac{x^2+y^2}{\sqrt{x^2+y^2}}$$

= $\sqrt{(x^2+y^2)}$

36. Question

If $\sin\theta = \frac{3}{5}$, find the value of $\tan^2\theta$ + sinθ cosθ + cotθ.

Answer



Given: $\text{Sin } \theta = \frac{3}{5}$

We know that,

$\sin\theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$

Or $\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

$\text{Sin } \theta = \frac{3}{5} \Rightarrow \frac{P}{H} = \frac{3}{5} \Rightarrow \frac{AB}{AC} = \frac{3}{5}$

Let,

Perpendicular =AB =3k

and Hypotenuse =AC =5k

where, k is any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

In right angled Δ ABC, we have

⇒ $(AB)^2 + (BC)^2 = (AC)^2$

⇒ $(3k)^2 + (BC)^2 = (5k)^2$

⇒ $9k^2 + (BC)^2 = 25k^2$

⇒ $(BC)^2 = 25\,k^2 - 9k^2$

⇒ $(BC)^2 = 16k^2$

⇒ $BC = \sqrt{16k^2}$

⇒ $BC = \pm 4k$

But side BC can't be negative. So, BC = 4k

Now, we have to find the value of cos θ and tan θ

We know that,

$$\cos\theta = \frac{\text{base}}{\text{hypotenuse}}$$

The side adjacent to angle θ or base = BC =4k

Hypotenuse = AC =5k

So, $\cos\theta = \frac{4k}{5k} = \frac{4}{5}$

Now,

We know that,

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}}$$

Perpendicular = AB =3k

Base = BC =4k

So, $\tan\theta = \frac{3k}{4k} = \frac{3}{4}$

$$\text{Cot}\,\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$$

Now, $\tan^2\theta + \sin\theta\cos\theta + \cot\theta$

$$= \left(\frac{3}{4}\right)^2 + \left(\frac{3}{5}\right)\left(\frac{4}{5}\right) + \left(\frac{4}{3}\right)$$

$$= \left(\frac{9}{16}\right) + \left(\frac{13}{25}\right) + \left(\frac{4}{3}\right)$$

$$= \frac{675 + 576 + 1600}{16 \times 25 \times 3}$$

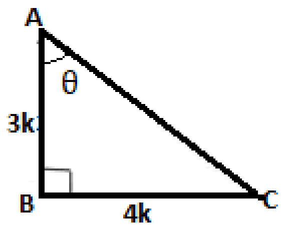
$$= \frac{2851}{1200}$$

37. Question

If 4cot θ = 3, show that $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = 7$.

Answer

Given: cot θ= $\frac{3}{4}$



We know that,

$$\cot\theta = \frac{\text{side adjacent to angle } \theta}{\text{side opposite to angle } \theta}$$

$$\text{Or } \cot\theta = \frac{\text{base}}{\text{perpendicular}}$$

$$\text{Cot}\,\theta = \frac{3}{4} \Rightarrow \frac{B}{P} = \frac{3}{4} \Rightarrow \frac{AB}{BC} = \frac{3}{4}$$

Let,

Side adjacent to angle θ =AB = 3k

The side opposite to angle θ =BC = 4k

where k is any positive integer

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (3k)^2 + (4k)^2 = (AC)^2$

$\Rightarrow (AC)^2 = 9k^2 + 16k^2$

$\Rightarrow (AC)^2 = 25k^2$

$\Rightarrow AC = \sqrt{25k^2}$

$\Rightarrow AC = \pm 5k$

But side AC can't be negative. So, AC = 5k

Now, we will find the sin θ

$$\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}}$$

Side opposite to angle θ = BC = 4k

and Hypotenuse = AC = 5k

So, $\text{Sin}\,\theta = \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5}$

Now, we know that,

$$\cos\theta = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}}$$

Side adjacent to angle θ = AB =3k

Hypotenuse = AC =5k

So, $\cos\theta = \frac{AB}{AC} = \frac{3k}{5k} = \frac{3}{5}$

Now, LHS $= \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$

$$= \frac{\frac{4}{5} + \frac{3}{5}}{\frac{4}{5} - \frac{3}{5}}$$

$$= \frac{7}{1}$$

= 7 = RHS

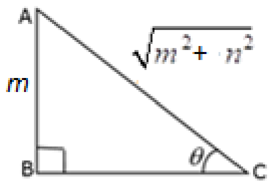
Hence Proved

38. Question

If $\sin\theta = \frac{m}{\sqrt{m^2+n^2}}$, prove that $m\sin\theta + n\cos\theta = \sqrt{m^2+n^2}$

Answer

Given: Sin θ = $\frac{m}{\sqrt{m^2+n^2}}$



We know that,

$$\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}}$$

Or $\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

$$\text{Sin }\theta = \frac{m}{\sqrt{m^2+n^2}} \Rightarrow \frac{P}{H} = \frac{m}{\sqrt{m^2+n^2}} \Rightarrow \frac{AB}{AC} = \frac{m}{\sqrt{m^2+n^2}}$$

Let,

Perpendicular =AB =m

and Hypotenuse =AC =$\sqrt{(m^2 + n^2)}$

where, k is any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

In right angled Δ ABC, we have

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (m)^2 + (BC)^2 = (\sqrt{(m^2 + n^2)})^2$

$\Rightarrow m^2 + (BC)^2 = m^2 + n^2$

$\Rightarrow (BC)^2 = m^2 + n^2 - m^2$

$\Rightarrow (BC)^2 = n^2$

$\Rightarrow BC = \sqrt{n^2}$

$\Rightarrow BC = \pm n$

But side BC can't be negative. So, BC = n

Now, we have to find the value of cos θ and tan θ

We know that,

$$\cos\theta = \frac{\text{base}}{\text{hypotenuse}}$$

Side adjacent to angle θ or base = BC = n

Hypotenuse = AC = $\sqrt{(m^2 + n^2)}$

So, $\cos\theta = \frac{n}{\sqrt{m^2+n^2}}$

Now, LHS = m sin θ + n cosθ

$$= m\left(\frac{m}{\sqrt{m^2+n^2}}\right) + n\left(\frac{n}{\sqrt{m^2+n^2}}\right)$$

$$= \frac{m^2+n^2}{\sqrt{m^2+n^2}}$$

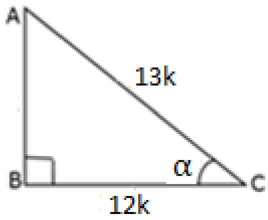
$= \sqrt{(m^2 + n^2)}$ = RHS

Hence Proved

39. Question

If $\cos\alpha = \frac{12}{13}$, show that $\sin\alpha(1 - \tan\alpha) = \frac{35}{156}$

Answer



We know that,

$$\cos\theta = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}}$$

Or $\cos\theta = \frac{\text{base}}{\text{Hypotenuse}}$

$$\text{Cos }\alpha = \frac{12}{13} \Rightarrow \frac{B}{H} = \frac{12}{13} \Rightarrow \frac{BC}{AC} = \frac{12}{13}$$

Let,

Base =BC = 12k

Hypotenuse =AC = 13k

Where, k ia any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (AB)^2 + (12k)^2 = (13k)^2$

$\Rightarrow (AB)^2 + 144k^2 = 169k^2$

$\Rightarrow (AB)^2 = 169\ k^2 - 144\ k^2$

$\Rightarrow (AB)^2 = 25\ k^2$

$\Rightarrow AB = \sqrt{25\ k^2}$

$\Rightarrow AB = \pm 5k$

But side AB can't be negative. So, AB = 5k

Now, we have to find sin α and tan α

We know that,

$$\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}}$$

Side opposite to angle α = AB =5k

And Hypotenuse = AC =13k

So, $\text{Sin}\ \alpha = \frac{5k}{13k} = \frac{5}{13}$

We know that,

$$\tan\theta = \frac{\text{side opposite to angle }\theta}{\text{side adjacent to angle }\theta}$$

Side opposite to angle α = AB =5k

Side adjacent to angle α = BC =12k

So, $\tan\alpha = \frac{5k}{12k} = \frac{5}{12}$

Now, LHS = sin α (1 – tan α)

$$= \frac{5}{13}\left(1 - \frac{5}{12}\right)$$

$$= \frac{5}{13}\left(\frac{12-5}{12}\right)$$

$= \frac{35}{156}$ = RHS

Hence Proved

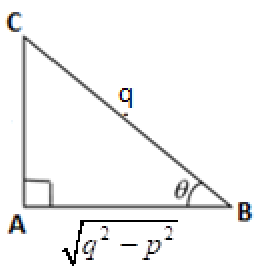
40. Question

If $q\cos\theta = \sqrt{q^2 - p^2}$, prove that q sin θ = p.

Answer

Given : q cos θ = $\sqrt{(q^2 - p^2)}$

$$\Rightarrow \cos\theta = \frac{\sqrt{q^2 - p^2}}{q}$$



We know that,

$$\cos\theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Or $\cos\theta = \frac{\text{base}}{\text{Hypotenuse}}$

$$\text{Cos } \theta = \frac{\sqrt{q^2 - p^2}}{q} \Rightarrow \frac{B}{H} = \frac{\sqrt{q^2 - p^2}}{q} \Rightarrow \frac{BC}{AC} = \frac{\sqrt{q^2 - p^2}}{q}$$

Let,

Base =BC = $\sqrt{(q^2 - p^2)}$

Hypotenuse =AC = q

Where, k ia any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (AB)^2 + (\sqrt{(q^2 - p^2)})^2 = (q)^2$

$\Rightarrow (AB)^2 + (q^2 - p^2) = q^2$

$\Rightarrow (AB)^2 = q^2 - q^2 + p^2)$

$\Rightarrow (AB)^2 = p^2$

$\Rightarrow AB = \sqrt{p^2}$

$\Rightarrow AB = \pm p$

But side AB can’t be negative. So, AB = p

Now, we have to find sin θ

We know that,

$$\sin\theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

The side opposite to angle θ = AB =p

And Hypotenuse = AC =q

So, Sin θ $= \left(\frac{p}{q}\right)$

Now, LHS = q sin θ

$= q\left(\frac{p}{q}\right)$

= q = RHS

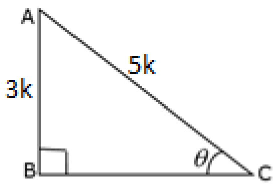
Hence Proved

41. Question

If $\sin\theta = \frac{3}{5}$, show that : $\frac{\cos\theta - \frac{1}{\tan\theta}}{2\cot\theta} = -\frac{1}{5}$

Answer

Given: Sin θ $= \frac{3}{5}$



We know that,

$$\sin\theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Or $\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

$$\text{Sin } \theta = \frac{3}{5} \Rightarrow \frac{P}{H} = \frac{3}{5} \Rightarrow \frac{AB}{AC} = \frac{3}{5}$$

Let,

Perpendicular =AB =3k

and Hypotenuse =AC =5k

where, k is any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

In right angled Δ ABC, we have

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (3k)^2 + (BC)^2 = (5k)^2$

$\Rightarrow 9k^2 + (BC)^2 = 25k^2$

$\Rightarrow (BC)^2 = 25\ k^2 - 9k^2$

$\Rightarrow (BC)^2 = 16k^2$

$\Rightarrow BC = \sqrt{16k^2}$

$\Rightarrow BC = \pm 4k$

But side BC can't be negative. So, BC = 4k

Now, we have to find the value of cos θ and tan θ

We know that,

$$\cos\theta = \frac{\text{base}}{\text{hypotenuse}}$$

The side adjacent to angle θ or base = BC =4k

Hypotenuse = AC =5k

So, $\cos\theta = \frac{4k}{5k} = \frac{4}{5}$

Now,

We know that,

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}}$$

Perpendicular = AB =3k

Base = BC =4k

So, $\tan\theta = \frac{3k}{4k} = \frac{3}{4}$

$$\text{Cot}\,\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$$

Now, LHS $= \frac{\cos\theta - \frac{1}{\tan\theta}}{2\cot\theta}$

$$= \frac{\left(\frac{4}{5}\right) - \left(\frac{1}{\frac{3}{4}}\right)}{2\left(\frac{4}{3}\right)}$$

$$= \frac{\left(\frac{4}{5}\right) - \left(\frac{4}{3}\right)}{\left(\frac{8}{3}\right)}$$

$$= \frac{\frac{12 - 20}{15}}{\left(\frac{8}{3}\right)}$$

$$= \frac{\left(-\frac{8}{15}\right)}{\left(\frac{8}{3}\right)}$$

$= -\frac{1}{5}$ = RHS

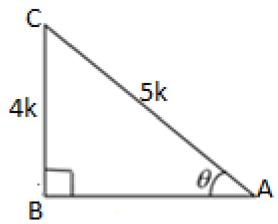
Hence Proved

42 A. Question

Find the value of

cos A sin B + sin A. cos B, if sin A= 4/5 and cos B = 12/13.

Answer



Given: $\sin A = \frac{4}{5}$ and $\cos B = \frac{12}{13}$

To find: cos A sin B + sin A cos B

As, we have the value of sin A and cos B but we don't have the value of cos A and sin B

So, First we find the value of cos A and sin B

$$\text{Sin A} = \frac{4}{5}$$

We know that,

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Or $\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

$\mathbf{Sin\ A} = \frac{\mathbf{4}}{\mathbf{5}} \Rightarrow \frac{P}{H} = \frac{4}{5}$

Let,

Side opposite to angle A = 4k

and Hypotenuse = 5k

where, k is any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

$\Rightarrow (P)^2 + (B)^2 = (H)^2$

$\Rightarrow (4k)^2 + (B)^2 = (5)^2$

$\Rightarrow 16\ k^2 + (B)^2 = 25\ k^2$

$\Rightarrow (B)^2 = 25\ k^2 - 16\ k^2$

$\Rightarrow (B)^2 = 9\ k^2$

$\Rightarrow B = \sqrt{9}\ k^2$

$\Rightarrow B = \pm 3k$ [taking positive square root since, side cannot be negative]

So, Base = 3k

Now, we have to find the value of cos A

We know that,

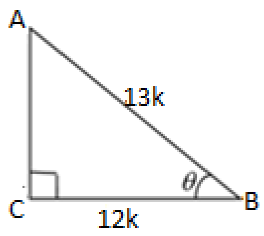
$\cos\theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$

Side adjacent to angle A =3k

Hypotenuse =5k

So, $\mathbf{cos\ A} = \frac{3k}{5k} = \frac{\mathbf{3}}{\mathbf{5}}$

Now, we have to find the sin B



We know that,

$$\cos\theta = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}}$$

$$\mathbf{Cos\ B} = \frac{\mathbf{12}}{\mathbf{13}} \Rightarrow \frac{\mathrm{B}}{\mathrm{H}} = \frac{12}{13}$$

Let,

Side adjacent to angle B =12k

Hypotenuse =13k

where, k is any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

$\Rightarrow (B)^2 + (P)^2 = (H)^2$

$\Rightarrow (12k)^2 + (P)^2 = (13)^2$

$\Rightarrow 144\ k^2 + (P)^2 = 169\ k^2$

$\Rightarrow (P)^2 = 169\ k^2 - 144\ k^2$

$\Rightarrow (P)^2 = 25\ k^2$

$\Rightarrow P = \sqrt{25}\ k^2$

$\Rightarrow P = \pm 5k$ [taking positive square root since, side cannot be negative]

So, Perpendicular = 5k

Now, we have to find the value of sin B

We know that,

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\mathbf{Sin\ B} = \frac{\mathrm{P}}{\mathrm{H}} = \frac{5\mathrm{k}}{13\mathrm{k}} = \frac{\mathbf{5}}{\mathbf{13}}$$

Now, cos A sin B + sin A cos B

Putting the values of sin A, sin B cos A and Cos B, we get

$$\Rightarrow \left(\frac{3}{5}\right)\left(\frac{5}{13}\right)+\left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$$

$$\Rightarrow \frac{15+48}{5\times 13}$$

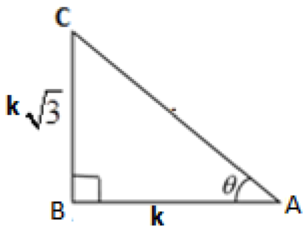
$$\Rightarrow \frac{63}{65}$$

42 B. Question

Find the value of

sin A. cos B – cos A. sin B, if tan A= √3 and sin B = 1/2.

Answer



Given: tan A =√3 and $\sin B = \frac{1}{2}$

We know that,

$$\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}}$$

Or $\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

$$\mathbf{Sin\ B} = \frac{\mathbf{1}}{\mathbf{2}} \Rightarrow \frac{P}{H} = \frac{1}{2}$$

Let,

Side opposite to angle θ = 1k

and Hypotenuse = 2k

where, k is any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

$\Rightarrow (AC)^2 + (BC)^2 = (AB)^2$

$\Rightarrow (1k)^2 + (BC)^2 = (2k)^2$

$\Rightarrow k^2 + (BC)^2 = 4k^2$

$\Rightarrow (BC)^2 = 4k^2 - k^2$

$\Rightarrow (BC)^2 = 3\,k^2$

$\Rightarrow BC = \sqrt{3}k^2$

$\Rightarrow BC = k\sqrt{3}$

So, BC = $k\sqrt{3}$

Now, we have to find the value of cos B

We know that,

$$\cos\theta = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}}$$

The side adjacent to angle B = BC = $k\sqrt{3}$

Hypotenuse = AB = 2k

So, $\mathbf{\cos B} = \frac{k\sqrt{3}}{2k} = \frac{\sqrt{3}}{2}$

We know that,

$$\tan\theta = \frac{\text{side opposite to angle }\theta}{\text{side adjacent to angle }\theta}$$

Or $\tan\theta = \frac{\text{perpendicular}}{\text{base}}$

Given: tan A = $\sqrt{3}$

$$\Rightarrow \tan A = \frac{\sqrt{3}}{1}$$

$$\tan A = \frac{\sqrt{3}}{1} \Rightarrow \frac{P}{B} = \frac{\sqrt{3}}{1} \Rightarrow \frac{BC}{AB} = \frac{\sqrt{3}}{1}$$

Let,

The side opposite to angle A = BC = $\sqrt{3}k$

The side adjacent to angle A = AB = 1k

where k is any positive integer

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (1k)^2 + (\sqrt{3}k)^2 = (AC)^2$

$\Rightarrow (AC)^2 = 1\ k^2 + 3\ k^2$

$\Rightarrow (AC)^2 = 4\ k^2$

$\Rightarrow AC = \sqrt{2\ k^2}$

$\Rightarrow AC = \pm 2k$

But side AC can't be negative. So, AC = 2k

Now, we will find the sin A and cos A

$$\sin\theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Side opposite to angle A = BC = $k\sqrt{3}$

and Hypotenuse = AC = 2k

So, $\mathbf{Sin\ A} = \frac{BC}{AC} = \frac{k\sqrt{3}}{2k} = \frac{\sqrt{3}}{2}$

Now, we know that,

$$\cos\theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

The side adjacent to angle A = AB =1k

Hypotenuse = AC =2k

So, $\cos A = \frac{AB}{AC} = \frac{1k}{2k} = \frac{1}{2}$

Now, sin A. cos B – cos A. sin B

Putting the values of sin A, sin B cos A and Cos B, we get

$$\Rightarrow \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$\Rightarrow \frac{3}{4} - \frac{1}{4}$

$\Rightarrow \frac{2}{4}$

$= \frac{1}{2}$

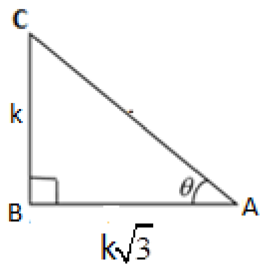
42 C. Question

Find the value of

sin A. cos B + cos A. sin B. if $\tan A = \frac{1}{\sqrt{3}}$ and tan B = √3.

Answer

Given:



$\tan A = \frac{1}{\sqrt{3}}$

$\tan A = \frac{1}{\sqrt{3}} \Rightarrow \frac{P}{B} = \frac{1}{\sqrt{3}}$

Let,

Side opposite to angle A =BC = 1k

Side adjacent to angle A =AB = k√3

where, k is any positive integer

Firstly we have to find the value of BC.

So, we can find the value of AC with the help of Pythagoras theorem

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (\sqrt{3}k)^2 + (1k)^2 = (AC)^2$

⇒ $(AC)^2 = 1\ k^2 + 3\ k^2$

⇒ $(AC)^2 = 4\ k^2$

⇒ $AC = \sqrt{2}\ k^2$

⇒ $AC = \pm 2k$

But side AC can't be negative. So, AC = 2k

Now, we will find the sin A and cos A

$$\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}}$$

Side opposite to angle A = BC = k

and Hypotenuse = AC = 2k

So, $\mathbf{Sin\ A} = \frac{BC}{AC} = \frac{1k}{2k} = \frac{1}{2}$

Now, we know that,

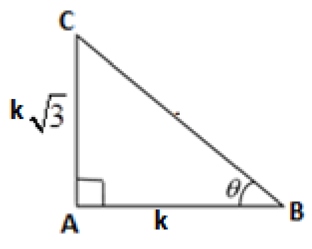
$$\cos\theta = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}}$$

Side adjacent to angle A = AB $= k\sqrt{3}$

Hypotenuse = AC =2k

So, $\mathbf{cos\ A} = \frac{AB}{BC} = \frac{k\sqrt{3}}{2k} = \frac{\sqrt{3}}{2}$

Now,



Given: tan B = $\sqrt{3}$

$$\Rightarrow \tan B = \frac{\sqrt{3}}{1}$$

$$\text{Tan B} = \frac{\sqrt{3}}{1} \Rightarrow \frac{P}{B} = \frac{\sqrt{3}}{1} \Rightarrow \frac{AC}{AB} = \frac{\sqrt{3}}{1}$$

Let,

Side opposite to angle B =AC = $\sqrt{3}$k

Side adjacent to angle B =AB = 1k

where, k is any positive integer

Firstly we have to find the value of BC.

So, we can find the value of AC with the help of Pythagoras theorem

$\Rightarrow (AB)^2 + (AC)^2 = (BC)^2$

$\Rightarrow (1k)^2 + (\sqrt{3}k)^2 = (BC)^2$

$\Rightarrow (BC)^2 = 1\,k^2 + 3\,k^2$

$\Rightarrow (BC)^2 = 4\,k^2$

$\Rightarrow BC = \sqrt{2}\,k^2$

$\Rightarrow BC = \pm 2k$

But side BC can't be negative. So, BC = 2k

Now, we will find the sin B and cos B

$$\sin\theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Side opposite to angle B = AC = k$\sqrt{3}$

and Hypotenuse = BC = 2k

So, $\mathbf{Sin\ B} = \frac{AC}{BC} = \frac{k\sqrt{3}}{2k} = \frac{\sqrt{3}}{2}$

Now, we know that,

$$\cos\theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Side adjacent to angle B = AB =1k

Hypotenuse = BC =2k

So, $\mathbf{cos\ B} = \frac{AB}{BC} = \frac{1k}{2k} = \frac{1}{2}$

Now, sin A. cos B + cos A. sin B

Putting the values of sin A, sin B cos A and Cos B, we get

$$\Rightarrow \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \frac{1}{4} + \frac{3}{4}$$

$$\Rightarrow \frac{4}{4}$$

=1

42 D. Question

Find the value of

$$\frac{\tan A + \tan B}{1 - \tan A . \tan B}, \text{ if } \sin A = \frac{1}{\sqrt{2}} \text{ and } \cos B = \frac{\sqrt{3}}{2}$$

Answer

Given: $\sin A = \frac{1}{\sqrt{2}}$ and $\cos B = \frac{\sqrt{3}}{2}$

$$\text{Sin A } = \frac{1}{\sqrt{2}}$$

We know that,

$$\sin\theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Or $\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

$$\textbf{Sin A} = \frac{1}{\sqrt{2}} \Rightarrow \frac{P}{H} = \frac{1}{\sqrt{2}}$$

Let,

Side opposite to angle A = k

and Hypotenuse = k√2

where, k is any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

$\Rightarrow (P)^2 + (B)^2 = (H)^2$

$\Rightarrow (k)^2 + (B)^2 = (k\sqrt{2})^2$

$\Rightarrow k^2 + (B)^2 = 2k^2$

$\Rightarrow (B)^2 = 2k^2 - k^2$

$\Rightarrow (B)^2 = k^2$

$\Rightarrow B = \sqrt{k^2}$

$\Rightarrow B = \pm k$ [taking positive square root since, side cannot be negative]

So, Base = k

Now, we have to find the value of tan A

We know that,

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}}$$

So, $\mathbf{\tan A} = \frac{k}{k} = \mathbf{1}$

Now, we have to find the tan B

We know that,

$$\cos\theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

$$\mathbf{Cos\ B} = \frac{\sqrt{3}}{2} \Rightarrow \frac{B}{H} = \frac{\sqrt{3}}{2}$$

Let,

Side adjacent to angle B $= k\sqrt{3}$

Hypotenuse $= 2k$

where, k is any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

$\Rightarrow (B)^2 + (P)^2 = (H)^2$

$\Rightarrow (k\sqrt{3})^2 + (P)^2 = (2k)^2$

$\Rightarrow 3k^2 + (P)^2 = 4k^2$

$\Rightarrow (P)^2 = 4k^2 - 3k^2$

$\Rightarrow (P)^2 = k^2$

⇒ P =$\sqrt{k^2}$

⇒ P =±k [taking positive square root since, side cannot be negative]

So, Perpendicular = k

Now, we have to find the value of sin B

We know that,

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}}$$

So, **tan B** $= \frac{k}{k\sqrt{3}} = \frac{1}{\sqrt{3}}$

Now, $\frac{\tan A + \tan B}{1 - \tan A \tan B}$

$$\Rightarrow \frac{(1) + \left(\frac{1}{\sqrt{3}}\right)}{1 - (1)\left(\frac{1}{\sqrt{3}}\right)}$$

$$\Rightarrow \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}}$$

$$\Rightarrow \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

Now, multiply and divide by the conjugate of √3 – 1, we get

$$\Rightarrow \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$\Rightarrow \frac{(\sqrt{3}+1)^2}{(\sqrt{3})^2 - (1)^2}$ [∵ (a – b)(a+b) = (a^2 – b^2)]

$$\Rightarrow \frac{3 + 1 + 2\sqrt{3}}{3 - 1}$$

$$\Rightarrow \frac{4 + 2\sqrt{3}}{2}$$

⇒ 2+√3

42 E. Question

Find the value of

sec A. tan A+$\tan^2$A – cosec A, if tan A =2

Answer

Given: tan A = 2 ⇒ $\tan^2$A = 4

We know that, $\sec^2$ A = 1+ $\tan^2$A

⇒ $\sec^2$ A = 1 + 4

⇒ $\sec^2$ A = 5

⇒ sec A =√5

$$\Rightarrow \cos A = \frac{1}{\sqrt{5}}$$

Now, we know that tan A= $\frac{\sin A}{\cos A}$

$$\Rightarrow 2 = \frac{\sin A}{\frac{1}{\sqrt{5}}}$$

⇒ 2 =√5 sin A

$$\Rightarrow \sin A = \frac{2}{\sqrt{5}}$$

$$\Rightarrow \operatorname{cosec} A = \frac{\sqrt{5}}{2}$$

Now, putting all the values in the given equation, we get

sec A. tan A+$\tan^2$A – cosec A

$$\Rightarrow (\sqrt{5})(2) + (4) - \left(\frac{\sqrt{5}}{2}\right)$$

$$\Rightarrow \frac{4\sqrt{5} + 8 - \sqrt{5}}{2}$$

$$\Rightarrow \frac{3\sqrt{5} + 8}{2}$$

42 F. Question

Find the value of

$\frac{1}{\tan A} + \frac{\sin A}{1+\cos A}$, if cosec A = 2

Answer

Given: cosec A =2

Now, we have to find $\frac{1}{\tan A} + \frac{\sin A}{1+\cos A}$

First, we simplify the above given trigonometry equation, we get

$$\frac{1}{\frac{\sin A}{\cos A}} + \frac{\sin A}{1 + \cos A}$$

$$\Rightarrow \frac{\cos A}{\sin A} + \frac{\sin A}{1 + \cos A}$$

Taking the LCM, we get

$$\Rightarrow \frac{\cos A(1 + \cos A) + \sin A(\sin A)}{(\sin A)(1 + \cos A)}$$

$\Rightarrow \frac{\cos A+\cos^2 A+\sin^2 A}{\sin A(1+\cos A)}$ [∵ $\cos^2\theta + \sin^2 \theta = 1$]

$$\Rightarrow \frac{\cos A + 1}{\sin A(1 + \cos A)}$$

$\Rightarrow \frac{1}{\sin A}$ [∵ $\text{cosec } \theta = \frac{1}{\sin \theta}$]

⇒ cosec A

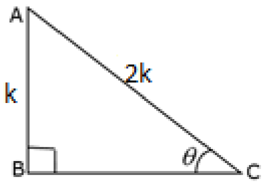
⇒ 2

43 A. Question

If $\sin B = \frac{1}{2}$, prove that : 3 cos B – $4\cos^3$ B = 0

Answer

Given: $\text{Sin B} = \frac{1}{2}$



We know that,

$$\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}}$$

Or $\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

$$\text{Sin B} = \frac{1}{2} \Rightarrow \frac{P}{H} = \frac{1}{2} \Rightarrow \frac{AB}{AC} = \frac{1}{2}$$

Let,

Perpendicular =AB =k

and Hypotenuse =AC =2k

where, k is any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

In right angled Δ ABC, we have

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (k)^2 + (BC)^2 = (2k)^2$

$\Rightarrow k^2 + (BC)^2 = 4k^2$

$\Rightarrow (BC)^2 = 4k^2 - k^2$

$\Rightarrow (BC)^2 = 3k^2$

$\Rightarrow BC = \sqrt{3}k^2$

$\Rightarrow BC = k\sqrt{3}$

So, BC = $k\sqrt{3}$

Now, we have to find the value of cos B

We know that,

$$\cos\theta = \frac{\text{base}}{\text{hypotenuse}}$$

Side adjacent to angle B or base = BC = $k\sqrt{3}$

Hypotenuse = AC =2k

So, $\cos B = \frac{k\sqrt{3}}{2k} = \frac{\sqrt{3}}{2}$

Now, LHS = $3 \cos B - 4\cos^3 B$

$$\Rightarrow 3\left(\frac{\sqrt{3}}{2}\right) - 4\left(\frac{\sqrt{3}}{2}\right)^3$$

$$\Rightarrow \frac{3\sqrt{3}}{2} - 4\left(\frac{3\sqrt{3}}{8}\right)$$

$$\Rightarrow \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = 0$$

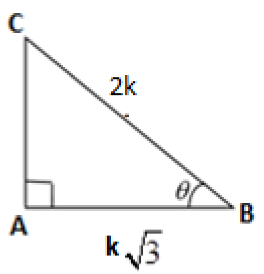
=RHS

Hence Proved

43 B. Question

If $\cos\theta = \frac{\sqrt{3}}{2}$, prove that: $3\sin\theta - 4\sin^3\theta = 1$.

Answer



We know that,

$$\cos\theta = \frac{\text{Base}}{\text{hypotenuse}}$$

$$\text{Cos } \theta = \frac{\sqrt{3}}{2} \Rightarrow \frac{B}{H} = \frac{\sqrt{3}}{2}$$

Let AB $=k\sqrt{3}$ and BC = 2k

In right angled ΔABC, we have

$B^2 + P^2 = H^2$

$\Rightarrow (k\sqrt{3})^2 + P^2 = (2k)^2$

$\Rightarrow P^2 + 3k^2 = 4k^2$

$\Rightarrow P^2 = 4k^2 - 3k^2$

$\Rightarrow P^2 = k^2$

$\Rightarrow P = \sqrt{k^2}$

$\Rightarrow P = \pm k$

$\Rightarrow$ P = k [taking positive square root since, side cannot be negative]

Now,

We know that,

$$\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}}$$

Or $\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

$$\therefore \sin\theta = \frac{P}{H} = \frac{k}{2k} = \frac{1}{2}$$

Now, LHS = $3\sin\theta - 4\sin^3\theta$

$$= 3\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)^3$$

$$\Rightarrow \frac{3}{2} - \frac{4}{8}$$

$$\Rightarrow \frac{3}{2} - \frac{1}{2}$$

$$\Rightarrow \frac{3-1}{2}$$

$$\Rightarrow \frac{2}{2}$$

$\Rightarrow$ 1 = RHS

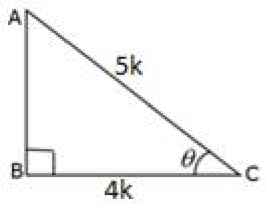
Hence Proved

43 C. Question

If $\sec\theta = \frac{5}{4}$, prove that : $\frac{\tan\theta}{1+\tan^2\theta} = \frac{\sin\theta}{\sec\theta}$

Answer

Given: $\sec\theta = \frac{5}{4}$



We know that,

$$\sec\theta = \frac{\text{hypotenuse}}{\text{base}}$$

$$\text{Sec}\,\theta = \frac{5}{4} \Rightarrow \frac{H}{B} = \frac{5}{4} \Rightarrow \frac{AC}{BC} = \frac{5}{4}$$

Let,

BC = 4k and AC = 5k

where, k is any positive integer.

In right angled ΔABC, we have

$(AB)^2 + (BC)^2 = (AC)^2$ [by using Pythagoras theorem]

$\Rightarrow (AB)^2 + (4k)^2 = (5k)^2$

$\Rightarrow (AB)^2 + 16k^2 = 25k^2$

$\Rightarrow (AB)^2 = 25k^2 – 16k^2$

$\Rightarrow (AB)^2 = 9k^2$

$\Rightarrow AB = \sqrt{9k^2}$

$\Rightarrow AB = \pm 3k$ [taking positive square root since, side cannot be negative]

Now, we have to find the value of other trigonometric ratios.

We, know that

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$= \frac{AB}{AC} = \frac{3k}{5k} = \frac{3}{5}$$

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$= \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5}$$

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$= \frac{AB}{BC} = \frac{3k}{4k} = \frac{3}{4}$$

Now, LHS $= \frac{\tan\theta}{1+\tan^2\theta}$

$$= \frac{\left(\frac{3}{4}\right)}{1 + \left(\frac{3}{4}\right)^2}$$

$$= \frac{\left(\frac{3}{4}\right)}{1 + \left(\frac{9}{16}\right)}$$

$$= \frac{\left(\frac{3}{4}\right)}{\left(\frac{16+9}{16}\right)}$$

$$= \frac{\left(\frac{3}{4}\right)}{\left(\frac{25}{16}\right)}$$

$$= \frac{3}{4} \times \frac{16}{25}$$

$$= \frac{12}{25}$$

Now, RHS $= \frac{\sin\theta}{\sec\theta}$

$$= \frac{\left(\frac{3}{5}\right)}{\left(\frac{5}{4}\right)}$$

$= \frac{3}{5} \times \frac{4}{5}$

$= \frac{12}{25}$

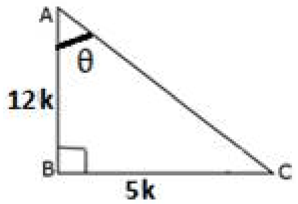
∴ LHS = RHS

Hence Proved

43 D. Question

$\cot B = \frac{12}{5}$, prove that : $\tan^2 B - \sin^2 B = \sin^4 B \sec^2 B$.

Answer



We know that,

$\cot\theta = \frac{\text{side adjacent to angle }\theta}{\text{side opposite to angle }\theta}$

Or $\cot\theta = \frac{\text{base}}{\text{perpendicular}}$

$\cot B = \frac{12}{5} \Rightarrow \frac{B}{P} = \frac{12}{5} \Rightarrow \frac{AB}{BC} = \frac{12}{5}$

Let,

Side adjacent to angle B =AB = 12k

Side opposite to angle B =BC = 5k

where, k is any positive integer

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (12k)^2 + (5k)^2 = (AC)^2$

$\Rightarrow (AC)^2 = 144\ k^2 + 25\ k^2$

$\Rightarrow (AC)^2 = 169\ k^2$

$\Rightarrow AC = \sqrt{169\ k^2}$

$\Rightarrow AC = \pm 13k$

But side AC can't be negative. So, AC = 13k

Now, we will find the sin θ

$$\sin\theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Side opposite to angle B = BC = 5k

and Hypotenuse = AC = 13k

So, $\text{Sin B} = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$

Now, we know that,

$$\cos\theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Side adjacent to angle B = AB =12k

Hypotenuse = AC =13k

So, $\cos B = \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13}$

$$\tan B = \frac{P}{B} = \frac{BC}{AB} = \frac{5}{12}$$

$$\sec B = \frac{1}{\cos B} = \frac{1}{\frac{12}{13}} = \frac{13}{12}$$

Now, LHS = $\tan^2 B - \sin^2 B$

$$= \left(\frac{5}{12}\right)^2 - \left(\frac{5}{13}\right)^2$$

$$= \frac{25}{144} - \frac{25}{169}$$

$$= \frac{4225 - 3600}{144 \times 169}$$

$$= \frac{625}{144 \times 169}$$

$$= \frac{625}{24336}$$

Now, RHS = $\sin^4 B \sec^2 B$

$$= \left(\frac{5}{13}\right)^4 \left(\frac{13}{12}\right)^2$$

$$= \left(\frac{5}{13}\right)^2 \left(\frac{5}{13}\right)^2 \left(\frac{13}{12}\right)^2$$

$$= \frac{625}{144 \times 169}$$

$$= \frac{625}{24336}$$

Now, LHS = RHS

Hence Proved

44. Question

If $\cos\theta = \frac{q}{\sqrt{p^2+q^2}}$, prove that $\left(\frac{\sqrt{p^2+q^2}}{p} + \frac{q}{p}\right)^2 = \frac{\sqrt{p^2+q^2}+q}{\sqrt{p^2+q^2}-q}$

Answer

Given: $\cos\theta = \frac{q}{\sqrt{p^2+q^2}}$

Now, squaring both the sides, we get

$$= \cos^2\theta = \frac{q^2}{p^2+q^2}$$

$$\Rightarrow\ p^2 + q^2 = \frac{q^2}{\cos^2\theta}$$

$$\Rightarrow\ p^2 = \frac{q^2}{\cos^2\theta} - q^2$$

$$\Rightarrow\ p^2 = \frac{q^2 - q^2\cos^2\theta}{\cos^2\theta}$$

$$\Rightarrow\ p^2 = \frac{q^2(1-\cos^2\theta)}{\cos^2\theta}$$

$$\Rightarrow\ p^2 = \frac{q^2 \sin^2\theta}{\cos^2\theta}$$

$\Rightarrow p^2 = q^2 \tan^2\theta$ ...(1)

Now, solving LHS $= \left(\frac{\sqrt{p^2+q^2}}{p} + \frac{q}{p}\right)^2$

Putting the value of p^2 in the above equation, we get

$$= \left(\frac{\sqrt{q^2\tan^2\theta + q^2}}{p} + \frac{q}{p}\right)^2$$

$$\Rightarrow \left(\frac{\sqrt{q^2(\tan^2\theta + 1)}}{p} + \frac{q}{p}\right)^2$$

$\Rightarrow \left(\frac{\sqrt{q^2\sec^2\theta}}{p} + \frac{q}{p}\right)^2$ [$\because$ 1+ $\tan^2\theta = \sec^2\theta$]

$$\Rightarrow \left(\frac{q\sec\theta}{p} + \frac{q}{p}\right)^2$$

$$\Rightarrow \frac{q^2(\sec\theta + 1)^2}{p^2}$$

$\Rightarrow \frac{q^2(\sec\theta+1)^2}{q^2\tan^2\theta}$ (from Eq. (1))

$$\Rightarrow \frac{(\sec\theta + 1)^2}{(\sec^2\theta - 1)}$$

$\Rightarrow \frac{(\sec\theta+1)(\sec\theta+1)}{(\sec\theta-1)(\sec\theta+1)}$ [$\because (a + b)(a - b) = (a^2 - b^2)$]

$$\Rightarrow \frac{\sec\theta + 1}{\sec\theta - 1}$$

Now, we solve the RHS

$$= \frac{\sqrt{p^2 + q^2} + q}{\sqrt{p^2 + q^2} - q}$$

$$= \frac{\sqrt{q^2\tan^2\theta + q^2} + q}{\sqrt{q^2\tan^2\theta + q^2} - q}$$

$$= \frac{\sqrt{q^2(\tan^2\theta + 1)} + q}{\sqrt{q^2(\tan^2\theta + 1)} - q}$$

$= \frac{\sqrt{q^2 \sec^2} + q}{\sqrt{q^2 \sec^2\theta} - q}$ [∵ 1+ $\tan^2\theta = \sec^2\theta$]

$$= \frac{q\sec\theta + q}{q\sec\theta - q}$$

$$\Rightarrow \frac{\sec\theta + 1}{\sec\theta - 1}$$

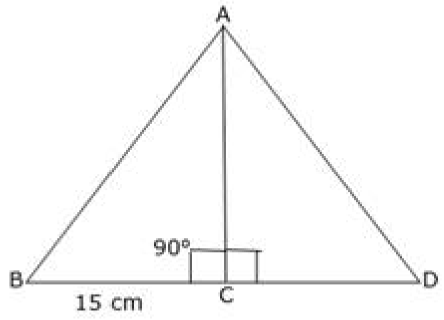
∴ LHS = RHS

Hence Proved

45. Question

In the given figure, BC = 15 cm and sin B = 4/5, show that

$$\tan^2 B - \frac{1}{\cos^2 B} = -1$$



Answer

Given: BC =15cm and $\sin B = \frac{4}{5}$

We know that,

$$\sin\theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Or $\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

$$\text{Sin B} = \frac{4}{5} \Rightarrow \frac{P}{H} = \frac{4}{5} \Rightarrow \frac{AC}{AB} = \frac{4}{5}$$

Let,

Side opposite to angle B = 4k

and Hypotenuse = 5k

where, k is any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

⇒ $(AC)^2 + (BC)^2 = (AB)^2$

⇒ $(4k)^2 + (BC)^2 = (5)^2$

⇒ $16k^2 + (BC)^2 = 25k^2$

⇒ $(BC)^2 = 25\ k^2 - 16\ k^2$

⇒ $(BC)^2 = 9\ k^2$

⇒ BC $=\sqrt{9}\ k^2$

⇒ BC $= \pm 3k$

But side BC can't be negative. So, BC = 3k

Now, we have to find the value of cos B and tan B

We know that,

$$\cos\theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Side adjacent to angle B = BC =3k

Hypotenuse = AB =5k

So, $\cos B = \frac{3k}{5k} = \frac{3}{5}$

Now, tan B

We know that,

$$\tan\theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

side opposite to angle B = AC =4k

Side adjacent to angle B = BC =3k

So, $\tan B = \frac{4k}{3k} = \frac{4}{3}$

Now, $\tan^2 B - \frac{1}{\cos^2 B}$

$= \left(\frac{4}{3}\right)^2 - \left(\frac{1}{\frac{3}{5}}\right)^2$

$= \frac{16}{9} - \frac{25}{9}$

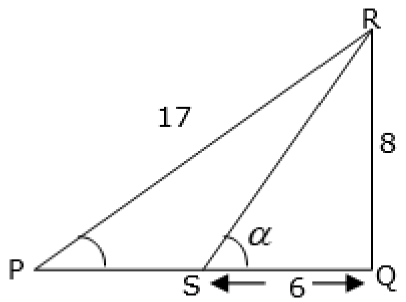
$= \frac{-9}{9}$

= –1 = RHS

Hence Proved

46. Question

In the given figure, find 3 tan θ – 2 sin α + 4 cos α.



Answer

First of all, we find the value of RS

In right angled ΔRQS, we have

$(RQ)^2 + (QS)^2 = (RS)^2$

$\Rightarrow (8)^2 + (6)^2 = (RS)^2$

$\Rightarrow 64 + 36 = (RS)^2$

$\Rightarrow RS = \sqrt{100}$

$\Rightarrow RS = \pm 10$ [taking positive square root, since side cannot be negative]

$\Rightarrow RS = 10$

$\therefore \sin\alpha = \frac{P}{H} = \frac{8}{10} = \frac{4}{5}$

$\cos\alpha = \frac{B}{H} = \frac{6}{10} = \frac{3}{5}$

Now, we find the value of QP

In right angled ΔRQP

$(RQ)^2 + (QP)^2 = (RP)^2$

$\Rightarrow (8)^2 + (QP)^2 = (17)^2$

$\Rightarrow 64 + (QP)^2 = 289$

$\Rightarrow (QP)^2 = 289 - 64$

$\Rightarrow (QP)^2 = 225$

$\Rightarrow QP = \sqrt{225}$

$\Rightarrow QP = \pm 15$ [taking positive square root, since side cannot be negative]

$\Rightarrow QP = 15$

$\tan\theta = \frac{P}{B} = \frac{8}{15}$

Now, $3\tan\theta - 2\sin\alpha + 4\cos\alpha$

$\Rightarrow 3\left(\frac{8}{15}\right) - 2\left(\frac{4}{5}\right) + 4\left(\frac{3}{5}\right)$

$\Rightarrow \frac{24}{15} - \frac{8}{5} + \frac{12}{5}$

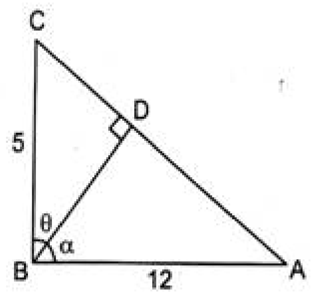
$\Rightarrow \frac{24-24+36}{15}$

$\Rightarrow \frac{36}{15}$

$\Rightarrow \frac{12}{5}$

47. Question

In the given figure ΔABC is right angled at B and BD is perpendicular to AC. Find (i) cos θ, (ii) cot α.



Answer

Firstly, we find the value of AC

In right angled ΔABC

$(AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (12)^2 + (5)^2 = (AC)^2$

$\Rightarrow 144+25 = (AC)^2$

$\Rightarrow (AC)^2 = 169$

$\Rightarrow AC = \sqrt{169}$

$\Rightarrow AC = \pm 13$

$\Rightarrow AC = 13$ [taking positive square root since, side cannot be negative]

(i) $\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{12}{13}$

(ii) $\cot\alpha = \frac{\text{Base}}{\text{Perpendicular}} = \frac{12}{5}$

48. Question

If $5\sin^2\theta + \cos^2\theta = 2$, find the value of $\sin\theta$.

Answer

Given: $5\sin^2\theta + \cos^2\theta = 2$

$\Rightarrow 5\sin^2\theta + (1 - \sin^2\theta) = 2$ [$\because \sin^2\theta + \cos^2\theta = 1$]

$\Rightarrow 5\sin^2\theta + 1 - \sin^2\theta = 2$

$\Rightarrow 4\sin^2\theta = 2 - 1$

$\Rightarrow 4\sin^2\theta = 1$

$\Rightarrow \sin^2\theta = \frac{1}{4}$

$\Rightarrow \sin\theta = \sqrt{\frac{1}{4}}$

$\Rightarrow \sin\theta = \pm\frac{1}{2}$

49. Question

If 7 $\sin^2\theta$ + 3 $\cos^2\theta$ =4, find the value of tan θ.

Answer

Given : 7 $\sin^2\theta$ + 3 $\cos^2\theta$ =4

⇒ 7 $\sin^2\theta$ + 3(1– $\sin^2\theta$) = 4 [∵ $\sin^2\theta + \cos^2\theta = 1$]

⇒ 7 $\sin^2\theta$ + 3 –3 $\sin^2\theta$ = 4

⇒ 4 $\sin^2\theta$ = 4 – 3

⇒ 4 $\sin^2\theta$ = 1

$$\Rightarrow \sin^2\theta = \frac{1}{4}$$

$$\Rightarrow \sin\theta = \sqrt{\frac{1}{4}}$$

$$\Rightarrow \sin\theta = \pm\frac{1}{2}$$

Put the value of $\sin^2\theta = \frac{1}{4}$ in given equation, we get

$$\Rightarrow 7\left(\frac{1}{2}\right)^2 + 3\cos^2\theta = 4$$

$$\Rightarrow \frac{7}{4} + 3\cos^2\theta = 4$$

$$\Rightarrow 3\cos^2\theta = 4 - \frac{7}{4}$$

$$\Rightarrow 3\cos^2\theta = \frac{16-7}{4}$$

$$\Rightarrow 3\cos^2\theta = \frac{9}{4}$$

$$\Rightarrow \cos^2\theta = \frac{3}{4}$$

$$\Rightarrow \cos\theta = \sqrt{\frac{3}{4}}$$

$$\Rightarrow \cos\theta = \pm\frac{\sqrt{3}}{2}$$

Now, we know that $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$$\Rightarrow \tan\theta = \frac{\left(\pm\frac{1}{2}\right)}{\left(\pm\frac{\sqrt{3}}{2}\right)}$$

$$\Rightarrow \tan\theta = \pm\frac{1}{\sqrt{3}}$$

50. Question

If 4 cos θ + 3 sin θ = 5, find the value of tan θ.

Answer

Given : 4 cos θ+ 3 sin θ = 5

Squaring both the sides, we get

$\Rightarrow (4\cos\theta + 3\sin\theta)^2 = 25$

$\Rightarrow 16\cos^2\theta + 9\sin^2\theta + 2(4\cos\theta)(3\sin\theta) = 25$ [$\because (a+b)^2 = a^2 + b^2 + 2ab$]

$\Rightarrow 16\cos^2\theta + 9\sin^2\theta + 24\cos\theta\sin\theta = 25$

Divide by $\cos^2\theta$, we get

$$\Rightarrow \frac{16\cos^2\theta}{\cos^2\theta} + \frac{9\sin^2\theta}{\cos^2\theta} + \frac{24\cos\theta\sin\theta}{\cos^2\theta} = \frac{25}{\cos^2\theta}$$

$\Rightarrow 16 + 9\tan^2\theta + 24\tan\theta = 25\sec^2\theta$

$\Rightarrow 16 + 9\tan^2\theta + 24\tan\theta = 25(1 + \tan^2\theta)$ [$\because 1 + \tan^2\theta = \sec^2\theta$]

$\Rightarrow 16 + 9\tan^2\theta + 24\tan\theta = 25 + 25\tan^2\theta$

$\Rightarrow 16\tan^2\theta - 24\tan\theta + 9 = 0$

$\Rightarrow 16\tan^2\theta - 12\tan\theta - 12\tan\theta + 9 = 0$

$\Rightarrow 4\tan\theta(4\tan\theta - 3) - 3(4\tan\theta - 3) = 0$

$\Rightarrow (4\tan\theta - 3)^2 = 0$

$\Rightarrow \tan\theta = \frac{3}{4}$

51. Question

If 7 sin A + 24 cos A = 25, find the value of tan A.

Answer

Given : 7 sin A + 24 cos A = 25

Squaring both the sides, we get

$\Rightarrow (7 \sin A + 24 \cos A)^2 = 625$

$\Rightarrow 49 \sin^2 A + 576 \cos^2 A + 2(7\sin A)(24\cos A) = 625$ [$\because (a + b)^2 = a^2 + b^2 + 2ab$]

$\Rightarrow 49 \sin^2 A + 576 \cos^2 A + 336 \cos A \sin A = 625$

Divide by $\cos^2 \theta$, we get

$$\Rightarrow \frac{49\sin^2 A}{\cos^2 A} + \frac{576\cos^2 A}{\cos^2 A} + \frac{336 \cos A \sin A}{\cos^2 A} = \frac{625}{\cos^2 A}$$

$\Rightarrow 49\tan^2 A + 576 + 336 \tan A = 625\sec^2 A$

$\Rightarrow 49\tan^2 A + 576 + 336 \tan A = 625(1 + \tan^2 A)$ [$\because 1 + \tan^2\theta = \sec^2 \theta$]

$\Rightarrow 49\tan^2 A + 576 + 336 \tan\theta A = 625 + 625 \tan^2 A$

$\Rightarrow 576\tan^2 A - 336\tan A + 49 = 0$

$\Rightarrow 576\tan^2 A - 168 \tan A - 168 \tan A + 49 = 0$

$\Rightarrow 24\tan\theta (24\tan A - 7) - 7(24\tan A - 7) = 0$

$\Rightarrow (24\tan A - 7)^2 = 0$

$\Rightarrow \tan A = \frac{7}{24}$

52. Question

If 9 sin θ + 40 cos θ= 41, find the value of cos θ and cosec θ

Answer

Given: 9 sin θ + 40 cos θ= 41

$\Rightarrow 9\sin\theta = 41 - 40 \cos\theta$ ...(i)

Squaring both sides, we get

$\Rightarrow 81\sin^2\theta = 1681 + 1600\cos^2\theta - 2(41)(40\cos\theta)$ [$\because (a-b)^2 = a^2 + b^2 - 2ab$]

$\Rightarrow 81(1 - \cos^2\theta) = 1681 + 1600\cos^2\theta - 3280\cos\theta$

$\Rightarrow 81 - 81\cos^2\theta = 1681 + 1600\cos^2\theta - 3280\cos\theta$

$\Rightarrow 1681\cos^2\theta - 3280\cos\theta + 1600 = 0$

$\Rightarrow (41)^2\cos^2\theta - 2(41)(40\cos\theta) + (40)^2 = 0$

$\Rightarrow (41\cos\theta - 40)^2 = 0$

$$\Rightarrow \cos\theta = \frac{40}{41}$$

Now, putting the value of cos θ in Eq. (i), we get

$$\Rightarrow 9\sin\theta = 41 - 40\left(\frac{40}{41}\right)$$

$$\Rightarrow 9\sin\theta = \left(\frac{1681 - 1600}{41}\right)$$

$$\Rightarrow \sin\theta = \left(\frac{81}{41 \times 9}\right)$$

$$\Rightarrow \frac{1}{\operatorname{cosec}\theta} = \left(\frac{9}{41}\right)$$

$$\Rightarrow \operatorname{cosec}\theta = \frac{41}{9}$$

53. Question

If tan A + sec A = 3, find the value of sin A.

Answer

tan A + sec A = 3

$\Rightarrow \tan A = 3 - \sec A$

Squaring both the sides, we get

$\Rightarrow \tan^2 A = (3 - \sec A)^2$

$\Rightarrow \tan^2 A = 9 + \sec^2 A - 6\sec A$

$\Rightarrow \sec^2 A - 1 = 9 + \sec^2 A - 6\sec A$ [$\because 1 + \tan^2 A = \sec^2 A$]

$\Rightarrow -1 - 9 = -6\sec A$

$\Rightarrow - 10 = -6\sec A$

$$\Rightarrow \sec A = \frac{10}{6}$$

$$\Rightarrow \frac{1}{\cos A} = \frac{5}{3} \left[\because \sec A = \frac{1}{\cos A}\right]$$

$$\Rightarrow \cos A = \frac{3}{5}$$

Now, tan A + sec A = 3

$$\Rightarrow \frac{\sin A}{\cos A} + \frac{1}{\cos A} = 3 \left[\because \tan A = \frac{\sin A}{\cos A}\right]$$

$$\Rightarrow \frac{\sin A}{\cos A} = \frac{3\cos A - 1}{\cos A}$$

$\Rightarrow \sin A = 3\cos A - 1$

$$\Rightarrow \sin A = 3\left(\frac{3}{5}\right) - 1$$

$$\Rightarrow \sin A = \left(\frac{9-5}{5}\right)$$

$$\Rightarrow \sin A = \left(\frac{4}{5}\right)$$

54. Question

If cosec A + cot A = 5, find the value of cos A.

Answer

cosec A + cot A = 5

$\Rightarrow \cot A = 5 - \operatorname{cosec} A$

Squaring both the sides, we get

$\Rightarrow \cot^2 A = (5 - \operatorname{cosec} A)^2$

$\Rightarrow \cot^2 A = 25 + \operatorname{cosec}^2 A - 10\operatorname{cosec} A$

$\Rightarrow \operatorname{cosec}^2 A - 1 = 25 + \operatorname{cosec}^2 A - 10\operatorname{cosec} A$ [$\because 1 + \cot^2 A = \operatorname{cosec}^2 A$]

$\Rightarrow -1 - 25 = -10\operatorname{cosec} A$

$\Rightarrow -26 = -10\text{cosec } A$

$\Rightarrow \text{cosec}A = \frac{26}{10}$

$\Rightarrow \frac{1}{\sin A} = \frac{13}{5}$ [$\because$ $\text{cosec}A = \frac{1}{\sin A}$]

$\Rightarrow \sin A = \frac{5}{13}$

Now, cosec A + cot A = 5

$\Rightarrow \frac{1}{\sin A} + \frac{\cos A}{\sin A} = 5$ [$\because$ $\cot A = \frac{\cos A}{\sin A}$]

$\Rightarrow \frac{13}{5} + \frac{\cos A}{\frac{5}{13}} = 5$

$\Rightarrow \frac{13}{5} + \frac{13\cos A}{5} = 5$

$\Rightarrow \frac{13\cos A}{5} = 5 - \frac{13}{5}$

$\Rightarrow \frac{13\cos A}{5} = \frac{25 - 13}{5}$

$\Rightarrow \cos A = \frac{12}{13}$

55. Question

If tan θ + sec θ = x, show that sin θ= $\frac{x^2 - 1}{x^2 + 1}$

Answer

tan θ+ sec θ = x

⇒ tan θ = x – sec θ

Squaring both sides, we get

$\Rightarrow \tan^2\theta = (x - \sec\theta)^2$

$\Rightarrow \tan^2\theta = x^2 + \sec^2\theta - 2x\sec\theta$

$\Rightarrow \sec^2\theta - 1 = x^2 + \sec^2\theta - 2x\sec\theta$ [$\because$ $1 + \tan^2 A = \sec^2 A$]

$\Rightarrow -1 - x^2 = -2x\sec\theta$

$$\Rightarrow \sec\theta = \frac{1+x^2}{2x}$$

Now,

$\tan\theta = x - \sec\theta$

$$\Rightarrow \frac{\sin\theta}{\cos\theta} = x - \sec\theta$$

$$\Rightarrow \sin\theta\left(\frac{1+x^2}{2x}\right) = x - \left(\frac{1+x^2}{2x}\right)$$

$$\Rightarrow \sin\theta\left(\frac{1+x^2}{2x}\right) = \left(\frac{2x^2-1+x^2}{2x}\right)$$

$$\Rightarrow \sin\theta\left(\frac{1+x^2}{2x}\right) = \left(\frac{x^2-1}{2x}\right)$$

$\Rightarrow \sin\theta = \frac{x^2-1}{x^2+1}$ = RHS

Hence Proved

56. Question

If $\cos\theta + \sin\theta = 1$, prove that $\cos\theta - \sin\theta = \pm 1$

Answer

Using the formula,

$(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$

$\Rightarrow (\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2 = 2(\cos^2\theta + \sin^2\theta)$

$\Rightarrow 1 + (\cos\theta - \sin\theta)^2 = 2(1)$

$\Rightarrow (\cos\theta - \sin\theta)^2 = 2 - 1$

$\Rightarrow (\cos\theta - \sin\theta)^2 = 1$

$\Rightarrow (\cos\theta - \sin\theta) = \sqrt{1}$

$\Rightarrow (\cos\theta - \sin\theta) = \pm 1$

Exercise 4.2

1. Question

Find the value of the following :

(i) sin 30^o + cos 60^o

(ii) $\sin^2 45^o + \cos^2 45^o$

(iii) sin 30^o + cos 60^o – tan45^o

(iv) $\sqrt{1+\tan^2 60^o}$

(v) tan 60^o x cos30^o

Answer

(i) sin 30^o + cos 60^o

We know that,

>$\sin(30°) = \frac{1}{2}$

$\cos(60°) = \frac{1}{2}$

So,sin(30^o) + cos(60^o)

$= \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)$

=1

(ii) $\sin^2 45^o + \cos^2 45^o$

We know that,

$\sin(45°) = \frac{1}{\sqrt{2}}$

$\cos(45°) = \frac{1}{\sqrt{2}}$

So, $\sin^2 45^o + \cos^2 45^o$

$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$

$= \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)$

=1

(iii) sin 30^o + cos 60^o – tan45^o

$\sin(30°) = \frac{1}{2}$

$\cos(60°) = \frac{1}{2}$

$\tan(45°) = 1$

So,sin 30^o + cos 60^o – tan 45^o

$= \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right) - 1$

$= \frac{1 + 1 - 2}{2}$

=0

(iv) $\sqrt{1 + \tan^2 60^o}$

We know that

tan(60^o) = √3

So,

$= \sqrt{1 + \tan^2 60}\,°$

$= \sqrt{1 + \left(\sqrt{3}\right)^2}$

$= \sqrt{1 + 3}$

=√4

= 2

(v) $\tan 60^{o} \times \cos 30^{o}$

$\tan(60^{o}) = \sqrt{3}$

$\cos(30°) = \frac{\sqrt{3}}{2}$

So,

$\tan 60^{o} \times \cos 30^{o}$

$= \sqrt{3} \times \frac{\sqrt{3}}{2}$

$= \frac{3}{2}$

2. Question

If θ = 45°, find the value of

(i) $\tan^2\theta + \frac{1}{\sin^2\theta}$

(ii) $\cos^2\theta - \sin^2\theta$

Answer

(i) $\tan^2\theta + \frac{1}{\sin^2\theta}$

Given θ =45°

We know that,

$\tan(45^{o}) = 1$

$\sin(45°) = \frac{1}{\sqrt{2}}$

$= (1)^2 + \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2}$

= 1+ 2

= 3

(ii) $\cos^2 \theta - \sin^2 \theta$

Given $\theta = 45°$

$$\sin(45°) = \frac{1}{\sqrt{2}}$$

$$\cos(45°) = \frac{1}{\sqrt{2}}$$

So, $\cos^2 45° - \sin^2 45°$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2$$

= 0

3 A. Question

Find the numerical value of the following :

$\sin45°.\cos45° - \sin^2 30°$.

Answer

We know that,

$$\sin(45°) = \frac{1}{\sqrt{2}}$$

$$\cos(45°) = \frac{1}{\sqrt{2}}$$

$$\sin(30°) = \frac{1}{2}$$

Now, putting the values

$$= \left(\frac{1}{\sqrt{2}}\right) \times \left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{2}\right)^2$$

$$= \left(\frac{1}{2}\right) - \left(\frac{1}{4}\right)$$

$$= \left(\frac{1}{4}\right)$$

3 B. Question

Find the numerical value of the following :

$$\frac{\tan 60^\circ}{\sin 60^\circ + \cos 60^\circ}$$

Answer

We know that,

$$\sin(60^\circ) = \frac{\sqrt{3}}{2}$$

$$\cos(60^\circ) = \frac{1}{2}$$

tan $(60^\circ) = \sqrt{3}$

Now putting the values;

$$= \frac{\sqrt{3}}{\frac{\sqrt{3}}{2} + \frac{1}{2}}$$

$$= \frac{\sqrt{3}}{\frac{1+\sqrt{3}}{2}}$$

$$= \sqrt{3} \times \frac{2}{1+\sqrt{3}}$$

$$= \frac{2\sqrt{3}}{1+\sqrt{3}}$$

Multiplying and dividing by the conjugate of $(1+\sqrt{3})$

$$= \frac{2\sqrt{3}}{1+\sqrt{3}} \times \frac{1-\sqrt{3}}{1-\sqrt{3}}$$

$$= \frac{2\sqrt{3}-6}{(1)^2-(\sqrt{3})^2}$$ [∵ $(a)^2 - (b)^2 = (a+b)(a-b)$]

$$= \frac{2\sqrt{3} - 6}{-2}$$

Multiplying and dividing by (-2)

$= 3 - \sqrt{3}$

3 C. Question

Find the numerical value of the following :

$$\frac{\tan 60^{\circ}}{\sin 60^{\circ} + \cos 30^{\circ}}$$

Answer

We know that,

$$\sin(60°) = \frac{\sqrt{3}}{2}$$

$$\cos(30°) = \frac{\sqrt{3}}{2}$$

$\tan(60^{\circ}) = \sqrt{3}$

Now putting the values;

$$= \frac{\sqrt{3}}{\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}}$$

$$= \frac{\sqrt{3}}{\sqrt{3}}$$

$= 1$

3 D. Question

Find the numerical value of the following :

$$\frac{4}{\sin^2 60^{\circ}} + \frac{3}{\cos^2 60^{\circ}}$$

Answer

We know that

$$\text{Sin}(60°) = \frac{\sqrt{3}}{2}$$

$\cos(60°) = \frac{1}{2}$

Now putting the value, we get

$= \frac{4}{\left(\frac{\sqrt{3}}{2}\right)^2} + \frac{3}{\left(\frac{1}{2}\right)^2}$

$= 4 \times \left(\frac{2}{\sqrt{3}}\right)^2 + 3 \times (2)^2$

$= 4\left(\frac{4}{3}\right) + 3 \times 4$

$= \frac{16}{3} + 12$

$= \frac{16 + 36}{3}$

$= \frac{52}{3}$

3 E. Question

Find the numerical value of the following :

$\sin^2 60° – \cos^2 60°$

Answer

We know that,

$\sin(60°) = \frac{\sqrt{3}}{2}$

$\cos(60°) = \frac{1}{2}$

Now putting the value;

$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$

$= \left(\frac{3}{4}\right) - \left(\frac{1}{4}\right)$

$= \frac{1}{2}$

3 F. Question

Find the numerical value of the following :

$4\sin^2 30° + 3 \tan 30° – 8 \sin 45° \cos 45°$

Answer

We know that,

$\sin(30°) = \frac{1}{2}$

$\tan(30°) = \frac{1}{\sqrt{3}}$

$\sin(45°) = \frac{1}{\sqrt{2}}$

$\cos(45°) = \frac{1}{\sqrt{2}}$

Now putting the value, we get

$= 4 \times \left(\frac{1}{2}\right)^2 + 3 \times \left(\frac{1}{\sqrt{3}}\right)^2 - 8 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$

$= 4 \times \frac{1}{4} + 3 \times \frac{1}{3} - 8 \times \frac{1}{2}$

$= 1 + 1 – 4$

$= -2$

3 G. Question

Find the numerical value of the following :

$2\sin^2 30° – 3\cos^2 45° + \tan^2 60°$

Answer

We know that,

$\text{Sin}(30°) = \frac{1}{2}$

$\cos(45°) = \frac{1}{\sqrt{2}}$

Tan $(60^{o}) = \sqrt{3}$

Now putting the value;

$= 2 \times \left(\frac{1}{2}\right)^2 - 3 \times \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\sqrt{3}\right)^2$

$= 2 \times \frac{1}{4} - 3 \times \frac{1}{2} + 3$

$= \frac{1}{2} - \frac{3}{2} + 3$

$= \frac{1 - 3 + 6}{2}$

$= \frac{4}{2}$

=2

3 H. Question

Find the numerical value of the following :

sin 90° + cos 0° + sin 30° + cos 60°

Answer

We know that,

Sin $(90^{o}) = 1$

Cos $(0^{o}) = 1$

$\text{Sin}(30°) = \frac{1}{2}$

$\cos(60°) = \frac{1}{2}$

Now putting the value;

$= 1 + 1 + \frac{1}{2} + \frac{1}{2}$

$= \frac{2+2+1+1}{2}$

$= \frac{6}{2}$

= 3

3 I. Question

Find the numerical value of the following :

sin 90° – cos 0° + tan 0° + tan 45°

Answer

We know that

Sin (90^o) = 1

Cos (0^o) = 1

Tan(0^o) = 0

Tan(45^o) = 1

Now putting the value, we get

= 1 – 1 + 0 + 1

= 1

3 J. Question

Find the numerical value of the following :

$\cos^2 0^\circ + \tan^2 \frac{\pi}{4} + \sin^2 \frac{\pi}{4}$, where π = 180°

Answer

We know that

Cos (0^o) = 1

Tan $(45^o) = 1 \left[\frac{\pi}{4} = \frac{180°}{4} = 45°\right]$

$$\text{Sin}(45°) = \frac{1}{\sqrt{2}} \left[\frac{\pi}{4} = \frac{180°}{4} = 45°\right]$$

Now putting the values;

$$= (1)^2 + (1)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 1 + 1 + \frac{1}{2}$$

$$= \frac{2 + 2 + 1}{2}$$

$$= \frac{5}{2}$$

3 K. Question

Find the numerical value of the following :

$$\frac{\cos 60^o}{\sin^2 45^o} - 3\cot 45^o + 2\sin 90^o$$

Answer

We know that,

$$\cos(60°) = \frac{1}{2}$$

$$\text{Sin}(45°) = \frac{1}{\sqrt{2}}$$

Cot $(45^o) = 1$

Sin $(90^o) = 1$

Now putting the values, we get

$$= \frac{\frac{1}{2}}{\left(\frac{1}{\sqrt{2}}\right)^2} - 3(1) + 2(1)$$

= 1-3+2

=0

3 L. Question

Find the numerical value of the following :

$$\frac{4}{\tan^2 60^o} + \frac{1}{\cos^2 30^o} - \sin^2 45^o$$

Answer

We can write the above equation as:

= 4 $\cot^2 60^o + \sec^2 30^o - \sin^2 45^o$ ...(a) $\left[\because \cos\theta = \frac{1}{\sec\theta} \text{ and } \tan\theta = \frac{1}{\cot\theta}\right]$

$$\text{Sin}(45°) = \frac{1}{\sqrt{2}}$$

$$\text{Cot}\,(60°) = \frac{1}{\sqrt{3}}$$

$$\text{Sec}\,(30°) = \frac{2}{\sqrt{3}}$$

Now putting the values in (a);

$$= 4\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{2}{\sqrt{3}}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 4 \times \frac{1}{3} + \frac{4}{3} - \frac{1}{2}$$

$$= \frac{8+8-3}{6}$$

$$= \frac{13}{6}$$

3 M. Question

Find the numerical value of the following :

$\cos 60° \,.\, \cos 30° - \sin 60° \,.\, \sin 30°$

Answer

We know that,

$$\cos(60°) = \frac{1}{2}$$

$$\cos(30°) = \frac{\sqrt{3}}{2}$$

$$\sin(60°) = \frac{\sqrt{3}}{2}$$

$$\sin(30°) = \frac{1}{2}$$

Now putting the values, we get

$$= \frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$$

= 0

3 N. Question

Find the numerical value of the following :

$$\frac{4(\sin^2 60^\circ - \cos^2 45^\circ)}{\tan^2 30^\circ + \cos^2 90^\circ}$$

Answer

We know that,

$$\sin(60°) = \frac{\sqrt{3}}{2}$$

$$\cos(45°) = \frac{1}{\sqrt{2}}$$

$$\text{Tan}(30°) = \frac{1}{\sqrt{3}}$$

$\cos(90^o) = 0$

Now putting the values;

$$= 4 \times \frac{\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2}{\left(\frac{1}{\sqrt{3}}\right)^2 - (0)^2}$$

$$= 4 \times \frac{\frac{3}{4} - \frac{1}{2}}{\frac{1}{3}}$$

$$= 4 \times \frac{1}{4} \times 3$$

= 3

4 A. Question

Evaluate the following :

sin30°.cos45° + cos30°.sin45°

Answer

We know that,

$$\sin(30°) = \frac{1}{2}$$

$$\cos(45°) = \frac{1}{\sqrt{2}}$$

$$\cos(30°) = \frac{\sqrt{3}}{2}$$

$$\sin(45°) = \frac{1}{\sqrt{2}}$$

Now putting the values, we get

$$= \frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$$

$$= \frac{1 + \sqrt{3}}{2\sqrt{2}}$$

4 B. Question

Evaluate the following :

$cosec^2 30°.tan^2 45° – sec^2 60°$

Answer

We know that

cosec $(30^o) = 2$

$Tan(45^o) = 1$

sec $(60\ ^o) = 2$

Now putting the values;

$= (2)^2 \times (1)^2 - (2)^2$

$= 4 – 4$

$= 0$

4 C. Question

Evaluate the following :

$2sin^2 30°.tan60° – 3cos^2 60°.sec^2 30°$

Answer

We know that

$\sin(30°) = \frac{1}{2}$

$\tan(60^o) = \sqrt{3}$

$\cos(60°) = \frac{1}{2}$

$\sec(30°) = \frac{2}{\sqrt{3}}$

Now putting the values;

$= 2 \times \left(\frac{1}{2}\right)^2 \times (\sqrt{3}) - \left(3 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{2}{\sqrt{3}}\right)^2\right)$

$= 2 \times \frac{1}{4} \times (\sqrt{3}) - \left(3 \times \frac{1}{4} \times \frac{4}{3}\right)$

$= \frac{\sqrt{3}}{2} - 1$

$= \frac{\sqrt{3} - 2}{2}$

4 D. Question

Evaluate the following :

$\tan 60° . \text{cosec}^2 45° + \sec^2 60° . \tan 45°$

Answer

We know that

$\tan(60^o) = \sqrt{3}$

$\text{cosec}(45^o) = \sqrt{2}$

$\sec(60^o) = 2$

$\tan(45^o) = 1$

Now putting the values;

$= (\sqrt{3}) \times (\sqrt{2})^2 + (2)^2 \times (1)$

$= 2\sqrt{3} + 4$

$= 2(\sqrt{3} + 2)$

4 E. Question

Evaluate the following :

$\tan 30° . \sec 45° + \tan 60° . \sin 30°$

Answer

We know that

$\tan(30°) = \frac{1}{\sqrt{3}}$

$\sec(45^o) = \sqrt{2}$

$\tan(60^o) = \sqrt{3}$

$$\sec(30°) = \frac{2}{\sqrt{3}}$$

Now putting the values, we get

$$= \frac{1}{\sqrt{3}} \times \sqrt{2} + \sqrt{3} \times \frac{2}{\sqrt{3}}$$

$$= \frac{\sqrt{2}}{\sqrt{3}} + 2$$

$$= 2 + \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= 2 + \frac{\sqrt{6}}{3}$$

4 F. Question

Evaluate the following :

cos30°.cos45° – sin30°.sin45°

Answer

We know that

$$\cos(30°) = \frac{\sqrt{3}}{2}$$

$$\cos(45°) = \frac{1}{\sqrt{2}}$$

$$\sin(30°) = \frac{1}{2}$$

$$\sin(45°) = \frac{1}{\sqrt{2}}$$

Now putting the values, we get

$$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

Multiplying and dividing by $(\sqrt{2})$, we get

$$= \frac{\sqrt{3}-1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{2}(\sqrt{3}-1)}{4}$$

4 G. Question

Evaluate the following :

$$\frac{4}{3}\tan^2 30^\circ + \sin^2 60^\circ - 3\cos^2 60^\circ + \frac{3}{4}\tan^2 60^\circ - 2\tan^2 45^\circ$$

Answer

We know that

$$\tan(30^\circ) = \frac{1}{\sqrt{3}}$$

$$\sin(60^\circ) = \frac{\sqrt{3}}{2}$$

$$\cos(60^\circ) = \frac{1}{2}$$

$\tan(60^\circ) = \sqrt{3}$

$\tan(45^\circ) = 1$

Now putting the values;

$$= \left(\frac{4}{3} \times \left(\frac{1}{\sqrt{3}}\right)^2\right) + \left[\left(\frac{\sqrt{3}}{2}\right)^2\right] - \left[3 \times \left(\frac{1}{2}\right)^2\right] + \left[\frac{3}{4}\left(\sqrt{3}\right)^2\right] - [2 \times (1)^2]$$

$$= [\frac{4}{3} \times \frac{1}{3}] + [\frac{3}{4}] - [3 \times \frac{1}{4}] + [\frac{3}{4} \times 3] - [2 \times (1)]$$

$$= \frac{4}{9} + \frac{3}{4} - \frac{3}{4} + \frac{9}{4} - 2$$

$$= \frac{16 + 27 - 27 + 81 - 72}{36}$$

$$= \frac{25}{36}$$

4 H. Question

Evaluate the following :

$$\frac{\tan^2 60^\circ + 4\cos^2 45^\circ + 3\sec^2 30^\circ + 5\cos^2 90^\circ}{\operatorname{cosec} 30^\circ + \sec 60^\circ - \cot^2 30^\circ}$$

Answer

We know that

tan $(60^\circ) = \sqrt{3}$

$\cos(45^\circ) = \frac{1}{\sqrt{2}}$

sec $(30^\circ) = \frac{2}{\sqrt{3}}$

$\cos(90^\circ) = 0$

cosec $(30^\circ) = 2$

sec $(60^\circ) = 2$

cot $(30^\circ) = \sqrt{3}$

Now putting the values, we get

$= \frac{(\sqrt{3})^2 + \left[4 \times \left(\frac{1}{\sqrt{2}}\right)^2\right] + \left[3 \times \left(\frac{2}{\sqrt{3}}\right)^2\right] + [5 \times (0)^2]}{(2)+(2)-(\sqrt{3})^2}$

$= \frac{(3) + \left[4 \times \frac{1}{2}\right] + \left[3 \times \frac{4}{3}\right] + [5 \times 0]}{2 + 2 - 3}$

$= \frac{(3)+[2]+[4]+[0]}{2+2-3}$

= 9

4 I. Question

Evaluate the following :

$$\frac{5\sin^2 30^\circ + \cos^2 45^\circ - 4\tan^2 30^\circ}{2\sin 30^\circ . \cos 30^\circ + \tan 45^\circ}$$

Answer

We know that

$$\sin(30°) = \frac{1}{2}$$

$$\cos(45°) = \frac{1}{\sqrt{2}}$$

$$\tan(30°) = \frac{1}{\sqrt{3}}$$

$$\cos(30°) = \frac{\sqrt{3}}{2}$$

tan (45$^{o)}$ = 1

Now putting the values, we get

$$= \frac{\left[5 \times \left(\frac{1}{2}\right)^2\right] + \left(\frac{1}{\sqrt{2}}\right)^2 - \left[4 \times \left(\frac{1}{\sqrt{3}}\right)^2\right]}{2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + (1)}$$

$$= \frac{\left(\frac{5}{4}\right) + \left(\frac{1}{2}\right) - \left(\frac{4}{3}\right)}{\left(\frac{\sqrt{3}}{2}\right) + 1}$$

$$= \frac{\left(\frac{15 + 6 - 16}{12}\right)}{\left(\frac{\sqrt{3} + 2}{2}\right)}$$

$$= \frac{5}{12} \times \frac{2}{\sqrt{3} + 1}$$

$$= \frac{5}{6} \times \frac{1}{\sqrt{3} + 2}$$

5 A. Question

Prove the following :

$$\frac{(1-\cos B)(1+\cos B)}{(1-\sin B)(1+\sin B)} = \frac{1}{3}$$ When B = 30°

Answer

Solving, L.H.S.

$= \frac{(1)^2 - (\cos B)^2}{(1)^2 - (\text{Sin } B)^2}$ $[(a)^2 - (b)^2 = (a+b)(a-b)]$

$= \frac{1 - \cos^2 B}{1 - \sin^2 B}$

We know that,

$\cos(30°) = \frac{\sqrt{3}}{2}$

$\sin(30°) = \frac{1}{2}$

Putting the values, we get

$= \frac{1 - \left(\frac{\sqrt{3}}{2}\right)^2}{1 - \left(\frac{1}{2}\right)^2}$

$= \frac{1 - \frac{3}{4}}{1 - \frac{1}{4}}$

$= \frac{4 - 3}{4 - 1}$

$= \frac{1}{3}$

=R.H.S.

Hence Proved

5 B. Question

Prove the following :

$\frac{(1 - \cos \alpha)(1 + \cos \alpha)}{(1 - \sin \alpha)(1 + \sin \alpha)} = 3$ When α =60°

Answer

Solving, L.H.S.

$= \frac{(1)^2 - (\cos\alpha)^2}{(1)^2 - (\text{Sin}\,\alpha)^2}$ $[(a)^2 - (b)^2 = (a+b)(a-b)]$

$= \frac{1-\cos^2\alpha}{1-\sin^2\alpha}$

We know that

$$\cos(60°) = \frac{1}{2}$$

$$\sin(60°) = \frac{\sqrt{3}}{2}$$

Putting the values, we get

$$= \frac{1 - \left(\frac{1}{2}\right)^2}{1 - \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{1 - \frac{1}{4}}{1 - \frac{3}{4}}$$

$$= \frac{4-1}{4-3}$$

= 3 = R.H.S.

5 C. Question

Prove the following :

$\cos(A - B) = \cos A.\cos B + \sin A.\sin B$ if $A=B=60^o$

Answer

Solving, L.H.S.

$= \cos(60^o - 60^o)$ [Putting the value $A=B=60^o$]

$= \cos(0^o)$

= 1

Solving, R.H.S.

$= \cos(60^o) \times \cos(60^o) + \sin(60^o) \times \sin(60^o)$ [Putting the value A=B=60^o]

$= \cos^2(60^o) + \sin^2(60^o)$

We know that,

$$\cos(60°) = \frac{1}{2}$$

$$\sin(60°) = \frac{\sqrt{3}}{2}$$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= \frac{1}{4} + \frac{3}{4}$$

$$= \frac{1+3}{4}$$

= 1

∴ LHS = RHS

Hence Proved

5 D. Question

Prove the following :

$4(\sin^4 30° + \cos^4 60°) - 3(\cos^2 45° - \sin^2 90°) = 2$

Answer

We know that,

$$\sin(30°) = \frac{1}{2}$$

$$\cos(60°) = \frac{1}{2}$$

$$\cos(45°) = \frac{1}{\sqrt{2}}$$

$\sin(90^o) = 1$

Now solving, L.H.S.

$= 4[\{(\sin 30^o)^2\}^2 + \{(\cos 60^o)^2\}^2] - 3[(\cos 45^o)^2 - (\sin 90^o)^2]$

Putting the values

$$= 4 \times \left[\left\{\left(\frac{1}{2}\right)^2\right\}^2 + \left\{\left(\frac{1}{2}\right)^2\right\}^2\right] - 3\left[\left(\frac{1}{\sqrt{2}}\right)^2 - 1\right]$$

$$= 4 \times \left[\left\{\frac{1}{4}\right\}^2 + \left\{\frac{1}{4}\right\}^2\right] - 3\left[\frac{1}{2} - 1\right]$$

$$= 4 \times \left[\frac{1}{16} + \frac{1}{16}\right] - 3\left[-\frac{1}{2}\right]$$

$$= 4 \times \left[\frac{1}{8}\right] - 3\left[-\frac{1}{2}\right]$$

$$= \left[\frac{1}{2}\right] + \left[\frac{3}{2}\right]$$

$$= \left[\frac{4}{2}\right]$$

=2 = R.H.S.

Hence Proved

5 E. Question

Prove the following :

$\sin 90° = 2\sin 45°.\cos 45°$

Answer

We know that,

$\sin(90^o) = 1$

$$\sin(45°) = \frac{1}{\sqrt{2}}$$

$$\cos(45°) = \frac{1}{\sqrt{2}}$$

Taking LHS = sin 90° = 1

Now, taking RHS

$= 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$

$= 2 \times \frac{1}{2}$

= 1

= R.H.S.

Hence Proved

5 F. Question

Prove the following :

cos60° = 2cos^{2}30° – 1 = 1 – 2 sin^{2}30°

Answer

We know that,

$\cos(60°) = \frac{1}{2}$

$\cos(30°) = \frac{\sqrt{3}}{2}$

$\sin(30°) = \frac{1}{2}$

Taking LHS = cos 60° $= \frac{1}{2}$

Now, solving RHS = 2cos^2 30° - 1 , we get

$= 2 \times \left(\frac{\sqrt{3}}{2}\right)^2 - 1$

$= 2 \times \frac{3}{4} - 1$

$= \frac{3}{2} - 1$

$= \frac{3}{2} - 1$

$= \frac{1}{2}$

= RHS

Now taking RHS = 1- 2sin^2 30°

$= 1 - 2\left(\frac{1}{2}\right)^2$

$= 1 - \frac{1}{2}$

$= \frac{2-1}{2}$

$= \frac{1}{2}$

= RHS

Hence, proved.

5 G. Question

Prove the following :

cos90° = 1 – 2 sin^{2}45° = 2cos^{2}45° – 1

Answer

We know that

cos(90^o) = 0

$\sin(45°) = \frac{1}{\sqrt{2}}$

$\cos(45°) = \frac{1}{\sqrt{2}}$

taking LHS = cos 90° = 0

Now solving RHS 1- 2sin^2 45°

$= 1 - 2\left(\frac{1}{\sqrt{2}}\right)^2$

$= 1 - 2 \times \frac{1}{2}$

= 1- 1

= 0

= RHS

Now, solving RHS = $2\cos^2 45° - 1$, we get

$= 1 \; - \; 2 \; \times \; \left(\frac{1}{\sqrt{2}}\right)^2$

$= 1 \; - \; 2 \; \times \; \frac{1}{2}$

= 1- 1

= 0

Hence, proved.

5 H. Question

Prove the following :

sin30°.cos60° + cos30°.sin60° = sin90°

Answer

We know that

$\sin(30°) = \frac{1}{2}$

$\cos(60°) = \frac{1}{2}$

$\cos(30°) \; = \; \frac{\sqrt{3}}{2}$

$\sin(60°) \; = \; \frac{\sqrt{3}}{2}$

$\sin(90^o) = 1$

Taking LHS =

$$= \left[\left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right)\right] + \left[\left(\frac{\sqrt{3}}{2}\right) \times \left(\frac{\sqrt{3}}{2}\right)\right]$$

$$= \left[\left(\frac{1}{4}\right)\right] + \left[\left(\frac{3}{4}\right)\right]$$

$$= \left[\frac{1+3}{4}\right]$$

= 1

Now, RHS = sin 90° = 1

∴ LHS = RHS

Hence, proved.

5 I. Question

Prove the following :

cos60°.cos30° – sin60°. sin30° = cos 90°

Answer

We know that

$$\cos(60°) = \frac{1}{2}$$

$$\cos(30°) = \frac{\sqrt{3}}{2}$$

$$\sin(60°) = \frac{\sqrt{3}}{2}$$

$$\sin(30°) = \frac{1}{2}$$

$\cos(90^o) = 0$

Taking LHS

$$= \left[\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)\right] - \left[\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)\right]$$

$$= \left[\left(\frac{\sqrt{3}}{4}\right)\right] - \left[\left(\frac{\sqrt{3}}{4}\right)\right]$$

= 0

Now, RHS = cos 90° = 0

∴ LHS =RHS

Hence, proved.

5 J. Question

Prove the following :

$$\cos 60^\circ = \frac{1 - \tan^2 30^\circ}{1 + \tan^2 30^\circ}$$

Answer

We know that,

$$\cos(60^\circ) = \frac{1}{2}$$

$$\tan(30^\circ) = \frac{1}{\sqrt{3}}$$

Taking LHS = $\cos 60^\circ = \frac{1}{2}$

Now, solving RHS

$$= \frac{1 - \left(\frac{1}{\sqrt{3}}\right)^2}{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}}$$

$$= \frac{\frac{3 - 1}{3}}{\frac{3 + 1}{3}}$$

$$= \frac{2}{4}$$

$$= \frac{1}{2}$$

∴ L.H.S. = R.H.S.

Hence, proved.

5 K. Question

Prove the following :

$$\frac{\tan 60^{o} - \tan 30^{o}}{1 + \tan 60^{o}.\tan 30^{o}} = \tan 30^{o}$$

Answer

We know that

$\tan(60^o) = \sqrt{3}$

$\tan(30°) = \frac{1}{\sqrt{3}}$

Taking LHS

$$= \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + (\sqrt{3})X\left(\frac{1}{\sqrt{3}}\right)}$$

$$= \frac{\frac{3 - 1}{\sqrt{3}}}{1 + 1}$$

$$= \frac{\frac{2}{\sqrt{3}}}{2}$$

$$= \frac{1}{\sqrt{3}}$$

Now, RHS = $\tan 30° = \frac{1}{\sqrt{3}}$

∴L.H.S. = R.H.S.

Hence, proved.

5 L. Question

Prove the following :

$$\frac{1-\tan 30^{\circ}}{1+\tan 30^{\circ}}=\frac{1-\sin 60^{\circ}}{\cos 60^{\circ}}$$

Answer

$$\tan\left(30^{\circ}\right) = \frac{1}{\sqrt{3}}$$

$$\sin(60^{\circ}) = \frac{\sqrt{3}}{2}$$

$$\cos(60^{\circ}) = \frac{1}{2}$$

Taking LHS

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{\sqrt{3} - 1}{\sqrt{3}}}{\frac{\sqrt{3} + 1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

Multiplying and Dividing, LHS by (√3- 1)

$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1}$$

$$= \frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-(1)^2}$$ $[(a)^2 - (b)^2 = (a+b)(a-b)]$

$$= \frac{(\sqrt{3} - 1)^2}{(\sqrt{3})^2 - (1)^2}$$

$$= \frac{3 + 1 - 2\sqrt{3}}{3 - 1}$$

$$= \frac{4 - 2\sqrt{3}}{2}$$

Multiplying and Dividing, LHS by 2

= 2- √3

Now, RHS

$$= \frac{1 - \frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

$$= \frac{\frac{2 - \sqrt{3}}{2}}{\frac{1}{2}}$$

= 2- √3

∴ LHS = RHS

Hence, proved.

5 M. Question

Prove the following :

$$\frac{\sin 60^\circ + \cos 30^\circ}{\sin 30^\circ + \cos 60^\circ + 1} = \cos 30^\circ$$

Answer

We know that

$$\sin(30°) = \frac{1}{2}$$

$$\sin(60°) = \frac{\sqrt{3}}{2}$$

$\cos(30°) = \frac{\sqrt{3}}{2}$

$\cos(60°) = \frac{1}{2}$

Taking LHS

$$= \frac{\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}}{\frac{1}{2} + \frac{1}{2} + 1}$$

$$= \frac{2 \times \frac{\sqrt{3}}{2}}{\frac{1 + 1 + 2}{2}}$$

$$= \frac{\sqrt{3}}{2}$$

Now, RHS= $\cos(30°) = \frac{\sqrt{3}}{2}$

∴ LHS =RHS

Hence Proved

5 N. Question

Prove the following :

$$\sin 60^\circ = 2\sin 30^\circ.\cos 30^\circ = \frac{2\tan 30^\circ}{1 + \tan^2 30^\circ}$$

Answer

We know that

$\sin(30°) = \frac{1}{2}$

$\sin(60°) = \frac{\sqrt{3}}{2}$

$\cos(30°) = \frac{\sqrt{3}}{2}$

$\tan(30°) = \frac{1}{\sqrt{3}}$

Taking LHS = $\sin 60° = \frac{\sqrt{3}}{2}$

Now, solving RHS = 2 sin 30° cos 30°

$= 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{2}$

= LHS

Now, RHS $= \frac{2\tan 30°}{1+\tan^2 30°}$

$$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}}$$

$$= \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{4}$$

$$= \frac{\sqrt{3}}{2}$$

∴ LHS =RHS

Hence proved

6 A. Question

If A=60^o and B = 30^o, verify that :

cos (A+B) = cos A cos B – sin A sin B

Answer

Given: $A=60^o$ and $B=30^o$

Now, LHS = Cos (A+B)

⇒ Cos ($60^o + 30^o$)

⇒ Cos (90^o)

⇒ 0 [∵ cos 90^o = 0]

Now, RHS = Cos A Cos B – Sin A Sin B

⇒ cos(60^o) cos(30^o) – sin(60^o) sin (30^o)

⇒ $\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)-\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$

⇒ 0

∴ LHS = RHS

Hence Proved

6 B. Question

If $A=60^o$ and $B = 30^o$, verify that :

sin (A – B) = sin A cos B – cos A sin B

Answer

Given: $A=60^o$ and $B=30^o$

Now, LHS = Sin (A-B)

⇒ Sin ($60^o - 30^o$)

⇒ Sin (30^o)

⇒ $\left(\frac{1}{2}\right)$

Now, RHS = Sin A Cos B – Cos A Sin B

⇒ sin(60^o) cos(30^o) – cos(60^o) sin (30^o)

$\Rightarrow \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$\Rightarrow \frac{3}{4} - \frac{1}{4}$

$\Rightarrow \left(\frac{1}{2}\right)$

∴ LHS = RHS

Hence Proved

6 C. Question

If A=60^{o} and B = 30^{o}, verify that :

tan (A – B) = $\frac{\tan A - \tan B}{1 + \tan A \tan B}$

Answer

Given: A=60^{o} and B =30^{o}

Now, LHS = tan (A-B)

$\Rightarrow$ tan (60 ^{o} - 30 ^{o})

$\Rightarrow$ tan (30 ^{o})

$\Rightarrow \left(\frac{1}{\sqrt{3}}\right)$

Now, RHS $= \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$= \frac{\tan 60^{o} - \tan 30^{o}}{1 + \tan 60^{o} \tan 30^{o}}$

$\Rightarrow \frac{\sqrt{3} - \left(\frac{1}{\sqrt{3}}\right)}{1 + (\sqrt{3})\left(\frac{1}{\sqrt{3}}\right)}$

$\Rightarrow \frac{\frac{3-1}{\sqrt{3}}}{1+1}$

$\Rightarrow \left(\frac{1}{\sqrt{3}}\right)$

∴ LHS = RHS

Hence Proved

7 A. Question

If A = 30^o, verify that :

sin 2A = 2 sin A cos A

Answer

Given: A =30^o

Now, LHS = sin $2(30^o)$

⇒ sin 60^o

$$\Rightarrow \frac{\sqrt{3}}{2}$$

Now, RHS = 2 sin A cos A

⇒ 2 sin (30^o) cos (30^o)

$$\Rightarrow 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \frac{\sqrt{3}}{2}$$

∴ LHS = RHS

Hence Proved

7 B. Question

If A = 30^o, verify that :

cos 2A = 1-2 $\sin^2 A = 2\cos^2 A - 1$

Answer

Given: A =30^o

Now, LHS = cos $2(30^o)$

⇒ cos 60^o

$\Rightarrow \frac{1}{2}$

Now, RHS = 1- 2sin^2 A

$\Rightarrow$ 1- 2sin^2 (30^o)

$\Rightarrow 1 - 2\left(\frac{1}{2}\right)^2$

$\Rightarrow 1 - 2\left(\frac{1}{4}\right)$

$\Rightarrow \frac{2-1}{2}$

$\Rightarrow \frac{1}{2}$

Now, RHS = 2cos^2 A – 1

$\Rightarrow$ 2cos^2 (30^o) - 1

$\Rightarrow 2\left(\frac{\sqrt{3}}{2}\right)^2 - 1$

$\Rightarrow 2\left(\frac{3}{4}\right) - 1$

$\Rightarrow \frac{3-2}{2}$

$\Rightarrow \frac{1}{2}$

∴ LHS = RHS

Hence Proved

8 A. Question

If θ = 30°, verify that :

sin 3θ = 3 sinθ – 4 sin^{3}θ

Answer

Given: θ =30^o

Now, LHS = sin 3(30^o)

$\Rightarrow$ sin 90^o

= 1

Now, RHS = 3 sin θ - 4 $\sin^3$ θ

$\Rightarrow$ 3 sin (30^o) - 4 $\sin^3$ (30^o)

$$\Rightarrow\ 3\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)^3$$

$$\Rightarrow\ \frac{3}{2} - \frac{1}{2}$$

= 1

∴ LHS = RHS

Hence Proved

8 B. Question

If θ = 30°, verify that :

cos3θ = $4\cos^3\theta$ – 3cosθ

Answer

Given: θ =30^o

Now, LHS = cos $3(30^o)$

$\Rightarrow$ cos 90^o

= 0

Now, RHS = 4 $\cos^3$ θ - 3 cos θ

$\Rightarrow$ 4 $\cos^3$ (30^o) - 3 cos (30^o)

$$\Rightarrow\ 4\left(\frac{\sqrt{3}}{2}\right)^3 - 3\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow\ \left(\frac{3\sqrt{3}}{2}\right) - \left(\frac{3\sqrt{3}}{2}\right)$$

= 0

∴ LHS = RHS

Hence Proved

9. Question

If sin (A + B) = 1 and cos (A – B) = $\frac{\sqrt{3}}{2}$, then find A and B.

Answer

Given : sin (A+B) =1

⇒ Sin(A+B) = sin (90 ^{o}) [∵ sin (90 ^{o})=1]

On equating both the sides, we get

A + B = 90 ^{o} ...(1)

And $\cos(A-B) = \frac{\sqrt{3}}{2}$

⇒ cos(A – B) = cos (30 ^{o}) [∵ $\cos(30°) = \frac{\sqrt{3}}{2}$]

On equating both the sides, we get

A – B = 30 ^{o} ...(2)

On Adding Eq. (1) and (2), we get

2A = 120 ^{o}

⇒ A = 60 ^{o}

Now, Putting the value of A in Eq.(1), we get

60 ^{o} + B =90 ^{o}

⇒ B = 30 ^{o}

Hence, A = 60 ^{o} and B = 30 ^{o}

10. Question

If sin (A + B) = 1 and cos (A – B) = 1, find A and B.

Answer

Given : sin (A+B) =1

⇒ Sin(A+B) = sin (90 ^{o}) [∵ sin (90 ^{o}) =1]

On equating both the sides, we get

$A + B = 90^{o}$ ...(1)

And cos (A – B) = 1

⇒ cos(A – B) = cos (0^{o}) [∵ cos(0^{o}) = 1]

On equating both the sides, we get

$A - B = 0^{o}$ ...(2)

On Adding Eq. (1) and (2), we get

$2A = 90^{o}$

⇒ $A = 45^{o}$

Now, Putting the value of A in Eq.(1), we get

$45^{o} + B = 90^{o}$

⇒ $B = 45^{o}$

Hence, $A = 45^{o}$ and $B = 45^{o}$

11. Question

If sin (A + B) = cos (A – B) = $\frac{\sqrt{3}}{2}$, fins A and B.

Answer

Given : $\sin(A + B) = \frac{\sqrt{3}}{2}$

⇒ Sin(A+B) = sin (60^{o}) [∵ $\sin(60\ o) = \frac{\sqrt{3}}{2}$]

On equating both the sides, we get

$A + B = 60^{o}$ ...(1)

And $\cos(A - B) = \frac{\sqrt{3}}{2}$

⇒ cos(A – B) = cos (30^{o}) [∵ $\cos(30\ o) = \frac{\sqrt{3}}{2}$]

On equating both the sides, we get

$A - B = 30^{o}$ ...(2)

On Adding Eq. (1) and (2), we get

$2A = 90^{o}$

$\Rightarrow A = 45^{o}$

Now, Putting the value of A in Eq.(1), we get

$45^{o} + B = 60^{o}$

$\Rightarrow B = 15^{o}$

Hence, $A = 45^{o}$ and $B = 15^{o}$

12. Question

If sin (A – B) = 1/2, cos(A + B) = 1/2; $0^{o} < A+B < 90^{o}$; A > B, find A and B.

Answer

Given : $\sin(A - B) = \frac{1}{2}$

$\Rightarrow$ Sin(A-B) = sin (30^{o}) [$\because \sin(30 o) = \frac{1}{2}$]

On equating both the sides, we get

$A - B = 30^{o}$ ...(1)

And $\cos(A + B) = \frac{1}{2}$

$\Rightarrow$ cos(A + B) = cos (60^{o}) [$\because \cos(60 o) = \frac{1}{2}$]

On equating both the sides, we get

$A + B = 60^{o}$ ...(2)

On Adding Eq. (1) and (2), we get

$2A = 90^{o}$

$\Rightarrow A = 45^{o}$

Now, Putting the value of A in Eq.(2), we get

$45^{o} + B = 60^{o}$

$\Rightarrow B = 15^{o}$

Hence, $A = 45^{o}$ and $B = 15^{o}$

13 A. Question

Show by an example that

cos A – cos B ≠ cos (A – B)

Answer

Let A = 60^{o} and B = 30^{o}, then

L.H.S. $= \cos A - \cos B = \cos 60° - \cos 30° = \frac{1}{2} - \frac{\sqrt{3}}{2} = \frac{1-\sqrt{3}}{2}$

R. H. S. $= \cos(A - B) = \cos(60° - 30°) = \cos 30° = \frac{\sqrt{3}}{2}$

∴ L.H.S. ≠R.H.S

13 B. Question

Show by an example that

cos C + cos D ≠ cos (C + D)

Answer

Let C = 60^{o} and D = 30^{o}, then

L.H.S. = cos C + cos D = cos 60^{o} + cos 30^{o}

$$= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$$

R. H. S. = cos (C+D) = cos (60^{o} + 30^{o}) = cos 90^{o}= 0

∴ L.H.S. ≠R.H.S

13 C. Question

Show by an example that

sin A + sin B ≠ sin (A + B)

Answer

Let A = 60^{o} and B = 30^{o}, then

L.H.S. = sin A + sin B = sin 60^{o} + sin 30^{o}

$$= \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3}+1}{2}$$

R. H. S. = sin (A + B) = sin (60^{o} + 30^{o}) = sin 90^{o} =1

∴ L.H.S. ≠R.H.S

13 D. Question

Show by an example that

sin A – sin B ≠ sin (A – B)

Answer

Let A = 60^o and B = 30^o, then

L.H.S. = sin A - sin B = sin 60^o - sin 30^o

$$= \frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{\sqrt{3}-1}{2}$$

R. H. S. = sin (A - B) = sin (60^o - 30^o) = sin 30^o

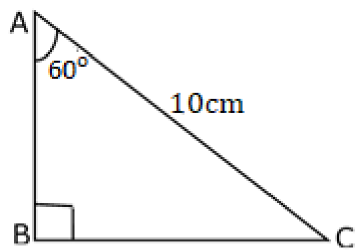
$$= \frac{1}{2}$$

∴ L.H.S. ≠R.H.S

14. Question

In a right ΔABC hypotenuse AC = 10 cm and ∠A = 60°, then find the length of the remaining sides.

Answer



Given: ∠A = 60^o and AC = 10cm

Now, $\text{Sin } 60° = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{BC}{10}$

Now, we know that $\sin 60° = \frac{\sqrt{3}}{2}$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{BC}{10}$$

⇒ BC = $5\sqrt{3}$ cm

In right angled ΔABC , we have

⇒ $(AB)^2 + (BC)^2 = (AC)^2$ [by using Pythagoras theorem]

⇒ $(AB)^2 + (5\sqrt{3})^2 = (10)^2$

⇒ $(AB)^2 + (25 \times 3) = 100$

⇒ $(AB)^2 + 75 = 100$

⇒ $(AB)^2 = 100 - 75$

⇒ $(AB)^2 = 25$

⇒ $AB = \sqrt{25}$

⇒ $AB = \pm 5$

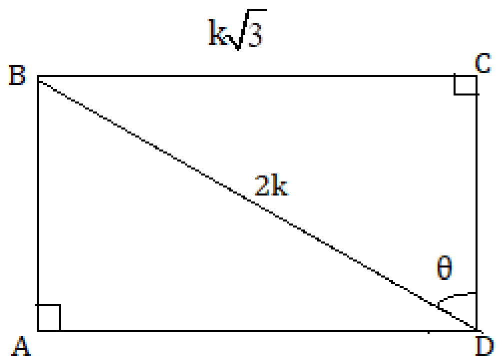
⇒ AB = 5cm [taking positive square root since, side cannot be negative]

∴ Length of the side AB = 5cm and BC = $5\sqrt{3}$ cm

15. Question

In a rectangle ABCD, BD : BC = 2 : $\sqrt{3}$, then find ∠BDC in degrees.

Answer



Given BD: BC = 2 : $\sqrt{3}$

We have to find the ∠BDC

We know that,

$$\text{Sin } \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\Rightarrow \sin\theta = \frac{k\sqrt{3}}{2k}$$

$\Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \sin \theta = \sin 60^o$

$\Rightarrow \theta = 60^o$

Exercise 4.3

1. Question

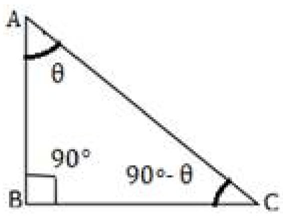
Express the following as trigonometric ratio of complementary angle of θ.

(i) cos θ (ii) sec θ

(iii) cot θ (iv) cosec θ

(v) tan θ

Answer



(i) We know that

$$\cos\theta = \frac{\text{base}}{\text{Hypotenuse}} = \frac{AB}{AC} = \sin(90° - \theta)$$

$\Rightarrow \cos\theta = \sin (90° - \theta)$

(ii) We know that

$$\sec\theta = \frac{\text{hypotenuse}}{\text{base}} = \frac{AC}{AB} = \text{cosec}(90° - \theta)$$

(iii) We know that

$$\cot\theta = \frac{\text{base}}{\text{perpendicular}} = \frac{AB}{BC} = \tan(90° - \theta)$$

(iv) We know that

$$\text{cosec}\,\theta = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{AC}{BC} = \sec(90° - \theta)$$

(v) We know that

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}} = \frac{BC}{AB} = \cot(90° - \theta)$$

2. Question

Express the following as trigonometric ratio of complementary angle of 90^{o}- θ.

(i) tan (90^{o}- θ)

(ii) cos (90^{o}- θ)

Answer

(i) We know that,

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\Rightarrow \tan(90° - \theta) = \frac{\sin(90° - \theta)}{\cos(90° - \theta)}$$

$$\Rightarrow \tan(90° - \theta) = \frac{\sin 90° \cos\theta - \cos 90° \sin\theta}{\cos 90° \cos\theta + \sin 90° \sin\theta}$$ [∵ sin 90° = 1 and cos 90° = 0]

$$\Rightarrow \tan(90° - \theta) = \frac{\cos\theta}{\sin\theta}$$

⇒ tan (90° - θ) = cot θ

(ii) We know that.

Cos(A - B) = cos A cos B + sin A sin B

⇒ cos (90^{o}- θ) = cos 90° cos θ + sin 90° sin θ

⇒ cos (90^{o}- θ) = (0) cos θ + (1) sin θ

⇒ cos (90^{o}- θ) = sin θ

3. Question

Fill up the blanks by an angle between $0^{o}$and 90^{o}:

(i) sin 70^{o} = cos(...) (ii) sin 35^{o} = cos(...)

(iii) cos 48^{o}=sin (...) (iv) cos 70^{o} = sin (...)

(v) cos 50^{o} = sin (...) (vi) sec 32^{o}=cosec(...)

Answer

(i) We know that

Sin $\theta = \cos(90° - \theta)$

Here, $\theta = 70°$

$\Rightarrow \sin 70° = \cos(90° - 70°)$

$\Rightarrow \sin 70° = \cos 20°$

(ii) We know that

Sin $\theta = \cos(90° - \theta)$

Here, $\theta = 35°$

$\Rightarrow \sin 35° = \cos(90° - 35°)$

$\Rightarrow \sin 35° = \cos 55°$

(iii) $\cos\theta = \sin(90° - \theta)$

Here, $\theta = 48°$

$\Rightarrow \cos 48° = \sin(90° - 48°)$

$\Rightarrow \cos 48° = \sin 42°$

(iv) $\cos\theta = \sin(90° - \theta)$

Here, $\theta = 70°$

$\Rightarrow \cos 70° = \sin(90° - 70°)$

$\Rightarrow \cos 70° = \sin 20°$

(v) $\cos\theta = \sin(90° - \theta)$

Here, $\theta = 50°$

$\Rightarrow \cos 50° = \sin(90° - 50°)$

$\Rightarrow \cos 50° = \sin 40°$

(vi) $\sec\theta = \text{cosec}(90° - \theta)$

Here, $\theta = 32°$

$\Rightarrow \sec 32° = \text{cosec}(90° – 32°)$

$\Rightarrow \sec 32° = \text{cosec } 58°$

4. Question

If A+B=90^{o}, then fill up the blanks with suitable trigonometric ratio of complementary angle of A or B.

(i) sin A =.... (ii) cos B =...

(iii) sec A =... (iv) tan B =...

(v) cosec B =... (vi) cot A=...

Answer

(i) Here, A+B = 90°

⇒ A = 90° - B

Multiplying both sides by Sin, we get

Sin A = Sin (90° - B)

⇒ sin A = Cos B [∵ cos θ = sin (90° - θ)]

(ii) Here, A+B = 90°

⇒ B = 90° - A

Multiplying both sides by cos, we get

Cos B = cos (90° - A)

⇒ cos B = sin A [∵ Sin θ = cos (90° - θ)]

(iii) Here, A+B = 90°

⇒ A = 90° - B

Multiplying both sides by sec, we get

Sec A = Sec (90° - B)

⇒ sec A = Cosec B [∵ cosec θ = sec (90° - θ)]

(iv) Here, A+B = 90°

⇒ B = 90° - A

Multiplying both sides by tan, we get

tan B = tan (90° - A)

⇒ tan B = cot A [∵ cot θ = tan (90° - θ)]

(v) Here, A+B = 90°

⇒ B = 90° - A

Multiplying both sides by cosec, we get

Cosec B = cosec (90° - A)

⇒ cosec B = sec A [∵ sec θ = cosec (90° - θ)]

(vi) Here, A+B = 90°

⇒ A = 90° - B

Multiplying both sides by Sin, we get

cotA = cot (90° - B)

⇒ cot A = tan B [∵ tan θ = cot (90° - θ)]

5 A. Question

If sin 37^o=a, then express cos 53^o in terms of a.

Answer

Given sin 37° = a

We know that sin θ = cos (90° - θ)

Here, θ = 37°

⇒ cos (90° - 37°) = a

⇒ cos 53° = a

5 B. Question

If cos 47^o=a, then express sin 43^o in terms of a.

Answer

Given cos 47° = a

We know that cos θ = sin (90° - θ)

Here, θ = 47°

⇒ sin (90° - 47°) = a

⇒ sin 43° = a

5 C. Question

If sin 52^o=a, then express sin 38^o in terms of a.

Answer

Given sin 52° = a

We know that sin θ = cos (90° - θ)

Here, θ = 52°

⇒ cos (90° - 52°) = a

⇒ cos 38° = a

5 D. Question

If sin 56^{O}=x, then express sin 34^{O} in terms of x.

Answer

Given sin 56° = x

We know that sin θ = cos (90° - θ)

Here, θ = 56°

⇒ cos (90° - 56°) = x

⇒ cos 34° = x

6. Question

Find the value of

(i) $\frac{\cos 59^{\circ}}{\sin 31^{\circ}}$ (ii) $\frac{\cos 53^{\circ}}{\sin 37^{\circ}}$

(iii) $\frac{\sin 20^{\circ}}{\cos 70^{\circ}}$ (iv) $\frac{\sqrt{2}\sin 22^{\circ}}{\cos 68^{\circ}}$

(v) $\frac{\sin 10^{\circ}}{\cos 80^{\circ}}$ (vi) $\frac{\sin 27^{\circ}}{\cos 63^{\circ}}$

(vii) $\frac{\sqrt{3}\cos 65^{\circ}}{\sin 25^{\circ}}$ (viii) $\frac{\cos 29^{\circ}}{\sin 61^{\circ}}$

(ix) sin 54° – cos 36° (x) $\frac{\tan 80^{\circ}}{\cot 10^{\circ}}$

(xi) cosec 31° – sec 59° (xii) $\frac{\sin 18^{\circ}}{\cos 72^{\circ}}$

(xiii) $\frac{\tan 65^{0}}{\cot 25^{0}}$

Answer

(i) $\frac{\cos 59°}{\sin 31°} = \frac{\sin(90° - 59°)}{\sin 31°} = \frac{\sin 31°}{\sin 31°} = 1$ [∵ cos θ = sin (90° - θ)]

(ii) $\frac{\cos 53°}{\sin 37°} = \frac{\sin(90° - 53°)}{\sin 37°} = \frac{\sin 37°}{\sin 37°} = 1$ [∵ cos θ = sin (90° - θ)]

(iii) $\frac{\sin 20°}{\cos 70°} = \frac{\cos(90° - 20°)}{\cos 70°} = \frac{\cos 70°}{\cos 70°} = 1$ [∵ Sin θ = cos (90° - θ)]

(iv) $\frac{\sqrt{2}\sin 22°}{\cos 68°} = \frac{\sqrt{2}\cos(90° - 22°)}{\cos 68°} = \frac{\sqrt{2}\cos 68°}{\cos 68°} = \sqrt{2}$ [∵ Sin θ = cos (90° - θ)]

(v) $\frac{\sin 10°}{\cos 80°} = \frac{\cos(90° - 10°)}{\cos 80°} = \frac{\cos 80°}{\cos 80°} = 1$ [∵ Sin θ = cos (90° - θ)]

(vi) $\frac{\sin 27°}{\cos 63°} = \frac{\cos(90° - 27°)}{\cos 63°} = \frac{\cos 63°}{\cos 63°} = 1$ [∵ Sin θ = cos (90° - θ)]

(vii) $\frac{\sqrt{3}\cos 65°}{\sin 25°} = \frac{\sqrt{3}\sin(90° - 65°)}{\sin 25°} = \frac{\sin 25°}{\sin 25°} = \sqrt{3}$ [∵ cos θ = sin (90° - θ)]

(viii) $\frac{\cos 29°}{\sin 61°} = \frac{\sin(90° - 29°)}{\sin 61°} = \frac{\sin 61°}{\sin 61°} = 1$ [∵ cos θ = sin (90° - θ)]

(ix) sin 54° - sin(90° - 36°) [∵ cos θ = sin (90° - θ)]

⇒ sin 54° - sin 54°

⇒ 0

(x) $\frac{\tan 80°}{\cot 10°} = \frac{\cot(90° - 80°)}{\cot 10°} = \frac{\cot 10°}{\cot 10°} = 1$ [∵ tan θ = cot (90° - θ)]

(xi) cosec 31° - cosec(90° - 59°) [∵ sec θ = cosec (90° - θ)]

⇒ cosec 31° - cosec 31°

⇒ 0

(xii) $\frac{\sin 18°}{\cos 72°} = \frac{\cos(90° - 18°)}{\cos 72°} = \frac{\cos 72°}{\cos 72°} = 1$ [∵ Sin θ = cos (90° - θ)]

(xiii) $\frac{\tan 65°}{\cot 25°} = \frac{\cot(90° - 65°)}{\cot 25°} = \frac{\cot 25°}{\cot 25°} = 1$ [∵ tan θ = cot (90° - θ)]

7. Question

Fill up the blanks :

(i) If sin 50^{0}=0.7660, then cos 40^{0}=……

(ii) If cos 44^{o} = 0.7193, then sin 46^{o}=.....

(iii) sin 50^{o}+cos 40^{o} = 2 sin (.........)

(iv) Value of $\frac{\sin 70^{\circ}}{\cos 20^{\circ}}$ is ....

Answer

(i) Given: sin 50^{o}=0.7660

We know that

Sin θ = cos (90° - θ)

⇒ cos (90° - 50°) = 0.7660

⇒ cos 40° = 0.7660

(ii) Given: cos 44° = 0.7193

We know that,

cos θ = sin (90° - θ)

⇒ sin (90° - 44°) = 0.7193

⇒ sin 46° = 0.7193

(iii) LHS = sin 50° + cos 40°

⇒ sin 50° + sin (90° - 40°) [∵ cos θ = sin (90° - θ)]

⇒ sin 50° + sin 50°

⇒ 2sin 50°

(iii) $\frac{\sin 70^{\circ}}{\cos 20^{\circ}} = \frac{\cos(90^{\circ}-70^{\circ})}{\cos 20^{\circ}} = \frac{\cos 20^{\circ}}{\cos 20^{\circ}} = 1$ [∵ Sin θ = cos (90° - θ)]

8 A. Question

If A + B = 90^{o}, then express cos B in terms of simplest trigonometric ratio of A.

Answer

Given: A+B =90°

⇒ B = 90° - A

Multiplying both side by cos, we get

= cos B = Cos (90° - A)

⇒ cos B = sin A [∵ Sin θ = cos (90° - θ)]

8 B. Question

If X + Y = 90°, then express cos X in terms of simplest trigonometric ratio of Y.

Answer

Given: X+Y =90°

⇒ X= 90° - Y

Multiplying both side by cos, we get

= cos X = Cos (90° - Y)

⇒ cos X = sin Y [∵ Sin θ = cos (90° - θ)]

9 A. Question

If A + B = 90°, sin A = a, sin B = b, then prove that

(a) $a^2 + b^2 = 1$

(b) $\tan A = \frac{a}{b}$

Answer

(a) LHS = $a^2 + b^2$

= $(\sin A)^2 + (\sin B)^2$

= $\sin^2 A + \sin^2 B$

= $\sin^2 A + \sin^2 (90° - A)$ [∵ cos θ = sin (90° - θ)]

= $\sin^2 A + \cos^2 A$

= 1 [∵ $\sin^2 \theta + \cos^2 \theta = 1$]

=RHS

Hence Proved

(b) LHS = tan A

Now, taking RHS = $\frac{a}{b}$

$\Rightarrow \frac{\sin A}{\sin B}$

$\Rightarrow \frac{\sin A}{\sin(90°-A)}$ {given, A +B = 90°)

$\Rightarrow \frac{\sin A}{\cos A}$ [∵ cos θ = sin (90° - θ)]

⇒ tan A

=LHS

∴ LHS = RHS

Hence Proved

9 B. Question

Show that sin(50° + θ) – cos (40° – θ) = 0.

Answer

LHS = sin (50° + θ) – cos (40° - θ)

We know that,

Sin A = cos (90° - A)

Here, A = 50° + θ

⇒ cos {90° -(50° + θ)} – cos (40° - θ)

⇒ cos (40° - θ) – cos (40° - θ)

= 0 = RHS

Hence Proved

10. Question

Prove that $\frac{\cos\theta}{\sin(90^{o}-\theta)}+\frac{\sin\theta}{\cos(90^{o}-\theta)}=2$

Answer

Taking LHS,

$\frac{\cos\theta}{\sin(90°-\theta)}+\frac{\sin\theta}{\cos(90°-\theta)}$ [∵ cos θ = sin (90° - θ) and Sin θ = cos (90° - θ)]

$\Rightarrow \frac{\cos\theta}{\cos\theta}+\frac{\sin\theta}{\sin\theta}$

$\Rightarrow$ 1+ 1

= 2 = RHS

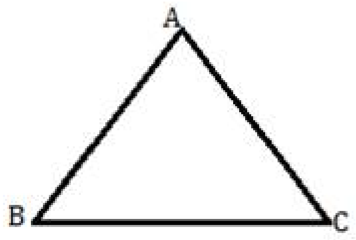
Hence Proved

11 A. Question

In a ΔABC prove that

$$\sin\frac{B+C}{2}=\cos\frac{A}{2}$$

Answer



In ΔABC,

Sum of angles of a triangle = 180°

A + B + C = 180°

$\Rightarrow$ B + C = 180° - A

Multiplying both sides by $\frac{1}{2}$

$$=\frac{B+C}{2}=\frac{180°-A}{2}$$

$$=\frac{B+C}{2}=\frac{180°}{2}-\frac{A}{2}$$

$$=\frac{B+C}{2}=90°-\frac{A}{2} \ldots(1)$$

Taking LHS

$$\sin\frac{B+C}{2}$$

$$=\sin\left(90°-\frac{A}{2}\right) \text{ (from eq (1))}$$

$$=\cos\frac{A}{2} \; [\because \sin(90° - \theta) = \cos\theta]$$

=RHS

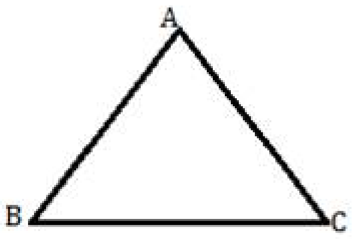
Hence Proved

11 B. Question

In a ΔABC prove that

$$\tan\frac{B+C}{2}=\cot\frac{A}{2}$$

Answer



In ΔABC,

Sum of angles of a triangle = 180°

A + B + C = 180°

⇒ B + C = 180° - A

Multiplying both sides by $\frac{1}{2}$

$$=\frac{B+C}{2}=\frac{180°-A}{2}$$

$$=\frac{B+C}{2}=\frac{180°}{2}-\frac{A}{2}$$

$=\frac{B+C}{2}=90°-\frac{A}{2}$ ...(2)

Taking LHS

$$\tan\frac{B+C}{2}$$

$=\tan\left(90°-\frac{A}{2}\right)$ (from eq (2))

$=\cot\frac{A}{2}$ [∵ tan (90° - θ) = cot θ]

=RHS

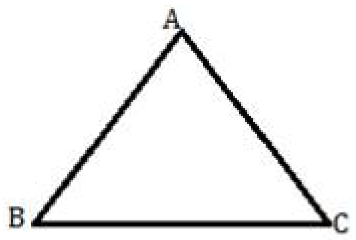
Hence Proved

11 C. Question

In a ΔABC prove that

$$\cos\frac{A+B}{2} = \sin\frac{C}{2}$$

Answer



In ΔABC,

Sum of angles of a triangle = 180°

A + B + C = 180°

⇒ A + B = 180° - C

Multiplying both sides by $\frac{1}{2}$

$$= \frac{A+B}{2} = \frac{180° - C}{2}$$

$$= \frac{A+B}{2} = \frac{180°}{2} - \frac{C}{2}$$

$$= \frac{A+B}{2} = 90° - \frac{C}{2} \text{...(3)}$$

Taking LHS

$$\cos\frac{A+B}{2}$$

$$= \cos\left(90° - \frac{C}{2}\right) \text{(from eq (3))}$$

$$= \sin\frac{C}{2} \text{ [∵ cos (90° - θ) = sin θ]}$$

=RHS

Hence Proved

12 A. Question

If sin 3A = cos(A – 26^{o}), where 3A is an acute angle, find the value of A.

Answer

sin 3A = cos (A-26°) …(i)

We know that

Sin θ = cos (90° - θ)

So, Eq. (i) become

Cos (90° - 3A) = cos (A -26°)

On Equating both the sides, we get

90° - 3A = A – 26°

⇒ -3A - A = -26° -90°

⇒ -4A = -116°

⇒ A = 29°

12 B. Question

Find θ if cos(2 θ +54^{o})= sin θ, where (2θ +54^{o}) is an acute angle.

Answer

cos(2 θ +54^{o})= sin θ …(i)

We know that

Sin θ = cos (90° - θ)

So, Eq. (i) become

cos(2 θ +54^{o}) = cos(90° - θ)

On Equating both the sides, we get

2θ + 54° = 90° - θ

⇒ 2θ + θ = 90° - 54°

⇒ 3θ = 36°

⇒ θ = 12°

12 C. Question

If tan 3 θ =cot (θ +18^{o}), where 3 θ and θ +18^{o} are acute angles, find the value of θ.

Answer

tan 3θ = cot (θ + 18°) ...(i)

We know that

tan θ = cot (90° - θ)

So, Eq. (i) become

Cot (90° - 3θ) = cot (θ + 18°)

On Equating both the sides, we get

90° - 3θ = θ + 18°

⇒ -3θ - θ = 18° -90°

⇒ -4θ = -72°

⇒ θ = 18°

12 D. Question

If sec 5 θ =cosec (θ -36^{o}), where 5 θ is an acute angle, find the value of θ.

Answer

sec 5θ = cosec (θ-36°) ...(i)

We know that

sec θ = cosec (90° - θ)

So, Eq. (i) become

Cosec (90° - 5θ) = cosec (θ -36°)

On Equating both the sides, we get

90° - 5θ = θ -36°

⇒ -5θ - θ = -36° -90°

⇒ -6θ = -126°

⇒ θ = 21°

13. Question

Prove that :

sin 70^{o}. sec 20^{o}=1

Answer

Taking LHS

sin 70° sec 20°

$\Rightarrow \sin 70° \times \frac{1}{\cos 20°}$

$\Rightarrow \sin 70° \times \frac{1}{\sin(90°-20°)}$ [∵ cos θ = sin (90° - θ)]

$\Rightarrow \frac{\sin 70°}{\sin 70°}$

= 1 = RHS

Hence Proved

14. Question

Prove that :

sin (90^{o}- θ) tan θ=sin θ

Answer

Taking LHS

Sin(90° - θ) tanθ [∵ cos θ = sin (90° - θ)]

⇒ cos θ tan θ

$\Rightarrow \cos\theta \times \frac{\sin\theta}{\cos\theta}$ [∵ $\tan\theta = \frac{\sin\theta}{\cos\theta}$]

= sin θ = RHS

Hence Proved

15. Question

Prove that :

tan 63^{o}. tan 27^{o}=1

Answer

Taking LHS

Tan 63° tan 27°

$\Rightarrow$ tan 63° cot (90° - 27°) [$\because$ tan θ = cot (90° - θ)]

$\Rightarrow$ tan 63° cot 63°

$$\Rightarrow \tan 63° \times \frac{1}{\tan 63°} \left[\because \tan\theta = \frac{1}{\cot\theta}\right]$$

= 1 =RHS

Hence Proved

16. Question

Prove that :

$$\frac{\sin(90^o - \theta)\sin\theta}{\tan\theta} - 1 = -\sin^2\theta$$

Answer

Taking LHS

$$= \frac{\sin(90° - \theta)\sin\theta}{\tan\theta} - 1$$

$$= \frac{\cos\theta\sin\theta}{\frac{\sin\theta}{\cos\theta}} - 1$$

$$= \frac{\cos\theta\sin\theta \times \cos\theta}{\sin\theta} - 1$$

= $\cos^2\theta - 1$

= $-\sin^2\theta$ [$\because \cos^2\theta + \sin^2\theta = 1$]

= RHS

Hence Proved

17. Question

Prove that :

$\sin 55^o . \cos 48^o = \cos 35^o . \sin 42^o$

Answer

Taking LHS = sin 55 ° cos 48°

We know that

$\cos\theta = \sin(90° - \theta)$

Here, $\theta = 48°$

⇒ $\sin 55° \sin(90° - 48°)$

⇒ $\sin 55° \sin 42°$

We also know that

$\text{Sin}\,\theta = \cos(90° - \theta)$

Here, $\theta = 55°$

⇒ $\cos(90° - 55°) \sin 42°$

⇒ $\cos 35° \sin 42° = \text{RHS}$

Hence Proved

18. Question

Prove that :

$\sin^2 25^o+\sin^2 65° = \cos^2 63°+\cos^2 39^o$

Answer

Taking LHS = $\sin 25^o+\sin 65^o$

We know that

$\text{Sin}\,\theta = \cos(90° - \theta)$

Here, $\theta = 25°$

⇒ $\cos^2(90° - 25°)+ \sin^2 65°$

⇒ $\cos^2 65° + \sin^2 65°$

$= 1$ [$\because \cos^2\theta + \sin^2\theta = 1$]

Now, RHS = $\cos^2 63^o+\cos^2 39^o$

We know that

$\cos\theta = \sin(90° - \theta)$

Here, $\theta = 39°$

⇒ $\cos^2 63° + \sin^2(90° - 39°)$

$\Rightarrow \cos^2 63° + \sin^2 63°$

$=1$ [$\because \cos^2 \theta + \sin^2 \theta = 1$]

LHS = RHS

Hence Proved

19. Question

Prove that :

$\sin 54^o + \cos 67^o = \sin 23^o + \cos 36^o$

Answer

Taking LHS = $\sin 54^o + \cos 67^o$

We know that

$\cos \theta = \sin (90° - \theta)$

Here, $\theta = 67°$

$\Rightarrow \sin 54° + \sin (90° - 67°)$

$\Rightarrow \sin 54° + \sin 23°$

We also know that

$\sin \theta = \cos (90° - \theta)$

Here, $\theta = 54°$

$\Rightarrow \cos (90° - 54°) + \sin 23°$

$\Rightarrow \cos 36° + \sin 23°$ = RHS

Hence Proved

20. Question

Prove that :

$\cos 27 + \sin 51^o = \sin 63^o + \cos 39^o$

Answer

Taking LHS = $\cos 27 + \sin 51^o$

We know that

$\cos \theta = \sin (90° - \theta)$

Here, θ = 27°

⇒ sin (90° - 27°)+ sin 51°

⇒ sin 63°+ sin 51°

We also know that

Sin θ = cos (90° - θ)

Here, θ = 51°

⇒ sin 63°+ cos (90° - 51°)

⇒ sin 63°+ cos 39° = RHS

Hence Proved

21. Question

Prove that :

$\sin^2 40^o + \sin^2 50^o = 1$

Answer

Taking LHS= $\sin^2 40^o + \sin^2 50^o$

⇒ $\cos^2 (90° - 40°) + \sin^2 50°$ [∵ Sin θ = cos (90° - θ)]

⇒ $\cos^2 50° + \sin^2 50°$

= 1 =RHS [∵ $\cos^2 \theta + \sin^2 \theta = 1$]

Hence Proved

22. Question

Prove that :

$\sin^2 29^o + \sin^2 61^o = 1$

Answer

Taking LHS= $\sin^2 29^o + \sin^2 61^o$

⇒ $\cos^2 (90° - 29°) + \sin^2 61°$ [∵ Sin θ = cos (90° - θ)]

⇒ $\cos^2 61° + \sin^2 61°$

= 1 =RHS [∵ $\cos^2 \theta + \sin^2 \theta = 1$]

Hence Proved

23. Question

Prove that :

sin θ .cos (90° - θ) + cos θ sin (90° - θ).

Answer

Taking LHS = sin θ cos (90° - θ) + cos θ sin (90° - θ)

⇒ sin θ × sin θ + cos θ × cos θ [∵ Sin θ = cos (90° - θ) and cos θ = sin (90° - θ)]

⇒ $\cos^2 \theta + \sin^2 \theta$ [∵ $\cos^2 \theta + \sin^2 \theta = 1$]

= 1 = RHS

Hence Proved

24. Question

Prove that :

cos θ . cos(90° – θ) + sin θ sin (90° – θ) = 0

Answer

Taking LHS = cos θ cos (90° - θ) + sin θ sin (90° - θ)

⇒ cos θ × sin θ – sinθ × cos θ [∵ Sin θ = cos (90° - θ) and cos θ = sin (90° - θ)]

= 0 = RHS

Hence Proved

25. Question

Prove that :

sin 42°. cos 48° + cos 42° . sin 48° = 1

Answer

Taking LHS

= sin 42° cos 48° + cos 42° sin 48°

= cos (90° - 42°) cos 48° + sin (90° - 42°) sin 48°

[∵ Sin θ = cos (90° - θ) and cos θ = sin (90° - θ)]

= cos 48° cos 48° + sin 48° sin 48°

$= \cos^2 48° + \sin^2 48°$

$= 1$ [∵ $\cos^2 \theta + \sin^2 \theta = 1$]

=LHS=RHS

Hence Proved

26. Question

Prove that :

$$\frac{\cos 20^o}{\sin 70^o} + \frac{\cos\theta}{\sin(90^o - \theta)} = 2$$

Answer

Taking LHS

$$= \frac{\cos 20°}{\sin 70°} + \frac{\cos\theta}{\sin(90° - \theta)}$$

$= \frac{\cos 20°}{\cos(90° - 70°)} + \frac{\cos\theta}{\cos\theta}$ [∵ Sin θ = cos (90° - θ) and cos θ = sin (90° - θ)]

$$= \frac{\cos 20°}{\cos 20°} + 1$$

= 1 + 1

= 2 = RHS

Hence Proved

27. Question

Prove that :

tan 27° tan 45° tan 63°

Answer

Taking LHS

= tan 27° tan 45° tan 63°

=tan (90° - 27°) tan 45° tan 63° [∵ tan θ = cot (90° - θ)]

=cot 63° tan 45° tan 63° $\left[\because \tan\theta = \frac{1}{\cot\theta}\right]$

$= \frac{1}{\tan 63°} \times \tan 45° \times \tan 63°$

= tan 45° [∵ tan 45° =1]

=1 =RHS

Hence Proved

28. Question

Prove that :

tan 9°. tan 27°. tan 45°. tan 63°. tan 81° = 1

Answer

Taking LHS

= tan 9° tan 27° tan 45° tan 63° tan 81°

=cot(90° - 9°) tan (90° - 27°) tan 45° tan 63° tan 81° [∵ tan θ = cot (90° - θ)]

=cot 81° cot 63° tan 45° tan 63° tan 81° $\left[\because \tan\theta = \frac{1}{\cot\theta}\right]$

$= \frac{1}{\tan 81°} \times \frac{1}{\tan 63°} \times \tan 45° \times \tan 63° \times \tan 81°$

= tan 45° [∵ tan 45° =1]

=1 =RHS

Hence Proved

29. Question

Prove that :

sin 9°. sin 27°. sin 63°. sin 81°

= cos9°.cos27°.cos63°.cos81°

Answer

Taking LHS

= sin 9° sin 27° sin 63° sin 81°

= cos (90° - 9°) cos (90° - 27°) cos (90° – 63°) cos (90°- 81°)

= cos 81° cos 63° cos 27° cos 9°

Or cos 9° cos 27° cos 63° cos 81° = RHS

Hence Proved

30 A. Question

Prove that :

$$\tan 7^{\circ}.\tan 23^{\circ}.\tan 60^{\circ}.\tan 67^{\circ}.\tan 83^{\circ} = \sqrt{3}$$

Answer

Taking LHS

= tan 7° tan 23° tan 60° tan 67° tan 83°

=cot(90° - 7°) tan (90° - 23°) tan 60° tan 67° tan 83° [∵ tan θ = cot (90° - θ)]

=cot 83° cot 67° tan 60° tan 67° tan 83° $\left[\because \tan\theta = \frac{1}{\cot\theta}\right]$

$$= \frac{1}{\tan 83^{\circ}} \times \frac{1}{\tan 67^{\circ}} \times \tan 60^{\circ} \times \tan 67^{\circ} \times \tan 83^{\circ}$$

= tan 60° [∵ tan 60° =√3]

=√3 =RHS

30 B. Question

Prove that :

$$\tan 15^{\circ} \tan 25^{\circ} \tan 60^{\circ} \tan 65^{\circ} \tan 75^{\circ} = \sqrt{3}$$

Answer

Taking LHS

= tan 15° tan 25° tan 60° tan 65° tan 75°

=cot(90° - 15°) tan (90° - 25°) tan 60° tan 65° tan 75° [∵ tan θ = cot (90° - θ)]

=cot 75° cot 65° tan 60° tan 65° tan 75° $\left[\because \tan\theta = \frac{1}{\cot\theta}\right]$

$$= \frac{1}{\tan 75^{\circ}} \times \frac{1}{\tan 65^{\circ}} \times \tan 60^{\circ} \times \tan 65^{\circ} \times \tan 75^{\circ}$$

= tan 60° [∵ tan 60° =√3]

=√3 =RHS

31. Question

Find the value off the following:

$$\frac{\sin 50^{\circ}}{\cos 40^{\circ}}+\frac{\operatorname{cosec} 40^{\circ}}{\sec 50^{\circ}}-4 \cos 50^{\circ} . \operatorname{cosec} 40^{\circ}$$

Answer

$$\frac{\sin 50^{\circ}}{\cos 40^{\circ}}+\frac{\operatorname{cosec} 40^{\circ}}{\sec 50^{\circ}}-4 \cos 50^{\circ}\ \operatorname{cosec} 40^{\circ}$$

$$=\frac{\sin 50^{\circ}}{\sin(90^{\circ}-40^{\circ})}+\frac{\operatorname{cosec} 40^{\circ}}{\operatorname{cosec}(90^{\circ}-50^{\circ})}-4 \sin(90^{\circ}-50^{\circ}) \operatorname{cosec} 40^{\circ}$$

[∵ cos θ = sin (90° - θ) and sec θ = cosec (90° - θ)]

$$=\frac{\sin 50^{\circ}}{\sin 50^{\circ}}+\frac{\operatorname{cosec} 40^{\circ}}{\operatorname{cosec} 40^{\circ}}-4 \sin 40^{\circ} \times \frac{1}{\sin 40^{\circ}}\left[\because \sin\theta=\frac{1}{\operatorname{cosec}\theta}\right]$$

= 1 + 1 – 4

= -2

32. Question

Find the value off the following:

$$\frac{\cos^{2} 20^{0}+\cos^{2} 70^{\circ}}{\sin^{2} 59^{\circ}+\sin^{2} 31^{\circ}}+\sin 35^{\circ} . \sec 55^{\circ}$$

Answer

$$\frac{\cos^{2} 20^{\circ}+\cos^{2} 70^{\circ}}{\sin^{2} 59^{\circ}+\sin^{2} 31^{\circ}}+\cos(90^{\circ}-35^{\circ}) \sec 55^{\circ}$$

$$=\frac{\cos^{2} 20^{\circ}+\sin^{2}(90^{\circ}-70^{\circ})}{\sin^{2}(59^{\circ})+\cos^{2}(90^{\circ}-31^{\circ})}+\cos 55^{\circ} \sec 55^{\circ}$$

[∵ cos θ = sin (90° - θ) and sec θ = cosec (90° - θ)]

$$=\frac{\cos^{2} 20^{\circ}+\sin^{2} 20^{\circ}}{\sin^{2}(59^{\circ})+\cos^{2} 59^{\circ}}+\cos 55^{\circ} \times \frac{1}{\cos 55^{\circ}}$$ [∵ $\cos^2\theta + \sin^2\theta = 1$]

= 1 + 1

=2

33. Question

Find the value off the following:

$$\frac{\tan 50^{\circ}+\sec 50^{\circ}}{\cot 40^{\circ}+\operatorname{cosec} 40^{\circ}}+\cos 40^{\circ} . \operatorname{cosec} 50^{\circ}$$

Answer

$$\frac{\tan 50°+\sec 50°}{\cot 40°+\operatorname{cosec} 40°}+\cos 40° \operatorname{cosec} 50°$$

$$=\frac{\cot(90°-50°)+\operatorname{cosec}(90°-50°)}{\cot 40°+\operatorname{cosec} 40°}+\sin(90°-40°) \operatorname{cosec} 50°$$

[∵ tan θ = cot (90° - θ) , sec θ = cosec (90° - θ) and cos θ = sin (90° - θ)]

$$=\frac{\cot 40°+\operatorname{cosec} 40°}{\cot 40°+\operatorname{cosec} 40°}+\sin 50° \operatorname{cosec} 50°$$

$$=1+\sin 50° \times \frac{1}{\sin 50°} \quad \because \ \sin\theta = \frac{1}{\operatorname{cosec}\theta}$$

= 1 + 1

= 2

34. Question

Find the value off the following:

cosec (65° + θ) – sec (25° - θ) – tan(55° - θ) + cot(35° +θ)

Answer

cosec (65° + θ) – sec (25° - θ) – tan(55° - θ) + cot(35° +θ)

= sec {90°-(65°+θ)} – sec (25° - θ) – tan(55° - θ) + tan {90°-(35° +θ)}

[∵ cosec θ = sec (90° - θ) and cot θ = tan (90° - θ)]

= sec (90° - 65°-θ) – sec (25° - θ) – tan(55° - θ) + tan (90°- 35° - θ)

= sec (25° - θ) - sec (25° - θ) – tan(55° - θ) + tan (55° - θ)

= 0

35. Question

Find the value off the following:

$$\frac{\cos 35^{o}}{\sin 55^{o}}+\frac{\sin 11^{o}}{\cos 79^{o}}-\cos 28^{o}.\operatorname{cosec} 62^{o}$$

Answer

$$\frac{\cos 35°}{\sin 55°}+\frac{\sin 11°}{\cos 79°}-\cos 28° \operatorname{cosec} 62°$$

$$= \frac{\sin(90° - 35°)}{\sin 55°} + \frac{\sin 11°}{\sin(90° - 79°)} - \sin(90° - 28°)\,\text{cosec}\,62°$$

$$= \frac{\sin 55°}{\sin 55°} + \frac{\sin 11°}{\sin 11°} - \sin 62°\,\text{cosec}\,62°$$

$$= 1 + 1 - \sin 62° \times \frac{1}{\sin 62°}$$

= 1 + 1 – 1

= 1

36. Question

Find the value off the following:

$$\frac{\cos^2 20° + \cos^2 70°}{\sin^2 59° + \sin^2 31°}$$

Answer

$$\frac{\cos^2 20° + \cos^2 70°}{\sin^2 59° + \sin^2 31°}$$

$$= \frac{\cos^2 20° + \sin^2(90° - 70°)}{\sin^2(59°) + \cos^2(90° - 31°)}$$

[∵ cos θ = sin (90° - θ) and sec θ = cosec (90° - θ)]

$$= \frac{\cos^2 20° + \sin^2 20°}{\sin^2(59°) + \cos^2 59°}$$

= 1

37. Question

Find the value off the following:

cosec (65° + θ) – sec (25° - θ)

Answer

cosec (65° + θ) – sec (25° - θ)

= sec {90°-(65°+θ)} – sec (25° - θ)

[∵ cosec θ = sec (90° - θ)]

= sec (90° - 65°-θ) – sec (25° - θ)

= sec (25° - θ) - sec (25° - θ)

= 0

38. Question

Find the value off the following:

cos (60° + θ) – sin (30° - θ)

Answer

cos (60° + θ) – sin (30° - θ)

= sin {90°-(60°+θ)} – sin (30° - θ) [∵ cos θ = sin (90° - θ)]

= sin (90° - 60°-θ) – sin (30° - θ)

= sin (30° - θ) - sin (30° - θ)

= 0

39. Question

Find the value off the following:

sec 70°. sin 20° - cos 20°. cosec 70°

Answer

sec 70° sin 20° - cos 20° cosec 70°

= cosec (90°-70°) cos (90° - 20°)- cos 20° cosec 70°

[∵ sec θ = cosec (90° - θ) and Sin θ = cos (90° - θ)]

= cosec 70° cos 20° - cos 20° cosec 70°

=0

40. Question

Find the value off the following:

(sin 72° + cos 18°)(sin 72° - cos 18°)

Answer

(sin 72° + cos 18°)(sin 72° - cos 18°)

Using the identity , $(a-b)(a+b) = a^2 - b^2$

= $(\sin 72°)^2 - (\cos 18°)^2$

$= \{\cos(90° - 72°)\}^2 - (\cos 18°)^2$ [∵ Sin θ = cos (90° - θ)]

$=(\cos 18°)^2 - (\cos 18°)^2$

= 0

41. Question

Find the value off the following:

$$\left(\frac{\sin 35^\circ}{\cos 55^\circ}\right)^2 + \left(\frac{\cos 55^\circ}{\sin 35^\circ}\right) - 2\cos 60^\circ$$

Answer

$$\left(\frac{\sin 35^\circ}{\cos 55^\circ}\right)^2 + \left(\frac{\cos 55^\circ}{\sin 35^\circ}\right)^2 - 2\cos 60^\circ$$

$= \left(\frac{\sin 35^\circ}{\sin(90^\circ - 55^\circ)}\right)^2 + \left(\frac{\sin(90^\circ - 55^\circ)}{\sin 35^\circ}\right)^2 - 2\cos 60^\circ$ [∵ cos θ = sin (90° - θ)]

$$= \left(\frac{\sin 35^\circ}{\sin 35^\circ}\right)^2 + \left(\frac{\sin 35^\circ}{\sin 35^\circ}\right)^2 - 2\left(\frac{1}{2}\right)$$

= 1 + 1 - 1

= 1

42. Question

Find the value off the following:

$$\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ . \operatorname{cosec} 31^\circ$$

Answer

$$\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec} 31^\circ$$

$$= \frac{\sin(90^\circ - 80^\circ)}{\sin 10^\circ} + \sin(90^\circ - 59^\circ) \operatorname{cosec} 31^\circ$$

$$= \frac{\sin 10^\circ}{\sin 10^\circ} + \sin 31^\circ \operatorname{cosec} 31^\circ$$

$$= 1 + \sin 31^\circ \times \frac{1}{\sin 31^\circ}$$

= 1 + 1

= 2

43. Question

Find the value off the following:

(sin 50° + θ)- cos (40° - θ) + tan 1°. tan 10° tan 20°. tan 70°. tan 80°. tan 89°

Answer

(sin 50° + θ)- cos (40° -θ) + tan 1° tan 10° tan 20° tan 70° tan 80° tan 89°

= cos {90° -(50° + θ)} - cos (40° -θ)+ cot(90° - 1°) tan (90° - 10°) cot(90° - 20°) tan 70° tan 80° tan 89° [∵ Sin θ = cos (90° - θ) & tan θ = cot (90° - θ)]

= cos (40° -θ) - cos (40° -θ) + cot89° cot80° cot70° tan 70° tan 80° tan 89°

$$= \frac{1}{\tan 89°} \times \frac{1}{\tan 80°} \times \frac{1}{\tan 70°} \times \tan 70° \tan 80° \tan 89° \left[\because \cot\theta = \frac{1}{\tan\theta}\right]$$

= 1

44. Question

Find the value off the following:

$$\sec^2 10^o - \cot^2 80^o + \frac{\sin 15^o \cos 75^o + \cos 15^o . \sin 75^o}{\cos\theta \sin(90-\theta) + \sin\theta \cos(90^o - \theta)}$$

Answer

$$\sec^2 10° - \cot^2 80° + \frac{\sin 15° \cos 75° + \cos 15° \sin 75°}{\cos\theta \sin(90°-\theta) + \sin\theta \cos(90°-\theta)}$$

$$= \sec^2 10° - \tan^2(90° - 80°) + \frac{\cos(90° - 15°)\cos 75° + \sin(90° - 15°)\sin 75°}{\cos\theta \sin(90° - \theta) + \sin\theta \cos(90° - \theta)}$$

[∵ cot θ = tan (90° - θ), cos θ = sin (90° - θ) and Sin θ = cos (90° - θ)]

$$= \sec^2 10° - \tan^2 10° + \frac{\cos 75° \cos 75° + \sin 75° \sin 75°}{\cos\theta \cos(\theta) + \sin\theta \sin\theta}$$

[∵ 1+$\tan^2$ θ =$\sec^2$ θ and $\cos^2$ θ +$\sin^2$θ = 1]

$$= 1 + \frac{\cos^2 75° + \sin^2 75^c}{\cos^2\theta + \sin^2\theta}$$

= 1 + 1

= 2

45. Question

Find the value off the following:

$$\cos(40° + \theta) - \sin(50° - \theta) + \frac{\cos^2 40° + \cos^2 50°}{\sin^2 40° + \sin^2 50°}$$

Answer

$$\cos(40° + \theta) - \sin(50° - \theta) + \frac{\cos^2 40° + \cos^2 50°}{\sin^2 40° + \sin^2 50°}$$

$$= \sin\{90° - (40° + \theta)\} - \sin(50° - \theta) + \frac{\cos^2 40° + \sin^2(90° - 50°)}{\sin^2 40° + \cos^2(90° - 50°)}$$

[∵ Sin θ = cos (90° - θ) and cos θ = sin (90° - θ)]

$$= \sin(50° - \theta) - \sin(50° - \theta) + \frac{\cos^2 40° + \sin^2(40°)}{\sin^2 40° + \cos^2(40°)}$$

= 0 + 1

= 1

46. Question

Find the value off the following:

$$\frac{\cos 70^0}{\sin 20^0} + \frac{\cos 55^0, \cos ec 35^0}{\tan 5^0 . \tan 25^0 . \tan 45^0 \tan 65^0 \tan 85^0}$$

Answer

$$\frac{\cos 70°}{\sin 20°} + \frac{\cos 55° \operatorname{cosec} 35°}{\tan 5° \tan 25° \tan 45° \tan 65° \tan 85°}$$

$$= \left(\frac{\sin(90° - 70°)}{\sin 20°}\right) + \frac{\sin(90° - 55°) \operatorname{cosec} 35°}{\cot(90° - 5°) \cot(90° - 25°) \tan 45° \tan 65° \tan 85°}$$

[∵ cos θ = sin (90° - θ) and tan θ = cot (90° - θ)]

$$= \frac{\sin 20°}{\sin 20°} + \frac{\sin 35° \operatorname{cosec} 35°}{\cot 85° \cot 65° \tan 45° \tan 65° \tan 85°}$$

$$= 1 + \frac{\sin 35° \times \frac{1}{\sin 35°}}{\frac{1}{\tan 85°} \times \frac{1}{\tan 65°} \times \tan 45° \times \tan 65° \times \tan 85°}$$

= 1 + 1 [∵ tan 45° = 1]

= 2

47. Question

Find the value off the following:

$$\left(\frac{\sin 27^{\circ}}{\cos 63^{\circ}}\right)^2 + \left(\frac{\cos 63^{\circ}}{\sin 27^{\circ}}\right)^2$$

Answer

$$\left(\frac{\sin 27^{\circ}}{\cos 63^{\circ}}\right)^2 + \left(\frac{\cos 63^{\circ}}{\sin 27^{\circ}}\right)^2$$

$= \left(\frac{\sin 27^{\circ}}{\sin(90^{\circ}-63^{\circ})}\right)^2 + \left(\frac{\sin(90^{\circ}-63^{\circ})}{\sin 27^{\circ}}\right)^2$ [∵ cos θ = sin (90° - θ)]

$$= \left(\frac{\sin 27^{\circ}}{\sin 27^{\circ}}\right)^2 + \left(\frac{\sin 27^{\circ}}{\sin 27^{\circ}}\right)^2$$

= 1 + 1

= 2

48 A. Question

Evaluate the following

$$\frac{3\sin 5^{\circ}}{\cos 85^{\circ}} + \frac{2\cos 33^{\circ}}{\sin 57^{\circ}}$$

Answer

$$\frac{3\sin 5^{\circ}}{\cos 85^{\circ}} + \frac{2\cos 33^{\circ}}{\sin 57^{\circ}}$$

$= \frac{3\sin 5^{\circ}}{\sin(90^{\circ}-85^{\circ})} + \frac{2\sin(90^{\circ}-33^{\circ})}{\sin 57^{\circ}}$ [∵ cos θ = sin (90° - θ)]

$$= \frac{3\sin 5^{\circ}}{\sin 5^{\circ}} + \frac{2\sin 57^{\circ}}{\sin 57^{\circ}}$$

= 3 + 2

= 5

48 B. Question

Evaluate the following

$$\frac{\cot 54^\circ}{\tan 36^\circ} + \frac{\tan 20^\circ}{\cot 70^\circ} - 2$$

Answer

$\frac{\cot 54^\circ}{\tan 36^\circ} + \frac{\tan 20^\circ}{\cot 70^\circ} - 2$

$= \frac{\cot 54^\circ}{\cot(90^\circ - 36^\circ)} + \frac{\cot(90^\circ - 20^\circ)}{\cot 70^\circ} - 2$ [∵ tan θ = cot (90° - θ)]

$= \frac{\cot 54^\circ}{\cot 54^\circ} + \frac{\cot 70^\circ}{\cot 70^\circ} - 2$

= 1 +1 – 2

= 0

48 C. Question

Evaluate the following

$$\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec} 31^\circ$$

Answer

$\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec} 31^\circ$

$= \frac{\cos 80^\circ}{\cos(90^\circ - 10^\circ)} + \sin(90^\circ - 59^\circ) \operatorname{cosec} 31^\circ$

[∵ cos θ = sin (90° - θ) and sec θ = cosec (90° - θ)]

$= \frac{\cos 80^\circ}{\cos 80^\circ} + \sin 31^\circ \operatorname{cosec} 31^\circ$

$= 1 + \sin 31^\circ \times \frac{1}{\sin 31^\circ} \quad \because \ \sin\theta = \frac{1}{\operatorname{cosec}\theta}$

= 1 + 1

= 2

48 D. Question

Evaluate the following

cos38° cos52° – sin38° sin 52°

Answer

We know that

cos θ = sin (90° - θ)

=sin (90° - 38°) sin (90° -52°) – sin 38° sin 52°

= sin 52° sin 38°– sin 38° sin 52°

=0

48 E. Question

Evaluate the following

sec41° sin49° + cos49° cosec 41°

Answer

We know that

sec θ = cosec (90° - θ) and cos θ = sin (90° - θ)

Cosec (90° – 41°) sin 49° + sin (90° – 49°) cosec 41°

= cosec 49° sin 49° + sin 41° cosec 41°

$$= \frac{1}{\sin 49°} \times \sin 49° + \sin 41° \times \frac{1}{\sin 41°} \left[\because \ \sin\theta = \frac{1}{\operatorname{cosec}\theta}\right]$$

= 1+ 1

= 2

Exercise 4.4

1. Question

Fill in the blanks

(i) $\sin^2 \theta \operatorname{cosec}^2 \theta$ = ……..

(ii) $1 + \tan^2 \theta$ = ……

(iii) Reciprocal sin θ. cot θ = ……

(iv) 1–…….= $\cos^2\theta$

(v) $\tan A = \frac{\ldots\ldots\ldots}{\cos A}$

(vi) ...... $= \frac{\cos A}{\sin A}$

(vii) cos θ is reciprocal of ....

(viii) Reciprocal of sin θ is.........

(ix) Value of sin θ in terms of cos θ is

(x) Value of cos θ in terms of sin θ is

Answer

(i) Given: $\sin^2 \theta \operatorname{cosec}^2 \theta$

$$\Rightarrow \sin^2\theta \times \frac{1}{\sin^2\theta} \left[\because \sin\theta = \frac{1}{\csc\theta}\right]$$

= 1

(ii) Given: $1 + \tan^2 \theta$

$$= 1 + \left(\frac{\sin\theta}{\cos\theta}\right)^2 \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$= 1 + \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta} [\because \cos^2\theta + \sin^2\theta = 1]$$

$$= \frac{1}{\cos^2\theta}$$

$$= \sec^2\theta \left[\because \cos\theta = \frac{1}{\sec\theta}\right]$$

(iii) Given : sin θ cot θ

Firstly, we simplify the given trigonometry

$$= \sin\theta \times \frac{\cos\theta}{\sin\theta} \left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

= cos θ

Now, the reciprocal of cos θ is

$$= \frac{1}{\cos\theta}$$

$=\sec\theta \left[\because \cos\theta = \frac{1}{\sec\theta}\right]$

(iv) Given: $1 - x = \cos^2\theta$

Subtracting 1 to both the sides, we get

$1 - x - 1 = \cos^2\theta - 1$

$\Rightarrow -x = -\sin^2\theta$

$\Rightarrow x = \sin^2\theta$

(v) $\tan A = \frac{\sin A}{\cos A}$

(vi) $\cot A = \frac{\cos A}{\sin A}$

(vii) $\cos\theta = \frac{1}{\sec\theta}$

(viii) $\sin\theta = \frac{1}{\csc\theta}$

(ix) We know that

$\cos^2\theta + \sin^2\theta = 1$

$\Rightarrow \sin^2\theta = 1 - \cos^2\theta$

$\Rightarrow \sin\theta = \sqrt{(1 - \cos^2\theta)}$

(x) We know that

$\cos^2\theta + \sin^2\theta = 1$

$\Rightarrow \cos^2\theta = 1 - \sin^2\theta$

$\Rightarrow \cos\theta = \sqrt{(1 - \sin^2\theta)}$

2. Question

If $\sin\theta = p$ and $\cos\theta = q$, what is the relation between p and q ?

Answer

We know that,

$\cos^2\theta + \sin^2\theta = 1$ ...(i)

Given : $\sin\theta = p$ and $\cos\theta = q$

Putting the values of $\sin\theta$ and $\cos\theta$ in eq. (i), we get

$(q)^2 + (p)^2 = 1$

$\Rightarrow p^2 + q^2 = 1$

3. Question

If cos A = x, express sin A in terms of x

Answer

We know that

$\cos^2 \theta + \sin^2 \theta = 1$

$\Rightarrow \sin^2 \theta = 1 - \cos^2 \theta$

$\Rightarrow \sin \theta = \sqrt{(1 - \cos^2 \theta)}$

And Given that cos θ = x

$\Rightarrow \sin \theta = \sqrt{(1 - x^2)}$

4. Question

If x cos θ = 1 and y sin θ = 1 find the value of tan θ.

Answer

Given x cosθ = 1 and y sinθ = 1

$$\Rightarrow \cos\theta = \frac{1}{x} \text{ and } \sin\theta = \frac{1}{y}$$

Now, we know that

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Putting the value of sin θ and cos θ, we get

$$\tan\theta = \frac{\frac{1}{y}}{\frac{1}{x}}$$

$$\Rightarrow \tan\theta = \frac{x}{y}$$

5. Question

If $\cos 40^o = p$, then write the value of $\sin 40^o$ in terms of p.

Answer

We know that

$\cos^2 \theta + \sin^2 \theta = 1$

$\Rightarrow \cos^2 40° + \sin^2 40° = 1$

$\Rightarrow \sin^2 40° = 1 - \cos^2 40°$

$\Rightarrow \sin 40° = \sqrt{(1 - \cos^2 40°)}$

And Given that $\cos 40° = p$

$\Rightarrow \sin 40° = \sqrt{(1 - p^2)}$

6. Question

If $\sin 77° = x$, then write the value of $\cos 77°$ in terms of x.

Answer

We know that

$\cos^2 \theta + \sin^2 \theta = 1$

$\Rightarrow \cos^2 77° + \sin^2 77° = 1$

$\Rightarrow \cos^2 77° = 1 - \sin^2 77°$

$\Rightarrow \cos 77° = \sqrt{(1 - \sin^2 77°)}$

And Given that $\sin 77° = x$

$\Rightarrow \cos 77° = \sqrt{(1 - x^2)}$

7. Question

If $\cos 55° = x^2$, then write the value of $\sin 55°$ in terms of x.

Answer

We know that

$\cos^2 \theta + \sin^2 \theta = 1$

$\Rightarrow \cos^2 55° + \sin^2 55° = 1$

$\Rightarrow \sin^2 55° = 1 - \cos^2 55°$

$\Rightarrow \sin 55° = \sqrt{(1 - \cos^2 55°)}$

And Given that cos 55° = x^2

⇒ sin 55° = √{1– $(x^2)^2$}

⇒ sin 55° = √(1 – x^4)

8. Question

If, sin 50° = α then write the value of cos 50° in terms of α.

Answer

We know that

$\cos^2 \theta + \sin^2 \theta = 1$

⇒ $\cos^2 50° + \sin^2 50° = 1$

⇒ $\cos^2 50° = 1 - \sin^2 50°$

⇒ cos 50° = √(1 – $\sin^2$ 50°)

And Given that sin 50° = a

⇒ cos 50° = √(1 – a^2)

9. Question

If x cos A = 1 and tan A = y, then what is the value of $x^2 - y^2$.

Answer

Given x cos A = 1 and tan A =y

$$\Rightarrow\ x = \frac{1}{\cos A} \text{ and } \frac{\sin A}{\cos A} = y \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

To find: $x^2 - y^2$

Putting tha values of x and y , we get

$$\left(\frac{1}{\cos A}\right)^2 - \left(\frac{\sin A}{\cos A}\right)^2$$

$$= \frac{1}{\cos^2 A} - \frac{\sin^2 A}{\cos^2 A}$$

$$= \frac{1 - \sin^2 A}{\cos^2 A}$$

$$= \frac{\cos^2 A}{\cos^2 A} [\because \cos^2\theta + \sin^2\theta = 1]$$

= 1

10. Question

Prove the followings identities:

(1 – sin θ)(1 + sin θ) = cso^2 θ

Answer

Taking LHS = (1 – sinθ)(1+ sinθ)

Using identity , (a + b) (a – b) = (a^2 – b^2) , we get

= $(1)^2$ – $(\sin\theta)^2$

= 1 – $\sin^2$ θ

= $\cos^2$ θ [∵ $\cos^2$ θ + $\sin^2$ θ = 1]

= RHS

Hence Proved

11. Question

Prove the followings identities:

(1 + cos θ)(1 – cos θ) = $\sin^2\theta$

Answer

Taking LHS =(1 – cosθ)(1+ cosθ)

Using identity , (a + b) (a – b) = (a^2 – b^2) , we get

= $(1)^2$ – $(\cos\theta)^2$

= 1 – $\cos^2$ θ

= $\sin^2$ θ [∵ $\cos^2$ θ + $\sin^2$ θ = 1]

= RHS

Hence Proved

12. Question

Prove the followings identities:

$$\frac{(1-\cos\theta)(1+\cos\theta)}{(1-\sin\theta)(1+\sin\theta)} = \tan^2\theta$$

Answer

Taking LHS $\frac{(1-\cos\theta)(1+\cos\theta)}{(1-\sin\theta)(1+\sin\theta)}$

$= \frac{(1)^2-(\cos\theta)^2}{(1)^2-(\sin\theta)^2}$ [Using identity , (a + b) (a – b) = (a^2 – b^2)]

$= \frac{\sin^2\theta}{\cos^2\theta}$ [∵ $\cos^2\theta + \sin^2\theta = 1$]

= $\tan^2\theta$ $\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$

= RHS

Hence Proved

13. Question

Prove the followings identities:

$$\frac{1}{\sec\theta + \tan\theta} = \sec\theta - \tan\theta$$

Answer

Taking LHS $= \frac{1}{\sec\theta+\tan\theta}$

Multiplying and divide by the conjugate of sec θ + tan θ

$$= \frac{1}{\sec\theta + \tan\theta} \times \frac{\sec\theta - \tan\theta}{\sec\theta - \tan\theta}$$

$= \frac{\sec\theta - \tan\theta}{\sec^2\theta-\tan^2\theta}$ [Using identity , (a + b) (a – b) = (a^2 – b^2)]

= sec θ – tan θ [∵ $1+\tan^2\theta = \sec^2\theta$]

= RHS

Hence Proved

14 A. Question

Prove the following identities :

sinθ. cotθ = cos θ

Answer

Taking LHS = sin θ cot θ

$$= \sin\theta \times \frac{\cos\theta}{\sin\theta} \left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

= cos θ

=RHS

Hence Proved

14 B. Question

Prove the following identities :

$\sin^2 \theta(1+ \cot^2 \theta) = 1$

Answer

Taking LHS = $\sin^2 \theta(1+ \cot^2 \theta)$

= $\sin^2 \theta (\text{cosec}^2 \theta)$ [∵ $\cot^2 \theta +1= \text{cosec}^2 \theta$]

$$= \sin^2\theta \times \frac{1}{\sin^2\theta} \left[\because \sin\theta = \frac{1}{\text{cscec}\,\theta}\right]$$

= 1

=RHS

Hence Proved

14 C. Question

Prove the following identities :

$\cos^2 A (\tan^2 A+1) = 1$

Answer

Taking LHS = $\cos^2 A (\tan^2 A+1)$

= $\cos^2 \theta (\sec^2 \theta)$ [∵ $1+ \tan^2 \theta = \sec^2 \theta$]

$$= \cos^2\theta \times \frac{1}{\cos^2\theta} \left[\because \cos\theta = \frac{1}{\sec\theta}\right]$$

= 1

=RHS

Hence Proved

14 D. Question

Prove the following identities :

$\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

Answer

Taking LHS = $\tan^4 \theta + \tan^2 \theta$

$= (\tan^2 \theta)^2 + \tan^2 \theta$

$= (\sec^2 \theta - 1)^2 + (\sec^2 \theta - 1)$ [$\because$ $1+ \tan^2 \theta = \sec^2 \theta$]

$= \sec^4 \theta + 1 - 2 \sec^2 \theta + \sec^2 \theta - 1$ [$\because$ $(a - b)^2 = (a^2 + b^2 - 2ab)$]

$= \sec^4 \theta - \sec^2 \theta$

=RHS

Hence Proved

14 E. Question

Prove the following identities :

$$\frac{(1+\tan^2\theta)\sin^2\theta}{\tan\theta} = \tan\theta$$

Answer

Taking LHS $= \frac{(1+\tan^2\theta)\sin^2\theta}{\tan\theta}$

$= \frac{(\sec^2\theta)\sin^2\theta}{\frac{\sin\theta}{\cos\theta}}$ [$\because$ $1+ \tan^2 \theta = \sec^2 \theta$]

$= \frac{1 \times \sin\theta \times \cos\theta}{\cos^2\theta}$ $\left[\because \cos\theta = \frac{1}{\sec\theta}\right]$

$= \frac{\sin\theta}{\cos\theta}$

$= \tan \theta$

=RHS

Hence Proved

14 F. Question

Prove the following identities :

$$\frac{\sin^2\theta}{\cos^2\theta}+1=\frac{\tan^2\theta}{\sin^2\theta}$$

Answer

Taking LHS$=\frac{\sin^2\theta}{\cos^2\theta}+1$

$=\frac{\sin^2\theta+\cos^2\theta}{\cos^2\theta}$ [∵ $\cos^2\theta + \sin^2\theta = 1$]

$=\frac{1}{\cos^2\theta}$

= $\sec^2\theta$ $\left[\because \cos\theta=\frac{1}{\sec\theta}\right]$

Now, RHS $=\frac{\tan^2\theta}{\sin^2\theta}$

$=\frac{\frac{\sin^2\theta}{\cos^2\theta}}{\sin^2\theta}$ $\left[\because \tan\theta=\frac{\sin\theta}{\cos\theta}\right]$

$=\frac{\sin^2\theta}{\sin^2\theta\times\cos^2\theta}$

$=\frac{1}{\cos^2\theta}$

= $\sec^2\theta$ $\left[\because \cos\theta=\frac{1}{\sec\theta}\right]$

∴ LHS = RHS

Hence Proved

14 G. Question

Prove the following identities :

$$\frac{3-4\sin^2\theta}{\cos^2\theta}=3-\tan^2\theta$$

Answer

Taking LHS $=\frac{3-4\sin^2\theta}{\cos^2\theta}$

$= \frac{3}{\cos^2\theta} - \frac{4\sin^2\theta}{\cos^2\theta}$

$= 3 \sec^2 \theta - 4 \tan^2 \theta$

We know that,

$1+ \tan^2 \theta = \sec^2 \theta$

$= 3(1+ \tan^2 \theta) - 4 \tan^2 \theta$

$= 3 + 3 \tan^2 \theta - 4 \tan^2 \theta$

$= 3 - \tan^2 \theta$

=RHS

Hence Proved

14 H. Question

Prove the following identities :

$(1+ \tan^2 \theta) \cos \theta. \sin \theta = \tan \theta$

Answer

Taking LHS = $(1+ \tan^2 \theta) \cos \theta \sin \theta$

$= (\sec^2 \theta) \cos \theta \sin \theta$ [$\because 1+ \tan^2 \theta = \sec^2 \theta$]

$= \frac{1}{\cos^2\theta} \times \cos\theta \times \sin\theta \left[\because \cos\theta = \frac{1}{\sec\theta}\right]$

$= \frac{\sin\theta}{\cos\theta}$

$= \tan \theta \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$

=RHS

Hence Proved

14 I. Question

Prove the following identities :

$\sin^2\theta - \cos^2 \phi = \sin^2\phi - \cos^2\theta$

Answer

Taking LHS = $\sin^2 \theta - \cos^2 \varphi$

$=(1 - \cos^2 \theta) - (1 - \sin^2 \varphi)$ [$\because \cos^2 \theta + \sin^2 \theta = 1$] & [$\because \cos^2 \varphi + \sin^2 \varphi = 1$]

$= 1 - \cos^2 \theta - 1 + \sin^2 \varphi$

$= \sin^2 \varphi - \cos^2 \theta$

=RHS

Hence Proved

14 J. Question

Prove the following identities :

$$\frac{1-\tan^2\theta}{\cot^2\theta-1} = \tan^2\theta$$

Answer

Taking LHS $= \frac{1-\tan^2\theta}{\cot^2\theta-1}$

$$= \frac{1 - \frac{\sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta}{\sin^2\theta} - 1}$$

$$= \frac{\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta - \sin^2\theta}{\sin^2\theta}}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta} \times \frac{\sin^2\theta}{\cos^2\theta - \sin^2\theta}$$

$= \tan^2 \theta$ $\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$

=RHS

Hence Proved

15 A. Question

Prove the following identities :

$(1 - \cos\theta)(1+ \cos\theta)(1+ \cot^2 \theta) = 1$

Answer

Taking LHS = (1 – cosθ)(1+ cosθ)(1+ $\cot^2\theta$)

Using identity , (a + b) (a – b) = ($a^2 - b^2$) in first two terms , we get

= $(1)^2 - (\cos\theta)^2 (\text{cosec}^2\theta)$ [∵ $\cot^2\theta + 1 = \text{cosec}^2\theta$]

= $(1 - \cos^2\theta)(\text{cosec}^2\theta)$

= $(\sin^2\theta)(\text{cosec}^2\theta)$ [∵ $\cos^2\theta + \sin^2\theta = 1$]

$$= \sin^2\theta \times \frac{1}{\sin^2\theta}\left[\because \sin\theta = \frac{1}{\csc\theta}\right]$$

=1

= RHS

Hence Proved

15 B. Question

Prove the following identities :

$$\frac{(1+\sin\theta)^2 + (1-\sin\theta)^2}{2\cos^2\theta} = \frac{1+\sin^2\theta}{1-\sin^2\theta}$$

Answer

Taking LHS $= \frac{(1+\sin\theta)^2 (1-\sin\theta)^2}{2\cos^2\theta}$

$$= \frac{1+\sin^2\theta + 2\sin\theta + 1 + \sin^2\theta - 2\sin\theta}{2\cos^2\theta}$$

[∵ $(a + b)^2 = (a^2 + b^2 + 2ab)$ and $(a - b)^2 = (a^2 + b^2 - 2ab)$]

$$= \frac{2 + 2\sin^2\theta}{2\cos^2\theta}$$

$= \frac{2(1+\sin^2\theta)}{2(1-\sin^2\theta)}$ [∵ $\cos^2\theta + \sin^2\theta = 1$]

$$= \frac{1+\sin^2\theta}{1-\sin^2\theta}$$

=RHS

Hence Proved

15 C. Question

Prove the following identities :

$$\frac{\cos^2\theta(1-\cos\theta)}{\sin^2\theta(1-\sin\theta)} = \frac{1+\sin\theta}{1+\cos\theta}$$

Answer

Taking LHS $= \frac{\cos^2\theta(1-\cos\theta)}{\sin^2\theta(1-\sin\theta)}$

Multiplying and divide by the conjugate of (1 – sinθ), we get

$$= \frac{\cos^2\theta(1-\cos\theta)}{\sin^2\theta(1-\sin\theta)} \times \frac{(1+\sin\theta)}{(1+\sin\theta)}$$

$$= \frac{\cos^2\theta(1-\cos\theta)(1+\sin\theta)}{\sin^2\theta[(1)^2-(\sin\theta)^2]}$$

$$= \frac{\cos^2\theta(1-\cos\theta)(1+\sin\theta)}{\sin^2\theta \times \cos^2\theta}$$

$$= \frac{(1-\cos\theta)(1+\sin\theta)}{\sin^2\theta}$$

Now, multiply and divide by conjugate of 1 – cos θ, we get

$$= \frac{(1-\cos\theta)(1+\sin\theta)}{\sin^2\theta} \times \frac{(1+\cos\theta)}{(1+\cos\theta)}$$

$$= \frac{(1^2-\cos^2\theta)(1+\sin\theta)}{\sin^2\theta(1+\cos\theta)}$$

$$= \frac{\sin^2\theta(1+\sin\theta)}{\sin^2\theta(1+\cos\theta)}$$

$$= \frac{1+\sin\theta}{1+\cos\theta}$$

=RHS

Hence Proved

15 D. Question

Prove the following identities :

$(\sin\theta - \cos\theta)^2 = 1 - 2\sin\theta \cdot \cos\theta$

Answer

Taking LHS = $(\sin\theta - \cos\theta)^2$

Using the identity,$(a - b)^2 = (a^2 + b^2 - 2ab)$

$= \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta$

$= 1 - 2\sin\theta\cos\theta$ [$\because \cos^2\theta + \sin^2\theta = 1$]

=RHS

Hence Proved

15 E. Question

Prove the following identities :

$(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$

Answer

Taking LHS = $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2$

Using the identity,$(a + b)^2 = (a^2 + b^2 + 2ab)$ and $(a - b)^2 = (a^2 + b^2 - 2ab)$

$= \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta$

$= 1 + 1$ [$\because \cos^2\theta + \sin^2\theta = 1$]

$= 2$

=RHS

Hence Proved

15 F. Question

Prove the following identities :

$(a\sin\theta + b\cos\theta)^2 + (a\cos\theta - b\sin\theta)^2 = a^2 + b^2$

Answer

Taking LHS = $(a\sin\theta + b\cos\theta)^2 + (a\cos\theta - b\sin\theta)^2$

Using the identity,$(a + b)^2 = (a^2 + b^2 + 2ab)$ and $(a - b)^2 = (a^2 + b^2 - 2ab)$

$= a^2\sin^2\theta + b^2\cos^2\theta + 2ab\sin\theta\cos\theta + a^2\cos^2\theta + b^2\sin^2\theta - 2ab\sin\theta\cos\theta$

$= a^2\sin^2\theta + a^2\cos^2\theta + b^2\sin^2\theta + b^2\cos^2\theta$

$= a^2 (\sin^2 \theta + \cos^2 \theta) + b^2 (\sin^2 \theta + \cos^2 \theta)$

$= a^2 + b^2$ [$\because \cos^2 \theta + \sin^2 \theta = 1$]

=RHS

Hence Proved

15 G. Question

Prove the following identities :

$\cos^4 A + \sin^4 A + 2 \sin^2 A. \cos^2 A = 1$

Answer

Taking LHS $= \cos^4 A + \sin^4 A + 2 \sin^2 A \cos^2 A$

Using the identity,$(a + b)^2 = (a^2 + b^2 + 2ab)$

Here, $a = \cos^2 A$ and $b = \sin^2 A$

$= (\cos^2 A + \sin^2 A)$ [$\because \cos^2 \theta + \sin^2 \theta = 1$]

$= 1$

15 H. Question

Prove the following identities :

$\sin^4 A - \cos^4 A = 2 \sin^2 A - 1 = 1 - 2 \cos^2 A = \sin^2 A - \cos^2 A$

Answer

Given:

$\underbrace{\sin^4 A - \cos^4 A}_{I} = \underbrace{2 \sin^2 A - 1}_{II} = \underbrace{1 - 2 \cos^2 A}_{III} = \underbrace{\sin^2 A - \cos^2 A}_{IV}$

Taking I term

$= \sin^4 A - \cos^4 A \rightarrow$ I term

$= (\sin^2 A)^2 - (\cos^2 A)^2$

$= (\sin^2 A - \cos^2 A)(\sin^2 A+ \cos^2 A)$

[$\because (a^2 - b^2) = (a + b) (a - b)$]

$= (\sin^2 A - \cos^2 A)(1)$ [$\because \cos^2 \theta + \sin^2 \theta = 1$]

$= (\sin^2 A - \cos^2 A)$ ...(i) → IV term

From Eq. (i)

$= \{\sin^2 A - (1 - \sin^2 A)\}$ [∵ $\cos^2 \theta + \sin^2 \theta = 1$]

$= \sin^2 A - 1 + \sin^2 A$

$= 2 \sin^2 A - 1$ → II term

Again, From Eq. (i)

$= \{(1 - \cos^2 A) - \cos^2 A\}$ [∵ $\cos^2 \theta + \sin^2 \theta = 1$]

$=1 - 2 \cos^2 A$ → III term

Hence, I = II = III = IV

Hence Proved

15 I. Question

Prove the following identities :

$\cos^4 \theta - \sin^4 \theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1$

Answer

Given:

$\underbrace{\cos^4 \theta - \sin^4 \theta}_{\text{I}} = \underbrace{\cos^2 \theta - \sin^2 \theta}_{\text{II}} = \underbrace{2 \cos^2 \theta - 1}_{\text{III}}$

Taking I term

$= \cos^4 \theta - \sin^4 \theta$ → I term

$= (\cos^2 \theta)^2 - (\sin^2 \theta)^2$

$= (\cos^2 \theta - \sin^2 \theta)(\cos^2 \theta + \sin^2 \theta)$

[∵ $(a^2 - b^2) = (a + b)(a - b)$]

$= (\cos^2 \theta - \sin^2 \theta)(1)$ [∵ $\cos^2 \theta + \sin^2 \theta = 1$]

$= (\cos^2 \theta - \sin^2 \theta)$ ...(i) → II term

From Eq. (i)

$= \{\cos^2 \theta - (1 - \cos^2 \theta)\}$ [∵ $\cos^2 \theta + \sin^2 \theta = 1$]

$= 2 \cos^2 \theta - 1 \rightarrow$ III term

Hence, I = II = III

Hence Proved

15 J. Question

Prove the following identities :

$2 \cos^2 \theta - \cos^4 \theta + \sin^4 \theta = 1$

Answer

Taking LHS $= 2 \cos^2 \theta - \cos^4 \theta + \sin^4 \theta$

$= 2 \cos^2 \theta - (\cos^4 \theta - \sin^4 \theta)$

$= 2 \cos^2 \theta - [(\cos^2 \theta)^2 - (\sin^2 \theta)^2]$

Using identity, $(a^2 - b^2) = (a + b)(a - b)$

$= 2 \cos^2 \theta - [(\cos^2 \theta - \sin^2 \theta)(\cos^2 \theta + \sin^2 \theta)]$

$= 2 \cos^2 \theta - [(\cos^2 \theta - \sin^2 \theta)(1)]$ $[\because \cos^2 \theta + \sin^2 \theta = 1]$

$=2 \cos^2 \theta - \cos^2 \theta + \sin^2 \theta$

$= \cos^2 \theta + \sin^2 \theta$

$= 1$ $[\because \cos^2 \theta + \sin^2 \theta = 1]$

= RHS

Hence Proved

15 K. Question

Prove the following identities :

$1 - 2 \cos^2 \theta + \cos^4 \theta = \sin^4\theta$

Answer

Taking LHS $= 1 - 2 \cos^2 \theta + \cos^4 \theta$

We know that,

$\cos^2 \theta + \sin^2 \theta = 1$

$= 1 - 2 \cos^2 \theta + (\cos^2 \theta)^2$

$= 1 - 2\cos^2\theta + (1 - \sin^2\theta)^2$

$= 1 - 2\cos^2\theta + 1 + \sin^4\theta - 2\sin^2\theta$

$= 2 - 2(\cos^2\theta + \sin^2\theta) + \sin^4\theta$

$= 2 - 2(1) + \sin^4\theta$

$= \sin^4\theta$

=RHS

Hence Proved

15 L. Question

Prove the following identities :

$1 - 2\sin^2\theta + \sin^4\theta = \cos^4\theta$

Answer

Taking LHS $= 1 - 2\sin^2\theta + \sin^4\theta$

We know that,

$\cos^2\theta + \sin^2\theta = 1$

$= 1 - 2\sin^2\theta + (\sin^2\theta)^2$

$= 1 - 2\sin^2\theta + (1 - \cos^2\theta)^2$

$= 1 - 2\sin^2\theta + 1 + \cos^4\theta - 2\cos^2\theta$

$= 2 - 2(\cos^2\theta + \sin^2\theta) + \cos^4\theta$

$= 2 - 2(1) + \cos^4\theta$

$= \cos^4\theta$

=RHS

Hence Proved

16 A. Question

Prove that the following identities :

$\sec^2\theta + \text{cosec}^2\theta = \sec^2\theta.\text{cosec}^2\theta$

Answer

Taking LHS = $\sec^2 \theta + \text{cosec}^2 \theta$

$= \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta} \because \left[\cos\theta = \frac{1}{\sec\theta}\right]$ and $\left[\sin\theta = \frac{1}{\csc\theta}\right]$

$= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta \sin^2\theta}$ [$\because \cos^2 \theta + \sin^2 \theta = 1$]

$= \frac{1}{\cos^2\theta \sin^2\theta}$

= $\sec^2 \theta \times \text{cosec}^2 \theta \because \left[\cos\theta = \frac{1}{\sec\theta}\right]$ and $\left[\sin\theta = \frac{1}{\csc\theta}\right]$

=RHS

Hence Proved

16 B. Question

Prove that the following identities :

$$\frac{\cos^2\theta}{\sin\theta} + \sin\theta = \text{cosec}\theta$$

Answer

Taking LHS $= \frac{\cos^2\theta}{\sin\theta} + \sin\theta$

$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta}$

$= \frac{1}{\sin\theta}$ [$\because \cos^2 \theta + \sin^2 \theta = 1$]

= $\text{cosec}\ \theta$

=RHS

Hence Proved

16 C. Question

Prove that the following identities :

$\cot\theta + \tan\theta = \text{cosec}\ \theta \cdot \sec\theta$

Answer

Taking LHS = $\cot\theta + \tan\theta$

$= \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$ $\left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$ and $\left[\tan\theta = \frac{\sin\theta}{\cos\theta}\right]$

$= \frac{\cos^2\theta+\sin^2\theta}{\cos\theta\sin\theta}$ [$\because \cos^2\theta + \sin^2\theta = 1$]

$$= \frac{1}{\cos\theta\sin\theta}$$

= cosec θ sec θ ∵ $\left[\cos\theta = \frac{1}{\sec\theta}\right]$ and $\left[\sin\theta = \frac{1}{\csc\theta}\right]$

=RHS

Hence Proved

17. Question

Prove that the following identities :

$$\frac{1-\sin\theta}{1+\sin\theta} = \left(\frac{1-\sin\theta}{\cos\theta}\right)^2$$

Answer

Taking LHS $= \frac{1-\sin\theta}{1+\sin\theta}$

Multiplying and divide by the conjugate of 1 + sin θ , we get

$$= \frac{1-\sin\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$$

$$= \frac{(1-\sin\theta)^2}{(1)^2-(\sin\theta)^2}$$

[$\because (a - b)(a - b) = (a - b)^2$ and $(a + b)(a - b) = (a^2 - b^2)$]

$$= \frac{(1-\sin\theta)^2}{1-\sin^2\theta}$$

$= \frac{(1-\sin\theta)^2}{\cos^2\theta}$ [$\because \cos^2\theta + \sin^2\theta = 1$]

$$= \left(\frac{1-\sin\theta}{\cos\theta}\right)^2$$

=RHS

Hence Proved

18. Question

Prove that the following identities :

$$\frac{1-\cos\theta}{1+\cos\theta}=\left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

Answer

Taking LHS $=\frac{1-\cos\theta}{1+\cos\theta}$

Multiplying and divide by the conjugate of 1 + cos θ , we get

$$=\frac{1-\cos\theta}{1+\cos\theta}\times\frac{1-\cos\theta}{1-\cos\theta}$$

$$=\frac{(1-\cos\theta)^2}{(1)^2-(\cos\theta)^2}$$

[∵ (a – b) (a – b) =(a – b)^2 and (a + b) (a – b) = (a^2 – b^2)]

$$=\frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$

$=\frac{(1-\cos\theta)^2}{\sin^2\theta}$ [∵ $\cos^2\theta+\sin^2\theta=1$]

$$=\left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

=RHS

Hence Proved

19. Question

Prove that the following identities :

$$\frac{1-\cos\theta}{1+\cos\theta}=\left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

Answer

Taking LHS $=\frac{1-\cos\theta}{1+\cos\theta}$

Multiplying and divide by the conjugate of 1 + cos θ , we get

$$= \frac{1 - \cos\theta}{1 + \cos\theta} \times \frac{1 - \cos\theta}{1 - \cos\theta}$$

$$= \frac{(1 - \cos\theta)^2}{(1)^2 - (\cos\theta)^2}$$

[∵ (a – b) (a – b) =(a – b)^2 and (a + b) (a – b) = (a^2 – b^2)]

$$= \frac{(1 - \cos\theta)^2}{1 - \cos^2\theta}$$

$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$ [∵ $\cos^2\theta + \sin^2\theta = 1$]

$$= \left(\frac{1 - \cos\theta}{\sin\theta}\right)^2$$

=RHS

Hence Proved

20. Question

Prove that the following identities :

$$\frac{\cos\theta}{1+\sin\theta} = \frac{1-\sin\theta}{\cos\theta}$$

Answer

Taking LHS $= \frac{\cos\theta}{1+\sin\theta}$

Multiplying and divide by the conjugate of 1 + sin θ , we get

$$= \frac{\cos\theta}{1 + \sin\theta} \times \frac{1 - \sin\theta}{1 - \sin\theta}$$

$= \frac{\cos\theta\,(1-\sin\theta)}{(1)^2-(\sin\theta)^2}$ [∵ (a + b) (a – b) = (a^2 – b^2)]

$$= \frac{\cos\theta\,(1 - \sin\theta)}{1 - \sin^2\theta}$$

$= \frac{\cos\theta\,(1-\sin\theta)}{\cos^2\theta}$ [∵ $\cos^2\theta + \sin^2\theta = 1$]

$$= \frac{1 - \sin\theta}{\cos\theta}$$

=RHS

Hence Proved

21. Question

Prove that the following identities :

$(\sin^8\theta - \cos^8\theta) = (\sin^2\theta - \cos^2\theta)(1 - 2\sin^2\theta .\cos^2\theta)$

Answer

Taking LHS

$= \sin^8 \theta - \cos^8 \theta$

$= (\sin^4 \theta)^2 - (\cos^4 \theta)^2$

$= (\sin^4 \theta - \cos^4 \theta)(\sin^4 \theta + \cos^4 \theta)$

$[\because (a^2 - b^2) = (a + b)(a - b)]$

$= \{(\sin^2 \theta)^2 - (\cos^2 \theta)^2\}\{(\sin^2 \theta)^2 + (\cos^2 \theta)^2\}$

$= (\sin^2 \theta + \cos^2 \theta)(\sin^2 \theta - \cos^2 \theta)[(\sin^2 \theta + \cos^2 \theta) - 2 \sin^2 \theta \cos^2 \theta]$

$[\because (a^2 + b^2) = (a + b)^2 - 2ab]$

$= (1)[\sin^2 \theta - \cos^2 \theta][(1) - 2 \sin^2 \theta \cos^2 \theta]$

$= (\sin^2 \theta - \cos^2 \theta)(1 - 2 \sin^2 \theta \cos^2 \theta)$

=RHS

Hence Proved

22. Question

Prove that the following identities :

$2(\sin^6 \theta - \cos^6 \theta) - 3(\sin^4 \theta + \cos^4 \theta) + (\sin^2 \theta + \cos^2 \theta)$

Answer

Taking LHS

$= 2(\sin^6 \theta - \cos^6 \theta) - 3(\sin^4 \theta + \cos^4 \theta) + (\sin^2 \theta + \cos^2 \theta)$

$= 2[(\sin^2 \theta)^3 - (\cos^2 \theta)^3] - 3[(\sin^2 \theta)^2 + (\cos^2 \theta)^2] + 1$ $[\because \cos^2 \theta + \sin^2 \theta = 1]$

Now, we use these identities, $(a^3 - b^3) = (a + b)^3 - 3ab(a+b)$ and $(a^2 + b^2) = (a + b)^2 - 2ab]$

$= 2[(\sin^2 \theta + \cos^2 \theta)^3 - 3\sin^2\theta \cos^2\theta (\sin^2 \theta + \cos^2 \theta)] - 3[(\sin^2 \theta + \cos^2 \theta) - 2 \sin^2 \theta \cos^2 \theta] + 1$

$= 2[(1) - 3\sin^2\theta \cos^2\theta (1)] - 3[(1) - 2 \sin^2 \theta \cos^2 \theta] + 1$ [$\because \cos^2 \theta + \sin^2 \theta = 1$]

$= 2(1 - 3 \sin^2 \theta \cos^2 \theta) - 3 + 6\sin^2 \theta \cos^2 \theta + 1$

$= 2 - 6\sin^2\theta \cos^2\theta - 2 + 6\sin^2 \theta \cos^2 \theta$

=0

=RHS

Hence Proved

23. Question

Prove the following identities

$$\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \sin A + \cos A$$

Answer

Taking LHS $= \frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A}$

$$= \frac{\cos A}{1 - \frac{\sin A}{\cos A}} + \frac{\sin A}{1 - \frac{\cos A}{\sin A}} \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta} \text{ and } \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

$$= \frac{\cos A}{\frac{\cos A - \sin A}{\cos A}} + \frac{\sin A}{\frac{\sin A - \cos A}{\sin A}}$$

$$= \frac{\cos^2 A}{\cos A - \sin A} + \frac{\sin^2 A}{\sin A - \cos A}$$

$$= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A}$$

Using the identity, $(a^2 - b^2) = (a + b)(a - b)$

$$= \frac{(\cos A - \sin A)(\cos A + \sin A)}{(\cos A - \sin A)}$$

= sin A + cos A

=RHS

Hence Proved

24. Question

Prove the following identities

$$\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=\frac{2}{\sin\theta}$$

Answer

Taking LHS $=\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}$

$$=\frac{\sin^2\theta+(1+\cos\theta)^2}{(1+\cos\theta)(\sin\theta)}$$

$$=\frac{\sin^2\theta+1+\cos^2\theta+2\cos\theta}{(1+\cos\theta)(\sin\theta)}$$

$=\frac{2+2\cos\theta}{(1+\cos\theta)(\sin\theta)}$ [$\because \cos^2\theta + \sin^2\theta = 1$]

$$=\frac{2(1+\cos\theta)}{(1+\cos\theta)(\sin\theta)}$$

$$=\frac{2}{\sin\theta}$$

=RHS

Hence Proved

25. Question

Prove the following identities

$$\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=2\sec^2\theta$$

Answer

Taking LHS $=\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}$

$$=\frac{1-\sin\theta+1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$$

Using the identity, $(a^2 - b^2) = (a + b)(a - b)$

$$=\frac{2}{(1)^2-(\sin\theta)^2}$$

$$= \frac{2}{1 - \sin^2\theta}$$

$= \frac{2}{\cos^2\theta}$ [∵ $\cos^2\theta + \sin^2\theta = 1$]

$= 2\sec^2\theta$

=RHS

Hence Proved

26. Question

Prove the following identities

$$\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta} = 2\sec\theta$$

Answer

Taking LHS $= \frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta}$

$$= \frac{(1+\sin\theta)^2 + \cos^2\theta}{(1+\sin\theta)(\cos\theta)}$$

$$= \frac{1+\sin^2\theta + 2\sin\theta + \cos^2\theta}{(\cos\theta)(1+\sin\theta)}$$

$= \frac{2+2\sin\theta}{(\cos\theta)(1+\sin\theta)}$ [∵ $\cos^2\theta + \sin^2\theta = 1$]

$$= \frac{2(1+\sin\theta)}{(\cos\theta)(1+\sin\theta)}$$

$$= \frac{2}{\cos\theta}$$

$=2\sec\theta$

=RHS

Hence Proved

27. Question

Prove the following identities

$$\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = \frac{2}{\cos\theta}$$

Answer

Taking LHS $= \frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta}$

$$= \frac{\cos\theta\,(1+\sin\theta) + \cos\theta(1-\sin\theta)}{(1-\sin\theta)\,(1+\sin\theta)}$$

$$= \frac{\cos\theta + \cos\theta\sin\theta + \cos\theta - \cos\theta\sin\theta}{(1-\sin\theta)\,(1+\sin\theta)}$$

$$= \frac{2\cos\theta}{(1-\sin\theta)\,(1+\sin\theta)}$$

Using the identity, $(a^2 - b^2) = (a + b)(a - b)$

$$= \frac{2\cos\theta}{(1)^2 - (\sin\theta)^2}$$

$$= \frac{2\cos\theta}{1 - \sin^2\theta}$$

$= \frac{2\cos\theta}{\cos^2\theta}$ [$\because \cos^2\theta + \sin^2\theta = 1$]

$$= \frac{2}{\cos\theta}$$

=RHS

Hence Proved

28. Question

Prove the following identities

$$\frac{1}{1+\cos\theta} + \frac{1}{1-\cos\theta} = \frac{2}{\sin^2\theta}$$

Answer

Taking LHS $= \frac{1}{1+\cos\theta} + \frac{1}{1-\cos\theta}$

$$= \frac{1 - \cos\theta + 1 + \cos\theta}{(1+\cos\theta)\,(1-\cos\theta)}$$

Using the identity, $(a^2 - b^2) = (a + b)(a - b)$

$$= \frac{2}{(1)^2 - (\cos\theta)^2}$$

$$= \frac{2}{1 - \cos^2\theta}$$

$= \frac{2}{\sin^2\theta}$ [$\because \cos^2\theta + \sin^2\theta = 1$]

=RHS

Hence Proved

29. Question

Prove the following identities

$$\frac{1}{1-\sin\theta} - \frac{1}{1+\sin\theta} = \frac{2\tan\theta}{\cos\theta}$$

Answer

Taking LHS $= \frac{1}{1-\sin\theta} - \frac{1}{1+\sin\theta}$

$$= \frac{1 + \sin\theta - 1 + \sin\theta}{(1 + \sin\theta)(1 - \sin\theta)}$$

Using the identity, $(a^2 - b^2) = (a + b)(a - b)$

$$= \frac{2\sin\theta}{(1)^2 - (\sin\theta)^2}$$

$$= \frac{2\sin\theta}{1 - \sin^2\theta}$$

$= \frac{2\sin\theta}{\cos^2\theta}$ [$\because \cos^2\theta + \sin^2\theta = 1$]

$$= \frac{2\tan\theta}{\cos\theta} \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

=RHS

Hence Proved

30. Question

Prove the following identities

$\cot^2\theta - \cos^2\theta = \cot^2\theta \cdot \cos^2\theta$

Answer

Taking LHS $= \cot^2\theta - \cos^2\theta$

$= \frac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta \left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$

$= \frac{\cos^2\theta - \sin^2\theta\cos^2\theta}{\sin^2\theta}$

$= \frac{\cos^2\theta(1-\sin^2\theta)}{\sin^2\theta}$ [$\because \cos^2\theta + \sin^2\theta = 1$]

$= \frac{\cos^2\theta\cos^2\theta}{\sin^2\theta}$

$= \cot^2\theta\cos^2\theta \left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$

=RHS

Hence Proved

31. Question

Prove the following identities

$\tan^2\varphi - \sin^2\varphi - \tan^2\varphi . \sin^2\varphi = 0$

Answer

Taking LHS = $\tan^2\varphi - \sin^2\varphi - \tan^2\varphi \sin^2\varphi$

$= \frac{\sin^2\varphi}{\cos^2\varphi} - \sin^2\varphi - \frac{\sin^2\varphi}{\cos^2\varphi}\sin^2\varphi$

$= \frac{\sin^2\varphi - \sin^2\varphi\cos^2\varphi - \sin^4\varphi}{\cos^2\varphi}$

$= \frac{\sin^2\varphi(1 - \cos^2\varphi - \sin^2\varphi)}{\cos^2\varphi}$

$= \frac{\sin^2\varphi\{1-(\cos^2\varphi+\sin^2\varphi)\}}{\cos^2\varphi}$ [$\because \cos^2\varphi + \sin^2\varphi = 1$]

$= \frac{\sin^2\varphi\{1 - 1\}}{\cos^2\varphi}$

= 0

=RHS

Hence Proved

32. Question

Prove the following identities

$\tan^2 \varphi + \cot^2 \varphi + 2 = \sec^2\phi . \text{cosec}^2\phi$

Answer

Taking LHS = $\tan^2 \varphi + \cot^2 \varphi + 2$

$= \frac{\sin^2\varphi}{\cos^2\varphi} + \frac{\cos^2\varphi}{\sin^2\varphi} + 2$

$= \frac{\sin^4\varphi + \cos^4\varphi + 2\sin^2\varphi\cos^2\varphi}{\cos^2\varphi\sin^2\varphi}$ [∵ $(a + b)^2 = (a^2 + b^2 + 2ab)$]

$= \frac{(\sin^2\varphi + \cos^2\varphi)^2}{\cos^2\varphi\sin^2\varphi}$ [∵ $\cos^2 \varphi + \sin^2 \varphi = 1$]

$= \frac{1}{\cos^2\varphi\sin^2\varphi}$

$= \sec^2 \varphi \text{ cosec}^2 \varphi \left[\because \cos\theta = \frac{1}{\sec\theta} \text{ and } \sin\theta = \frac{1}{\csc\theta}\right]$

=RHS

Hence Proved

33. Question

Prove the following identities

$$\frac{\text{cosec}\theta + \cot\theta - 1}{\cot\theta - \text{cosec}\theta + 1} = \frac{1 + \cos\theta}{\sin\theta}$$

Answer

Taking LHS $= \frac{\text{cosec }\theta + \cot\theta - 1}{\cot\theta - \text{cosec }\theta + 1}$

$= \frac{(\cot\theta + \text{cosec }\theta) - (\text{cosec}^2\theta - \cot^2\theta)}{\cot\theta - \text{cosec }\theta + 1}$ [∵ $\cot^2 \theta - \text{cosec}^2 \theta = 1$]

$= \frac{(\cot\theta + \text{cosec }\theta) - \{(\text{cosec }\theta + \cot\theta)(\text{cosec}\theta - \cot\theta)\}}{\cot\theta - \text{cosec }\theta + 1}$

$= \frac{(\cot\theta + \text{cosec }\theta)(1 - \text{cosec }\theta + \cot\theta)}{\cot\theta - \text{cosec }\theta + 1}$

$= \cot \theta + \text{cosec } \theta$

$= \frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta} \left[\because \cot\theta = \frac{\cos\theta}{\sin\theta} \text{ and } \sin\theta = \frac{1}{\csc\theta}\right]$

$$= \frac{1 + \cos\theta}{\sin\theta}$$

=RHS

Hence Proved

34. Question

Prove the following identities

$$\frac{\tan\theta}{1 - \cot\theta} + \frac{\cot\theta}{1 - \tan\theta} = 1 + \tan\theta + \cot\theta$$

Answer

Taking LHS $= \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{1 - \frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1 - \frac{\sin\theta}{\cos\theta}} \left[\because \cot\theta = \frac{\cos\theta}{\sin\theta} \text{ and } \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta - \cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta - \sin\theta}{\cos\theta}}$$

$$= \frac{\sin\theta}{\cos\theta} \times \frac{\sin\theta}{\sin\theta - \cos\theta} + \frac{\cos\theta}{\sin\theta} \times \frac{\cos\theta}{\cos\theta - \sin\theta}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta - \sin\theta)}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} + \frac{\cos^2\theta}{\sin\theta\{-(\sin\theta - \cos\theta)\}}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta - \cos\theta)}$$

$$= \frac{\sin^3\theta - \cos^3\theta}{(\cos\theta\sin\theta)(\sin\theta - \cos\theta)}$$

$$= \frac{(\sin\theta - \cos\theta)(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta)}{(\cos\theta\sin\theta)(\sin\theta - \cos\theta)} \left[\because (a^3 - b^3) = (a - b)(a^2 + b^2 + ab)\right]$$

$$= \frac{1 + \sin\theta\cos\theta}{\cos\theta\sin\theta}$$

$$= \frac{1}{\cos\theta\sin\theta} + \frac{\sin\theta\cos\theta}{\cos\theta\sin\theta}$$

$= \tan\theta \cot\theta + 1$

=RHS

Hence Proved

35. Question

Prove the following identities

$$\frac{1-\cos\theta}{1+\cos\theta} = (\cot\theta - \text{cosec}\theta)^2$$

Answer

Taking LHS $= \frac{1-\cos\theta}{1+\cos\theta}$

Multiplying and divide by the conjugate of $1 + \cos\theta$, we get

$$= \frac{1-\cos\theta}{1+\cos\theta} \times \frac{1-\cos\theta}{1-\cos\theta}$$

$$= \frac{(1-\cos\theta)^2}{(1)^2 - (\cos\theta)^2}$$

[∵ $(a - b)(a - b) = (a - b)^2$ and $(a + b)(a - b) = (a^2 - b^2)$]

$$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$

$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$ [∵ $\cos^2\theta + \sin^2\theta = 1$]

$$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$

$= (\text{cosec}\,\theta - \cot\theta)^2$

$= \{-(\cot\theta - \text{cosec}\,\theta)\}^2$

$= (\cot\theta - \text{cosec}\,\theta)^2$

=RHS

Hence Proved

36. Question

Prove the following identities

$$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \frac{1+\cos\theta}{\sin\theta}$$

Answer

Taking LHS $= \sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$

$= \sqrt{\frac{1+\cos\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}}$ [multiplying and divide by conjugate of 1– cosθ]

$$= \sqrt{\frac{(1+\cos\theta)^2}{(1)^2-(\cos\theta)^2}}$$

$$= \sqrt{\frac{(1+\cos\theta)^2}{(1-\cos^2\theta)}}$$

$= \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}}$ [$\because \cos^2\theta + \sin^2\theta = 1$]

$= \frac{1+\cos\theta}{\sin\theta}$

=RHS

Hence Proved

37. Question

Prove the following identities

$$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \frac{\sin\theta}{1-\cos\theta}$$

Answer

Taking LHS $= \sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$

$= \sqrt{\frac{1+\cos\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}}$ [multiplying and divide by conjugate of 1– cosθ]

$$= \sqrt{\frac{(1+\cos\theta)^2}{(1)^2-(\cos\theta)^2}}$$

$$= \sqrt{\frac{(1+\cos\theta)^2}{(1-\cos^2\theta)}}$$

$= \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}}$ [∵ $\cos^2\theta + \sin^2\theta = 1$]

$$= \frac{1+\cos\theta}{\sin\theta}$$

Multiply and divide by conjugate of 1+ cosθ, we get

$$= \frac{1+\cos\theta}{\sin\theta} \times \frac{1-\cos\theta}{1-\cos\theta}$$

$$= \frac{1-\cos^2\theta}{\sin\theta \times (1-\cos\theta)}$$

$$= \frac{\sin^2\theta}{\sin\theta \times (1-\cos\theta)}$$

$$= \frac{\sin\theta}{1-\cos\theta}$$

=RHS

Hence Proved

38. Question

Prove the following identities

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$$

Answer

Taking LHS $= \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$

$= \sqrt{\frac{1-\sin\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}}$ [multiplying and divide by conjugate of 1+sinθ]

$$= \sqrt{\frac{(1-\sin\theta)^2}{(1)^2-(\sin\theta)^2}}$$

$= \sqrt{\frac{(1 - \sin\theta)^2}{(1 - \sin^2\theta)}}$

$= \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$ [$\because \cos^2\theta + \sin^2\theta = 1$]

$= \frac{1-\sin\theta}{\cos\theta}$

$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$

$= \sec\theta - \tan\theta \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta} \text{ and } \cos\theta = \frac{1}{\sec\theta}\right]$

=RHS

Hence Proved

39. Question

Prove the following identities

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \frac{\cos\theta}{1+\sin\theta}$$

Answer

Taking LHS $= \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$

$= \sqrt{\frac{1-\sin\theta}{1+\sin\theta} \times \frac{1+\sin\theta}{1+\sin\theta}}$ [multiplying and divide by of 1+ sin θ]

$= \sqrt{\frac{(1)^2 - (\sin\theta)^2}{(1 + \sin\theta)^2}}$

$= \sqrt{\frac{1 - \sin^2\theta}{(1 + \sin\theta)^2}}$

$= \sqrt{\frac{\cos^2\theta}{(1+\sin\theta)^2}}$ [$\because \cos^2\theta + \sin^2\theta = 1$]

$= \frac{\cos\theta}{1 + \sin\theta}$

=RHS

Hence Proved

40. Question

Prove the following identities

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}=2\sec\theta$$

Answer

Taking LHS $=\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$

[multiplying and divide by conjugate of 1– sinθ in 1st term and 1+sinθ in 2nd term]

$$=\sqrt{\frac{1+\sin\theta}{1-\sin\theta}\times\frac{1+\sin\theta}{1+\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta}}$$

$$=\sqrt{\frac{(1+\sin\theta)^2}{(1)^2-(\sin\theta)^2}}+\sqrt{\frac{(1-\sin\theta)^2}{(1)^2-(\sin\theta)^2}}$$

$$=\sqrt{\frac{(1+\sin\theta)^2}{(1-\sin^2\theta)}}+\sqrt{\frac{(1-\sin\theta)^2}{(1-\sin^2\theta)}}$$

$=\sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}+\sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$ [$\because \cos^2\theta+\sin^2\theta=1$]

$=\frac{1+\sin\theta}{\cos\theta}+\frac{1-\sin\theta}{\cos\theta}$

$$=\frac{1+\sin\theta+1-\sin\theta}{\cos\theta}$$

$$=\frac{2}{\cos\theta}$$

= 2 sec θ

=RHS

Hence Proved

41. Question

If secθ + tanθ=m and sec θ – tanθ = n, then prove that $\sqrt{mn}=1$

Answer

Given : $\sec\theta + \tan\theta = m$ and $\sec\theta - \tan\theta = n$

To Prove : $\sqrt{mn} = 1$

Taking LHS = $\sqrt{mn}$

Putting the value of m and n, we get

$$= \sqrt{(\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}$$

Using the identity, $(a + b)(a - b) = (a^2 - b^2)$

$$= \sqrt{\sec^2\theta - \tan^2\theta}$$

$=\sqrt{(1)}$ [$\because 1 + \tan^2\theta = \sec^2\theta$]

$=\pm 1$

=RHS

Hence Proved

42. Question

If $\cos\theta + \sin\theta = 1$, then prove that $\cos\theta - \sin\theta = \pm 1$.

Answer

Given: $\cos\theta + \sin\theta = 1$

On squaring both the sides, we get

$(\cos\theta + \sin\theta)^2 = (1)^2$

$\Rightarrow \cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta = 1$

$\Rightarrow \cos^2\theta + \sin^2\theta = \cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta$

[$\because \cos^2\theta + \sin^2\theta = 1$]

$\Rightarrow \cos^2\theta + \sin^2\theta = (\cos\theta - \sin\theta)^2$

[$\because (a - b)^2 = (a^2 + b^2 - 2ab)$]

$\Rightarrow 1 = (\cos\theta - \sin\theta)^2$

$\Rightarrow (\cos\theta - \sin\theta) = \pm 1$

Hence Proved

43. Question

If $\sin\theta + \sin^2\theta = 1$, then prove that $\cos^2\theta + 1 \cos^4\theta = 1$

Answer

Given : $\sin\theta + \sin^2\theta = 1$

$\Rightarrow \sin\theta = 1 - \sin^2\theta$

Taking LHS = $\cos^2\theta + \cos^4\theta$

$= \cos^2\theta + (\cos^2\theta)^2$

$= (1 - \sin^2\theta) + (1 - \sin^2\theta)^2$ ...(i)

Putting $\sin\theta = 1 - \sin^2\theta$ in Eq. (i), we get

$= \sin\theta + (\sin\theta)^2$

$= \sin\theta + \sin^2\theta$

= 1 [Given: $\sin\theta + \sin^2\theta = 1$]

=RHS

Hence Proved

44. Question

If $\tan\theta + \sec\theta = x$, show that $\sin\theta = \frac{x^2 - 1}{x^2 + 1}$

Answer

To show : $\sin\theta = \frac{x^2-1}{x^2+1}$

Taking RHS $= \frac{x^2-1}{x^2+1}$

Given $\tan\theta + \sec\theta = x$

$$= \frac{(\tan\theta + \sec\theta)^2 - 1}{(\tan\theta + \sec\theta)^2 + 1}$$

$$= \frac{\tan^2\theta + \sec^2\theta + 2\tan\theta\sec\theta - 1}{\tan^2\theta + \sec^2\theta + 2\tan\theta\sec\theta + 1}$$

$= \frac{\tan^2\theta + \tan^2\theta + 2\tan\theta\sec\theta}{\sec^2\theta - 1 + \sec^2\theta + 2\tan\theta\sec\theta + 1}$ [$\because 1 + \tan^2\theta = \sec^2\theta$]

$$= \frac{2\tan^2\theta + 2\tan\theta\sec\theta}{2\sec^2\theta + 2\tan\theta\sec\theta}$$

$$= \frac{\frac{\sin^2\theta}{\cos^2\theta} + \frac{\sin\theta}{\cos\theta} \times \frac{1}{\cos\theta}}{\frac{1}{\cos^2\theta} + \frac{\sin\theta}{\cos\theta} \times \frac{1}{\cos\theta}} \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta} \text{ and } \cos\theta = \frac{1}{\sec\theta}\right]$$

$$= \frac{\sin^2\theta + \sin\theta}{1 + \sin\theta}$$

$$= \frac{\sin\theta(\sin\theta + 1)}{1 + \sin\theta}$$

=sin θ

=LHS

Hence Proved

45. Question

If sinθ + cosθ = p and secθ + cosecθ = q, then show $q(p^2-1) = 2p$

Answer

Given: sin θ + cos θ = p and sec θ + cosec θ = q

To show $q(p^2 - 1) = 2p$

Taking LHS = $q(p^2 - 1)$

Putting the value of sin θ + cos θ = p and sec θ + cosec θ = q, we get

$=(\sec\theta + \text{cosec}\,\theta)\{(\sin\theta + \cos\theta)^2 - 1)$

$=(\sec\theta + \text{cosec}\,\theta)\{(\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta) - 1)\}$

$[\because (a + b)^2 = (a^2 + b^2 + 2ab)]$

=(sec θ + cosec θ)(1+2sin θ cos θ – 1)

=(sec θ + cosec θ)(2sin θcosθ)

$$= \left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right) \times (2\sin\theta\cos\theta) \left[\because \cos\theta = \frac{1}{\sec\theta} \text{ and } \sin\theta = \frac{1}{\csc\theta}\right]$$

$$= \frac{\sin\theta + \cos\theta}{\cos\theta\sin\theta} \times 2\sin\theta\cos\theta$$

= 2(sin θ +cos θ)

= 2p [given sin θ + cos θ = p]

=RHS

Hence Proved

46. Question

If x cosθ =a and y = a tanθ, then prove that $x^2 - y^2 = a^2$

Answer

Given: x cosθ = a and y = a tanθ

$$\Rightarrow \ x = \frac{a}{\cos\theta} \text{ and } y = a\tan\theta$$

To Prove : $x^2 - y^2 = a^2$

Taking LHS = $x^2 - y^2$

Putting the values of x and y, we get

$$= \left(\frac{a}{\cos\theta}\right)^2 - (a\tan\theta)^2$$

$$= \frac{a^2}{\cos^2\theta} - a^2\tan^2\theta$$

$$= \frac{a^2}{\cos^2\theta} - a^2\frac{\sin^2\theta}{\cos^2\theta}\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$= \frac{a^2 - a^2\sin^2\theta}{\cos^2\theta}$$

$$= \frac{a^2(1 - \sin^2\theta)}{\cos^2\theta}$$

$$= \frac{a^2\cos^2\theta}{\cos^2\theta} \left[\because \cos^2\theta + \sin^2\theta = 1\right]$$

= a^2

= RHS

Hence Proved

47. Question

If x= r cos α sin β, y = r sin α sin β and z = r cos α then prove that $x^2 + y^2 + z^2 = r^2$.

Answer

Taking LHS = $x^2 + y^2 + z^2$

Putting the values of x, y and z , we get

$=(r \cos \alpha \sin \beta)^2 + (r \sin \alpha \sin \beta)^2 + (r \cos \alpha)^2$

$= r^2 \cos^2\alpha \sin^2\beta + r^2 \sin^2\alpha \sin^2\beta + r^2 \cos^2\alpha$

Taking common $r^2 \sin^2 \alpha$, we get

$= r^2 \sin^2\alpha (\cos^2\beta + \sin^2 \beta) + r^2\cos^2 \alpha$

$= r^2 \sin^2\alpha + r^2 \cos^2 \alpha$ [$\because \cos^2 \beta + \sin^2 \beta = 1$]

$=r^2 (\sin^2 \alpha + \cos^2 \alpha)$

$= r^2$ [$\because \cos^2 \alpha + \sin^2 \alpha = 1$]

=RHS

Hence Proved

48. Question

If $\sec\theta - \tan\theta = x$, then prove that

(i) $\cos\theta = \frac{2x}{1+x^2}$

(ii) $\sin\theta = \frac{1-x^2}{1+x^2}$

Answer

(i) Given $\sec \theta - \tan \theta = x$

Taking RHS $= \frac{2x}{1+x^2}$

Putting the value of x, we get

$$= \frac{2(\sec\theta - \tan\theta)}{1 + (\sec\theta - \tan\theta)^2}$$

$$= \frac{2(\sec\theta - \tan\theta)}{1 + \sec^2\theta + \tan^2\theta - 2\sec\theta \tan\theta}$$

$= \frac{2(\sec\theta - \tan\theta)}{\sec^2\theta + \sec^2\theta - 2\sec\theta\tan\theta}$ [$\because 1+ \tan^2 \theta = \sec^2 \theta$]

$$= \frac{2(\sec\theta - \tan\theta)}{2\sec\theta(\sec\theta - \tan\theta)}$$

$$= \frac{1}{\sec\theta}$$

$= \cos\theta \left[\because \cos\theta = \frac{1}{\sec\theta}\right]$

=RHS

Hence Proved

(ii) Given sec θ – tan θ = x

Taking RHS $= \frac{1-x^2}{1+x^2}$

Putting the value of x, we get

$$= \frac{1 - (\sec\theta - \tan\theta)^2}{1 + (\sec\theta - \tan\theta)^2}$$

$$= \frac{1 - \sec^2\theta - \tan^2\theta + 2\sec\theta\tan\theta}{1 + \sec^2\theta + \tan^2\theta - 2\sec\theta\tan\theta}$$

$= \frac{-\tan^2\theta - \tan^2\theta + 2\sec\theta\tan\theta)}{\sec^2\theta + \sec^2\theta - 2\sec\theta\tan\theta}$ [$\because 1+\tan^2\theta = \sec^2\theta$]

$$= \frac{2\tan\theta(\sec\theta - \tan\theta)}{2\sec\theta(\sec\theta - \tan\theta)}$$

$$= \frac{\sin\theta}{\cos\theta} \times \cos\theta$$

$= \sin\theta \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$

=RHS

Hence Proved

49. Question

If a cos θ + b sinθ = c, then prove that a sinθ – b cos θ = ± $\sqrt{a^2 + b^2 - c^2}$

Answer

Let

$(a \cos\theta + b \sin\theta)^2 + (a \sin\theta - b \cos\theta)^2 = a^2\cos^2\theta + b^2 \sin^2\theta + 2ab \cos\theta \sin\theta + a^2\sin^2\theta$

$+ b^2 \cos^2\theta - 2ab \cos\theta \sin\theta$

$\Rightarrow c^2 + (a \sin\theta - b \cos\theta)^2 = a^2 (\cos^2\theta + \sin^2\theta) + b^2 (\cos^2\theta + \sin^2\theta)$

$\Rightarrow c^2 + (a \sin\theta - b \cos\theta)^2 = a^2 + b^2$

$\Rightarrow (a \sin\theta - b \cos\theta)^2 = a^2 + b^2 - c^2$

$\Rightarrow (a \sin\theta - b \cos\theta) = \pm\sqrt{(a^2 + b^2 - c^2)}$

50. Question

If $1+\sin^2\theta = 3 \sin\theta . \cos\theta$, then prove that $\tan\theta = 1$ or $1/2$.

Answer

Given: $1+\sin^2\theta = 3 \sin\theta \cos\theta$

Divide by $\cos^2\theta$ to both the sides, we get

$$\Rightarrow \frac{1}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{3\sin\theta\cos\theta}{\cos^2\theta}$$

$\Rightarrow \sec^2\theta + \tan^2\theta = 3 \tan\theta$

$\Rightarrow 1+ \tan^2\theta + \tan^2\theta = 3\tan\theta$

$\Rightarrow 2 \tan^2\theta - 3\tan\theta + 1 = 0$

Let $\tan\theta = x$

$\Rightarrow 2x^2 - 3x + 1 = 0$

$\Rightarrow 2x^2 - 2x - x + 1 = 0$

$\Rightarrow 2x (x - 1) - 1(x - 1) = 0$

$\Rightarrow (2x - 1)(x - 1) = 0$

Putting each of the factor = 0, we get

$\Rightarrow x = 1$ or $\frac{1}{2}$

And above, we let $\tan\theta = x$

$$\Rightarrow \tan\theta = 1 \text{ or } \frac{1}{2}$$

Hence Proved

51. Question

If a cos θ – b sinθ = x and a sinθ + b cosθ = y that $a^2 + b^2 = x^2 + y^2$.

Answer

Taking RHS $=x^2 + y^2$

Putting the values of x and y, we get

$(a \cos \theta - b \sin \theta)^2 + (a \sin \theta + b \cos \theta)^2$

$= a^2\cos^2\theta + b^2 \sin^2\theta - 2ab \cos \theta \sin \theta + a^2\sin^2\theta + b^2 \cos^2\theta + 2ab \cos \theta \sin \theta$

$= a^2 (\cos^2 \theta + \sin^2 \theta) + b^2 (\cos^2 \theta + \sin^2 \theta)$

$= a^2 + b^2$ [$\because \cos^2 \theta + \sin^2 \theta = 1$]

=RHS

Hence Proved

52. Question

If x = a sec θ + b tan θ a and y = a tan θ + b sec θ, then prove that $x^2 - y^2 = a^2 - b^2$.

Answer

Taking LHS $=x^2 - y^2$

Putting the values of x and y, we get

$(a \sec \theta + b \tan \theta)^2 - (a \tan \theta + b \sec \theta)^2$

$= a^2 \sec^2\theta + b^2 \tan^2\theta + 2ab \sec \theta \tan \theta - a^2\tan^2\theta - b^2 \sec^2\theta - 2ab \sec \theta \tan \theta$

$= a^2 (\sec^2\theta - \tan^2\theta) - b^2 (\sec^2\theta - \tan^2\theta)$

$= a^2 - b^2$ [$\because 1+ \tan^2 \theta = \sec^2 \theta$]

=RHS

Hence Proved

53. Question

If $(a^2 - b^2) \sin \theta + 2ab \cos\theta = a^2 + b^2$, then prove that $\tan \theta = \frac{a^2 - b^2}{2ab}$.

Answer

Taking $(a^2 - b^2) \sin \theta + 2ab \cos \theta = a^2 + b^2$

We know that $\sin \theta = \frac{2 \tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$ and $\cos \theta = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$

Then, substituting the above values in the given equation, we get

$$= a^2 - b^2 \frac{2 \tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} + 2ab \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} = a^2 + b^2$$

Now, substituting, $t = \tan\frac{\theta}{2}$, we hwve

$$a^2 - b^2 \frac{2t}{1+t^2} + 2ab \frac{1-t^2}{1+t^2} = a^2 + b^2$$

$\Rightarrow (a^2 - b^2)2t - 2ab(1 - t^2) = (a^2 + b^2)(1+t^2)$

Simplify, we get

$(a^2 + 2ab + b^2)t^2 - 2(a^2 - b^2)t + (a^2 - 2ab + b^2)=0$

$\Rightarrow (a+b)^2 t^2 - 2(a^2 - b^2)t + (a - b)^2 = 0$

$\Rightarrow (a+b)^2 t^2 - 2 (a - b)(a+b)t + (a - b)^2 = 0$

$\Rightarrow [(a+b)t - (a - b)]^2 = 0$ [$\because (a - b)^2 = (a^2 + b^2 - 2ab)$]

$\Rightarrow [(a+b)t - (a - b)] = 0$

$\Rightarrow (a+b)t = (a - b)$

$$\Rightarrow t = \frac{a-b}{a+b}$$

$$\Rightarrow \tan\frac{\theta}{2} = \frac{a-b}{a+b}$$

We know that, $\frac{2t}{1-t^2} = \tan\theta$, where $t = \tan\frac{\theta}{2}$

$$\Rightarrow \tan\theta = \frac{2\left(\frac{a-b}{a+b}\right)}{1-\left(\frac{a-b}{a+b}\right)^2}$$

$$\Rightarrow \tan\theta = \frac{2(a+b)(a-b)}{(a+b)^2-(a-b)^2}$$

$$\Rightarrow \tan\theta = \frac{a^2-b^2}{2ab}$$

Hence Proved