8. Arithmetic Progressions (AP)

Exercise 8.1

1 A. Question

Write the first three terms of the following sequences defined by:

$t_n = 3n + 1$

Answer

Given: $t_n = 3n + 1$

Taking n = 1, we get

$t_1 = 3(1) + 1 = 3 + 1 = 4$

Taking n = 2, we get

$t_2 = 3(2) + 1 = 6 + 1 = 7$

Taking n = 3, we get

$t_3 = 3(3) + 1 = 9 + 1 = 10$

Hence, the first three terms are 4, 7 and 10.

1 B. Question

Write the first three terms of the following sequences defined by:

$$t_n = 2^n$$

Answer

Given: $t_n = 2^n$

Taking n = 1, we get

$t_1 = 2^1 = 2$

Taking n = 2, we get

$t_2 = 2^2 = 2 \times 2 = 4$

Taking n = 3, we get

$t_3 = 2^3 = 2 \times 2 \times 2 = 8$

Hence, the first three terms are 2, 4 and 8.

1 C. Question

Write the first three terms of the following sequences defined by:

$t_n = n^2 + 1$

Answer

Given: $t_n = n^2 + 1$

Taking n = 1, we get

$t_1 = (1)^2 + 1 = 1 + 1 = 2$

Taking n = 2, we get

$t_2 = (2)^2 + 1 = 4 + 1 = 5$

Taking n = 3, we get

$t_3 = (3)^2 + 1 = 9 + 1 = 10$

Hence, the first three terms are 2, 5 and 10.

1 D. Question

Write the first three terms of the following sequences defined by:

$t_n = n(n + 2)$

Answer

Given: $t_n = n(n+2)$

Taking n = 1, we get

$t_1 = (1)(1+2) = (1)(3) = 3$

Taking n = 2, we get

$t_2 = (2)(2+2) = (2)(4) = 8$

Taking n = 3, we get

$t_3 = (3)(3+2) = (3)(5) = 15$

Hence, the first three terms are 3, 8 and 15.

1 E. Question

Write the first three terms of the following sequences defined by:

$t_n = 2n+5$

Answer

Given: $t_n = 2n + 5$

Taking n = 1, we get

$t_1 = 2(1) + 5 = 2 + 5 = 7$

Taking n = 2, we get

$t_2 = 2(2) + 5 = 4 + 5 = 9$

Taking n = 3, we get

$t_3 = 2(3) + 5 = 6 + 5 = 11$

Hence, the first three terms are 7, 9 and 11.

1 F. Question

Write the first three terms of the following sequences defined by:

$$t_n = \frac{n-3}{4}$$

Answer

Given: $t_n = \frac{n-3}{4}$

Taking n = 1, we get

$$t_1 = \frac{1-3}{4} = \frac{-2}{4} = \frac{-1}{2}$$

Taking n = 2, we get

$$t_2 = \frac{2-3}{4} = \frac{-1}{4}$$

Taking n = 3, we get

$$t_3 = \frac{3-3}{4} = 0$$

Hence, the first three terms are $\frac{-1}{2}, \frac{-1}{4}$ and 0

2 A. Question

Find the indicated terms in each of the following sequence whose nth terms are:

$$t_n = \frac{n(n-2)}{n+3}; t_{20}$$

Answer

Given: $t_n = \frac{n^2(n+1)}{3}$

Now, we have to find t_1 and t_2.

So, in t_1, n = 1

$$\therefore t_1 = \frac{(1)^2(1+1)}{3} = \frac{(1)(2)}{3} = \frac{2}{3}$$

Now, t_2, n = 2

$$\therefore t_2 = \frac{(2)^2(2+1)}{3} = \frac{(4)(3)}{3} = 4$$

2 B. Question

Find the indicated terms in each of the following sequence whose nth terms are:

$$t_n = \frac{n(n-2)}{n+3}; t_{20}$$

Answer

Given: $t_n = \frac{n(n-2)}{n+3}$

So, $t_{20} = \frac{20(20-2)}{20+3} = \frac{20\times18}{23} = \frac{360}{23}$

2 C. Question

Find the indicated terms in each of the following sequence whose nth terms are:

$t_n = (n - 1)(2 - n)(3 + n)$; t_{20}

Answer

Given: $t_n = (n - 1)(2 - n)(3+n)$

So, $t_{20} = (20 - 1)(2 - 20)(3+20)$

$= (19)(-18)(23)$

$= -7866$

2 D. Question

Find the indicated terms in each of the following sequence whose nth terms are:

$$t_n = \frac{t_{n-1}}{n^2}, t_1 = 3; t_2, t_3, (n \geq 2)$$

Answer

Given: $t_n = \frac{t_{n-1}}{n^2}$

So, $t_2 = \frac{t_{2-1}}{(2)^2} = \frac{t_1}{4} = \frac{3}{4}$ [given: $t_1 = 3$]

and $t_3 = \frac{t_{3-1}}{(3)^2} = \frac{t_2}{9} = \frac{\frac{3}{4}}{9} = \frac{3}{4\times 9} = \frac{1}{12}$ $\left[\because t_2 = \frac{3}{4}\right]$

3 A. Question

Write the next three terms of the following sequences:

$$t_1 = 3, t_n = 3t_{n-1} + 2$$

Answer

Given: $t_2 = 2$ and $t_n = t_{n-1} + 1$

Now, we have to find next three terms i.e. t_3, t_4 and t_5

Taking n = 3, we get

$t_3 = t_{3-1} + 1$

$= t_2 + 1$

$= 2 + 1$ [given: $t_2 = 2$]

$t_3 = 3$ ...(i)

Taking n = 4, we get

$t_4 = t_{4-1} + 1$

$= t_3 + 1$

= 3 + 1 [from (i)]

$t_4 = 4$ ...(ii)

Taking n = 5, we get

$t_5 = t_{5-1} + 1$

$= t_4 + 1$

= 4 +1

$t_5 = 5$ [from (ii)]

Hence, the next three terms are 3, 4 and 5.

3 B. Question

Write the next three terms of the following sequences:

$t_1 = 3, t_n = 3t_{n-1} + 2$ for all $n > 1$

Answer

Given: $t_1 = 3$ and $t_n = 3t_{n-1} + 2$

Now, we have to find next three terms i.e. t_2, t_3 and t_4

Taking n = 2, we get

$t_2 = 3t_{2-1} + 2$

$= 3t_1 + 2$

= 3(3) + 2 [given: $t_1 = 3$]

$t_3 = 9 + 2$

$t_3 = 11$ ...(i)

Taking n = 3, we get

$t_3 = 3t_{3-1} + 2$

$= 3t_2 + 2$

$= 3(11) + 2$ [from (i)]

$= 33 + 2$

$t_3 = 35$ …(ii)

Taking n = 4, we get

$t_4 = 3t_{4-1} + 2$

$= 3t_3 + 2$

$= 3(35) +2$

$t_5 = 105 + 2$

$t_5 = 107$ [from (ii)]

Hence, the next three terms are 11, 35 and 107.

4 A. Question

Write the first four terms of the A.P. when first term a and common difference d are given as follows:

a = 1, d = 1

Answer

Given: a = 1 and d = 1

The general form of an A.P is a, a+d, a+2d, a+3d,…

So, the first term a is 1 and the common difference d is 1, then the first four terms of the AP is

1, (1+1), (1+2×1), (1+3×1)

⇒ 1, 2, 3, 4

Hence, the first four terms of the A.P. is 1, 2, 3 and 4.

4 B. Question

Write the first four terms of the A.P. when first term a and common difference d are given as follows:

a= 3, d=0

Answer

Given: a = 3 and d = 0

The general form of an A.P is a, a+d, a+2d, a+3d,…

So, the first term a is 3 and the common difference d is 0, then the first four terms of the AP is

3, (3+0), (3+2×0), (3+3×0)

⇒ 3, 3, 3, 3

Hence, first four terms of the A.P. is 3, 3, 3 and 3.

4 C. Question

Write the first four terms of the A.P. when first term a and common difference d are given as follows:

a = 10, d = 10

Answer

Given: a = 10 and d = 10

The general form of an A.P is a, a+d, a+2d, a+3d,…

So, the first term a is 10 and the common difference d is 10, then the first four terms of the AP is

10, (10+10), (10+2×10), (10+3×10)

⇒ 10, (20), (10+20), (10+30)

⇒ 10, 20, 30, 40

Hence, first four terms of the A.P. is 10, 20, 30 and 40.

4 D. Question

Write the first four terms of the A.P. when first term a and common difference d are given as follows:

a= -2, d=0

Answer

Given: a = -2 and d = 0

The general form of an A.P is a, a+d, a+2d, a+3d,…

So, the first term a is -2 and the common difference d is 0, then the first four terms of the AP is

-2, (-2+0), (-2+2×0), (-2+3×0)

⇒ -2, -2, -2, -2

Hence, the first four terms of the A.P. is -2, -2, -2 and -2.

4 E. Question

Write the first four terms of the A.P. when first term a and common difference d are given as follows:

a = 100, d = -30

Answer

Given: a = 100 and d = -30

The general form of an A.P is a, a+d, a+2d, a+3d,...

So, the first term a is 100 and the common difference d is -30, then the first four terms of the AP is

100, (100+(-30)), (100+2×(-30)),(100+3×(-30))

⇒ 100, (100 – 30), (100 – 60), (100 – 90)

⇒ 100, 70, 40, 10

Hence, first four terms of the A.P. is 100, 70, 40 and 10.

4 F. Question

Write the first four terms of the A.P. when first term a and common difference d are given as follows:

a= -1, d= 1/2

Answer

Given: a = -1 and d = $\frac{1}{2}$

The general form of an A.P is a, a+d, a+2d, a+3d,...

So, the first term a is 1 and the common difference d is $\frac{1}{2}$, then the first four terms of the AP is

$$-1, \left(-1+\frac{1}{2}\right), \left(-1+2\times\frac{1}{2}\right), \left(-1+3\times\frac{1}{2}\right)$$

$$\Rightarrow -1, \left(\frac{-2+1}{2}\right), (-1+1), \left(\frac{-2+3}{2}\right)$$

$$\Rightarrow -1, \frac{-1}{2}, 0, \frac{1}{2}$$

Hence, first four terms of the A.P. is $-1, \frac{-1}{2}, 0, \frac{1}{2}$.

4 G. Question

Write the first four terms of the A.P. when first term a and common difference d are given as follows:

a = -7, d = -7

Answer

Given: a = -7 and d = -7

The general form of an A.P is a, a+d, a+2d, a+3d,…

So, the first term a is -7, and the common difference d is -7, then the first four terms of the AP is

-7, (-7+(-7)), (-7+2×(-7)), (-7+3×(-7))

⇒ -7, (-7 – 7), (-7 – 14), (-7 – 21)

⇒ -7, -14, -21, -28

Hence, the first four terms of the A.P. is -7, -14, -21 and -28.

4 H. Question

Write the first four terms of the A.P. when first term a and common difference d are given as follows:

a = 1, d = 0.1

Answer

Given: a = 1 and d = 0.1

The general form of an A.P is a, a+d, a+2d, a+3d,…

So, the first term a is 1, and the common difference d is 0.1, then the first four terms of the AP is

1, (1+0.1), (1+2×(0.1)), (1+3×(0.1))

⇒ 1, 1.1, 1.2, 1.3

Hence, the first four terms of the A.P. is 1, 1.1, 1.2 and 1.3.

5 A. Question

For the following A.P's write the first term and common difference:

6, 3, 0, - 3, ...

Answer

In general, for an AP $a_1, a_2, \ldots, a_n$, we have

$d = a_{k+1} - a_k$

where a_{k+1} and a_k are the $(k+1)^{th}$ and k^{th} terms respectively.

For the list of numbers: 6, 3, 0, -3, . . .

$a_2 - a_1 = 3 - 6 = -3$

$a_3 - a_2 = 0 - 3 = -3$

$a_4 - a_3 = -3 - 0 = -3$

Here, the difference of any two consecutive terms in each case is -3.

So, the given list is an AP whose first term a is 6, and common difference d is -3.

5 B. Question

For the following A.P's write the first term and common difference:

- 3.1, - 3.0, - 2.9, - 2.8, ...

Answer

In general, for an AP $a_1, a_2, \ldots, a_n$, we have

$d = a_{k+1} - a_k$

where a_{k+1} and a_k are the $(k+1)^{th}$ and k^{th} terms respectively.

For the list of numbers: - 3.1, - 3.0, - 2.9, - 2.8, ...

$a_2 - a_1 = -3.0 - (-3.1) = -3.0 + 3.1 = 0.1$

$a_3 - a_2 = -2.9 - (-3.0) = -2.9 + 3.0 = 0.1$

$a_4 - a_3 = -2.8 - (-2.9) = -2.8 + 2.9 = 0.1$

Here, the difference of any two consecutive terms in each case is 0.1. So, the given list is an AP whose first term a is -3.1 and common difference d is 0.1.

5 C. Question

For the following A.P's write the first term and common difference:

147, 148, 149, 150, ...

Answer

In general, for an AP $a_1, a_2, \dots, a_n$, we have

$d = a_{k+1} - a_k$

where a_{k+1} and a_k are the $(k+1)^{th}$ and k^{th} terms respectively.

For the list of numbers: 147, 148, 149, 150, ...

$a_2 - a_1 = 148 - 147 = 1$

$a_3 - a_2 = 149 - 148 = 1$

$a_4 - a_3 = 150 - 149 = 1$

Here, the difference of any two consecutive terms in each case is -1. So, the given list is an AP whose first term a is 147 and common difference d is 1.

5 D. Question

For the following A.P's write the first term and common difference:

- 5, - 1, 3, 7, ...

Answer

In general, for an AP $a_1, a_2, \dots, a_n$, we have

$d = a_{k+1} - a_k$

where a_{k+1} and a_k are the $(k+1)^{th}$ and k^{th} terms respectively.

For the list of numbers: - 5, - 1, 3, 7, ...

$a_2 - a_1 = -1 - (-5) = -1 + 5 = 4$

$a_3 - a_2 = 3 - (-1) = 3 + 1 = 4$

$a_4 - a_3 = 7 - 3 = 4$

Here, the difference of any two consecutive terms in each case is -4. So, the given list is an AP whose first term a is -5 and common difference d is 4.

5 E. Question

For the following A.P's write the first term and common difference:

3, 1, - 1, - 3, ...

Answer

In general, for an AP $a_1, a_2, \dots, a_n$, we have

$d = a_{k+1} - a_k$

where a_{k+1} and a_k are the $(k+1)^{th}$ and k^{th} terms respectively.

For the list of numbers: 3, 1, - 1, - 3, ...

$a_2 – a_1 = 1 – 3 = -2$

$a_3 – a_2 = -1 – 1 = -1 - 1 = -2$

$a_4 – a_3 = -3 – (-1) = -3 + 1 = -2$

Here, the difference of any two consecutive terms in each case is --2. So, the given list is an AP whose first term a is 3 and common difference d is -2.

5 F. Question

For the following A.P's write the first term and common difference:

$$2, 2\frac{1}{3}, 2\frac{2}{3}, -3,$$

Answer

In general, for an AP $a_1, a_2, \dots, a_n$, we have

$d = a_{k+1} - a_k$

where a_{k+1} and a_k are the $(k+1)^{th}$ and k^{th} terms respectively.

For the list of numbers: $2, \frac{7}{3}, \frac{8}{3}, 3, \dots$

$$a_2 - a_1 = \frac{7}{3} - 2 = \frac{7-6}{3} = \frac{1}{3}$$

$$a_3 - a_2 = \frac{8}{3} - \frac{7}{3} = \frac{1}{3}$$

$$a_4 - a_3 = 3 - \frac{8}{3} = \frac{9-8}{3} = \frac{1}{3}$$

Here, the difference of any two consecutive terms in each case is $\frac{1}{3}$. So, the given list is an AP whose first term a is 2 and common difference d is $\frac{1}{3}$.

5 G. Question

For the following A.P's write the first term and common difference:

$$\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}, \ldots$$

Answer

In general, for an AP $a_1, a_2, \ldots, a_n$, we have

$d = a_{k+1} - a_k$

where a_{k+1} and a_k are the $(k+1)^{th}$ and k^{th} terms respectively.

For the list of numbers: $\frac{3}{2}, \frac{1}{2}, \frac{-1}{2}, \frac{-3}{2}, \ldots$

$$a_2 - a_1 = \frac{1}{2} - \frac{3}{2} = \frac{-2}{2} = -1$$

$$a_3 - a_2 = \frac{-1}{2} - \frac{1}{2} = \frac{-2}{2} = -1$$

$$a_4 - a_3 = -\frac{3}{2} - \left(\frac{-1}{2}\right) = \frac{-3+1}{2} = \frac{-2}{2} = -1$$

Here, the difference of any two consecutive terms in each case is -1. So, the given list is an AP whose first term a is $\frac{3}{2}$ and common difference d is -1.

6 A. Question

Which of the following list of numbers form A.R's? If they form an A.P., find the common difference d and also write its next three terms.

1, - 1, - 3, - 5,

Answer

We have,

$a_2 - a_1 = -1 - 1 = -2$

$a_3 - a_2 = -3 - (-1) = -3 + 1 = -2$

$a_4 - a_3 = -5 - (-3) = -5 + 3 = -2$

i.e. $a_{k+1} - a_k$ is the same every time.

So, the given list of numbers forms an AP with the common difference d = -2

Now, we have to find the next three terms.

We have $a_1 = 1$, $a_2 = -1$, $a_3 = -3$ and $a_4 = -5$

Now, we will find a_5, a_6 and a_7

So, $a_5 = -5 + (-2) = -5 - 2 = -7$

$a_6 = -7 + (-2) = -7 - 2 = -9$

and $a_7 = -9 + (-2) = -9 - 2 = -11$

Hence, the next three terms are -7, -9 and -11

6 B. Question

Which of the following list of numbers form A.R's? If they form an A.P., find the common difference d and also write its next three terms.

2, 4, 8, 16, ...

Answer

We have,

$a_2 - a_1 = 4 - 2 = 2$

$a_3 - a_2 = 8 - 4 = 4$

$a_4 - a_3 = 16 - 8 = 8$

i.e. $a_{k+1} - a_k$ is not same every time.

So, the given list of numbers do not form an AP.

6 C. Question

Which of the following list of numbers form A.R's? If they form an A.P., find the common difference d and also write its next three terms.

- 2, 2, - 2, 2, - 2, ...

Answer

We have,

$a_2 - a_1 = 2 - (-2) = 2 + 2 = 4$

$a_3 - a_2 = -2 - 2 = -4$

$a_4 - a_3 = 2 - (-2) = 2 + 2 = 4$

i.e. $a_{k+1} - a_k$ is not same every time.

So, the given list of numbers do not form an AP.

6 D. Question

Which of the following list of numbers form A.R's? If they form an A.P., find the common difference d and also write its next three terms.

$$-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},\ldots$$

Answer

We have,

$$a_2-a_1=-\frac{1}{2}-\left(-\frac{1}{2}\right)=\frac{-1+1}{2}=0$$

$$a_3-a_2=-\frac{1}{2}-\left(-\frac{1}{2}\right)=\frac{-1+1}{2}=0$$

$$a_4-a_3=-\frac{1}{2}-\left(-\frac{1}{2}\right)=\frac{-1+1}{2}=0$$

i.e. $a_{k+1} - a_k$ is the same every time.

So, the given list of numbers forms an AP with the common difference d = 0

Now, we have to find the next three terms.

We have $a_1=-\frac{1}{2}, a_2=-\frac{1}{2}, a_3=-\frac{1}{2}, a_4=-\frac{1}{2}$

Now, we will find a_5, a_6 and a_7

So, $a_5=-\frac{1}{2}+0=-\frac{1}{2}$

$$a_6=-\frac{1}{2}+0=-\frac{1}{2}$$

and $a_7=-\frac{1}{2}+0=-\frac{1}{2}$

Hence, the next three terms are $-\frac{1}{2},-\frac{1}{2}$ and $-\frac{1}{2}$

6 E. Question

Which of the following list of numbers form A.R's? If they form an A.P., find the common difference d and also write its next three terms.

$$2, \frac{5}{2}, 3 \frac{7}{2}, \ldots\ldots$$

Answer

We have,

$$a_2 - a_1 = \frac{5}{2} - 2 = \frac{5-4}{2} = \frac{1}{2}$$

$$a_3 - a_2 = 3 - \left(\frac{5}{2}\right) = \frac{6-5}{2} = \frac{1}{2}$$

$$a_4 - a_3 = \frac{7}{2} - 3 = \frac{7-6}{2} = \frac{1}{2}$$

i.e. $a_{k+1} - a_k$ is the same every time.

So, the given list of numbers forms an AP with the common difference $d = \frac{1}{2}$

Now, we have to find the next three terms.

We have $a_1 = 2, a_2 = \frac{5}{2}, a_3 = 3, a_4 = \frac{7}{2}$

Now, we will find a_5, a_6 and a_7

So, $a_5 = \frac{7}{2} + \frac{1}{2} = \frac{8}{2} = 4$

$$a_6 = 4 + \frac{1}{2} = \frac{8+1}{2} = \frac{9}{2}$$

and $a_7 = \frac{9}{2} + \frac{1}{2} = \frac{10}{2} = 5$

Hence, the next three terms are $4, \frac{9}{2}$ and 5.

6 F. Question

Which of the following list of numbers form A.R's? If they form an A.P., find the common difference d and also write its next three terms.

0, - 4, - 8, - 12,

Answer

We have,

$a_2 - a_1 = -4 - 0 = -4$

$a_3 - a_2 = -8 - (-4) = -8 + 4 = -4$

$a_4 - a_3 = -12 - (-8) = -12 + 8 = -4$

i.e. $a_{k+1} - a_k$ is the same every time.

So, the given list of numbers forms an AP with the common difference d = -4

Now, we have to find the next three terms.

We have $a_1 = 0$, $a_2 = -4$, $a_3 = -8$ and $a_4 = -12$

Now, we will find a_5, a_6 and a_7

So, $a_5 = -12 + (-4) = -12 - 4 = -16$

$a_6 = -16 + (-4) = -16 - 4 = -20$

and $a_7 = -20 + (-4) = -20 - 4 = -24$

Hence, the next three terms are -16, -20 and -24

6 G. Question

Which of the following list of numbers form A.R's? If they form an A.P., find the common difference d and also write its next three terms.

4, 10, 16, 22, ...

Answer

We have,

$a_2 - a_1 = 10 - 4 = 6$

$a_3 - a_2 = 16 - 10 = 6$

$a_4 - a_3 = 22 - 16 = 6$

i.e. $a_{k+1} - a_k$ is the same every time.

So, the given list of numbers forms an AP with the common difference d = 6

Now, we have to find the next three terms.

We have $a_1 = 4$, $a_2 = 10$, $a_3 = 16$ and $a_4 = 22$

Now, we will find a_5, a_6 and a_7

So, $a_5 = 22 + 6 = 28$

$a_6 = 28 + 6 = 34$

and $a_7 = 34 + 6 = 40$

Hence, the next three terms are 28, 34 and 40

6 H. Question

Which of the following list of numbers form A.R's? If they form an A.P., find the common difference d and also write its next three terms.

a, 2a, 3a, 4a, ...

Answer

We have,

$a_2 - a_1 = 2a - a = a$

$a_3 - a_2 = 3a - 2a = a$

$a_4 - a_3 = 4a - 3a = a$

i.e. $a_{k+1} - a_k$ is the same every time.

So, the given list of numbers forms an AP with the common difference d = a

Now, we have to find the next three terms.

We have $a_1 = a$, $a_2 = 2a$, $a_3 = 3a$ and $a_4 = 4a$

Now, we will find a_5, a_6 and a_7

So, $a_5 = 4a + a = 5a$

$a_6 = 5a + a = 6a$

and $a_7 = 6a + a = 7a$

Hence, the next three terms are 5a, 6a and 7a

6 I. Question

Which of the following list of numbers form A.R's? If they form an A.P., find the common difference d and also write its next three terms.

- 1.2, - 3.2, - 5.2, - 7.2, ...

Answer

We have,

$a_2 - a_1 = -3.2 - (-1.2) = -3.2 + 1.2 = -2.0$

$a_3 - a_2 = -5.2 - (-3.2) = -5.2 + 3.2 = -2.0$

$a_4 - a_3 = -7.2 - (-5.2) = -7.2 + 5.2 = -2.0$

i.e. $a_{k+1} - a_k$ is the same every time.

So, the given list of numbers forms an AP with the common difference d = -2

Now, we have to find the next three terms.

We have $a_1 = -1.2$, $a_2 = -3.2$, $a_3 = -5.2$ and $a_4 = -7.2$

Now, we will find a_5, a_6 and a_7

So, $a_5 = -7.2 + (-2) = -7.2 - 2.0 = -9.2$

$a_6 = -9.2 + (-2) = -9.2 - 2.0 = -11.2$

and $a_7 = -11.2 + (-2) = -11.2 - 2.0 = -13.2$

Hence, the next three terms are -9.2, -11.2 and -13.2

6 J. Question

Which of the following list of numbers form A.R's? If they form an A.P., find the common difference d and also write its next three terms.

$\sqrt{3}, \sqrt{12}, \sqrt{48}, \sqrt{192}$

Answer

We have,

$a_2 - a_1 = \sqrt{12} - \sqrt{3} = 2\sqrt{3} - \sqrt{3} = \sqrt{3}$

$a_3 - a_2 = \sqrt{48} - \sqrt{12} = 4\sqrt{3} - 2\sqrt{3} = 2\sqrt{3}$

$a_4 - a_3 = \sqrt{192} - \sqrt{48} = 8\sqrt{3} - 4\sqrt{3} = 4\sqrt{3}$

i.e. $a_{k+1} - a_k$ is not same every time.

So, the given list of numbers do not form an AP.

6 K. Question

Which of the following list of numbers form A.R's? If they form an A.P., find the common difference d and also write its next three terms.

$a, a^2, a^3, a^4, \ldots$

Answer

We have,

$a_2 - a_1 = a^2 - a = a(a - 1)$

$a_3 - a_2 = a^3 - a^2 = a^2(a - 1)$

$a_4 - a_3 = a^4 - a^3 = a^3(a - 1)$

i.e. $a_{k+1} - a_k$ is not same every time.

So, the given list of numbers does not form an AP.

6 L. Question

Which of the following list of numbers form A.R's? If they form an A.P., find the common difference d and also write its next three terms.

1, 3, 9, 27, ...

Answer

We have,

$a_2 - a_1 = 3 - 1 = 2$

$a_3 - a_2 = 9 - 3 = 6$

$a_4 - a_3 = 27 - 9 = 18$

i.e. $a_{k+1} - a_k$ is not same every time.

So, the given list of numbers does not form an AP.

6 M. Question

Which of the following list of numbers form A.R's? If they form an A.P., find the common difference d and also write its next three terms.

$1^2, 2^2, 3^2, 4^2, \ldots$

Answer

We have,

$a_2 - a_1 = 2^2 - (1)^2 = 4 - 1 = 3$

$a_3 - a_2 = 3^2 - (2)^2 = 9 - 4 = 5$

$a_4 - a_3 = 4^2 - (3)^2 = 16 - 9 = 7$

i.e. $a_{k+1} - a_k$ is not same every time.

So, the given list of numbers does not form an AP.

6 N. Question

Which of the following list of numbers form A.R's? If they form an A.P., find the common difference d and also write its next three terms.

$1^2, 5^2, 7^2, 7^2, \ldots$

Answer

We have,

$a_2 - a_1 = 5^2 - (1)^2 = 25 - 1 = 24$

$a_3 - a_2 = 7^2 - (5)^2 = 49 - 25 = 24$

$a_4 - a_3 = 7^2 - (7)^2 = 0$

i.e. $a_{k+1} - a_k$ is not same every time.

So, the given list of numbers do not form an AP.

6 O. Question

Which of the following list of numbers form A.R's? If they form an A.P., find the common difference d and also write its next three terms.

$1^2, 3^2, 5^2, 7^2, \ldots$

Answer

We have,

$a_2 - a_1 = 3^2 - (1)^2 = 9 - 1 = 8$

$a_3 - a_2 = 5^2 - (3)^2 = 25 - 9 = 16$

$a_4 - a_3 = 7^2 - (5)^2 = 49 - 25 = 24$

i.e. $a_{k+1} - a_k$ is not same every time.

So, the given list of numbers does not form an AP.

7 A. Question

In which of the following situations does the list of numbers involved arithmetic progression, and why?

The salary of a teacher in successive years when starting salary is Rs. 8000, with an annual increment of Rs. 500.

Answer

Salary for the 1st year = Rs. 8000

and according to the question,

There is an annual increment of Rs. 500

⇒ The salary for the 2nd year = Rs. 8000 + 500 = Rs.8500

Now, again there is an increment of Rs. 500

⇒ The salary for the 3rd year = Rs. 8500 +500 = Rs. 9000

Therefore, the series is

8000 , 8500 , 9000 , …

Difference between 2nd term and 1st term = 8500 – 8000 = 500

Difference between 3rd term and 2nd term = 9000 – 8500 = 500

Since, the difference is same.

Hence, salary in successive years are in AP with common difference d = 500 and first term a is 8000.

7 B. Question

In which of the following situations does the list of numbers involved arithmetic progression, and why?

The taxi fare after each km when the fare is Rs. 15 for the first km and Rs. 8 for each additional km.

Answer

Taxi fare for 1km = 15

According to question, Rs. 8 for each additional km

⇒ Taxi fare for 2km = 15 + 8 =23

and Taxi fare for 3km = 23 + 8 =31

Therefore, series is

15, 23, 31 ,…

Difference between 2nd and 1st term = 23 – 15 = 8

Difference between 3^{rd} and 2^{nd} term = 31 – 23 = 8

Since, difference is same.

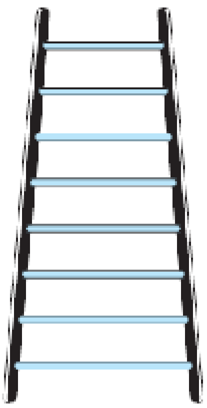
Hence, the taxi fare after each km form an AP with the first term, a = Rs. 15 and common difference, d = Rs. 8

7 C. Question

In which of the following situations does the list of numbers involved arithmetic progression, and why?

The lengths of the rungs of a ladder when the bottom rung is 45cm, and length of rungs decrease by 2 cm from bottom to top.

Answer



The length of the bottom rung = 45cm

According to the question,

Length of rungs decreases by 2cm from bottom to top. The lengths (in cm) of the 1^{st}, 2^{nd}, 3^{rd}, … from the bottom to top respectively are 45, 43, 41, …

Difference between 2^{nd} and 1^{st} term = 43 – 45 = -2

Difference between 3^{rd} and 2^{nd} term = 41 – 43 = -2

Since, the difference is same.

Hence, the length of the rungs form an AP with a = 45cm and d = -2 cm.

7 D. Question

In which of the following situations does the list of numbers involved arithmetic progression, and why?

The amount of money in the account every year when Rs. 10000 is deposited at compound interest 8% per annum.

Answer

Original Amount = Rs. 10,000

Interest earned in first year = 10,000 × 8%

$$= 10,000 \times \frac{8}{100}$$

= Rs 800

Total amount outstanding after one year = Rs 10000 + 800

= Rs 10800

Now, interest earned in 2^{nd} year $= 10800 \times \frac{8}{100} = \text{Rs}.864$

Total amount outstanding after 2^{nd} year = Rs10800 + 864

= Rs 11664

Interest earned in 3^{rd} year $= 11664 \times \frac{8}{100}$

= Rs 933.12

Total amount outstanding after 3^{rd} year = Rs 11664 + 933.12

= Rs 12597.12

Therefore, the series is

10800, 11664, 12597.12,...

Difference between second and first term = 11664 – 10800

= 864

Difference between third and second term = 12597.12 – 11664

= 933.12

Since the difference is not same

Therefore, it doesn't form an AP.

7 E. Question

In which of the following situations does the list of numbers involved arithmetic progression, and why?

The money saved by Sudha in successive years when she saves Rs. 100 the first year and increased the amount by Rs. 50 every year.

Answer

The money saved by Sudha in the first year = Rs. 100

According to the question,

Sudha increased the amount by Rs. 50 every year

⇒ The money saved by Sudha in a 2^{nd} year = Rs. 100 +50

= Rs. 150

The money saved by Sudha in a 3^{rd} year = Rs. 150 + 50

= Rs. 200

Therefore, the series is

100, 150, 200, 250,…

Difference in the 2^{nd} term and 1^{st} term = 150 – 100 = 50

Difference in the 3^{rd} term and 2^{nd} term = 200 – 150 = 50

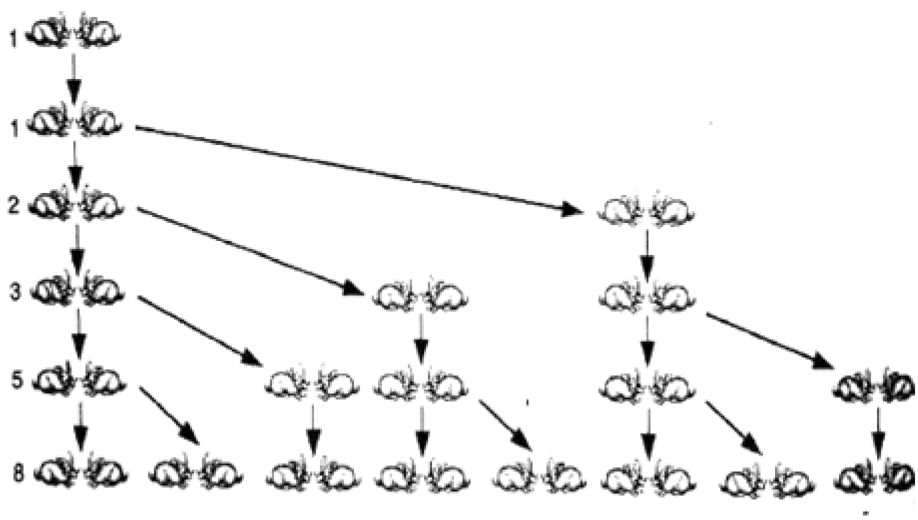
Since the difference is the same.

Therefore, the money saved by Sudha in successive years form an AP with a = Rs 100 and d =Rs 50

7 F. Question

In which of the following situations does the list of numbers involved arithmetic progression, and why?

Number of pairs of rabbits in successive months when the pair of rabbits is too young to produce in their first month. In the second month and every subsequent month, they produce a new pair. Each new pair of rabbits pr a new pair in their second months and every subsequent month (see Fig.) (assume that no rabbit dies).



Answer

Assuming that no rabbit dies,

the number of pairs of rabbits at the start of the 1^{st} month = 1

the number of pairs of rabbits at the start of the 2^{nd} month = 1

the number of pairs of rabbits at the start of the 3^{rd} month = 2

the number of pairs of rabbits at the start of the 4^{th} month = 3

the number of pairs of rabbits at the start of the 5^{th} month = 5

Therefore, the series is

1, 1, 2, 3, 5, 8,…

Difference between 2^{nd} and 1^{st} term = 1 – 1 = 0

Difference between 3^{rd} and 2^{nd} term = 2 – 1 = 1

Since, the difference is not same.

Therefore, the number of pair of rabbits in successive months are 1,1,2,3,5,8,… and they don't form an AP.

7 G. Question

In which of the following situations does the list of numbers involved arithmetic progression, and why?

The values of an investment after 1, 2, 3, 4, ... years if after each subsequent year it increases by 5/4 times the initial investment.

Answer

Let the initial investment be I,

After one year it increases by $\frac{5}{4}$ I,

So the investment becomes, I + $\frac{5}{4}$ I

$=\frac{9}{4}$ I

At the end of 2^{nd} year it again increase to $\frac{5}{4}$ I,

So the investment becomes, (I + $\frac{5}{4}$ I + $\frac{5}{4}$ I)

$=\left(\frac{9}{4}I+\frac{5}{4}I\right)$

$=\frac{14}{4}I$

At the end of the 3rd year it again increases to $\frac{5}{4}I$

So the investment becomes $\left(I+\frac{5}{4}I+\frac{5}{4}I+\frac{5}{4}I\right)$

$=\frac{9}{4}I+\frac{5}{4}I+\frac{5}{4}I$

$=\frac{14}{4}I+\frac{5}{4}I$

$=\frac{19}{4}I$

Therefore the series is:

$I, \frac{9}{4}I, \frac{14}{4}I, \frac{19}{4}I, \ldots\ldots\ldots$

Now difference between 2^{nd} and 1^{st} term is $\frac{9}{4}I - I = \frac{5}{4}I$

difference between 3^{rd} and 2^{nd} term is $\frac{14}{4}I - \frac{9}{4}I = \frac{5}{4}I$

Since the difference is same,

Hence the obtained series is an A.P.

Exercise 8.2

1 A. Question

Find the indicated terms in each of the following arithmetic progression:

$1, 6, 11, 16, \ldots, t_{16},$

Answer

Given: 1, 6, 11, 16, …

Here, a = 1

$d = a_2 - a_1 = 6 - 1 = 5$

and n = 16

We have,

$t_n = a + (n - 1)d$

So, $t_{16} = 1 + (16 - 1)5$

$= 1 + 15\times5$

$t_{16} = 1 + 75$

$t_{16} = 76$

1 B. Question

Find the indicated terms in each of the following arithmetic progression:

$a = 3, d = 2;$; t_n, t_{10}

Answer

Given: a = 3 , d = 2

To find: t_n and t_{10}

We have,

$t_n = a + (n - 1)d$

$t_n = 3 + (n - 1)\, 2$

$= 3 + 2n - 2$

$t_n = 2n + 1$

Now, n = 10

So, $t_{10} = 3 + (10 - 1)2$

$= 3 + 9\times2$

$t_{10} = 3 + 18$

$t_{10} = 21$

1 C. Question

Find the indicated terms in each of the following arithmetic progression:

— 3, — 1/2, 2, ... ; t_{10},

Answer

Given: $-3, -\frac{1}{2}, 2, \ldots$

Here, a = −3

$$d = a_2 - a_1 = -\frac{1}{2} - (-3) = \frac{-1+6}{2} = \frac{5}{2}$$

and n = 10

We have,

t_n = a + (n – 1)d

So, $t_{10} = -3 + (10-1)\frac{5}{2}$

$$= -3 + 9 \times \frac{5}{2}$$

$$= \frac{-6+45}{2}$$

$$t_{10} = \frac{39}{2}$$

1 D. Question

Find the indicated terms in each of the following arithmetic progression:

a = 21, d = — 5; t_n, t_{25}

Answer

Given: a = 21 , d = −5

To find: t_n and t_{25}

We have,

t_n = a + (n – 1)d

t_n = 21 + (n – 1)(−5)

= 21 – 5n + 5

t_n = 26 – 5n

Now, n = 25

So, t_{25} = 21 + (25 – 1)(−5)

= 21 + 24 × (−5)

t_{25} = 21 – 120

$t_{25} = -99$

2. Question

Find the 10th term of the A.P. 10, 5, 0, — 5, — 10, ...

Answer

Given: 10, 5, 0, –5, –10,…

To find: 10^{th} term i.e. t_{10}

Here, a = 10

$d = a_2 - a_1 = 5 - 10 = -5$

and n = 10

We have,

$\mathbf{t_n = a + (n - 1)d}$

$t_{10} = 10 + (10 - 1)(-5)$

$= 10 + 9 \times -5$

$t_{10} = 10 - 45$

$t_{10} = -35$

Therefore, the 10^{th} term of the given is –35.

3. Question

Find the 10th term of the A.P. $\frac{13}{5}, \frac{7}{5}, \frac{1}{5}, -1, \ldots$

Answer

Given: $\frac{13}{5}, \frac{7}{5}, \frac{1}{5}, -1$

Here, $a = \frac{13}{5}$

$$d = a_2 - a_1 = \frac{7}{5} - \frac{13}{5} = -\frac{6}{5}$$

and n = 10

We have,

t_n = a + (n – 1)d

$$t_{10} = \frac{13}{5} + (10 - 1)\left(-\frac{6}{5}\right)$$

$$t_{10} = \frac{13}{5} + 9 \times \frac{-6}{5}$$

$$t_{10} = \frac{13 - 54}{5}$$

$$t_{10} = -\frac{41}{5}$$

Therefore, the 10^{th} term of the given AP is $-\frac{41}{5}$

4. Question

Find the sum of 20th and 25th terms of A.P. 2, 5, 8, 11, ...

Answer

Given: 2, 5, 8, 11, ...

Here, a = 2

d = $a_2 - a_1$ = 5 – 2 = 3

and n = 20

We have,

t_n = a + (n – 1)d

t_{20} = 2 + (20 – 1)(3)

t_{20} = 2 + 19 × 3

= 2 + 57

t_{20} = 59

Now, n = 25

t_{25} = 2 + (25 – 1)(3)

t_{25} = 2 + 24 × 3

t_{25} = 2 + 72

$t_{25} = 74$

The sum of 20th and 25th terms of AP = $t_{20} + t_{25} = 59 + 74 = 133$

5 A. Question

Find the number of terms in the following A.P.'s

6, 3, 0, — 3,.....,−36

Answer

Here, a = 6 , d = 3 − 6 = −3 and l = −36

where **l = a + (n − 1)d**

⇒ −36 = 6 + (n − 1)(−3)

⇒ −36 = 6 −3n + 3

⇒ −36 = 9 − 3n

⇒ −36 − 9 = −3n

⇒ −45 = −3n

$$\Rightarrow n = \frac{-45}{-3} = 15$$

Hence, the number of terms in the given AP is 15

5 B. Question

Find the number of terms in the following A.P.'s

$$\frac{5}{6}, 1, 1\frac{1}{6}, \ldots, 3\frac{1}{3}$$

Answer

Here, $a = \frac{5}{6}$

$$d = a_2 - a_1 = 1 - \frac{5}{6} = \frac{6-5}{6} = \frac{1}{6}$$

And $l = \frac{10}{3}$

We have,

l = a + (n − 1)d

$$\Rightarrow \frac{10}{3} = \frac{5}{6} + (n-1) \times \frac{1}{6}$$

$$\Rightarrow \frac{10}{3} - \frac{5}{6} = (n-1) \times \frac{1}{6}$$

$$\Rightarrow 6 \times \left(\frac{20-5}{6}\right) = n - 1$$

⇒ 15 = n – 1

⇒ n = 16

Hence, the number of terms in the given AP is 16.

6. Question

Determine the number of terms in the A.P. 3, 7, 11, ..., 399. Also, find its 20th term from the end.

Answer

Here, a = 3, d = 7 – 3 = 4 and l = 399

To find : n and 20^{th} term from the end

We have,

l = a + (n – 1)d

⇒ 399 = 3 + (n – 1) × 4

⇒ 399 – 3 = 4n – 4

⇒ 396 + 4 = 4n

⇒ 400 = 4n

⇒ n = 100

So, there are 100 terms in the given AP

Last term = 100^{th}

Second Last term = 100 – 1 = 99^{th}

Third last term = 100 – 2 = 98^{th}

And so, on

20^{th} term from the end = 100 – 19 = 81^{st} term

The 20^{th} term from the end will be the 81^{st} term.

So, $t_{81} = 3 + (81 - 1)(4)$

$t_{81} = 3 + 80 \times 4$

$t_{81} = 3 + 320$

$t_{81} = 323$

Hence, the number of terms in the given AP is 100, and the 20^{th} term from the last is 323.

7 A. Question

Which term of the A.P. 5, 9, 13, 17, ... is 81?

Answer

Here, $a = 5$, $d = 9 - 5 = 4$ and $a_n = 81$

To find : n

We have,

$\mathbf{a_n = a + (n - 1)d}$

$\Rightarrow 81 = 5 + (n - 1) \times 4$

$\Rightarrow 81 = 5 + 4n - 4$

$\Rightarrow 81 = 4n + 1$

$\Rightarrow 80 = 4n$

$\Rightarrow n = 20$

Therefore, the 20^{th} term of the given AP is 81.

7 B. Question

Which term of the A.P. 14, 9, 4, – I, – 6, ... is – 41 ?

Answer

Here, $a = 14$, $d = 9 - 14 = -5$ and $a_n = -41$

To find : n

We have,

$\mathbf{a_n = a + (n - 1)d}$

$\Rightarrow -41 = 14 + (n - 1) \times (-5)$

$\Rightarrow -41 = 14 - 5n + 5$

$\Rightarrow -41 = 19 - 5n$

$\Rightarrow -41 - 19 = -5n$

$\Rightarrow -60 = -5n$

$\Rightarrow n = 12$

Therefore, the 12^{th} term of the given AP is −41.

7 C. Question

Which term of A.P. 3, 8, 13, 18, ... is 88?

Answer

Here, $a = 3$, $d = 8 - 3 = 5$ and $a_n = 88$

To find : n

We have,

$\mathbf{a_n = a + (n - 1)d}$

$\Rightarrow 88 = 3 + (n - 1) \times (5)$

$\Rightarrow 88 = 3 + 5n - 5$

$\Rightarrow 88 = -2 + 5n$

$\Rightarrow 88 + 2 = 5n$

$\Rightarrow 90 = 5n$

$\Rightarrow n = 18$

Therefore, the 18^{th} term of the given AP is 88.

7 D. Question

Which term of A.P. $\frac{5}{6}, 1, 1\frac{1}{6}, 1\frac{1}{3}, \ldots$ is 3?

Answer

Here, $a = \frac{5}{6}$

$$d = a_2 - a_1 = 1 - \frac{5}{6} = \frac{6-5}{6} = \frac{1}{6}$$

and $a_n = 3$

We have,

$a_n = a + (n - 1)d$

$$\Rightarrow 3 = \frac{5}{6} + (n-1) \times \frac{1}{6}$$

$$\Rightarrow 3 - \frac{5}{6} = (n-1) \times \frac{1}{6}$$

$$\Rightarrow 6 \times \left(\frac{18-5}{6}\right) = n - 1$$

$\Rightarrow 13 = n - 1$

$\Rightarrow n = 14$

Therefore, the 14^{th} term of a given AP is 3.

7 E. Question

Which term of A.P. 3, 8, 13, 18, ..., is 248 ?

Answer

Here, $a = 3$, $d = 8 - 3 = 5$ and $a_n = 248$

To find : n

We have,

$\mathbf{a_n = a + (n - 1)d}$

$\Rightarrow 248 = 3 + (n - 1) \times (5)$

$\Rightarrow 248 = 3 + 5n - 5$

$\Rightarrow 248 = -2 + 5n$

$\Rightarrow 248 + 2 = 5n$

$\Rightarrow 250 = 5n$

$\Rightarrow n = 50$

Therefore, the 50^{th} term of the given AP is 248.

8 A. Question

Find the 6th term from end of the A.P. 17, 14, 11,… – 40.

Answer

Here, a = 17, d = 14 – 17 = –3 and l = –40

where l = a + (n – 1)d

Now, to find the 6^{th} term from the end, we will find the total number of terms in the AP.

So, –40 = 17 + (n – 1)(–3)

⇒ –40 = 17 –3n + 3

⇒ –40 = 20 – 3n

⇒ –60 = –3n

⇒ n = 20

So, there are 20 terms in the given AP.

Last term = 20^{th}

Second last term = 20 – 1 = 19^{th}

Third last term = 20 – 2 = 18^{th}

And so, on

So, the 6^{th} term from the end = 20 – 5 = 15^{th} term

So, $a_n = a + (n - 1)d$

⇒ $a_{15} = 17 + (15 - 1)(-3)$

⇒ $a_{15} = 17 + 14 \times -3$

⇒ $a_{15} = 17 - 42$

⇒ $a_{15} = -25$

8 B. Question

Find the 8th term from end of the A.P. 7, 10, 13, ..., 184.

Answer

Here, a = 7, d = 10 – 7 = 3 and l = 184

where l = a + (n – 1)d

Now, to find the 8^{th} term from the end, we will find the total number of terms in the AP.

So, 184 = 7 + (n – 1)(3)

⇒ 184 = 7 + 3n – 3

⇒ 184 = 4 + 3n

⇒ 180 = 3n

⇒ n = 60

So, there are 60 terms in the given AP.

Last term = 60^{th}

Second last term = 60 – 1 = 59^{th}

Third last term = 60 – 2 = 58^{th}

And so, on

So, the 8^{th} term from the end = 60 – 7 = 53^{th} term

So, a_n = a + (n – 1)d

⇒ a_{53} = 7 + (53 – 1)(3)

⇒ a_{53} = 7 + 52 × 3

⇒ a_{53} = 7 + 156

⇒ a_{53} = 163

9 A. Question

Find the number of terms of the A.P.

6, 10, 14, 18, ..., 174?

Answer

Here, a = 6 , d = 10 – 6 = 4 and l = 174

where **l = a + (n – 1)d**

⇒ 174 = 6 + (n – 1)(4)

⇒ 174 = 6 + 4n – 4

$\Rightarrow 174 = 2 + 4n$

$\Rightarrow 174 - 2 = 4n$

$\Rightarrow 172 = 4n$

$$\Rightarrow n = \frac{172}{4} = 43$$

Hence, the number of terms in the given AP is 43

9 B. Question

Find the number of terms of the A.P.

7, 11, 15, ..., 139?

Answer

Here, a = 7 , d = 11 – 7 = 4 and l = 139

where **l = a + (n – 1)d**

$\Rightarrow 139 = 7 + (n - 1)(4)$

$\Rightarrow 139 = 7 + 4n - 4$

$\Rightarrow 139 = 3 + 4n$

$\Rightarrow 139 - 3 = 4n$

$\Rightarrow 136 = 4n$

$$\Rightarrow n = \frac{136}{4} = 34$$

Hence, the number of terms in the given AP is 34

9 C. Question

Find the number of terms of the A.P.

41, 38, 35, ..., 8?

Answer

Here, a = 41 , d = 38 – 41 = –3 and l = 8

where **l = a + (n – 1)d**

$\Rightarrow 8 = 41 + (n - 1)(-3)$

$\Rightarrow 8 = 41 - 3n + 3$

⇒ 8 = 44 – 3n

⇒ 8 – 44 = –3n

⇒ –36 = –3n

$$\Rightarrow n = \frac{-36}{-3} = 12$$

Hence, the number of terms in the given AP is 12

10. Question

Find the first negative term of sequence 999, 995, 991, 987, ...

Answer

AP = 999, 995, 991, 987,...

Here, a = 999, d = 995 – 999 = –4

$a_n < 0$

⇒ a + (n – 1)d < 0

⇒ 999 + (n – 1)(–4) < 0

⇒ 999 – 4n + 4 < 0

⇒ 1003 – 4n < 0

⇒ 1003 < 4n

$$\Rightarrow \frac{1003}{4} < n$$

⇒ n > 250.75

Nearest term greater than 250.75 is 251

So, 251st term is the first negative term

Now, we will find the 251st term

$a_n = a + (n - 1)d$

$\Rightarrow a_{251} = 999 + (251 - 1)(-4)$

$\Rightarrow a_{251} = 999 + 250 \times -4$

$\Rightarrow a_{251} = 999 - 1000$

$\Rightarrow a_{251} = -1$

∴, −1 is the first negative term of the given AP.

11. Question

Is 51 a term of the A.P. 5, 8, 11, 14, ...?

Answer

AP = 5, 8, 11, 14, …

Here, a = 5 and d = 8 – 5 = 3

Let 51 be a term, say, nth term of this AP.

We know that

$a_n = a + (n - 1)d$

So, 51 = 5 + (n – 1)(3)

⇒ 51 = 5 + 3n – 3

⇒ 51 = 2 + 3n

⇒ 51 – 2 = 3n

⇒ 49 = 3n

$\Rightarrow n = \frac{49}{3}$

But n should be a positive integer because n is the number of terms. So, 51 is not a term of this given AP.

12. Question

Is 56 a term of the A.P. $4, 4\frac{1}{2}, 5, 5\frac{1}{2}, 6, \ldots?$

Answer

$AP = 4, \frac{9}{2}, 5, \frac{11}{2}, 6, \ldots.$

Here, a = 4 and $d = 5 - \frac{9}{2} = \frac{10-9}{2} = \frac{1}{2}$

Let 56 be a term, say, nth term of this AP.

We know that

$a_n = a + (n - 1)d$

So, $56 = 4 + (n - 1) \times \frac{1}{2}$

$\Rightarrow 2 \times (56 - 4) = n - 1$

$\Rightarrow 2 \times 52 = n - 1$

$\Rightarrow 104 = n - 1$

$\Rightarrow 105 = n$

Hence, 56 is the 105^{th} term of this given AP.

13. Question

The 7th term of an A.P. is 20 and its 13th term is 32. Find the A.P. [CBSE 20041

Answer

We have

$a_7 = a + (7 - 1)d = a + 6d = 20$ ...(1)

and $a_{13} = a + (13 - 1)d = a + 12d = 32$ ...(2)

Solving the pair of linear equations (1) and (2), we get

$a + 6d - a - 12d = 20 - 32$

$\Rightarrow - 6d = -12$

$\Rightarrow d = 2$

Putting the value of d in eq (1), we get

$a + 6(2) = 20$

$\Rightarrow a + 12 = 20$

$\Rightarrow a = 8$

Hence, the required AP is 8, 10, 12, 14,...

14. Question

The 7th term of an A.P. is – 4 and its 13th term is – 16. Find the A.P. [CBSE 20041

Answer

We have

$a_7 = a + (7 - 1)d = a + 6d = -4$ …(1)

and $a_{13} = a + (13 - 1)d = a + 12d = -16$ …(2)

Solving the pair of linear equations (1) and (2), we get

$a + 6d - a - 12d = -4 - (-16)$

$\Rightarrow -6d = -4 + 16$

$\Rightarrow -6d = 12$

$\Rightarrow d = -2$

Putting the value of d in eq (1), we get

$a + 6(-2) = -4$

$\Rightarrow a - 12 = -4$

$\Rightarrow a = 8$

Hence, the required AP is 8, 6, 4, 2,…

15. Question

The 8th term of an A.P. is 37, and its 12th term is 57. Find the A.P.

Answer

We have

$a_8 = a + (8 - 1)d = a + 7d = 37$ …(1)

and $a_{12} = a + (12 - 1)d = a + 11d = 57$ …(2)

Solving the pair of linear equations (1) and (2), we get

$a + 7d - a - 11d = 37 - 57$

$\Rightarrow -4d = -20$

$\Rightarrow d = 5$

Putting the value of d in eq (1), we get

$a + 7(5) = 37$

$\Rightarrow a + 35 = 37$

$\Rightarrow a = 2$

Hence, the required AP is 2, 7, 12, 17,…

16. Question

Find the 10th term of the A.P. whose 7th and 12th terms are 34 and 64 respectively.

Answer

We have

$a_7 = a + (7 - 1)d = a + 6d = 34$ …(1)

and $a_{12} = a + (12 - 1)d = a + 11d = 64$ …(2)

Solving the pair of linear equations (1) and (2), we get

$a + 6d - a - 11d = 34 - 64$

$\Rightarrow -5d = -30$

$\Rightarrow d = 6$

Putting the value of d in eq (1), we get

$a + 6(6) = 34$

$\Rightarrow a + 36 = 34$

$\Rightarrow a = -2$

Hence, the required AP is –2, 4, 10, 16,…

Now, we to find the 10^{th} term

So, $a_n = a + (n - 1)d$

$a_{10} = -2 + (10 - 1)6$

$a_{10} = -2 + 9 \times 6$

$a_{10} = 52$

17 A. Question

For what value of n are the nth term of the following two A.P's the same. Also find this term

13, 19, 25, ... and 69, 68, 67, ...

Answer

1^{st} AP = 13, 19, 25, …

Here, a = 13, d = 19 – 13 = 6

and 2^{nd} AP = 69, 68, 67, …

Here, a = 69, d = 68 – 69 = –1

According to the question,

13 + (n – 1)6 = 69 + (n – 1)(–1)

⇒ 13 + 6n – 6 = 69 – n + 1

⇒ 7 + 6n = 70 – n

⇒ 6n + n = 70 – 7

⇒ 7n = 63

⇒ n = 9

9^{th} term of the given AP's are same.

Now, we will find the 9^{th} term

We have,

$a_n = a + (n - 1)d$

$a_9 = 13 + (9 - 1)6$

$a_9 = 13 + 8 \times 6$

$a_9 = 13 + 48$

$a_9 = 61$

17 B. Question

For what value of n are the nth term of the following two A.P's the same. Also find this term

23, 25, 27, 29, ... and – 17, – 10, – 3, 4, ...

Answer

1^{st} AP = 23, 25, 27, 29, ...

Here, a = 23, d = 25 – 23 = 2

and 2^{nd} AP = – 17, – 10, – 3, 4, ...

Here, $a = -17$, $d = -10 - (-17) = -10 + 17 = 7$

According to the question,

$23 + (n - 1)2 = -17 + (n - 1)7$

$\Rightarrow 23 + 2n - 2 = -17 + 7n - 7$

$\Rightarrow 21 + 2n = -24 + 7n$

$\Rightarrow 2n - 7n = -24 - 21$

$\Rightarrow -5n = -45$

$\Rightarrow n = 9$

9^{th} term of the given AP’s are same.

Now, we will find the 9^{th} term

We have,

$a_n = a + (n - 1)d$

$a_9 = 23 + (9 - 1)2$

$a_9 = 23 + 8 \times 2$

$a_9 = 23 + 16$

$a_9 = 39$

17 C. Question

For what value of n are the nth term of the following two A.P's the same. Also find this term

24, 20, 16, 12, ... and – 11, – 8, – 5, – 2, ...

Answer

1^{st} AP = 24, 20, 16, 12, ...

Here, $a = 24$, $d = 20 - 24 = -4$

and 2^{nd} AP = – 11, – 8, – 5, – 2, ...

Here, $a = -11$, $d = -8 - (-11) = -8 + 11 = 3$

According to the question,

$24 + (n - 1)(-4) = -11 + (n - 1)3$

$\Rightarrow 24 - 4n + 4 = -11 + 3n - 3$

$\Rightarrow 28 - 4n = -14 + 3n$

$\Rightarrow 28 + 14 = 3n + 4n$

$\Rightarrow 7n = 42$

$\Rightarrow n = 6$

6^{th} term of the given AP's are same.

Now, we will find the 6^{th} term

We have,

$a_n = a + (n - 1)d$

$a_6 = 24 + (6 - 1)(-4)$

$a_6 = 24 + 5 \times -4$

$a_6 = 24 - 20$

$a_6 = 4$

17 D. Question

For what value of n are the nth term of the following two A.P's the same. Also find this term

63, 65, 67, ... and 3, 10, 17, ...

Answer

1^{st} AP = 63, 65, 67, ...

Here, a = 63, d = 65 − 63 = 2

and 2^{nd} AP = 3, 10, 17, ...

Here, a = 3, d = 10 − 3 = 7

According to the question,

$63 + (n - 1)2 = 3 + (n - 1)7$

$\Rightarrow 63 + 2n - 2 = 3 + 7n - 7$

$\Rightarrow 61 + 2n = 7n - 4$

$\Rightarrow 65 = 7n - 2n$

$\Rightarrow$ 5n = 65

$\Rightarrow$ n = 13

13^{th} term of the given AP's are same.

Now, we will find the 13^{th} term

We have,

$a_n = a + (n - 1)d$

$a_{13} = 63 + (13 - 1)2$

$a_{13} = 63 + 12 \times 2$

$a_{13} = 63 + 24$

$a_{13} = 87$

18 A. Question

In the following A.P., find the missing terms:

5, □,□, $9\frac{1}{2}$

Answer

Here, a = 5 , n = 4 and $l = \frac{19}{2}$

We have,

$l = a + (n - 1)d$

$\Rightarrow \frac{19}{2} = 5 + (4 - 1)d$

$\Rightarrow 19 = 10 + 6d$

$\Rightarrow 9 = 6d$

$\Rightarrow d = \frac{9}{6} = \frac{3}{2}$

So, the missing terms are –

$a_2 = a + d = 5 + \frac{3}{2} = \frac{10+3}{2} = \frac{13}{2}$

a_3 = a + 2d $= 5 + 2 \times \frac{3}{2} = 5 + 3 = 8$

Hence, the missing terms are $\frac{13}{2}$ and 8

18 B. Question

In the following A.P., find the missing terms:

54, □,□, 42

Answer

Here, a = 54 , n = 4 and l = 42

We have,

l = a + (n – 1)d

⇒42 = 54 + (4 – 1)d

⇒42 = 54 + 3d

⇒–12 = 3d

$\Rightarrow d = \frac{-12}{3} = -4$

So, the missing terms are –

a_2 = a + d = 54 – 4 = 50

a_3 = a + 2d = 54 + 2(–4) = 54 – 8 = 46

Hence, the missing terms are 50 and 46

18 C. Question

In the following A.P., find the missing terms:

– 4, □,□,□,□, 6

Answer

Here, a = –4, n = 6 and l = 6

We have,

l = a + (n – 1)d

⇒6 = –4 + (6 – 1)d

⇒6 = –4 + 5d

$\Rightarrow 10 = 5d$

$$\Rightarrow d = \frac{10}{5} = 2$$

So, the missing terms are –

$a_2 = a + d = -4 + 2 = -2$

$a_3 = a + 2d = -4 + 2(2) = -4 + 4 = 0$

$a_4 = a + 3d = -4 + 3(2) = -4 + 6 = 2$

$a_5 = a + 4d = -4 + 4(2) = -4 + 8 = 4$

Hence, the missing terms are −2, 0, 2 and 4

18 D. Question

In the following A.P., find the missing terms:

□, 13, □, 3

Answer

Given: $a_2 = 13$ and $a_4 = 3$

We know that,

$a_n = a + (n - 1)d$

$a_2 = a + (2 - 1)d$

$13 = a + d$ ...(i)

and $a_4 = a + (4 - 1)d$

$3 = a + 3d$ ...(ii)

Solving linear equations (i) and (ii), we get

$a + d - a - 3d = 13 - 3$

$\Rightarrow -2d = 10$

$\Rightarrow d = -5$

Putting the value of d in eq. (i), we get

$a - 5 = 13$

$\Rightarrow a = 18$

Now, $a_3 = a + 2d = 18 + 2(-5) = 18 - 10 = 8$

Hence, the missing terms are 18 and 8

18 E. Question

In the following A.P., find the missing terms:

7, □,□,□,27

Answer

Here, a = 7, n = 5 and l = 27

We have,

$l = a + (n - 1)d$

$\Rightarrow 27 = 7 + (5 - 1)d$

$\Rightarrow 27 = 7 + 4d$

$\Rightarrow 20 = 4d$

$$\Rightarrow d = \frac{20}{4} = 5$$

So, the missing terms are –

$a_2 = a + d = 7 + 5 = 12$

$a_3 = a + 2d = 7 + 2(5) = 7 + 10 = 17$

$a_4 = a + 3d = 7 + 3(5) = 7 + 15 = 22$

Hence, the missing terms are 12, 17 and 22

18 F. Question

In the following A.P., find the missing terms:

2, □, 26

Answer

Here, a = 2, n = 3 and l = 26

We have,

$l = a + (n - 1)d$

$\Rightarrow 26 = 2 + (3 - 1)d$

$\Rightarrow 26 = 2 + 2d$

$\Rightarrow 24 = 2d$

$$\Rightarrow d = \frac{24}{2} = 12$$

So, the missing terms are –

$a_2 = a + d = 2 + 12 = 14$

Hence, the missing terms is 14

18 G. Question

In the following A.P., find the missing terms:

□, □, 13, □, □, 22

Answer

Given: $a_3 = 13$ and $a_6 = 22$

We know that,

$a_n = a + (n - 1)d$

$a_3 = a + (3 - 1)d$

$13 = a + 2d$ ...(i)

and $a_6 = a + (6 - 1)d$

$22 = a + 5d$ ...(ii)

Solving linear equations (i) and (ii), we get

$a + 2d - a - 5d = 13 - 22$

$\Rightarrow -3d = 9$

$\Rightarrow d = 3$

Putting the value of d in eq. (i), we get

$a + 2(3) = 13$

$\Rightarrow a + 6 = 13$

$\Rightarrow a = 7$

Now, $a_2 = a + d = 7 + 3 = 10$

a_4 = a + 3d = 7 + 3(3) = 7 + 9 = 16

a_5 = a + 4d = 7 + 4(3) = 7 + 12 = 19

Hence, the missing terms are 7, 10, 16 and 19

18 H. Question

In the following A.P., find the missing terms:

– 4, □, □, □, 6

Answer

Here, a = –4, n = 5 and l = 6

We have,

l = a + (n – 1)d

⇒6 = –4 + (5 – 1)d

⇒6 = –4 + 4d

⇒10 = 4d

$$\Rightarrow d = \frac{10}{4} = \frac{5}{2}$$

So, the missing terms are –

$a_2 = a + d = -4 + \frac{5}{2} = \frac{-8+5}{2} = \frac{-3}{2}$

$a_3 = a + 2d = -4 + 2 \times \frac{5}{2} = -4 + 5 = 1$

$a_4 = a + 3d = = -4 + 3 \times \frac{3}{2} = \frac{-8+9}{2} = \frac{1}{2}$

Hence, the missing terms are $\frac{-3}{2}$, 1 and $\frac{1}{2}$

18 I. Question

In the following A.P., find the missing terms:

□, 38, □,□,□, – 22

Answer

Given: a_2 = 38 and a_6 = –22

We know that,

$a_n = a + (n - 1)d$

$a_2 = a + (2 - 1)d$

$38 = a + d$ ...(i)

and $a_6 = a + (6 - 1)d$

$-22 = a + 5d$ ...(ii)

Solving linear equations (i) and (ii), we get

$a + d - a - 5d = 38 - (-22)$

$\Rightarrow -4d = 60$

$\Rightarrow d = -15$

Putting the value of d in eq. (i), we get

$a + (-15) = 38$

$\Rightarrow a - 15 = 38$

$\Rightarrow a = 53$

Now, $a_3 = a + 2d = 53 + 2(-15) = 53 - 30 = 23$

$a_4 = a + 3d = 53 + 3(-15) = 53 - 45 = 8$

$a_5 = a + 4d = 53 + 4(-15) = 53 - 60 = -7$

Hence, the missing terms are 53, 23, 8 and –7

19 A. Question

If 10th term of an A.P. is 52 and 17th term is 20 more than the 13th term, find the A.P.

Answer

Given: $a_{10} = 52$ and $a_{17} = 20 + a_{13}$

Now, $a_n = a + (n - 1)d$

$a_{10} = a + (10 - 1)d$

$52 = a + 9d$ ...(i)

and $a_{17} = 20 + a_{13}$

$a + (17 - 1)d = 20 + a + (13 - 1)d$

$\Rightarrow$ a + 16d = 20 + a + 12d

$\Rightarrow$ 16d –12d = 20

$\Rightarrow$ 4d = 20

$\Rightarrow$ d = 5

Putting the value of d in eq. (i), we get

a + 9(5) = 52

$\Rightarrow$ a + 45 = 52

$\Rightarrow$ a = 52 – 45

$\Rightarrow$ a = 7

Therefore, the AP is 7, 12, 17, …

19 B. Question

Which term of the A.P. 3, 15, 27, 39, ... will be 132 more than its 54th term?

Answer

Given: 3, 15, 27, 39, …

First we need to calculate 54^{th} term.

We know that

$a_n = a + (n - 1)d$

Here, a = 3, d = 15 – 3 = 12 and n = 54

So, $a_{54} = 3 + (54 - 1)12$

$\Rightarrow a_{54} = 3 + 53 \times 12$

$\Rightarrow a_{54} = 3 + 636$

$\Rightarrow a_{54} = 639$

Now, the term is 132 more than a_{54} is 132 + 639 = 771

Now,

a + (n – 1)d = 771

$\Rightarrow$ 3 + (n – 1)12 = 771

$\Rightarrow$ 3 + 12n – 12 = 771

$\Rightarrow 12n = 771 + 12 - 3$

$\Rightarrow 12n = 780$

$\Rightarrow n = 65$

Hence, the 65^{th} term is 132 more than the 54^{th} term.

20. Question

Which term of the A.P. 3, 10, 17, 24, ... will be 84 more than its 13th term ?

Answer

Given: 3, 10, 17, 24, ...

First we need to calculate 13^{th} term.

We know that

$a_n = a + (n - 1)d$

Here, $a = 3$, $d = 10 - 3 = 7$ and $n = 13$

So, $a_{13} = 3 + (13 - 1)7$

$\Rightarrow a_{13} = 3 + 12 \times 7$

$\Rightarrow a_{13} = 3 + 84$

$\Rightarrow a_{13} = 87$

Now, the term is 84 more than a_{13} is $84 + 87 = 171$

Now,

$a + (n - 1)d = 171$

$\Rightarrow 3 + (n - 1)7 = 171$

$\Rightarrow 3 + 7n - 7 = 171$

$\Rightarrow 7n = 171 + 7 - 3$

$\Rightarrow 7n = 175$

$\Rightarrow n = 25$

Hence, the 25^{th} term is 84 more than the 13^{th} term.

21. Question

The 4th term of an A.P. is zero. Prove that its 25th term is triple its 11th term.

Answer

Given: $a_4 = 0$

To Prove: $a_{25} = 3 \times a_{11}$

Now, $a_4 = 0$

$\Rightarrow a + 3d = 0$

$\Rightarrow a = -3d$

We know that,

$a_n = a + (n - 1)d$

$a_{11} = -3d + (11 - 1)d$ [from (i)]

$a_{11} = -3d + 10d$

$a_{11} = 7d$ …(ii)

Now,

$a_{25} = a + (25 - 1)d$

$a_{25} = -3d + 24d$ [from(i)]

$a_{25} = 21d$

$a_{25} = 3 \times 7d$

$a_{25} = 3 \times a_{11}$ [from(ii)]

Hence Proved

22. Question

If 10 times the 10th term of an A.P. is equal to 15 times the 15th term, show that its 25th term is zero.

Answer

Given: $10 \times a_{10} = 15 \times a_{15}$

To Prove: $a_{25} = 0$

Now,

$10 \times (a + 9d) = 15 \times (a + 14d)$

$\Rightarrow 10a + 90d = 15a + 210d$

$\Rightarrow 10a - 15a = 210d - 90d$

$\Rightarrow -5a = 120d$

$\Rightarrow a = -24d$ ...(i)

Now,

$a_n = a + (n - 1)d$

$a_{25} = -24d + (25 - 1)d$ [from (i)]

$a_{25} = -24d + 24d$

$a_{25} = 0$

Hence Proved

23. Question

If (m + 1)th term of an A.P. is twice the (n + 1)th term, prove that (3m + 1)th term is twice the (m + n + 1)th term.

Answer

Given: $a_{m+1} = 2a_{n+1}$

To Prove: $a_{3m+1} = 2a_{m+n+1}$

Now,

$a_n = a + (n - 1)d$

$\Rightarrow a_{m+1} = a + (m + 1 - 1)d$

$\Rightarrow a_{m+1} = a + md$

and $a_{n+1} = a + (n + 1 - 1)d$

$\Rightarrow a_{n+1} = a + nd$

Given: $a_{m+1} = 2a_{n+1}$

$a + md = 2(a + nd)$

$\Rightarrow a + md = 2a + 2nd$

$\Rightarrow md - 2nd = 2a - a$

$\Rightarrow d(m - 2n) = a$ ...(i)

Now,

a_{m+n+1} = a + (m + n + 1 – 1)d

= a + (m + n)d

= md – 2nd + md + nd [from (i)]

= 2md – nd

a_{m+n+1} = d (2m – n) ...(ii)

a_{3m+1} = a + (3m + 1 – 1)d

= a + 3md

= md – 2nd + 3md [from (i)]

= 4md – 2nd

= 2d(2m – n)

$a_{3m+1} = 2a_{m+n+1}$ [from (ii)]

Hence Proved

24. Question

If t_n be the nth term of an A.P. such that $\frac{t_4}{t_7} = \frac{2}{3}$ find $\frac{t_8}{t_9}$.

Answer

Given: $\frac{t_4}{t_7} = \frac{2}{3}$

To find: $\frac{t_8}{t_9}$

We know that,

t_n = a + (n – 1)d

So,

$$\frac{t_4}{t_7} = \frac{a + (4-1)d}{a + (7-1)d} = \frac{2}{3}$$

$$\Rightarrow \frac{a + 3d}{a + 6d} = \frac{2}{3}$$

$\Rightarrow 3(a+3d) = 2(a+6d)$

$\Rightarrow 3a + 9d = 2a + 12d$

$\Rightarrow 3a - 2a = 12d - 9d$

$\Rightarrow a = 3d$ ...(i)

Now, $\frac{t_8}{t_9} = \frac{a+(8-1)d}{a+(9-1)d} = \frac{3d+7d}{3d+8d} = \frac{10d}{11d} = \frac{10}{11}$ [from (i)]

25. Question

Find the number of all positive integers of 3 digits which are divisible by 5.

Answer

The list of 3 digit numbers divisible by 5 is:

100, 105, 110,...,995

Here a = 100, d = 105 – 100 = 5, $a_n = 995$

We know that

$a_n = a + (n - 1)d$

$995 = 100 + (n - 1)5$

$\Rightarrow 895 = (n - 1)5$

$\Rightarrow 179 = n - 1$

$\Rightarrow 180 = n$

So, there are 180 three– digit numbers divisible by 5.

26. Question

How many three digit numbers are divisible by 7.

Answer

The list of 3 digit numbers divisible by 7 is:

105, 112, 119,...,994

Here a = 105, d = 112 – 105 = 7, $a_n = 994$

We know that

$a_n = a + (n - 1)d$

994 = 105 + (n – 1)7

⇒ 889 = (n – 1)7

⇒ 127 = n – 1

⇒ 128 = n

So, there are 128 three– digit numbers divisible by 7.

27. Question

If t_n denotes the nth term of an A.P., show that $t_m + t_{2n+m} = 2\ t_{m+n}$.

Answer

To show: $t_m + t_{2n+m} = 2\ t_{m+n}$

Taking LHS

$t_m + t_{2n+m}$ = a + (m – 1)d + a + (2n + m – 1)d

= 2a + md – d + 2nd + md – d

= 2a + 2md + 2nd – 2d

= 2 {a + (m + n – 1)d}

= $2t_{m+n}$

= RHS

∴LHS = RHS

Hence Proved

28. Question

Find a if 5a + 2, 4a – I, a + 2 are in A.P.

Answer

Let 5a + 2, 4a – 1, a + 2 are in AP

So, first term a = 5a + 2

d = 4a – 1 – 5a – 2 = – a – 3

n = 3

l = a + 2

So,

l = a + (n – 1)d

⇒ a + 2 =5a + 2 + (3 – 1)(–a – 3)

⇒ a + 2 – 5a – 2 = –3a – 9 + a + 3

⇒ – 4a = –2a – 6

⇒ – 4a + 2a = – 6

⇒ –2a = – 6

⇒ a = 3

29. Question

nth term of a sequence is 2n + 1. Is this sequence an A.P.? If so find its first term and common difference.

Answer

We know that nth term of an A.P is given by,

$a_n = a + (n - 1)\ d$

Now equating it with the expression given we get,

2 n + 1 = a + (n – 1) d

2 n + 1 = a + nd – d

2 n + 1 = nd + (a – d)

Equating both sides we get,

d = 2 and a – d = 1

So we get,

a = 3 and d = 2.

So the first term of this sequence is 3, and the common difference is 2.

30. Question

The sum of the 4th and Sth terms of an A.P. is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of A.P.

Answer

Given: $a_4 + a_8 = 24$

⇒ a +3d + a + 7d = 24

⇒ $2a + 10d = 24$ ...(i)

and $a_6 + a_{10} = 44$

⇒ $a + 5d + a + 9d = 44$

⇒ $2a + 14d = 44$ ...(ii)

Solving Linear equations (i) and (ii), we get

$2a + 10d - 2a - 14d = 24 - 44$

⇒ $-4d = -20$

⇒ $d = 5$

Putting the value of d in eq. (i), we get

$2a + 10\times5 = 24$

⇒ $2a + 50 = 24$

⇒ $2a = 24 - 50$

⇒ $2a = -26$

⇒ $a = -13$

So, the first three terms are −13, −8, −3.

31. Question

A person was appointed in the pay scale of Rs. 700–40–1500. Find in how many years he will reach maximum of the scale.

Answer

Let the required number of years = n

Given $t_n = 1500$, $a = 700$, $d = 40$

We know that,

$t_n = a + (n - 1)d$

⇒ $1500 = 700 + (n - 1)40$

⇒ $800 = (n - 1)40$

⇒ $20 = n - 1$

⇒ $n = 21$

Hence, in 21years he will reach maximum of the scale.

32. Question

A sum of money kept in a hank amounts to Rs. 600/– in 4 years and Rs. 800/– in 12 years. Find the sum and interest carried every year.

Answer

Let the required sum = a

and the interest carried every year = d

According to question,

In 4years, a sum of money kept in bank account = Rs. 600

i.e. $t_5 = 600 \Rightarrow a + 4d = 600$ ...(i)

and in 12 years , sum of money kept = Rs. 800

i.e. $t_{13} = 800 \Rightarrow a + 12d = 800$ ...(ii)

Solving linear equations (i) and (ii), we get

$a + 4d - a - 12d = 600 - 800$

$\Rightarrow - 8d = -200$

$\Rightarrow d = 25$

Putting the value of d in eq.(i), we get

$a + 4(25) = 600$

$\Rightarrow a + 100 = 600$

$\Rightarrow a = 500$

Hence, the sum and interest carried every year is Rs 500 and Rs 25 respectively.

33. Question

A man starts repaying, a loan with the first instalment of Rs. 100. If he increases the installment by Rs. 5 every month, what amount he will pay in the 30th instalment?

Answer

The first instalment of the loan = Rs. 100

The 2^{nd} instalment of the loan = Rs. 105

The 3rd instalment of the loan = Rs. 110

and so, on

The amount that the man repays every month forms an AP.

Therefore, the series is

100, 105, 110, 115,…

Here, a = 100, d = 105 – 100 = 5

We know that,

$a_n = a + (n - 1)d$

$a_{30} = 100 + (30 - 1)5$

$\Rightarrow a_{30} = 100 + 29 \times 5$

$\Rightarrow a_{30} = 100 + 145$

$\Rightarrow a_{30} = 245$

Hence, the amount he will pay in the 30th installment is Rs 245.

Exercise 8.3

1. Question

Three numbers are in A.P. Their sum is 27 and the sum of their squares is 275. Find the numbers.

Answer

Let the three numbers are in AP = a, a + d, a + 2d

According to the question,

The sum of three terms = 27

$\Rightarrow a + (a + d) + (a + 2d) = 27$

$\Rightarrow 3a + 3d = 27$

$\Rightarrow a + d = 9$

$\Rightarrow a = 9 - d$ …(i)

and the sum of their squares = 275

$\Rightarrow a^2 + (a + d)^2 + (a + 2d)^2 = 275$

$\Rightarrow (9 - d)^2 + (9)^2 + (9 - d + 2d)^2 = 275$ [from(i)]

$\Rightarrow 81 + d^2 - 18d + 81 + 81 + d^2 + 18d = 275$

$\Rightarrow 243 + 2d^2 = 275$

$\Rightarrow 2d^2 = 275 - 243$

$\Rightarrow 2d^2 = 32$

$\Rightarrow d^2 = 16$

$\Rightarrow d = \sqrt{16}$

$\Rightarrow d = \pm 4$

Now, if $d = 4$, then $a = 9 - 4 = 5$

and if $d = -4$, then $a = 9 - (-4) = 9 + 4 = 13$

So, the numbers are →

if $a = 5$ and $d = 4$

5, 9, 13

and if $a = 13$ and $d = -4$

13, 9, 5

2. Question

The sum of three numbers in A.P. is 12 and the sum of their cubes is 408. Find the numbers.

Answer

Let the three numbers are in AP = a, a + d, a + 2d

According to the question,

The sum of three terms = 12

$\Rightarrow a + (a + d) + (a + 2d) = 12$

$\Rightarrow 3a + 3d = 12$

$\Rightarrow a + d = 4$

$\Rightarrow a = 4 - d$ …(i)

and the sum of their cubes = 408

$\Rightarrow a^3 + (a + d)^3 + (a + 2d)^3 = 408$

$\Rightarrow (4 - d)^3 + (4)^3 + (4 - d + 2d)^3 = 408$ [from(i)]

$\Rightarrow (4 - d)^3 + (4)^3 + (4 + d)^3 = 408$

$\Rightarrow 64 - d^3 + 12d^2 - 48d + 64 + 64 + d^3 + 12d^2 + 48d = 408$

$\Rightarrow 192 + 24d^2 = 408$

$\Rightarrow 24d^2 = 408 - 192$

$\Rightarrow 24d^2 = 216$

$\Rightarrow d^2 = 9$

$\Rightarrow d = \sqrt{9}$

$\Rightarrow d = \pm 3$

Now, if d = 3, then a = 4 – 3 = 1

and if d = – 3, then a = 4 – (– 3) = 4 + 3 = 7

So, the numbers are →

if a = 1 and d = 3

1, 4, 7

and if a = 7 and d = – 3

7, 4, 1

3 A. Question

Divide 15 into three parts which are in A.P. and the sum of their squares is 83.

Answer

Let the middle term = a and the common difference = d

The first term = a – d and the succeeding term = a + d

So, the three parts are a – d, a, a + d

According to the question,

Sum of these three parts = 15

$\Rightarrow a - d + a + a + d = 15$

$\Rightarrow 3a = 15$

$\Rightarrow a = 5$

and the sum of their squares = 83

$\Rightarrow (a - d)^2 + a^2 + (a + d)^2 = 83$

$\Rightarrow (5 - d)^2 + (5)^2 + (5 + d)^2 = 83$ [from(i)]

$\Rightarrow 25 + d^2 - 10d + 25 + 25 + d^2 + 10d = 83$

$\Rightarrow 75 + 2d^2 = 83$

$\Rightarrow 2d^2 = 83 - 75$

$\Rightarrow 2d^2 = 8$

$\Rightarrow d^2 = 4$

$\Rightarrow d = \sqrt{4}$

$\Rightarrow d = \pm 2$

Case: (i) If d = 2, then

$a - d = 5 - 2 = 3$

$a = 5$

$a + d = 5 + 2 = 7$

Hence, the three parts are

3, 5, 7

Case: (ii) If d = – 2, then

$a - d = 5 - (- 2) = 7$

$a = 5$

$a + d = 5 + (- 2) = 3$

Hence, the three parts are

7, 5, 3

3 B. Question

Divide 20 into four parts which are in A.P. such that the ratio of the product of the first and fourth is to the product of the second and third is 2 : 3.

Answer

Let the four parts which are in AP are

$(a - 3d), (a - d), (a + d), (a + 3d)$

According to question,

The sum of these four parts = 20

$\Rightarrow (a - 3d) + (a - d) + (a + d) + (a + 3d) = 20$

$\Rightarrow 4a = 20$

$\Rightarrow a = 5$ ...(i)

Now, it is also given that

product of the first and fourth : product of the second and third = 2 : 3

i.e. $(a - 3d) \times (a + 3d) : (a - d) \times (a + d) = 2 : 3$

$$\Rightarrow \frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{2}{3}$$

$\Rightarrow \frac{a^2-9d^2}{a^2-d^2} = \frac{2}{3}$ $[\because (a - b)(a + b) = a^2 - b^2]$

$\Rightarrow 3(a^2 - 9d^2) = 2(a^2 - d^2)$

$\Rightarrow 3a^2 - 27d^2 = 2a^2 - 2d^2$

$\Rightarrow 3a^2 - 2a^2 = - 2d^2 + 27d^2$

$\Rightarrow (5)^2 = - 2d^2 + 27d^2$ [from (i)]

$\Rightarrow 25 = 25d^2$

$\Rightarrow 1 = d^2$

$\Rightarrow d = \pm 1$

Case I: if d = 1 and a = 5

$a - 3d = 5 - 3(1) = 5 - 3 = 2$

$a - d = 5 - 1 = 4$

$a + d = 5 + 1 = 6$

$a + 3d = 5 + 3(1) = 5 + 3 = 8$

Hence, the four parts are

2, 4, 6, 8

Case II: if $d = -1$ and $a = 5$

$a - 3d = 5 - 3(-1) = 5 + 3 = 8$

$a - d = 5 - (-1) = 5 + 1 = 6$

$a + d = 5 + (-1) = 5 - 1 = 4$

$a + 3d = 5 + 3(-1) = 5 - 3 = 2$

Hence, the four parts are

8, 4, 6, 2

4 A. Question

Sum of three numbers in A.P. is 21 and their product is 231. Find the numbers.

Answer

Let the three numbers are $(a - d)$, a and $(a + d)$

According to question,

Sum of these three numbers = 21

$\Rightarrow a - d + a + a + d = 21$

$\Rightarrow 3a = 21$

$\Rightarrow a = 7$ ...(i)

and it is also given that

Product of these numbers = 231

$\Rightarrow (a - d) \times a \times (a + d) = 231$

$\Rightarrow (7 - d) \times 7 \times (7 + d) = 231$

$\Rightarrow 7 \times (7^2 - d^2) = 231$ [$\because (a - b)(a + b) = a^2 - b^2$]

$\Rightarrow 7 \times (49 - d^2) = 231$

$\Rightarrow 343 - 7d^2 = 231$

$\Rightarrow -7d^2 = 231 - 343$

$\Rightarrow -7d^2 = -112$

$\Rightarrow d^2 = 16$

$\Rightarrow$ d = $\sqrt{16}$

$\Rightarrow$ d = ± 4

Case I: If d = 4 and a = 7

a – d = 7 – 4 = 3

a = 7

a + d = 7 + 4 = 11

So, the numbers are

3, 7, 11

Case II: If d = – 4 and a = 7

a – d = 7 – (– 4) = 7 + 4 = 11

a = 7

a + d = 7 + (– 4) = 7 – 4 = 3

So, the numbers are

11, 7, 3

4 B. Question

Sum of three numbers in A.P. is 3 and their product is — 35. Find the numbers.

Answer

Let the three numbers are (a – d), a and (a + d)

According to question,

Sum of these three numbers = 3

$\Rightarrow$ a – d + a + a + d = 3

$\Rightarrow$ 3a = 3

$\Rightarrow$ a = 1 …(i)

and it is also given that

Product of these numbers = – 35

$\Rightarrow$(a – d) × a × (a + d) = – 35

$\Rightarrow$(1 – d) × 1 × (1 + d) = – 35

$\Rightarrow 1 \times (1^2 - d^2) = -35$ [$\because (a - b)(a + b) = a^2 - b^2$]

$\Rightarrow 1 \times (1 - d^2) = -35$

$\Rightarrow 1 - d^2 = -35$

$\Rightarrow -d^2 = -35 - 1$

$\Rightarrow -d^2 = -36$

$\Rightarrow d^2 = 36$

$\Rightarrow d = \sqrt{36}$

$\Rightarrow d = \pm 6$

Case I: If d = 6 and a = 1

$a - d = 1 - 6 = -5$

$a = 1$

$a + d = 1 + 6 = 7$

So, the numbers are

$-5, 1, 7$

Case II: If d = – 6 and a = 1

$a - d = 1 - (-6) = 1 + 6 = 7$

$a = 1$

$a + d = 1 + (-6) = 1 - 6 = -5$

So, the numbers are

$7, 1, -5$

5. Question

If $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in A.P. and $a + b + c \neq 0$, prove that $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.

Answer

Given: $a + b + c \neq 0$

and $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in AP

To Prove: $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in AP

if $\frac{a+b+c}{b+c}, \frac{a+b+c}{c+a}, \frac{a+b+c}{a+b}$ are in AP

[multiplying each term by a + b + c]

i.e. if $\frac{a}{b+c} + 1, \frac{b}{c+a} + 1, \frac{c}{a+b} + 1$ are in AP

which is given to be true

Hence, $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in AP

6. Question

If a^2, b^2, c^2 are in A.P., show that $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in A.P.

Answer

a^2, b^2, c^2 are in AP

$\therefore b^2 - a^2 = c^2 - b^2$

$\Rightarrow (b - a)(b + a) = (c - b)(c + b)$

$$\Rightarrow \frac{b-a}{c+b} = \frac{c-b}{b+a}$$

$$\Rightarrow \frac{b-a}{(c+b)(c+a)} = \frac{c-b}{(b+a)(c+a)}$$

$$\Rightarrow \frac{b+c-c-a}{(c+b)(c+a)} = \frac{c+a-a-b}{(c+a)(b+a)}$$

$$\Rightarrow \frac{(b+c)-(c+a)}{(c+b)(c+a)} = \frac{(c+a)-(a+b)}{(c+a)(b+a)}$$

$$\Rightarrow \frac{(b+c)}{(c+b)(c+a)} - \frac{(c+a)}{(c+b)(c+a)} = \frac{(c+a)}{(c+a)(b+a)} - \frac{(a+b)}{(c+a)(b+a)}$$

$$\Rightarrow \frac{1}{c+a} - \frac{1}{c+b} = \frac{1}{b+a} - \frac{1}{c+a}$$

$$\Rightarrow \frac{1}{b + c}, \frac{1}{c + a}, \frac{1}{a + b} \text{ are in AP}$$

$$\Rightarrow \frac{a + b + c}{b + c}, \frac{a + b + c}{c + a}, \frac{a + b + c}{a + b} \text{ are in AP}$$

$$\Rightarrow \frac{a}{b + c} + 1, \frac{b}{c + a} + 1, \frac{c}{c + a} + 1 \text{ are in AP}$$

$$\Rightarrow \frac{a}{b + c}, \frac{b}{c + a}, \frac{c}{a + b} \text{ are in AP}$$

Hence Proved

7 A. Question

If a, b, c are in A.P., prove that

$\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are in A.P.

Answer

Given: a, b, c are in AP

∴ b – a = c – b ...(i)

To Prove: $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are in AP

$$a_2 - a_1 = \frac{1}{ca} - \frac{1}{bc} = \frac{bc - ca}{(ca)(bc)} = \frac{c(b - a)}{(ca)(bc)}$$

$$a_3 - a_2 = \frac{1}{ab} - \frac{1}{ca} = \frac{ca - ab}{(ca)(ab)} = \frac{a(c - b)}{(ca)(ab)}$$

$$\Rightarrow \frac{c(b - a)}{(ca)(bc)} = \frac{a(c - b)}{(ca)(ab)}$$

$$\Rightarrow \frac{c(b - a)}{bc} = \frac{a(c - b)}{ab}$$

⇒ b – a = c – b

∴a, b, c are in AP

∴ $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are in AP

7 B. Question

If a, b, c are in A.P., prove that

$(b + c)^2 — a^2, (c + a)^2 — b^2, (a + b)^2 — c^2$ are in A.P.

Answer

Given: a, b, c are in AP

Since, a, b, c are in AP, we have a + c = 2b ...(i)

Now, $(b + c)^2 - a^2, (c + a)^2 - b^2, (a + b)^2 - c^2$ will be in A.P

If $(b + c - a)(b + c + a), (c + a - b)(c + a + b), (a + b - c)(a + b + c)$ are in AP

i.e. if $b + c - a, c + a - b, a + b - c$ are in AP

[dividing by (a + b + c)]

if $(b + c - a) + (a + b - c) = 2(c + a - b)$

if $2b = 2(c + a - b)$

if $b = c + a - b$

if $a + c = 2b$ which is true by (i)

Hence, $(b + c)^2 - a^2, (c + a)^2 - b^2, (a + b)^2 - c^2$ are in A.P

7 C. Question

If a, b, c are in A.P., prove that

$\frac{1}{\sqrt{b}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}}$ are in A.P.

Answer

Given: a, b, c are in AP

Since, a, b, c are in AP, we have a + c = 2b ...(i)

To Prove : $\frac{1}{\sqrt{b}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}}$ are in AP

$\Rightarrow \frac{1}{\sqrt{c}+\sqrt{a}} - \frac{1}{\sqrt{b}+\sqrt{c}} = \frac{1}{\sqrt{a}+\sqrt{b}} - \frac{1}{\sqrt{c}+\sqrt{a}}$

$\Rightarrow \frac{\sqrt{b} + \sqrt{c} - \sqrt{c} - \sqrt{a}}{(\sqrt{c} + \sqrt{a})(\sqrt{b} + \sqrt{c})} = \frac{\sqrt{c} + \sqrt{a} - \sqrt{a} - \sqrt{b}}{(\sqrt{a} + \sqrt{b})(\sqrt{c} + \sqrt{a})}$

$$\Rightarrow \frac{\sqrt{b}-\sqrt{a}}{(\sqrt{b}+\sqrt{c})} = \frac{\sqrt{c}-\sqrt{b}}{(\sqrt{a}+\sqrt{b})}$$

$\Rightarrow (\sqrt{b} - \sqrt{a})(\sqrt{b} + \sqrt{a}) = (\sqrt{c} - \sqrt{b})(\sqrt{c} + \sqrt{b})$

$\Rightarrow b - a = c - b$

$\Rightarrow 2b = a + c$, which is True ... from (i)

Hence, the result.

8. Question

$\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in A.P., show that $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P. provided $a + b + c \neq 0$

Answer

Given: $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in AP

$$\therefore \frac{b+c-a}{a} + \frac{a+b-c}{c} = 2\left(\frac{c+a-b}{b}\right)$$

$$\Rightarrow \frac{b}{a} + \frac{c}{a} - 1 + \frac{a}{c} + \frac{b}{c} - 1 = \frac{2c}{b} + \frac{2a}{b} - 2$$

$$\Rightarrow \frac{b}{a} + \frac{c}{a} + \frac{a}{c} + \frac{b}{c} - \frac{2c}{b} - \frac{2a}{b} = 0$$

Taking LCM

$\Rightarrow b^2c + c^2b + a^2b + ab^2 - 2ac^2 - 2a^2c = 0$

$\Rightarrow b^2c + c^2b + a^2b + ab^2 - ac^2 - ac^2 - a^2c - a^2c = 0$

$\Rightarrow (b^2c - a^2c) + (c^2b - ac^2) + (a^2b - a^2c) + (ab^2 - ac^2) = 0$

$\Rightarrow c\,(b - a)(b + a) + c^2(b - a) + a^2\,(b - c) + a(b + c)(b - c) = 0$

$\Rightarrow c(b - a)\,\{(b + a) + c\} + a(b - c)\,\{a + (b + c)\} = 0$

$\Rightarrow (a + b + c)\{cb - ca + ab - ca\} = 0$

Given $a + b + c \neq 0$

$\Rightarrow cb - ca + ab - ca = 0$

$\Rightarrow cb - 2ca + ab = 0$

$$\Rightarrow \frac{1}{a} - \frac{2}{b} + \frac{1}{c} = 0$$

$$\Rightarrow \frac{1}{a} + \frac{1}{c} = \frac{2}{b}$$

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP

Hence Proved

9. Question

If $(b - c)^2, (c - a)^2, (a - b)^2$ are in A.P., then show that: $\frac{1}{b-c}, \frac{1}{c-a}, \frac{1}{a-b}$ are in A.P.

[Hint: Add ab + bc + ca — a^2 — b^2 — c^2 to each term or let α = b — c, β = c — a, γ = a — b, then $\alpha + \beta + \gamma = 0$]

Answer

Given: $(b - c)^2, (c - a)^2, (a - b)^2$ are in A.P

$\therefore 2(c - a)^2 = (b - c)^2 + (a - b)^2$ ...(i)

To Prove: $\frac{1}{b-c}, \frac{1}{c-a}, \frac{1}{a-b}$ are in AP

or $\frac{2}{c-a} = \frac{1}{b-c} + \frac{1}{a-b}$

$$\Rightarrow \frac{2}{c-a} = \frac{a-b+b-c}{(b-c)(a-b)}$$

$$\Rightarrow \frac{2}{c-a} = \frac{a-c}{(b-c)(a-b)}$$

$\Rightarrow 2(b - c)(a - b) = (a - c)(c - a)$

$\Rightarrow 2[ab - b^2 - ca + cb] = ac - a^2 - c^2 + ac$

$\Rightarrow 2ab - 2b^2 - 2ac + 2cb = 2ac - a^2 - c^2$

$\Rightarrow a^2 + c^2 - 4ac = 2b^2 - 2ab - 2cb$

Adding both sides, $a^2 + c^2$, we get

$\Rightarrow 2(a^2 + c^2) - 4ac = a^2 + b^2 - 2ab + c^2 + b^2 - 2cb$

$\Rightarrow 2\ (a - c)^2 = (b - a)^2 + (b - c)^2$ which is true from (i)

$\therefore (b - c)^2, (c - a)^2, (a - b)^2$ are in A.P

$\therefore \frac{1}{b-c}, \frac{1}{c-a}, \frac{1}{a-b}$ are in AP

Hence Proved

10 A. Question

If a, b, c are in A.P., prove that:

$(a - c)^2 = 4\ (a - b)(b - c)$

Answer

Given: a, b, c are in AP

$\therefore a + c = 2b$

$\Rightarrow b = \frac{a+c}{2}$ ...(i)

Now taking RHS i.e. $4(a - b)(b - c)$

$\Rightarrow 4\left(a - \frac{a+c}{2}\right)\left(\frac{a+c}{2} - c\right)$ [from(i)]

$\Rightarrow 4\left(\frac{2a - a - c}{2}\right)\left(\frac{a + c - 2c}{2}\right)$

$\Rightarrow 4\left(\frac{a - c}{2}\right)\left(\frac{a - c}{2}\right)$

$\Rightarrow (a - c)^2$

= LHS

Hence Proved

10 B. Question

If a, b, c are in A.P., prove that:

$a^3 + c^3 + 6abc = 8b^3$

Answer

Given: a, b, c are in AP

$\therefore a + c = 2b$ ...(i)

$\Rightarrow b = \frac{a+c}{2}$ ...(ii)

Taking Lhs i.e. $a^3 + c^3 + 6abc$

$\Rightarrow a^3 + c^3 + 6ac\left(\frac{a+c}{2}\right)$ [from (i)]

$\Rightarrow a^3 + c^3 + 3ac(a + c)$

$\Rightarrow a^3 + c^3\ 3a^2c + 3ac^2$

$\Rightarrow (a + c)^3$

$\Rightarrow (2b)^3$ [from (ii)]

$= 8b^3 =$ RHS

Hence Proved

10 C. Question

If a, b, c are in A.P., prove that:

$(a + 2b — c)(2b + c — a)(c + a — b) = 4abc$

[Hint: Put $b = \frac{a+c}{2}$ on L.H.S. and R.H.S.]

Answer

Given: a, b, c are in AP

$\therefore a + c = 2b$ ...(i)

$\Rightarrow b = \frac{a+c}{2}$ ...(ii)

Now, taking LHS i.e. $(a + 2b — c)(2b + c — a)(c + a — b)$

$$\Rightarrow \left(a + 2 \times \frac{a+c}{2} - c\right)\left(2 \times \frac{a+c}{2} + c - a\right)\left(c + a - \frac{a+c}{2}\right)$$

[from (ii)]

$$\Rightarrow (a + a + c - c)(a + c + c - a)\left(\frac{2c + 2a - a - c}{2}\right)$$

$$\Rightarrow (2a)(2c)\left(\frac{a+c}{2}\right)$$

$\Rightarrow 4abc$

[from (ii)]

= RHS

Hence Proved

Exercise 8.4

1. Question

The sum of n terms of an A.P. is $\left(\frac{5n^2}{2}+\frac{3n}{2}\right)$. Find its 20th term.

Answer

$$S=\left(\frac{5n^2}{2}+\frac{3n}{2}\right)$$

Taking n = 1, we get

$$S_1=\left(\frac{5(1)^2}{2}+\frac{3(1)}{2}\right)$$

$$\Rightarrow\ S_1=\left(\frac{5}{2}+\frac{3}{2}\right)$$

$\Rightarrow S_1 = 4$

$\Rightarrow a_1 = 4$

Taking n = 2, we get

$$S_2=\left(\frac{5(2)^2}{2}+\frac{3(2)}{2}\right)$$

$$\Rightarrow\ S_2=\left(\frac{20}{2}+\frac{6}{2}\right)$$

$\Rightarrow S_2 = 13$

$\therefore a_2 = S_2 - S_1 = 13 - 4 = 9$

Taking n = 3, we get

$$S_3=\left(\frac{5(3)^2}{2}+\frac{3(3)}{2}\right)$$

$\Rightarrow\ S_3 = \left(\frac{45}{2} + \frac{9}{2}\right)$

$\Rightarrow S_3 = 27$

$\therefore a_3 = S_3 - S_2 = 27 - 13 = 14$

So, a = 4,

$d = a_2 - a_1 = 9 - 4 = 5$

Now, we have to find the 20^{th} term

$a_n = a + (n - 1)d$

$a_{20} = 4 + (20 - 1)5$

$a_{20} = 4 + 19 \times 5$

$a_{20} = 4 + 95$

$a_{20} = 99$

Hence, the 20^{th} term is 99.

2. Question

The sum of first n terms of an A.P. is given by $S_n = 3n^2 + 2n$. Determine the A.P. and its 15th term.

Answer

$S_n = 3n^2 + 2n$

Taking n = 1, we get

$S_1 = 3(1)^2 + 2(1)$

$\Rightarrow S_1 = 3 + 2$

$\Rightarrow S_1 = 5$

$\Rightarrow a_1 = 5$

Taking n = 2, we get

$S_2 = 3(2)^2 + 2(2)$

$\Rightarrow S_2 = 12 + 4$

$\Rightarrow S_2 = 16$

$\therefore a_2 = S_2 - S_1 = 16 - 5 = 11$

Taking n = 3, we get

$S_3 = 3(3)^2 + 2(3)$

$\Rightarrow S_3 = 27 + 6$

$\Rightarrow S_3 = 33$

$\therefore a_3 = S_3 - S_2 = 33 - 16 = 17$

So, a = 5,

$d = a_2 - a_1 = 11 - 5 = 6$

Now, we have to find the 15th term

$a_n = a + (n - 1)d$

$a_{15} = 5 + (15 - 1)6$

$a_{15} = 5 + 14 \times 6$

$a_{15} = 5 + 84$

$a_{15} = 89$

Hence, the 15th term is 89 and AP is 5, 11, 17, 23,…

3 A. Question

The sum of the first n terms of an A.P. is given by $S_n = 2n^2 + 5n$, find the nth term of the A.P.

Answer

$S_n = 2n^2 + 5n$

Taking n = 1, we get

$S_1 = 2(1)^2 + 5(1)$

$\Rightarrow S_1 = 2 + 5$

$\Rightarrow S_1 = 7$

$\Rightarrow a_1 = 7$

Taking n = 2, we get

$S_2 = 2(2)^2 + 5(2)$

$\Rightarrow S_2 = 8 + 10$

$\Rightarrow S_2 = 18$

$\therefore a_2 = S_2 - S_1 = 18 - 7 = 11$

Taking n = 3, we get

$S_3 = 2(3)^2 + 5(3)$

$\Rightarrow S_3 = 18 + 15$

$\Rightarrow S_3 = 33$

$\therefore a_3 = S_3 - S_2 = 33 - 18 = 15$

So, a = 7,

$d = a_2 - a_1 = 11 - 7 = 4$

Now, we have to find the 15^{th} term

$a_n = a + (n - 1)d$

$a_n = 7 + (n - 1)4$

$a_n = 7 + 4n - 4$

$a_n = 3 + 4n$

Hence, the n^{th} term is 4n + 3.

3 B. Question

The sum of n terms of an A.P. is $3n^2 + 5n$. Find the A.P. Hence, find its 16th term.

Answer

$S_n = 3n^2 + 5n$

Taking n = 1, we get

$S_1 = 3(1)^2 + 5(1)$

$\Rightarrow S_1 = 3 + 5$

$\Rightarrow S_1 = 8$

$\Rightarrow a_1 = 8$

Taking n = 2, we get

$S_2 = 3(2)^2 + 5(2)$

$\Rightarrow S_2 = 12 + 10$

$\Rightarrow S_2 = 22$

$\therefore a_2 = S_2 - S_1 = 22 - 8 = 14$

Taking n = 3, we get

$S_3 = 3(3)^2 + 5(3)$

$\Rightarrow S_3 = 27 + 15$

$\Rightarrow S_3 = 42$

$\therefore a_3 = S_3 - S_2 = 42 - 22 = 20$

So, a = 8,

$d = a_2 - a_1 = 14 - 8 = 6$

Now, we have to find the 15^{th} term

$a_n = a + (n - 1)d$

$a_{16} = 8 + (16 - 1)6$

$a_{16} = 8 + 15 \times 6$

$a_{16} = 8 + 90$

$a_{16} = 98$

Hence, the 16^{th} term is 98.

4. Question

If the sum of the first n terms of an A.P. is given by $S_n = (3n^2 - n)$, find its

(i) first term (ii) common difference

(iii) nth term.

Answer

$S_n = 3n^2 - n$

Taking n = 1, we get

$S_1 = 3(1)^2 - (1)$

$\Rightarrow S_1 = 3 - 1$

$\Rightarrow S_1 = 2$

$\Rightarrow a_1 = 2$

Taking n = 2, we get

$S_2 = 3(2)^2 - 2$

$\Rightarrow S_2 = 12 - 2$

$\Rightarrow S_2 = 10$

$\therefore a_2 = S_2 - S_1 = 10 - 2 = 8$

Taking n = 3, we get

$S_3 = 3(3)^2 - 3$

$\Rightarrow S_3 = 27 - 3$

$\Rightarrow S_3 = 24$

$\therefore a_3 = 24 - 10 = 14$

So, a = 1,

$d = a_2 - a_1 = 8 - 2 = 6$

Now, we have to find the 15th term

$a_n = a + (n - 1)d$

$a_n = 2 + (n - 1)6$

$a_n = 2 + 6n - 6$

$a_n = -4 + 6n$

Hence, the nth term is 4n - 3.

5. Question

If the sum to first n terms of an A.P. is $\left(\frac{3n^2}{2}+\frac{5n}{2}\right)$, find its 25th term.

Answer

$S_n = \left(\frac{3n^2}{2}+\frac{5n}{2}\right)$

Taking n = 1, we get

$S_1 = \left(\frac{3(1)^2}{2}+\frac{5(1)}{2}\right)$

$\Rightarrow\ S_1 = \left(\frac{3}{2}+\frac{5}{2}\right)$

$\Rightarrow S_1 = 4$

$\Rightarrow a_1 = 4$

Taking n = 2, we get

$S = \left(\frac{3(2)^2}{2}+\frac{5(2)}{2}\right)$

$\Rightarrow\ S_2 = \left(\frac{12}{2}+\frac{10}{2}\right)$

$\Rightarrow S_2 = 11$

$\therefore a_2 = S_2 - S_1 = 11 - 4 = 7$

Taking n = 3, we get

$S_3 = \left(\frac{3(3)^2}{2}+\frac{5(3)}{2}\right)$

$\Rightarrow\ S_3 = \left(\frac{27}{2}+\frac{15}{2}\right)$

$\Rightarrow S_3 = 21$

$\therefore a_3 = S_3 - S_2 = 21 - 11 = 10$

So, a = 4,

$d = a_2 - a_1 = 7 - 4 = 3$

Now, we have to find the 25^{th} term

$a_n = a + (n - 1)d$

$a_{25} = 4 + (25 - 1)3$

$a_{25} = 4 + 24 \times 3$

$a_{25} = 4 + 72$

$a_{25} = 76$

Hence, the 25^{th} term is 76.

6. Question

If the nth term of an A.P. is (2n + 1), find the sum of first n terms of the A.P.

Answer

Given: $a_n = 2n + 1$

Taking n = 1,

$a_1 = 2(1) + 1 = 2 + 1 = 3$

Taking n = 2,

$a_2 = 2(2) + 1 = 4 + 1 = 5$

Taking n = 3,

$a_3 = 2(3) + 1 = 6 + 1 = 7$

Therefore the series is 3, 5, 7, …

So, a = 3, $d = a_2 - a_1 = 5 - 3 = 2$

Now, we have to find the sum of first n terms of the AP

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\Rightarrow S_n = \frac{n}{2}[2 \times 3 + (n - 1)2]$$

$$\Rightarrow S_n = \frac{n}{2}[6 + 2n - 2]$$

$$\Rightarrow\ S_n = \frac{n}{2}[4 + 2n]$$

$\Rightarrow S_n = 2n + n^2$

Hence, the sum of n terms is $n^2 + 2n$.

7 A. Question

If the nth term of an A.P. is 9 — 5n, find the sum to first 15 terms.

Answer

Given: $a_n = 9 - 5n$

Taking n = 1,

$a_1 = 9 - 5(1) = 9 - 5 = 4$

Taking n = 2,

$a_2 = 9 - 5(2) = 9 - 10 = -1$

Taking n = 3,

$a_3 = 9 - 5(3) = 9 - 15 = -6$

Therefore the series is 4, -1, -6, …

So, $a = 4$, $d = a_2 - a_1 = -1 - 4 = -5$

Now, we have to find the sum of the first 15 terms of the AP

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\Rightarrow\ S_{15} = \frac{15}{2}[2 \times 4 + (15 - 1)(-5)]$$

$$\Rightarrow\ S_{15} = \frac{15}{2}[8 - 70]$$

$$\Rightarrow\ S_{15} = \frac{15}{2}[-62]$$

$\Rightarrow S_{15} = 15 \times (-31)$

$\Rightarrow S_{15} = -465$

Hence, the sum of 15 terms is -465.

7 B. Question

Find the sum of first 25 terms of an A.P. whose nth term is 1 — 4n.

Answer

Given: $a_n = 1 - 4n$

Taking n = 1,

$a_1 = 1 - 4(1) = 1 - 4 = -3$

Taking n = 2,

$a_2 = 1 - 4(2) = 1 - 8 = -7$

Taking n = 3,

$a_3 = 1 - 4(3) = 1 - 12 = -11$

Therefore the series is -3, -7, -11, …

So, a = -3, $d = a_2 - a_1 = -7 - (-3) = -7 + 3 = -4$

Now, we have to find the sum of the first 25 terms of the AP

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\Rightarrow S_{25} = \frac{25}{2}[2 \times (-3) + (25 - 1)(-4)]$$

$$\Rightarrow S_{25} = \frac{25}{2}[-6 - 96]$$

$$\Rightarrow S_{25} = \frac{25}{2}[-102]$$

$\Rightarrow S_{25} = 25 \times (-51)$

$\Rightarrow S_{25} = -1275$

Hence, the sum of 25 terms is -1275.

8. Question

If the sum to n terms of a sequence be $n^2 + 2n$, then prove that the sequence is an A.P.

Answer

Given: $S_n = n^2 + 2n$ ...(i)

$S_{n-1} = (n - 1)^2 + 2(n - 1) = n^2 + 1 - 2n + 2n - 2 = n^2 - 1$ ...(ii)

Subtracting eq (ii) from (i), we get

$t_n = S_n - S_{n-1} = n^2 + 2n - n^2 + 1 = 2n + 1$

The nth term of an AP is 2n + 1.

9. Question

Find the sum to first n terms of an A.P. whose kth term is 5k + 1.

Answer

As it is given that kth term of the AP = 5k + 1

$\therefore a_k = a + (k - 1)d$

$\Rightarrow 5k + 1 = a + (k - 1)d$

$\Rightarrow 5k + 1 = a + kd - d$

Now, on comparing the coefficient of k, we get

$d = 5$

and $a - d = 1$

$\Rightarrow a - 5 = 1$

$\Rightarrow a = 6$

We know that,

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\Rightarrow S_n = \frac{n}{2}[2 \times 6 + (n - 1)5]$$

$$\Rightarrow S_n = \frac{n}{2}[12 + 5n - 5]$$

$$\Rightarrow S_n = \frac{n}{2}[7 + 5n]$$

10. Question

If the sum of n terms of an A.P. is $3n^2 + 5n$ and its mth term is 164, find the value of m.

[Hint: $t_m = S_m — S_{m-1} = 3m^2 + 5m — 3(m— 1)^2 — 5(m— 1) = 3(2m — 1) + 5 = 6m + 2$]

Answer

$S_n = 3n^2 + 5n$

Taking n = 1, we get

$S_1 = 3(1)^2 + 5(1)$

$\Rightarrow S_1 = 3 + 5$

$\Rightarrow S_1 = 8$

$\Rightarrow a_1 = 8$

Taking n = 2, we get

$S_2 = 3(2)^2 + 5(2)$

$\Rightarrow S_2 = 12 + 10$

$\Rightarrow S_2 = 22$

$\therefore a_2 = S_2 - S_1 = 22 - 8 = 14$

Taking n = 3, we get

$S_3 = 3(3)^2 + 5(3)$

$\Rightarrow S_3 = 27 + 15$

$\Rightarrow S_3 = 42$

$\therefore a_3 = S_3 - S_2 = 42 - 22 = 20$

So, a = 8,

$d = a_2 - a_1 = 14 - 8 = 6$

Now, we have to find the value of m

$a_n = a + (n - 1)d$

$\Rightarrow a_m = 8 + (m - 1)6$

$\Rightarrow 164 = 8 + 6m - 6$

$\Rightarrow 164 = 2 + 6m$

$\Rightarrow 162 = 6m$

$\Rightarrow m = 27$

11. Question

If the sum of n terms of an A.P. is $pn + qn^2$, where p and q are constants, find the common difference.

Answer

$S_n = qn^2 + pn$

Taking n = 1, we get

$S_1 = q(1)^2 + p(1)$

$\Rightarrow S_1 = q + p$

$\Rightarrow a_1 = q + p$

Taking n = 2, we get

$S_2 = q(2)^2 + p(2)$

$\Rightarrow S_2 = 4q + 2p$

$\therefore a_2 = S_2 - S_1 = 4q + 2p - q - p = 3q + p$

Taking n = 3, we get

$S_3 = q(3)^2 + p(3)$

$\Rightarrow S_3 = 9q + 3p$

$\therefore a_3 = S_3 - S_2 = 9q + 3p - 4q - 2p = 5q + p$

So, $a = q + p$,

$d = a_2 - a_1 = 3q + p - (q + p) = 3q + p - q - p = 2q$

Hence, the common difference is 2q.

12. Question

If the sum of n terms of an A.P. is $nP + 1/2\ n(n - 1)Q$, where P and Q are constants, find the common difference of the A.P.

Answer

$$S_n = nP + \frac{1}{2}n(n - 1)Q$$

Taking n = 1, we get

$$S_1 = (1)P + \frac{1}{2}(1)(1 - 1)Q$$

$\Rightarrow S_1 = P$

$\Rightarrow a_1 = P$

Taking n = 2, we get

$$S_2 = (2)P + \frac{1}{2} \times 2(2 - 1)Q$$

$\Rightarrow S_2 = 2P + Q$

$\therefore a_2 = S_2 - S_1 = 2P + Q - P = P + Q$

Taking n = 3, we get

$$S_3 = (3)P + \frac{1}{2}(3)(3 - 1)Q$$

$\Rightarrow S_3 = 3P + 3Q$

$\therefore a_3 = S_3 - S_2 = 3P + 3Q - 2P - Q = P + 2Q$

So, a = P,

$d = a_2 - a_1 = P + Q - (P) = Q$

$= a_3 - a_2 = P + 2Q - (P + Q) = P + 2Q - P - Q = Q$

Hence, the common difference is Q.

13. Question

Find the sum : 25 + 28 + 31 +... + 100

Answer

Here, a = 25, d = 28 – 25 = 3 and $a_n = 100$

We know that,

$a_n = a + (n - 1)d$

$\Rightarrow 100 = 25 + (n - 1)3$

$\Rightarrow 75 = (n - 1)3$

$\Rightarrow 25 = n - 1$

$\Rightarrow 26 = n$

Now,

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\Rightarrow S_{26} = \frac{26}{2}[2 \times 25 + (26 - 1)3]$$

$\Rightarrow S_{26} = 13[50 + 25 \times 3]$

$\Rightarrow S_{26} = 13[50 + 75]$

$\Rightarrow S_{26} = 13 \times 125$

$\Rightarrow S_{26} = 1625$

14. Question

Which term of the A.P. 4, 9, 14, ... is 89? Also, find the sum 4 + 9 + 14 + + 89.

Answer

Let $a_n = 89$

AP = 4, 9, 14, ...89

Here, a = 4, d = 14 – 9 = 5

We know that

$a_n = a + (n - 1)d$

$\Rightarrow 89 = 4 + (n - 1)5$

$\Rightarrow 85 = (n - 1)5$

$\Rightarrow 17 = n - 1$

$\Rightarrow 18 = n$

So, 89 is the 18^{th} term of the given AP

Now, we find the sum of 4 + 9 + 14 + ... + 89

We know that,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow \; S_{18} = \frac{18}{2}[2 \times 4 + (18-1)5]$$

$\Rightarrow S_{18} = 9[8 + 17 \times 5]$

$\Rightarrow S_{18} = 9[8 + 85]$

$\Rightarrow S_{18} = 9 \times 93$

$\Rightarrow S_{18} = 837$

Hence, the sum of the given AP is 837.

15 A. Question

Solve for x

1 + 6+11 + 16 +...+x= 148

Answer

Here, a = 1, d = 6 – 1 = 5 and S_n = 148

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow \; S_n = \frac{n}{2}[2 \times 1 + (n-1)5]$$

$$\Rightarrow \; 148 = \frac{n}{2}[2 + 5n - 5]$$

$$\Rightarrow \; 148 = \frac{n}{2}[5n - 3]$$

$\Rightarrow 296 = n[5n - 3]$

$\Rightarrow 5n^2 - 3n - 296 = 0$

$\Rightarrow 5n^2 - 40n + 37n - 296 = 0$

$\Rightarrow 5n(n - 8) + 37(n - 8) = 0$

$\Rightarrow (5n + 37)(n - 8) = 0$

$\Rightarrow 5n + 37 = 0$ or $n - 8 = 0$

$\Rightarrow n = -\frac{37}{5}$ or $n = 8$

But $n = -\frac{37}{5}$ is not a positive integer.

$\therefore$ n = 8

$\Rightarrow$ x = a_8 = a + 7d = 1 + 7 × 5 = 1 + 35 = 36

Hence, x = 36

15 B. Question

Solve for x

25+22+19+ 16+...+x= 115

Answer

Here, a = 25, d = 22 – 25 = -3 and S_n = 115

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow\ S_n = \frac{n}{2}[2 \times 25 + (n-1)(-3)]$$

$$\Rightarrow\ 115 = \frac{n}{2}[50 - 3n + 3]$$

$$\Rightarrow\ 115 = \frac{n}{2}[53 - 3n]$$

$\Rightarrow$ 230 = n[53 – 3n]

$\Rightarrow$ $3n^2$ – 53n + 230 = 0

$\Rightarrow$ $3n^2$ – 30n - 23n + 230 = 0

$\Rightarrow$ 3n(n – 10) - 23(n – 10) = 0

$\Rightarrow$ (3n – 23)(n – 10) = 0

$\Rightarrow$ 3n – 23 = 0 or n – 10 = 0

$\Rightarrow n = \frac{23}{3}$ or n = 10

But $n = \frac{23}{3}$ is not an integer.

$\therefore$ n = 10

$\Rightarrow$ x = a_{10} = a + 9d = 25 + 9 × (-3) = 25 – 27 = -2

Hence, x = -2

16. Question

Find the number of terms of the A.P. 64, 60, 56, ... so that their sum is 544. Explain the double answer.

Answer

AP = 64, 60, 56, …

Here, a = 64, d = 60 – 64 = -4

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow\ 544 = \frac{n}{2}[2 \times 64 + (n-1)(-4)]$$

$$\Rightarrow\ 544 = \frac{n}{2}[128 - 4n + 4]$$

$$\Rightarrow\ 544 = \frac{n}{2}[132 - 4n]$$

$\Rightarrow$ 1088 = n[132 – 4n]

$\Rightarrow 4n^2 - 132n + 1088 = 0$

$\Rightarrow n^2 - 33n + 272 = 0$

$\Rightarrow n^2 - 16n - 17n + 272 = 0$

$\Rightarrow$ n(n – 16) - 17(n – 16) = 0

$\Rightarrow$ (n – 16)(n – 17) = 0

$\Rightarrow$ n – 16 = 0 or n – 17 = 0

$\Rightarrow$ n = 16 or n = 17

If n = 16, a = 64 and d = -4

a_{16} = 64 + (16 – 1)(-4)

a_{16} = 64 + 15 × -4

a_{16} = 64 – 60

a_{16} = 4

and If n = 17, a = 64 and d = -4

$a_{17} = 64 + (17 - 1)(-4)$

$a_{17} = 64 + 16 \times -4$

$a_{17} = 64 - 64$

$a_{17} = 0$

Now, we will check at which term the sum of the AP is 544.

$$S_n = \frac{n}{2}[a + a_n]$$

$$\Rightarrow \ S_{16} = \frac{16}{2}[64 + 4]$$

$\Rightarrow S_{16} = 8[68]$

$\Rightarrow S_{16} = 544$

and $S_{17} = \frac{17}{2}[64 + 0]$

$\Rightarrow S_{17} = 17 \times 32$

$\Rightarrow S_{17} = 544$

So, the terms may be either 17 or 16 both holds true.

We get a double answer because the 17^{th} term is zero and when we add this in the sum, the sum remains the same.

17. Question

How many terms of the A.P. 3, 5, 7, 9, ... must be added to get the sum 120?

Answer

AP = 3, 5, 7, 9, ...

Here, a = 3, d = 5 – 3 = 2 and $S_n = 120$

We know that,

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\Rightarrow \ 120 = \frac{n}{2}[2 \times 3 + (n - 1)(2)]$$

$$\Rightarrow \ 120 = \frac{n}{2}[6 + 2n - 2]$$

$\Rightarrow\ 120 = \frac{n}{2}[4 + 2n]$

$\Rightarrow$ 120 = n[2+n]

$\Rightarrow n^2 + 2n - 120 = 0$

$\Rightarrow n^2 + 12n - 10n - 120 = 0$

$\Rightarrow$ n(n + 12) - 10(n + 12) = 0

$\Rightarrow$ (n – 10)(n + 12) = 0

$\Rightarrow$ n – 10 = 0 or n + 12 = 0

$\Rightarrow$ n = 10 or n = -12

But number of terms can't be negative. So, n = 10

Hence, for n = 10 the sum is 120 for the given AP.

18. Question

Find the number of terms of the A.P. 63, 60, 57, ... so that their sum is 693. Explain the double answer.

Answer

AP = 63, 60, 57,...

Here, a = 63, d = 60 – 63 = -3 and S_n = 693

We know that,

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow\ 693 = \frac{n}{2}[2 \times 63 + (n - 1)(-3)]$

$\Rightarrow\ 693 = \frac{n}{2}[126 - 3n + 3]$

$\Rightarrow\ 693 = \frac{n}{2}[129 - 3n]$

$\Rightarrow$ 1386 = n[129 – 3n]

$\Rightarrow 3n^2 - 129n + 1386 = 0$

$\Rightarrow n^2 - 43n + 462 = 0$

$\Rightarrow n^2 - 22n - 21n + 462 = 0$

$\Rightarrow n(n - 22) - 21(n - 22) = 0$

$\Rightarrow (n - 21)(n - 22) = 0$

$\Rightarrow n - 21 = 0$ or $n - 22 = 0$

$\Rightarrow n = 21$ or $n = 22$

So, n = 21 and 22

If n = 21, a = 63 and d = -3

$a_{21} = 63 + (21 - 1)(-3)$

$a_{21} = 63 + 20 \times -3$

$a_{21} = 63 - 60$

$a_{21} = 3$

and If n = 22, a = 63 and d = -3

$a_{22} = 63 + (22 - 1)(-3)$

$a_{22} = 63 + 21 \times -3$

$a_{22} = 63 - 63$

$a_{22} = 0$

Now, we will check at which term the sum of the AP is 693.

$$S_n = \frac{n}{2}[a + a_n]$$

$$\Rightarrow S_{21} = \frac{21}{2}[63 + 3]$$

$\Rightarrow S_{21} = 21 \times 33$

$\Rightarrow S_{21} = 693$

and $S_{22} = \frac{22}{2}[63 + 0]$

$\Rightarrow S_{22} = 11 \times 63$

$\Rightarrow S_{22} = 693$

So, the terms may be either 21 or 22 both holds true.

We get the double answer because here the 22^{nd} term is zero and it does not affect the sum.

19. Question

How many terms of the series 15 + 12 + 9 + ... must be taken to make 15? Explain the double answer.

Answer

Here, a = 15, d = 12 – 15 = -3 and S_n = 15

We know that,

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\Rightarrow \; 15 = \frac{n}{2}[2 \times 15 + (n - 1)(-3)]$$

$$\Rightarrow \; 15 = \frac{n}{2}[30 - 3n + 3]$$

$$\Rightarrow \; 15 = \frac{n}{2}[33 - 3n]$$

⇒ 30 = n[33 – 3n]

⇒ $3n^2$ – 33n + 30 = 0

⇒ $3n^2$ – 30n – 3n + 30 = 0

⇒ 3n(n – 10) -3(n – 10) = 0

⇒ (n – 10)(3n – 3) = 0

⇒ n – 10 = 0 or 3n – 3 = 0

⇒ n = 10 or n = 1

The number of terms can be 1 or 10.

Here, the common difference is negative.

∴ The AP starts from a positive term, and its terms are decreasing.

∴ All the terms after 6^{th} term are negative.

We get a double answer because these positive terms from 2^{nd} to 5^{th} term when added to negative terms from 7^{th} to 10^{th} term, they cancel out each other and the sum remains same.

20 A. Question

Find the sum of all the odd numbers lying between 100 and 200.

Answer

The odd numbers lying between 100 and 200 are

101, 103, 105,…, 199

$a_2 - a_1 = 103 - 101 = 2$

$a_3 - a_2 = 105 - 103 = 2$

$\because a_3 - a_2 = a_2 - a_1 = 2$

Therefore, the series is in AP

Here, a = 101, d = 2 and $a_n = 199$

We know that,

$a_n = a + (n - 1)d$

$\Rightarrow 199 = 101 + (n - 1)2$

$\Rightarrow 199 - 101 = (n - 1)2$

$\Rightarrow 98 = (n - 1)2$

$\Rightarrow 49 = (n - 1)$

$\Rightarrow n = 50$

Now, we have to find the sum of this AP

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow S_{50} = \frac{50}{2}[2 \times 101 + (50-1)2]$$

$\Rightarrow S_{50} = 25[202 + 49 \times 2]$

$\Rightarrow S_{50} = 25[300]$

$\Rightarrow S_{50} = 7500$

Hence, the sum of all odd numbers lying between 100 and 200 is 7500.

20 B. Question

Find the sum of all odd integers from 1 to 2001.

Answer

The odd numbers lying between 1 and 2001 are

1, 3, 5,..., 2001

$a_2 - a_1 = 3 - 1 = 2$

$a_3 - a_2 = 5 - 3 = 2$

$\because a_3 - a_2 = a_2 - a_1 = 2$

Therefore, the series is in AP

Here, a = 1, d = 2 and $a_n = 2001$

We know that,

$a_n = a + (n - 1)d$

$\Rightarrow 2001 = 1 + (n - 1)2$

$\Rightarrow 2001 - 1 = (n - 1)2$

$\Rightarrow 2000 = (n - 1)2$

$\Rightarrow 1000 = (n - 1)$

$\Rightarrow n = 1001$

Now, we have to find the sum of this AP

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\Rightarrow S_{1001} = \frac{1001}{2}[2 \times 1 + (1001 - 1)2]$$

$\Rightarrow S_{1001} = 1001[1 + 1000]$

$\Rightarrow S_{1001} = 1001\ [1001]$

$\Rightarrow S_{1001} = 1002001$

Hence, the sum of all odd numbers lying between 1 and 2001 is 1002001.

21. Question

Determine the sum of first 35 terms of an A.P., if the second term is 2 and the seventh term is 22.

Answer

Given: $a_2 = 2$ and $a_7 = 22$ and $n = 35$

We know that,

$a_2 = a + d = 2$ ...(i)

and $a_7 = a + 6d = 22$ ...(ii)

Solving the linear equations (i) and (ii), we get

$a + d - a - 6d = 2 - 22$

$\Rightarrow - 5d = -20$

$\Rightarrow d = 4$

Putting the value of d in eq. (i), we get

$a + 4 = 2$

$\Rightarrow a = 2 - 4 = -2$

Now, we have to find the sum of first 35 terms.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow S_{35} = \frac{35}{2}[2 \times (-2) + (35-1)4]$$

$$\Rightarrow S_{35} = \frac{35}{2}[-4 + 34 \times 4]$$

$\Rightarrow S_{35} = 35\ [-2 + 34 \times 2]$

$\Rightarrow S_{35} = 35\ [66]$

$\Rightarrow S_{35} = 2310$

22. Question

If the sum of the first p terms of an A.P. is q and the sum of first q terms is p, then find the sum of first (p + q) terms.

Answer

Given: $S_p = q$ and $S_q = p$

To find: S_{p+q}

We know that,

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\Rightarrow S_p = \frac{p}{2}[2a + (p - 1)d]$$

$$\Rightarrow q = \frac{p}{2}[2a + (p - 1)d]$$

$$\Rightarrow \frac{2q}{p} = 2a + (p - 1)d$$

$$\Rightarrow \frac{2q}{p} - (p - 1)d = 2a \text{ ...(i)}$$

Now,

$$\Rightarrow S_q = \frac{q}{2}[2a + (q - 1)d]$$

$$\Rightarrow p = \frac{q}{2}[2a + (q - 1)d]$$

$$\Rightarrow \frac{2p}{q} = 2a + (q - 1)d$$

$$\Rightarrow \frac{2p}{q} - (q - 1)d = 2a \text{ ...(ii)}$$

From eq. (i) and (ii), we get

$$\frac{2q}{p} - (p - 1)d = \frac{2p}{q} - (q - 1)d$$

$$\Rightarrow \frac{2q}{p} - \frac{2p}{q} = (p - 1)d - (q - 1)d$$

$$\Rightarrow \frac{2q^2 - 2p^2}{pq} = d[p - 1 - q + 1]$$

$$\Rightarrow \frac{2(q - p)(q + p)}{pq} = d(p - q)$$

[$\because$, $a^2 - b^2 = (a - b)(a + b)$]

$$\Rightarrow \frac{2(q - p)(q + p)}{-pq(q - p)} = d$$

$\Rightarrow \ d = \frac{-2(q+p)}{pq}$ ...(iii)

Now, putting the value of d in eq. (i), we get

$$\Rightarrow \frac{2q}{p} - (p-1)\left(\frac{-2(q+p)}{pq}\right) = 2a$$

$$\Rightarrow \frac{q}{p} + \frac{(p-1)(q+p)}{pq} = a$$

$$\Rightarrow \frac{q^2+pq+p^2-q-p}{pq} = a \text{ ...(iv)}$$

Now, we to find S_{p+q}

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow S_{p+q} = \frac{p+q}{2}\left[2\left(\frac{q^2+pq+p^2-q-p}{pq}\right) + (p+q-1)\left(\frac{-2(q+p)}{pq}\right)\right]$$

[from (iii) & (iv)]

$$\Rightarrow S_{p+q} = (p+q)\left[\left(\frac{q^2+pq+p^2-q-p}{pq}\right) + (p+q-1)\left(\frac{-(q+p)}{pq}\right)\right]$$

$$\Rightarrow S_{p+q} = \frac{p+q}{pq}[q^2+pq+p^2-q-p+(p+q-1)(-q-p)]$$

$$\Rightarrow S_{p+q} = \frac{p+q}{pq}[q^2+pq+p^2-q-p-pq-p^2-q^2-pq+q+p]$$

$$\Rightarrow S_{p+q} = \frac{p+q}{pq}[-pq]$$

$\Rightarrow S_{p+q}$ = - (p+q)

Hence, the sum of first (p+q) terms is $-(p+q)$

23. Question

How many terms of the A.P. -6,-$\frac{11}{2}$,-5 ... are needed to get the sum - 25?

Answer

$AP = -6, \frac{-11}{2}, -5, \ldots$

Here, a = -6,

$$d = -\frac{11}{2} - (-6) = \frac{-11 + 12}{2} = \frac{1}{2}$$

and S_n = -25

We know that,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow -25 = \frac{n}{2}[2 \times (-6) + (n-1)\left(\frac{1}{2}\right)]$$

$$\Rightarrow -25 = \frac{n}{2}[\frac{-24 + n - 1}{2}]$$

$$\Rightarrow -25 = \frac{n}{4}[-25 + n]$$

⇒ -100 = n[-25 + n]

⇒ n^2 – 25n + 100 = 0

⇒ n^2 – 20n – 5n + 100 = 0

⇒ n(n – 20) - 5(n – 20) = 0

⇒ (n – 20)(n – 5) = 0

⇒ n – 5 = 0 or n – 20 = 0

⇒ n = 5 or n = 20

So, n = 5 or 20

24 A. Question

Find the sum of the numbers lying between 107 and 253 that are multiples of 5.

Answer

The numbers lying between 107 and 253 that are multiples of 5 are

110, 115, 120,…, 250

$a_2 - a_1$ = 115 – 110 = 5

$a_3 - a_2$ = 120 – 115 = 5

$\because a_3 - a_2 = a_2 - a_1 = 5$

Therefore, the series is in AP

Here, a = 110, d = 5 and $a_n = 250$

We know that,

$a_n = a + (n - 1)d$

$\Rightarrow 250 = 110 + (n - 1)5$

$\Rightarrow 250 - 110 = (n - 1)5$

$\Rightarrow 140 = (n - 1)5$

$\Rightarrow 28 = (n - 1)$

$\Rightarrow n = 29$

Now, we have to find the sum of this AP

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\Rightarrow S_{29} = \frac{29}{2}[2 \times 110 + (29 - 1)5]$$

$\Rightarrow S_{29} = 29[110 + 14 \times 5]$

$\Rightarrow S_{29} = 29[180]$

$\Rightarrow S_{29} = 5220$

Hence, the sum of all numbers lying between 107 and 253 is 5220.

24 B. Question

Find the sum of all natural numbers lying between 100 and 1000 which are multiples of 5.

Answer

The numbers lying between 100 and 1000 that are multiples of 5 are

105, 110, 115, 120,…, 995

$a_2 - a_1 = 110 - 105 = 5$

$a_3 - a_2 = 115 - 110 = 5$

$\because a_3 - a_2 = a_2 - a_1 = 5$

Therefore, the series is in AP

Here, $a = 105$, $d = 5$ and $a_n = 995$

We know that,

$a_n = a + (n - 1)d$

$\Rightarrow 995 = 105 + (n - 1)5$

$\Rightarrow 995 - 105 = (n - 1)5$

$\Rightarrow 890 = (n - 1)5$

$\Rightarrow 178 = (n - 1)$

$\Rightarrow n = 179$

Now, we have to find the sum of this AP

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\Rightarrow S_{179} = \frac{179}{2}[2 \times 105 + (179 - 1)5]$$

$\Rightarrow S_{179} = 179[105 + 89 \times 5]$

$\Rightarrow S_{179} = 179\ [550]$

$\Rightarrow S_{179} = 98450$

Hence, the sum of all numbers lying between 100 and 1000 that are multiples of 5 is 98450.

25. Question

Find the sum of all the two digit odd positive integers.

Answer

The two digit odd positive integers are

11, 13, 15,…, 99

$a_2 - a_1 = 13 - 11 = 2$

$a_3 - a_2 = 15 - 13 = 2$

$\because a_3 - a_2 = a_2 - a_1 = 2$

Therefore, the series is in AP

Here, $a = 11$, $d = 2$ and $a_n = 99$

We know that,

$a_n = a + (n - 1)d$

$\Rightarrow 99 = 11 + (n - 1)2$

$\Rightarrow 99 - 11 = (n - 1)2$

$\Rightarrow 88 = (n - 1)2$

$\Rightarrow 44 = (n - 1)$

$\Rightarrow n = 45$

Now, we have to find the sum of this AP

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\Rightarrow \ S_{45} = \frac{45}{2}[2 \times 11 + (45 - 1)2]$$

$\Rightarrow S_{45} = 45[11 + 44]$

$\Rightarrow S_{45} = 45[55]$

$\Rightarrow S_{45} = 2475$

Hence, the sum of all two digit odd numbers are 2475.

26. Question

Find the sum of all multiplies of 9 lying between 300 and 700.

Answer

The numbers lying between 300 and 700 which are multiples of 9 are

306, 315, 324,…, 693

$a_2 - a_1 = 315 - 306 = 9$

$a_3 - a_2 = 324 - 315 = 9$

$\because a_3 - a_2 = a_2 - a_1 = 9$

Therefore, the series is in AP

Here, $a = 306$, $d = 9$ and $a_n = 693$

We know that,

$a_n = a + (n - 1)d$

$\Rightarrow 693 = 306 + (n - 1)9$

$\Rightarrow 693 - 306 = (n - 1)9$

$\Rightarrow 387 = (n - 1)9$

$\Rightarrow 43 = (n - 1)$

$\Rightarrow n = 44$

Now, we have to find the sum of this AP

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\Rightarrow S_{44} = \frac{44}{2}[2 \times 306 + (44 - 1)9]$$

$\Rightarrow S_{44} = 22[612 + 387]$

$\Rightarrow S_{44} = 22[999]$

$\Rightarrow S_{44} = 21978$

Hence, the sum of all numbers lying between 300 and 700 is 21978.

27. Question

Find the sum of all the three digit natural numbers which are multiples of 7.

Answer

The three digit natural numbers which are multiples of 7 are

105, 112, 119,..., 994

$a_2 - a_1 = 112 - 105 = 7$

$a_3 - a_2 = 112 - 105 = 7$

$\because a_3 - a_2 = a_2 - a_1 = 7$

Therefore, the series is in AP

Here, $a = 105$, $d = 7$ and $a_n = 994$

We know that,

$a_n = a + (n - 1)d$

$\Rightarrow 994 = 105 + (n - 1)7$

$\Rightarrow 994 - 105 = (n - 1)7$

$\Rightarrow 889 = (n - 1)7$

$\Rightarrow 127 = (n - 1)$

$\Rightarrow n = 128$

Now, we have to find the sum of this AP

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow S_{128} = \frac{128}{2}[2 \times 105 + (128-1)7]$$

$\Rightarrow S_{128} = 64[210 + 127 \times 7]$

$\Rightarrow S_{128} = 64[1099]$

$\Rightarrow S_{128} = 70336$

Hence, the sum of all three digit numbers which are multiples of 7 are 70336.

28. Question

Find the sum of all natural numbers lying between 100 and 500, which are divisible by 8.

Answer

The numbers lying between 100 and 500 which are divisible by 8 are

104, 112, 120, 128, 136,…, 496

$a_2 - a_1 = 112 - 104 = 8$

$a_3 - a_2 = 120 - 112 = 8$

$\because a_3 - a_2 = a_2 - a_1 = 8$

Therefore, the series is in AP

Here, $a = 120$, $d = 8$ and $a_n = 496$

We know that,

$a_n = a + (n - 1)d$

$\Rightarrow 496 = 104 + (n - 1)8$

$\Rightarrow 496 - 104 = (n - 1)8$

$\Rightarrow 392 = (n - 1)8$

$\Rightarrow 49 = (n - 1)$

$\Rightarrow n = 50$

Now, we have to find the sum of this AP

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\Rightarrow \ S_{50} = \frac{50}{2}[2 \times 104 + (50 - 1)8]$$

$\Rightarrow S_{50} = 25[208 + 49 \times 8]$

$\Rightarrow S_{50} = 25[600]$

$\Rightarrow S_{50} = 15000$

Hence, the sum of all numbers lying between 100 and 500 and divisible by 8 is 15000.

29. Question

Find the sum of all the 3 digit natural numbers which are divisible by 13.

Answer

The three digit natural numbers which are divisible by 13 are

104, 117, 130,..., 988

$a_2 - a_1 = 117 - 104 = 13$

$a_3 - a_2 = 130 - 117 = 13$

$\because a_3 - a_2 = a_2 - a_1 = 13$

Therefore, the series is in AP

Here, $a = 104$, $d = 13$ and $a_n = 988$

We know that,

$a_n = a + (n - 1)d$

$\Rightarrow 988 = 104 + (n - 1)13$

$\Rightarrow 988 - 104 = (n - 1)13$

$\Rightarrow 884 = (n - 1)13$

$\Rightarrow 68 = (n - 1)$

$\Rightarrow n = 69$

Now, we have to find the sum of this AP

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\Rightarrow S_{69} = \frac{69}{2}[2 \times 104 + (69 - 1)13]$$

$$\Rightarrow S_{69} = \frac{69}{2}[2 \times 104 + 68 \times 13]$$

$\Rightarrow S_{69} = 69[104 + 34 \times 13]$

$\Rightarrow S_{69} = 69[546]$

$\Rightarrow S_{69} = 37674$

Hence, the sum of three digit natural numbers which are divisible by 13 are 37674.

30. Question

The 5th and 15th terms of an A.P. are 13 and - 17 respectively. Find the sum of first 21 terms of the A.P.

Answer

Given: $a_5 = 13$ and $a_{15} = -17$ and $n = 21$

We know that,

$a_5 = a + 4d = 13$ …(i)

and $a_{15} = a + 14d = -17$ …(ii)

Solving the linear equations (i) and (ii), we get

$a + 4d - a - 14d = 13 - (-17)$

$\Rightarrow -10d = 13 + 17$

$\Rightarrow -10d = 30$

$\Rightarrow d = -3$

Putting the value of d in eq. (i), we get

a + 4(-3) = 13

⇒ a = 13 + 12 = 25

Now, we have to find the sum of first 21 terms.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow \ S_{21} = \frac{21}{2}[2 \times (25) + (21-1)(-3)]$$

$$\Rightarrow \ S_{21} = \frac{21}{2}[50 + 20 \times (-3)]$$

⇒ S_{21} = 21 [25 + 10 × (-3)]

⇒ S_{21} = 21 [-5]

⇒ S_{21} =-105

31. Question

Find the sum of first 21 terms of the A.P. whose 2nd term is 8 and 4th term is 14.

Answer

Given: a_2 = 8 and a_4 = 14 and n = 21

We know that,

a_2 = a + d = 8 ...(i)

and a_4 = a + 3d = 14 ...(ii)

Solving the linear equations (i) and (ii), we get

a + d – a – 3d = 8 – 14

⇒ -2d = -6

⇒ d = 3

Putting the value of d in eq. (i), we get

a + 3 = 8

⇒ a = 8 – 3 = 5

Now, we have to find the sum of first 21 terms.

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\Rightarrow S_{21} = \frac{21}{2}[2 \times (5) + (21 - 1)(3)]$$

$$\Rightarrow S_{21} = \frac{21}{2}[10 + 20 \times (3)]$$

$\Rightarrow S_{21} = 21 [5 + 10 \times (3)]$

$\Rightarrow S_{21} = 21 [35]$

$\Rightarrow S_{21} = 735$

32. Question

Find the sum of 51 terms of the A.P. whose second term is 2 and the 4th term is 8.

Answer

Given: $a_2 = 2$ and $a_4 = 8$ and $n = 51$

We know that,

$a_2 = a + d = 2$ ...(i)

and $a_4 = a + 3d = 8$ ...(ii)

Solving the linear equations (i) and (ii), we get

$a + d - a - 3d = 2 - 8$

$\Rightarrow -2d = -6$

$\Rightarrow d = 3$

Putting the value of d in eq. (i), we get

$a + 3 = 2$

$\Rightarrow a = 2 - 3 = -1$

Now, we have to find the sum of first 51 terms.

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\Rightarrow \ S_{51} = \frac{51}{2}[2 \times (-1) + (51 - 1)(3)]$$

$$\Rightarrow \ S_{51} = \frac{51}{2}[-1 + 50 \times (3)]$$

$\Rightarrow S_{51} = 51 [-1 + 25 \times (3)]$

$\Rightarrow S_{51} = 51 [74]$

$\Rightarrow S_{51} = 3774$

33. Question

Find the sum of the first 25 terms of the A.P. whose 2nd term is 9 and 4th term is 21.

Answer

Given: $a_2 = 9$ and $a_4 = 21$ and $n = 25$

We know that,

$a_2 = a + d = 9$ ...(i)

and $a_4 = a + 3d = 21$ ...(ii)

Solving the linear equations (i) and (ii), we get

$a + d – a – 3d = 9 – 21$

$\Rightarrow -2d = -12$

$\Rightarrow d = 6$

Putting the value of d in eq. (i), we get

$a + 6 = 9$

$\Rightarrow a = 9 – 6 = 3$

Now, we have to find the sum of first 25 terms.

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\Rightarrow \ S_{25} = \frac{25}{2}[2 \times (3) + (25 - 1)(6)]$$

$$\Rightarrow \ S_{25} = \frac{25}{2}[6 + 24 \times (6)]$$

$\Rightarrow S_{25} = 25 [3 + 12 \times (6)]$

$\Rightarrow S_{25} = 25 [75]$

$\Rightarrow S_{25} = 1875$

34 A. Question

If the sum of 8 terms of an A.P. is 64 and the sum of 19 terms is 361, find the sum of n terms.

Answer

Given: $S_8 = 64$ and $S_{19} = 361$

We know that,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow S_8 = \frac{8}{2}[2a + (8-1)d]$$

$\Rightarrow 64 = 4 [2a + 7d]$

$\Rightarrow 16 = 2a + 7d$ ...(i)

Now,

$$\Rightarrow S_{19} = \frac{19}{2}[2a + (19-1)d]$$

$$\Rightarrow 361 = \frac{19}{2}[2a + 18d]$$

$\Rightarrow 38 = 2a + 18d$ ...(ii)

Solving linear equations (i) and (ii), we get

$2a + 7d - 2a - 18d = 16 - 38$

$\Rightarrow -11d = -22$

$\Rightarrow d = 2$ ...(iii)

Putting the value of d in eq. (i), we get

$2a + 7(2) = 16$

$\Rightarrow 2a = 16 - 14$

$\Rightarrow 2a = 2$ ...(iv)

Now, we have to find the S_n

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$\Rightarrow S_n = \frac{n}{2}[2 + (n - 1)2]$ [from (iii) and (iv)]

$\Rightarrow S_n = n\,[1 + n - 1]$

$\Rightarrow S_n = n^2$

34 B. Question

The first and the last terms of an A.P. are 17 and 350 respectively. If the common difference is 9, how many terms are there in the A.P. and what is their sum?

Answer

Given: First term, a = 17

Last term, l = 350

common difference, d = 9

We know that,

$l = a + (n - 1)d$

$\Rightarrow 350 = 17 + (n - 1)9$

$\Rightarrow 333 = (n - 1)9$

$\Rightarrow 37 = n - 1$

$\Rightarrow n = 38$

So, there are 38 terms in the AP

Now, we have to find the sum of this AP

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\Rightarrow S_{38} = \frac{38}{2}[2 \times 17 + (38 - 1)9]$$

$\Rightarrow S_{38} = 19\,[34 + 37 \times 9]$

$\Rightarrow S_{38} = 19\,[34 + 333]$

$\Rightarrow S_{38} = 19 \times 367$

$\Rightarrow S_{38} = 6973$

Hence, the sum of 38 terms is 6973.

35. Question

If a, b, c be the 1st, 3rd and nth terms respectively of an A.P., there prove that the sum to n terms is $\frac{c+a}{2} + \frac{c^2 - a^2}{b - a}$

Answer

Given: $a_1 = a$

$a_3 = a + 2d = b$

$\Rightarrow 2d = b - a$

$$\Rightarrow d = \frac{b-a}{2}$$

and $a_n = a + (n - 1)d$

$$c = a + (n-1)\left(\frac{b-a}{2}\right)$$

$$\Rightarrow c - a = (n-1)\left(\frac{b-a}{2}\right)$$

$$\Rightarrow \frac{2(c-a)}{(b-a)} = (n-1)$$

$$\Rightarrow \frac{2(c-a)}{(b-a)} + 1 = n$$

$$\Rightarrow \frac{2c - 2a + b - a}{b - a} = n$$

$$\Rightarrow \frac{b + 2c - 3a}{b - a} = n$$

We know that,

$$S_n = \frac{n}{2}[a + l]$$

$$\Rightarrow S_n = \frac{\frac{b+2c-3a}{b-a}}{2}[a+c]$$

$$\Rightarrow S_n = \frac{(b+2c-3a)(a+c)}{2(b-a)}$$

$$\Rightarrow S_n = \frac{(c+a)[b-a+2(c-a)]}{2(b-a)}$$

$$\Rightarrow S_n = \frac{c+a}{2} + \frac{(c+a)(c-a)}{b-a}$$

$$\Rightarrow S_n = \frac{c+a}{2} + \frac{c^2-a^2}{b-a}$$

36. Question

If the mth term of an A.P. is $\frac{1}{n}$ and the nth term is $\frac{1}{m}$, then prove that the sum to mn terms is $\frac{mn+1}{2}$, where in $m \neq n$.

Answer

Given: $a_m = \frac{1}{n}$

Now, $a_m = a + (m - 1)d$

$$\Rightarrow \frac{1}{n} = a + (m-1)d$$

$\Rightarrow$ an + n(m – 1)d = 1

$\Rightarrow$ an + mnd – nd = 1 ...(i)

and $a_n = \frac{1}{m}$

$$\Rightarrow a + (n-1)d = \frac{1}{m}$$

$\Rightarrow$ am + mnd – md = 1 ...(ii)

From eq. (i) and (ii), we get

an + mnd – nd = am + mnd – md

$\Rightarrow$ a(n – m) –d (n – m) = 0

$\Rightarrow$ a = d

Now, putting the value of a in eq. (i), we get

dn + mnd – nd = 1

$\Rightarrow$ mnd = 1

$$\Rightarrow \ d = \frac{1}{mn}$$

Hence, $a = \frac{1}{mn}$

Sum of mn terms of AP is

$$S_{mn} = \frac{mn}{2}[2a + (mn - 1)d]$$

$$\Rightarrow \ S_{mn} = \frac{mn}{2}\left[2 \times \frac{1}{mn} + (mn - 1)\left(\frac{1}{mn}\right)\right]$$

$$\Rightarrow \ S_{mn} = \frac{mn}{2}\left[\frac{2}{mn} + 1 - \frac{1}{mn}\right]$$

$$\Rightarrow \ S_{mn} = \frac{mn}{2}\left[\frac{1}{mn} + 1\right]$$

$$\Rightarrow \ S_{mn} = \frac{1}{2}[mn + 1]$$

Hence Proved

37. Question

If the 12th term of an A.P. is - 13 and the sum of the first four terms is 24, what is the sum of the first 10 terms?

Answer

Given: a_{12} = -13

$\Rightarrow$ a + 11d = -13

$\Rightarrow$ a = -13 – 11d ...(i)

and S_4 = 24

$\frac{4}{2}[2(-13 - 11d) + (4 - 1)d] = 24$ [from(i)]

$\Rightarrow$ 2[-26 -22d + 3d] = 24

$\Rightarrow$ -26 – 19d = 12

$\Rightarrow$ -19d = 12 + 26

$\Rightarrow$ -19d = 38

$\Rightarrow$ d = -2

Putting the value of d in eq. (i), we get

a = -13 – 11(-2) = -13 + 22 = 9

So, a = 9 , d = -2 and n = 10

Now, we have to find the S_{10}

$$S_{10} = \frac{10}{2}[2a + (10 - 1)(-2)]$$

$\Rightarrow S_{10} = 5[2\times9 + 9(-2)]$

$\Rightarrow S_{10} = 5[18 - 18]$

$\Rightarrow S_{10} = 0$

Hence, the sum of first 10 terms is 0

38. Question

If the number of terms of an A.P. be 2n + 3, then find the ratio of sum of the odd terms to the sum of even terms.

Answer

Given: Total number of terms = 2n + 3

Let the first term = a

and the common difference = d

Then, $a_k = a + (k - 1)d$ ...(i)

Let S_1 and S_2 denote the sum of all odd terms and the sum of all even terms respectively.

Then,

$S_1 = a_1 + a_3 + a_5 \ldots + a_{2n+3}$

$$= \frac{n+2}{2}\{a_1 + a_{2n+3}\}$$

$= \frac{n+2}{2}\{a + a + (2n + 3 - 1)d\}$ [using (i)]

$= \frac{n+2}{2}\{2a + 2nd + 2d\}$

= (n + 2)(a + nd + d) ...(ii)

And, $S_2 = a_2 + a_4 + a_6 \ldots + a_{2n+2}$

$= \frac{n+1}{2}\{a_2 + a_{2n+2}\}$

$= \frac{n+1}{2}[(a + d) + \{a + (2n + 2 - 1)d\}]$ [using (i)]

$= \frac{n+1}{2}\{2a + 2nd + 2d\}$

= (n+1)(a + nd + d) ...(iii)

$\therefore \frac{S_1}{S_2} = \frac{(n\ +\ 1)(a\ +\ nd)}{(n\,)(a\ +\ nd)} = \frac{n+2}{n+1}$

39. Question

If the sum of first m terms of an A.P. is the same as the sum of its first n terms, show that the sum of its first (m + n) terms is zero.

Answer

Let the first term be a and common difference of the given AP is d.

Given: $S_m = S_n$

$\Rightarrow \frac{m}{2}[2a + (m - 1)d] = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow 2am + md(m - 1) = 2an + nd(n - 1)$

$\Rightarrow 2am - 2an + m^2d - md - n^2d + nd = 0$

$\Rightarrow 2a(m - n) + d[(m^2 - n^2) - (m - n)] = 0$

$\Rightarrow 2a(m - n) + d[(m - n)(m + n) - (m - n)] = 0$

$\Rightarrow (m - n)[2a + \{(m + n) - 1\}d] = 0$

$\Rightarrow 2a + (m + n - 1)d = 0$ [$\because m - n \neq 0$]...(i)

Now,

$S_{m+n} = \frac{m+n}{2}[2a + \{(m + n) - 1\}d] = 0$

$\Rightarrow S_{m+n} = \frac{m+n}{2} \times 0$ [using (i)]

$\Rightarrow S_{m+n} = 0$

Hence Proved

40. Question

In an A.P. the first term is 2, and the sum of the first five terms is one-fourth of the next five terms. Show that its 20th term is — 112.

Answer

Given: first term, a = 2

And

Sum of first five terms $= \frac{1}{4}$(sum of next 5 terms)

Sum of next 5 terms $= \frac{1}{4}$(Sum of 6^{th} to 10^{th} terms)

$\Rightarrow$ Sum of first 5 terms $= \frac{1}{4}$(Sum of first 10 terms − sum of first five terms)

$\Rightarrow S_5 = \frac{1}{4}(S_{10} - S_5)$

$\Rightarrow 4S_5 = S_{10} - S_5$

$\Rightarrow 5S_5 = S_{10}$

$\Rightarrow 5 \times \frac{5}{2}[2 \times 2 + (5 - 1)d] = \frac{10}{2}[2 \times 2 + (10 - 1)d]$

$\Rightarrow \frac{25}{2}[4 + 4d] = 5[4 + 9d]$

$\Rightarrow 20 + 20d = 8 + 18d$

$\Rightarrow 20d - 18d = 8 - 20$

$\Rightarrow 2d = -12$

$\Rightarrow d = -6$

Thus, a = 2 and d = -6

$\therefore a_{20} = a + (n - 1)d$

$\Rightarrow a_{20} = 2 + (20 - 1)(-6)$

$\Rightarrow a_{20} = 2 + (19)(-6)$

$\Rightarrow a_{20} = 2 - 114$

$\Rightarrow a_{20} = -112$

Hence Proved

41. Question

If d be the common difference of an A.P. and S_n be the sum of its n terms, then prove that $d = S_n - 2S_{n-1} + S_{n-2}$

Answer

Given: S_n be the sum of n terms and d be the common difference.

To Prove: $d = S_n - 2S_{n-1} + S_{n-2}$

Taking RHS

$S_n - 2S_{n-1} + S_{n-2}$

$$= \frac{n}{2}[2a + (n-1)d] - 2 \times \frac{n-1}{2}[2a + (n-1-1)d] + \frac{n-2}{2}[2a + (n-2-1)d]$$

$$= \frac{n}{2}[2a + (n-1)d] - [\frac{2(n-1)}{2}[2a + (n-2)d]] + \frac{n-2}{2}[2a + (n-3)d]$$

$$= \frac{2an + n(n-1)d - 4a(n-1) - 2(n-1)(n-2)d + 2a(n-2) + (n-2)(n-3)d}{2}$$

$$= \frac{1}{2}[2an + n^2d - nd - 4an + 4a - 2n^2d + 4nd + 2nd - 4d + 2an - 4a + n^2d - 3nd - 2nd + 6d]$$

$$= \frac{1}{2}[2d]$$

= d

=LHS

Hence Proved

42. Question

The sum of the first 7 terms of an A.P. is 10, and that of the next 7 terms is 17. Find the progression.

Answer

Given: Sum of first 7 terms, $S_7 = 10$

and Sum of the next 7 terms = 17

⇒ Sum of 8^{th} to 14^{th} terms = 17

⇒ Sum of first 14 terms – Sum of first 7 terms = 17

⇒ $S_{14} - S_7 = 17$

⇒ $S_{14} - 10 = 17$

⇒ $S_{14} = 27$

Sum of 7 terms, $S_7 = \frac{7}{2}[2a + (7-1)d]$

$$\Rightarrow \ 10 = \frac{7}{2}[2a + (7-1)d$$

⇒ 20 = 7[2a + 6d]

⇒ 20 = 14a + 42d ...(i)

Sum of 14 terms, $S_{14} = \frac{14}{2}[2a + (14-1)d]$

⇒ 27 = 7[2a + 13d]

⇒ 27 = 14a + 91d ...(ii)

Solving the linear equations (i) and (ii), we get

14a + 42d – 14a – 91d = 20 – 27

⇒ -49d = -7

$$\Rightarrow \ d = \frac{1}{7}$$

Putting the value of d in eq. (i), we get

20 = 14a + 42d

$$\Rightarrow \ 20 = 14a + 42 \times \frac{1}{7}$$

$\Rightarrow 20 = 14a + 6$

$\Rightarrow 20 - 6 = 14a$

$\Rightarrow 14 = 14a$

$\Rightarrow a = 1$

Thus, a = 1 and $d = \frac{1}{7}$

So, AP is

$a_1 = 1$

$a_2 = a + d = 1 + \frac{1}{7} = 1\frac{1}{7}$

$a_3 = a + 2d = 1 + 2 \times \frac{1}{7} = 1\frac{2}{7}$

Hence, AP is $1, 1\frac{1}{7}, 1\frac{2}{7}, \ldots$

43. Question

If the pth term of an A.P. is x and qth term is y, show that the sum of (p + q) terms is $\frac{p+q}{2}\left[x + y + \left(\frac{x-y}{p-q}\right)\right]$

Answer

Given: $a_p = x$ and $a_q = y$

We know that,

$a_n = a + (n - 1)d$

$a_p = a + (p - 1)d$

$\Rightarrow x = a + (p - 1)d$ ...(i)

Now,

$a_q = a + (q - 1)d$

$\Rightarrow y = a + (q - 1)d$ ...(ii)

From eq. (i) and (ii), we get

$x - (p - 1)d = y - (q - 1)d$

$\Rightarrow x - y = (p - 1)d - (q - 1)d$

$\Rightarrow$ x – y = d [p – 1 – q + 1]

$\Rightarrow$ x – y = d[p – q]

$\Rightarrow\ d = \frac{x-y}{p-q}$ ...(iii)

Adding, Eq (i) and (ii), we get

x + y = 2a + (p – 1) + (q – 1)d

$\Rightarrow$ x + y = 2a + d[p + q – 1 – 1]

$\Rightarrow$ x + y = 2a + d (p + q – 1) –d

$\Rightarrow$ x + y + d = 2a + (p + q – 1)d ...(iv)

We know that,

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\Rightarrow\ S_{p+q} = \frac{p+q}{2}[2a + (p + q - 1)d]$$

$\Rightarrow\ S_{p+q} = \frac{p+q}{2}[x + y + d]$ [using (iv)]

$\Rightarrow\ S_{p+q} = \frac{p+q}{2}[x + y + \frac{x-y}{p-q}]$ [using (iii)]

Hence Proved

44 A. Question

The sum of 17 terms of two series in A.P. are in the ratio (3n + 8) : (7n + 15). Find the ratio of their 12th terms.

Answer

There are two AP with different first term and common difference.

For the First AP

Let first term be a

Common difference = d

Sum of n terms = $S_n = \frac{n}{2}[2a + (n - 1)d]$

and n^{th} term = a_n = a + (n – 1)d

For the second AP

Let first term be A

Common difference = D

Sum of n terms = $S_n = \frac{n}{2}[2A + (n - 1)D]$

and n^{th} term = A_n = A + (n – 1)D

It is given that

$$\frac{\text{Sum of n terms of first A. P}}{\text{Sum of n terms os second A. P}} = \frac{3n + 8}{7n + 15}$$

$$\Rightarrow \frac{\frac{n}{2}[2a + (n - 1)d}{\frac{n}{2}[2A + (n - 1)D} = \frac{3n + 8}{7n + 15}$$

$$\Rightarrow \frac{n[a + \left(\frac{n - 1}{2}\right)d]}{n[A + \left(\frac{n - 1}{2}\right)D]} = \frac{3n + 8}{7n + 15}$$

$$\Rightarrow \frac{a+\left(\frac{n-1}{2}\right)d}{A+\left(\frac{n-1}{2}\right)D} = \frac{3n+8}{7n+15} \quad ...(i)$$

Now, we need to find ratio of their 12^{th} term

$$\text{i. e.} \frac{12^{th}\text{term of first AP}}{12^{th}\text{ term of second AP}}$$

$$= \frac{a_{12}\text{ of first AP}}{A_{12}\text{ of second AP}}$$

$$= \frac{a + (12 - 1)d}{A + (12 - 1)D}$$

$$= \frac{a + 11d}{A + 11D}$$

Hence, $\frac{n-1}{2} = 11$

n – 1 = 11 × 2

⇒ n = 22 + 1

⇒ n = 23

Putting n = 23 in eq. (i), we get

$$\frac{a+\left(\frac{23-1}{2}\right)d}{A+\left(\frac{23-1}{2}\right)D}=\frac{3(23)+8}{7(23)+15}$$

$$\Rightarrow \frac{a+11d}{A+11D}=\frac{69+8}{161+15}$$

$$\Rightarrow \frac{12^{th}\text{term of first AP}}{12^{th}\text{term of second AP}}=\frac{77}{176}$$

$$\Rightarrow \frac{12^{th}\text{term of first AP}}{12^{th}\text{term of second AP}}=\frac{7}{16}$$

Hence the ratio of 12^{th} term of 1^{st} AP and 12^{th} term if 2^{nd} AP is 7:16

44 B. Question

The sum of 11 terms of two A.P.'s are in the ratio (5n + 4) : (9n + 6), find the ratio of their 18th terms.

Answer

There are two AP with different first term and common difference.

For the First AP

Let first term be a

Common difference = d

Sum of n terms = $S_n = \frac{n}{2}[2a+(n-1)d]$

and n^{th} term = $a_n = a + (n - 1)d$

For the second AP

Let first term be A

Common difference = D

Sum of n terms = $S_n = \frac{n}{2}[2A+(n-1)D]$

and n^{th} term = $A_n = A + (n - 1)D$

It is given that

$$\frac{\text{Sum of n terms of first A. P}}{\text{Sum of n terms os second A. P}}=\frac{5n+4}{9n+6}$$

$$\Rightarrow \frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}[2A+(n-1)D]} = \frac{5n+4}{9n+6}$$

$$\Rightarrow \frac{n\left[a+\left(\frac{n-1}{2}\right)d\right]}{n\left[A+\left(\frac{n-1}{2}\right)D\right]} = \frac{5n+4}{9n+6}$$

$$\Rightarrow \frac{a+\left(\frac{n-1}{2}\right)d}{A+\left(\frac{n-1}{2}\right)D} = \frac{5n+4}{9n+6} \ldots(i)$$

Now, we need to find ratio of their 18^{th} term

$$\text{i. e.} \frac{18^{th}\text{term of first AP}}{18^{th}\text{ term of second AP}}$$

$$= \frac{a_{18}\text{ of first AP}}{A_{18}\text{ of second AP}}$$

$$= \frac{a+(18-1)d}{A+(18-1)D}$$

$$= \frac{a+17d}{A+17D}$$

Hence, $\frac{n-1}{2} = 17$

n – 1 = 17 × 2

⇒ n = 34 + 1

⇒ n = 35

Putting n = 35 in eq. (i), we get

$$\frac{a+\left(\frac{35-1}{2}\right)d}{A+\left(\frac{35-1}{2}\right)D} = \frac{5(35)+4}{9(35)+6}$$

$$\Rightarrow \frac{a+17d}{A+17D} = \frac{175+4}{315+6}$$

$$\Rightarrow \frac{18^{th}\text{term of first AP}}{18^{th}\text{term of second AP}} = \frac{179}{321}$$

Hence the ratio of 18^{th} term of 1^{st} AP and 18^{th} term if 2^{nd} AP is 179:321

45. Question

In an A.P. S_n denotes the sum to first n terms, if $S_n = n^2p$ and $S_m = m^2p$ ($m \neq n$) prove that $S_p = p^3$.

Answer

Given: $S_n = n^2p$ and $S_m = m^2p$

To Prove: $S_p = p^3$

We know that,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow n^2p = \frac{n}{2}[2a + (n-1)d]$$

$\Rightarrow 2np = [2a + (n - 1)d]$

$\Rightarrow 2np - (n - 1)d = 2a$ ...(i)

and $S_m = \frac{m}{2}[2a + (m-1)d]$

$$\Rightarrow m^2p = \frac{m}{2}[2a + (m-1)d]$$

$\Rightarrow 2mp = 2a + (m - 1)d$

$\Rightarrow 2mp - (m - 1)d = 2a$ ...(ii)

From eq. (i) and (ii), we get

$\Rightarrow 2np - (n - 1)d = 2mp - (m - 1)d$

$\Rightarrow 2np - nd + d = 2mp - md + d$

$\Rightarrow 2np - nd = 2mp - md$

$\Rightarrow md - nd = 2mp - 2np$

$\Rightarrow d(m - n) = 2p(m - n)$

$\Rightarrow d = 2p$ ...(iii)

Putting the value of d in eq. (i), we get

$\Rightarrow 2np - (n - 1)(2p) = 2a$

$\Rightarrow 2pn - 2pn + 2p = 2a$

⇒ 2p = 2a ...(iv)

Now, we have to find the S_p

$S_p = \frac{p}{2}[2p + (p-1)2p]$ [from (iii) & (iv)]

$$\Rightarrow S_p = \frac{p}{2}[2p + 2p^2 - 2p]$$

$$\Rightarrow S_p = \frac{p}{2}[2p^2]$$

⇒ $S_p = p^3$

Hence Proved

46. Question

The income of a person is Rs. 300000 in the first year and he receives an increase of Rs. 10000 to his income per year for the next 19 years. Find the total amount he received in 20 years.

Answer

The income of a person in 1st year = Rs 300000

The income of a person in 2nd year = Rs 300000 + 10000

= Rs 310000

The income of a person in 3rd year = Rs 310000 + 10000

= Rs 320000

and so,on

Therefore, the AP is

300000, 310000, 320000,...

Here a = 300000, d = 310000 – 300000 = 10000

and n = 20

We know that,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow S_{20} = \frac{20}{2}[2(300000) + (20-1)(10000)]$$

$\Rightarrow S_{20} = 10\ [600000 + 190000]$

$\Rightarrow S_{20} = 10[790000]$

$\Rightarrow S_{20} = 7900000$

Hence, the total amount he received in 20 years is Rs 7900000.

47. Question

A man starts repaying a loan as first installment of Rs. 100. If he increases the installments by Rs. 5 every month, what amount he will pay in 30 installments?

Answer

The 1^{st} installment of the loan = Rs. 100

the 2^{nd} installment of the loan = Rs 100 + 5 = Rs 105

The 3^{rd} installment of the loan = Rs 105 + 5 = Rs 110

Therefore, the AP is 100, 105, 110, …

Here, a = 100, d = 105 – 100 = 5 and n = 30

We know that,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow\ S_{30} = \frac{30}{2}[2 \times 100 + (30-1)(5)]$$

$\Rightarrow S_{30} = 15\ [200 + 29 \times 5]$

$\Rightarrow S_{30} = 15\ [200+145]$

$\Rightarrow S_{30} = 15\ [345]$

$\Rightarrow S_{30} = 5175$

Hence, the amount he will pay in 30^{th} installments is Rs. 5175

48. Question

The interior angles of a polygon are in A.P., the smallest angle is 75° and the common difference is 10°. Find the number of sides of the polygon.

Answer

Given: The smallest angle is 75°

i.e. a = 75

and common difference = 10°

i.e. d = 10

Therefore, the series is

75, 85, 95, 105, …

and the sum of interior angles of a polygon =(n – 2) 180°

i.e. S_n = 180

We know that,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow\ (n-2)180 = \frac{n}{2}[2 \times (75) + (n-1)(10)]$$

$$\Rightarrow\ (n-2)180 = \frac{n}{2}[150 + 10n - 10]$$

⇒ (n – 2)360 = n [140+10n]

⇒ 360n – 720 = 140n + $10n^2$

⇒ 36n – 72 – 14n – n^2 = 0

⇒ n^2 – 22n + 72 = 0

⇒ n^2 – 18n – 4n + 72 = 0

⇒ n(n – 18) – 4(n – 18) = 0

⇒ (n – 4)(n – 18) = 0

Putting both the factor equal to 0, we get

n – 4 = 0 or n – 18 = 0

⇒ n = 4 or n = 18

Hence, the number of sides of a polygon can be 4 or 18.