10. Coordinates Geometry

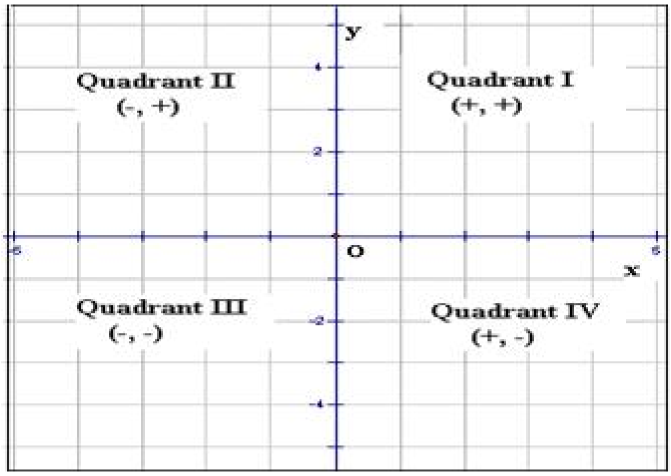
Exercise 10.1

1 A. Question

In which quadrants do the following points lie:

(10, -3)

Answer



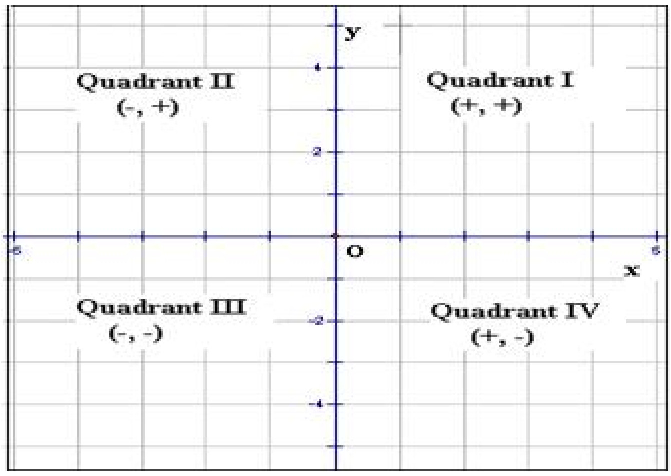
Given coordinate (10,-3) lies in Quadrant IV because as shown in the figure that those coordinate who have (+, -) sign lies in IV quadrants, or can say whose x-axis is "+" and y-axis is "–" lies in IV Quadrant.

1 B. Question

In which quadrants do the following points lie:

(-4, -6)

Answer



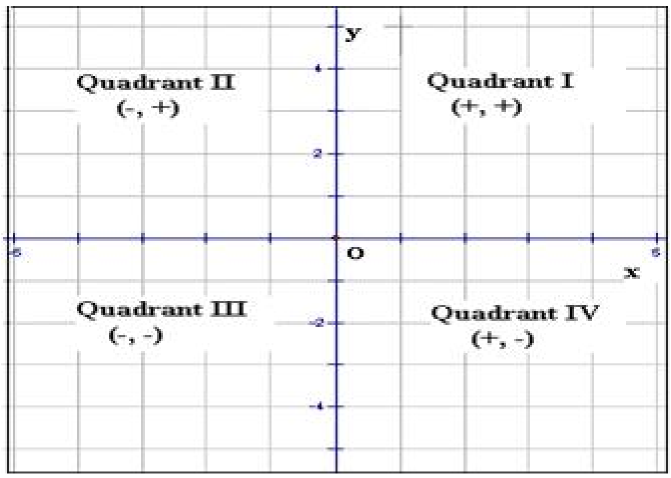
Given coordinate (-4,-6) lies in Quadrant III because as shown in the figure that those points which have a sign like this (-, -) or can say whose both x-axis and y-axis is "–" lies in III quadrants. So (-4,-6) lies in III Quadrant.

1 C. Question

In which quadrants do the following points lie:

(-8, 6)

Answer



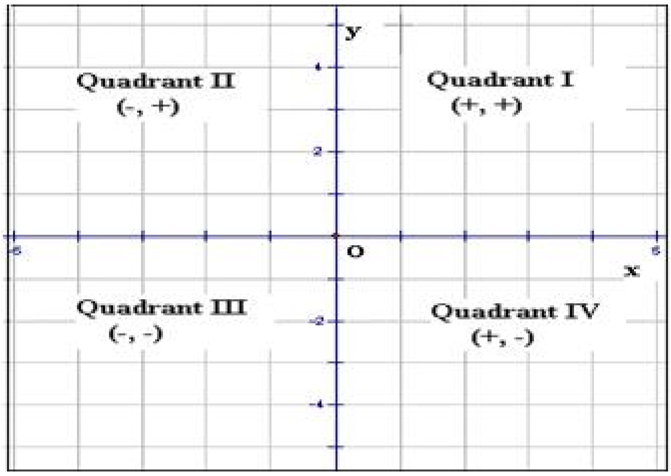
Given coordinate (-8,6) lies in Quadrant II because as shown in the figure that those points which have a sign like this (-, +) lies in II quadrants. So (-8,6) lies in III Quadrant. Here also x-axis point is "-" and y-axis point is "+" so (-8,6) lies in Quadrant II.

1 D. Question

In which quadrants do the following points lie:

$$\left(\frac{3}{2}, 5\right)$$

Answer



Given coordinate $\left(\frac{3}{2}, 5\right)$ lies, in Quadrant I because as shown in the figure

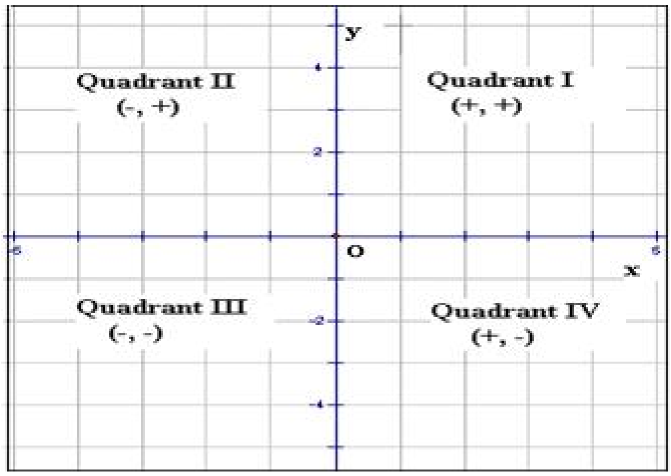
that those coordinate who have (+, +) sign lies in IV quadrants or can say whose x-axis is "+" and y-axis is also "+" lies in I Quadrant.

1 E. Question

In which quadrants do the following points lie:

(3, 0)

Answer



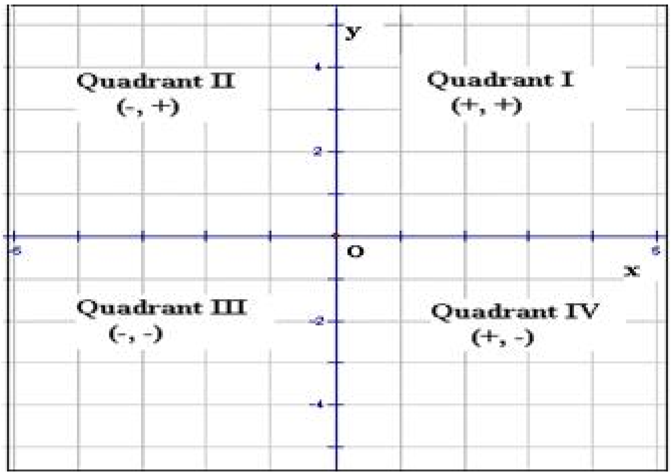
Given coordinate is (3,0). This point lies on the x-axis because its y-axis is on origin. Therefore, it lies on the x-axis between the Quadrant I and Quadrant IV.

1 F. Question

In which quadrants do the following points lie:

(0, -5)

Answer



Given coordinate is (0, -5). This point lies on the y-axis because its x-axis is on origin. Therefore, it lies on the y-axis between the Quadrant III and Quadrant IV.

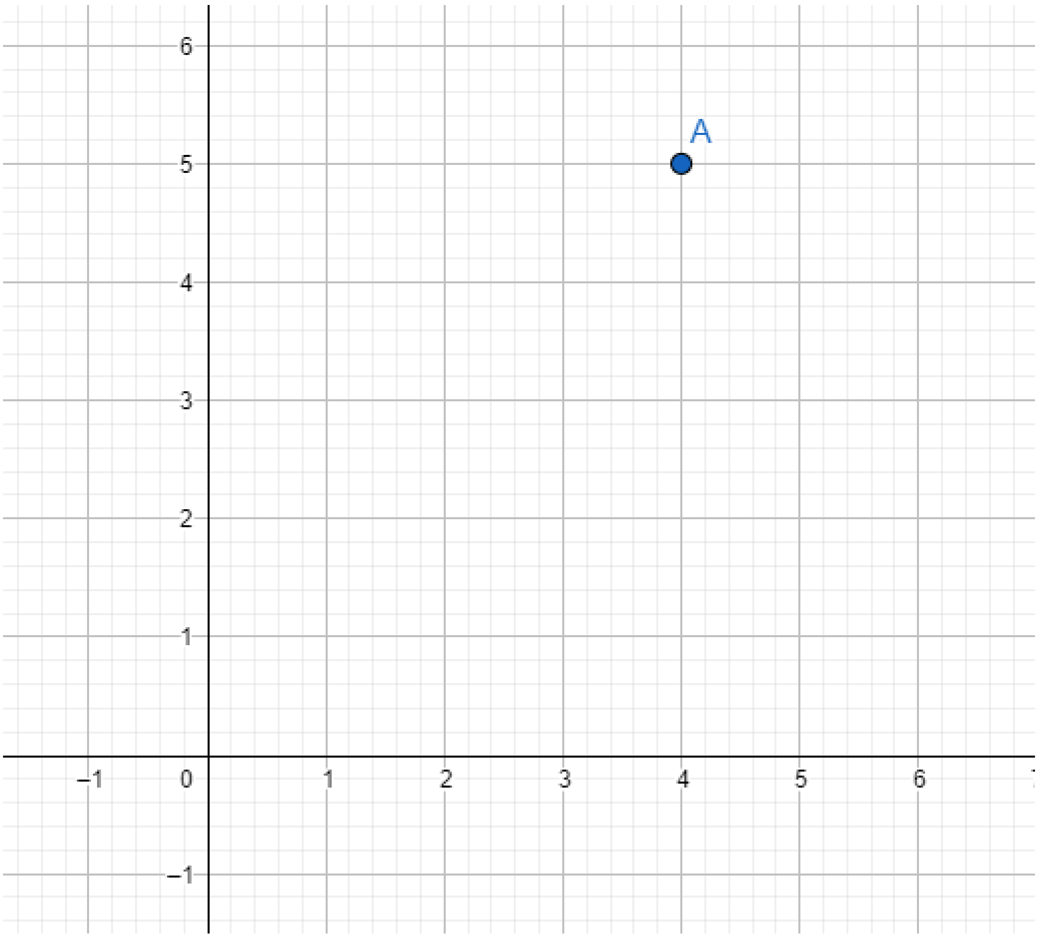
2 A. Question

Plot the following points in a rectangular coordinate system:

(4, 5)

Answer

Here is the graph for coordinate (4,5)

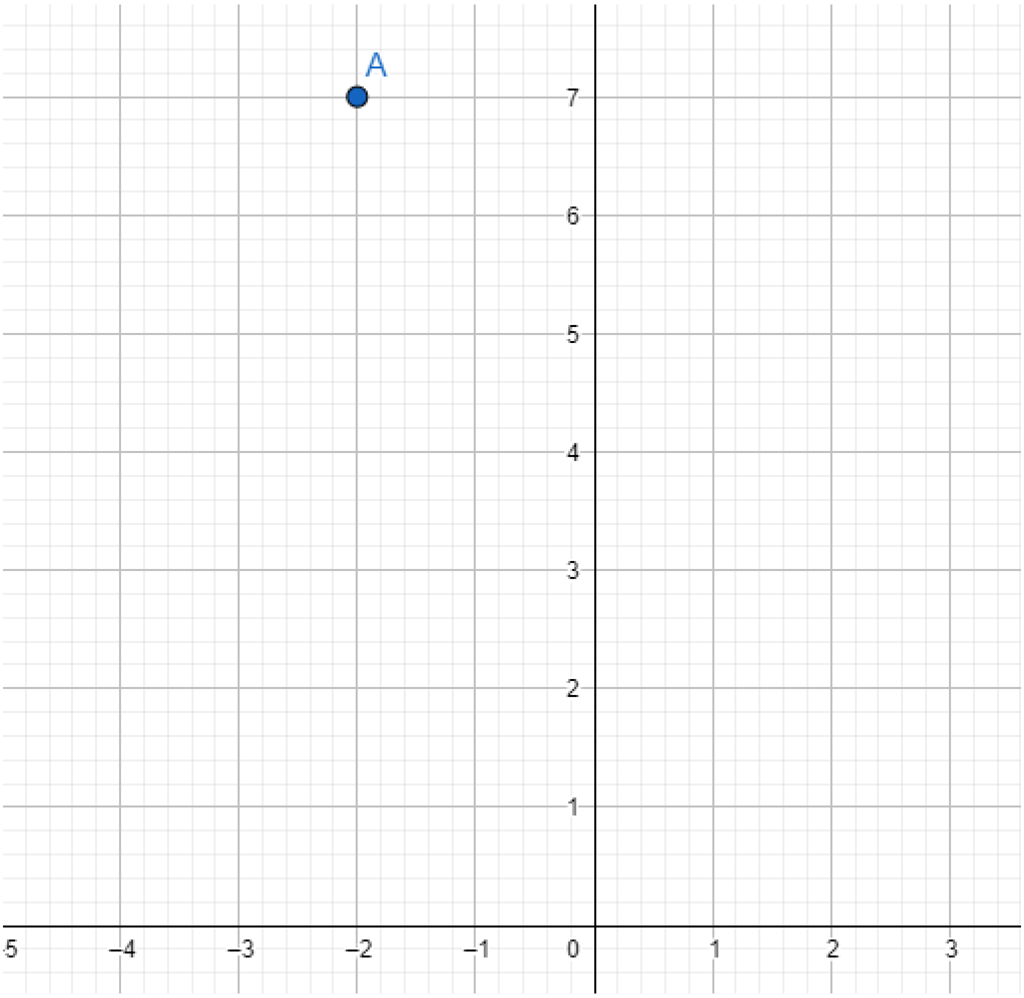


2 B. Question

Plot the following points in a rectangular coordinate system:

(-2,-7)

Answer

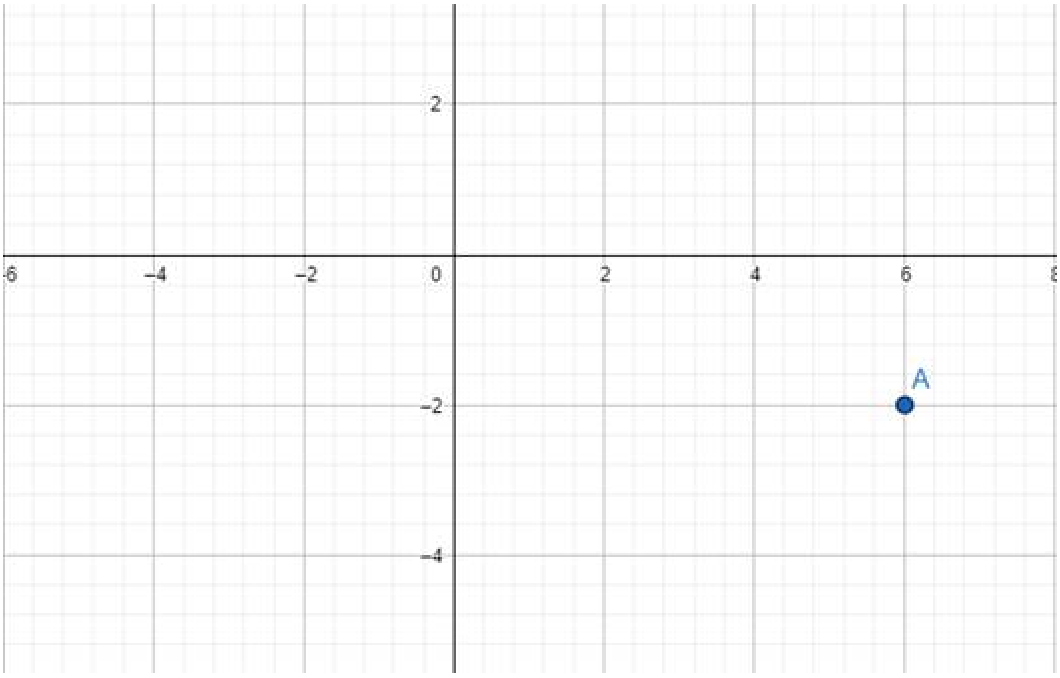


2 C. Question

Plot the following points in a rectangular coordinate system:

(6,-2)

Answer

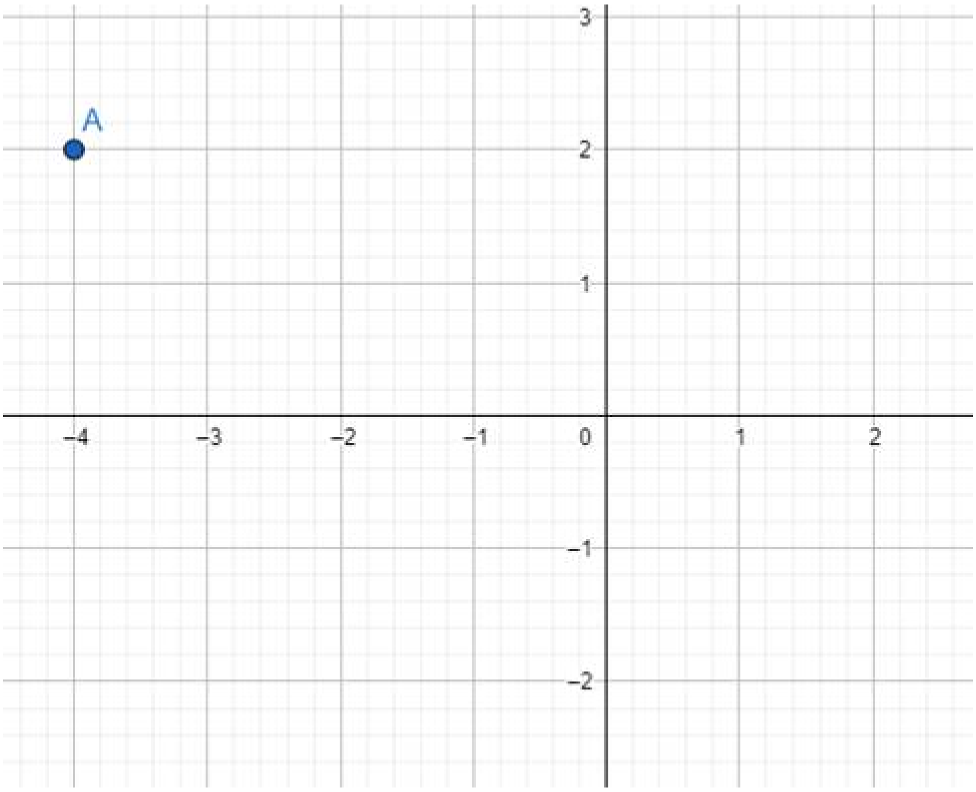


2 D. Question

Plot the following points in a rectangular coordinate system:

(-4, 2)

Answer



2 E. Question

Plot the following points in a rectangular coordinate system:

(4, 0)

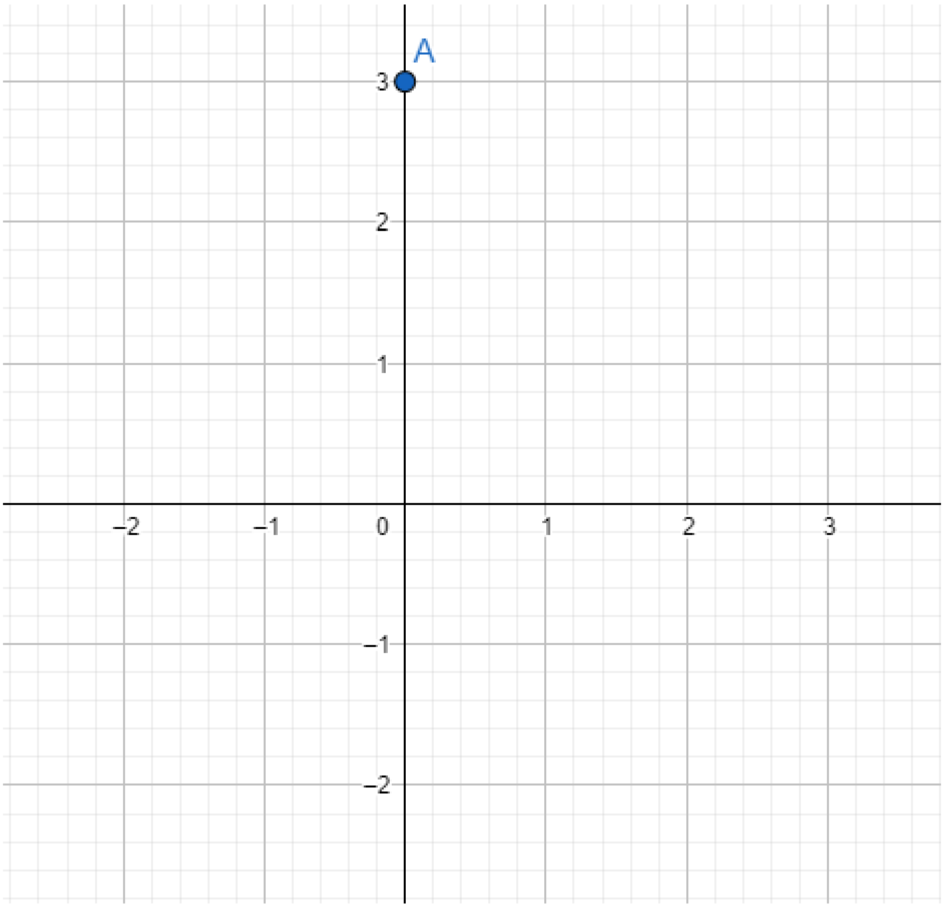
Answer

2 F. Question

Plot the following points in a rectangular coordinate system:

(0, 3)

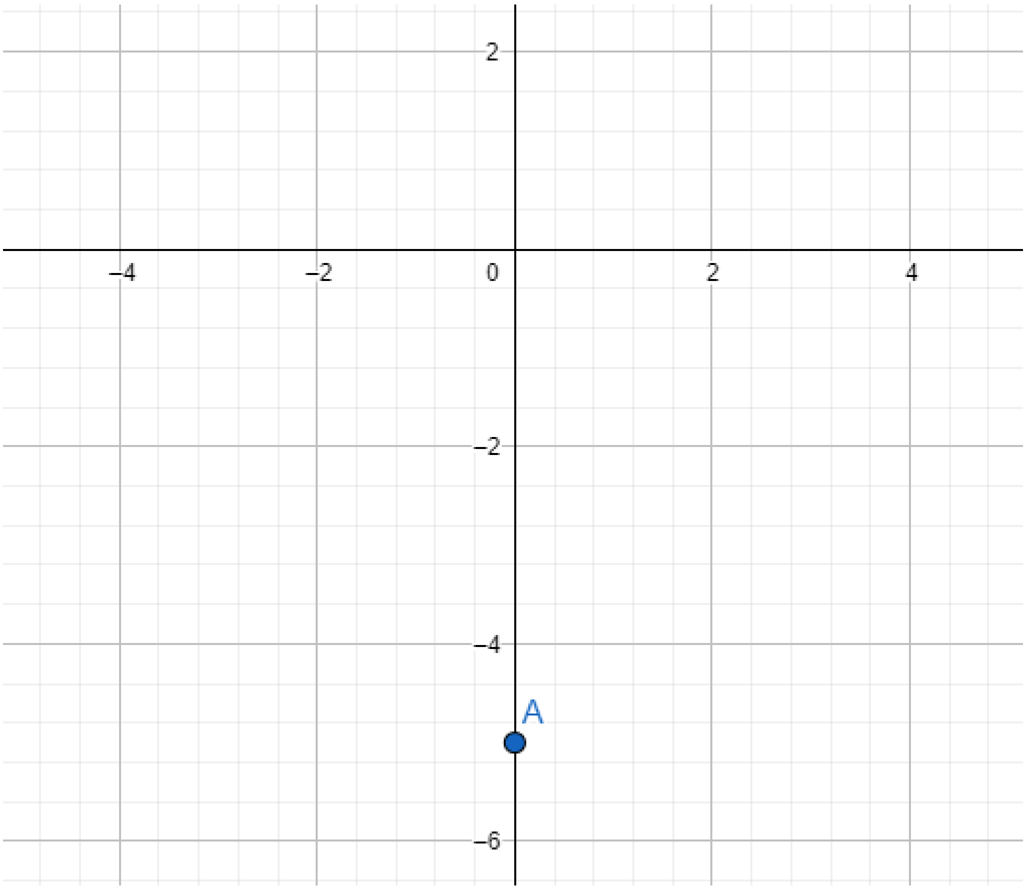
Answer



3. Question

Where does the point having y-coordinate -5 lie?

Answer

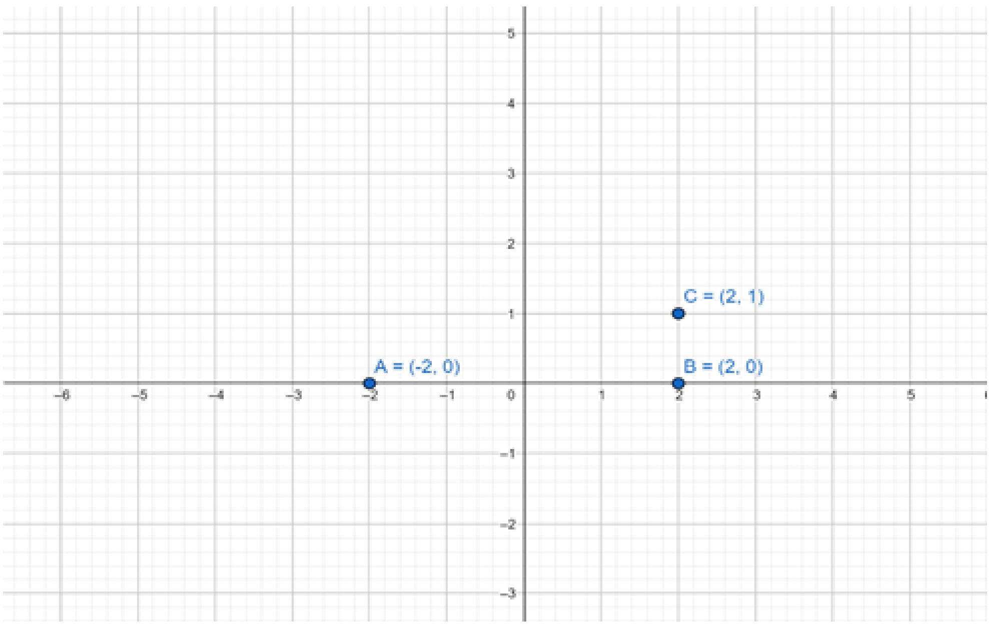


The point having -5 lies on the on the y-axis on the negative side because here x-axis is 0 and when x-axis is 0 then points lies on the y-axis and when y-axis is 0 then point lies on the x-axis.

We can show it on graph with points (0, -5).

4. Question

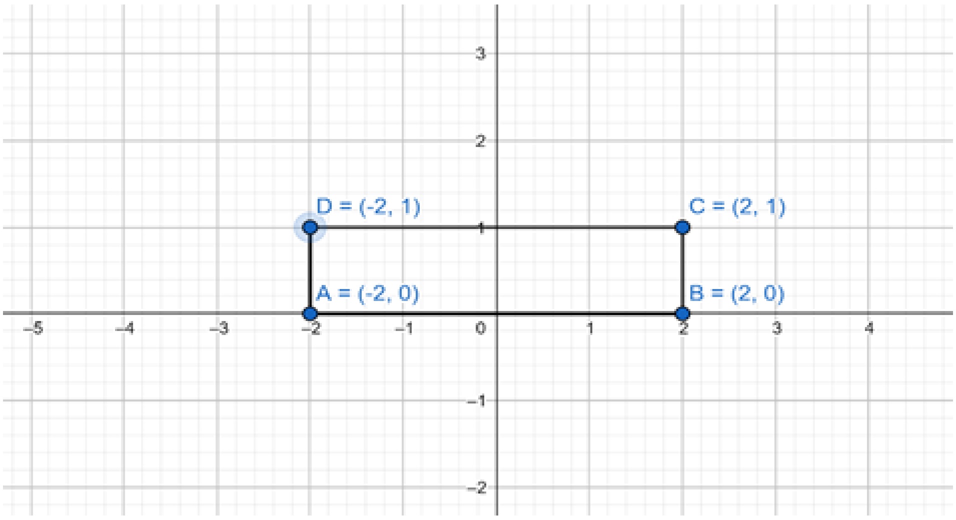
If three vertices of a rectangle are (-2, 0), (2, 0), (2, 1) find the coordinates of the fourth vertex.



Answer

Here we have three vertices of rectangle say A (-2, 0) B (2, 0) and C (2, 1) so when we start graphing it on the graph as shown in the graph below then after plotting all three vertices you will get something like this.

Therefore, after joining all the vertices with a line segment, we will get our fourth vertex because in rectangle opposites sides are parallel and all the angles are right angle so, by joining all the lines according to properties of the rectangle you will get the fourth vertex. So fourth vertex of a rectangle is (-2, 1).



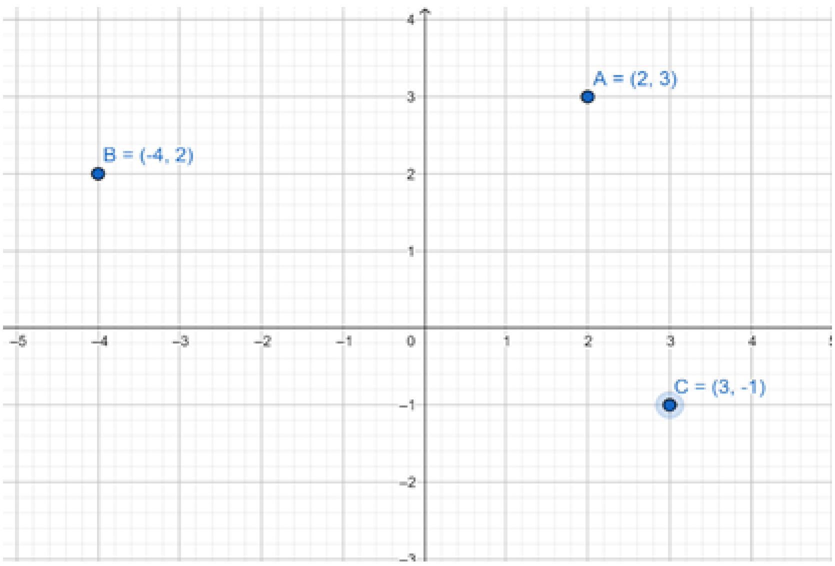
5. Question

Draw the triangle whose vertices are (2, 3), (-4, 2) and (3, -1).

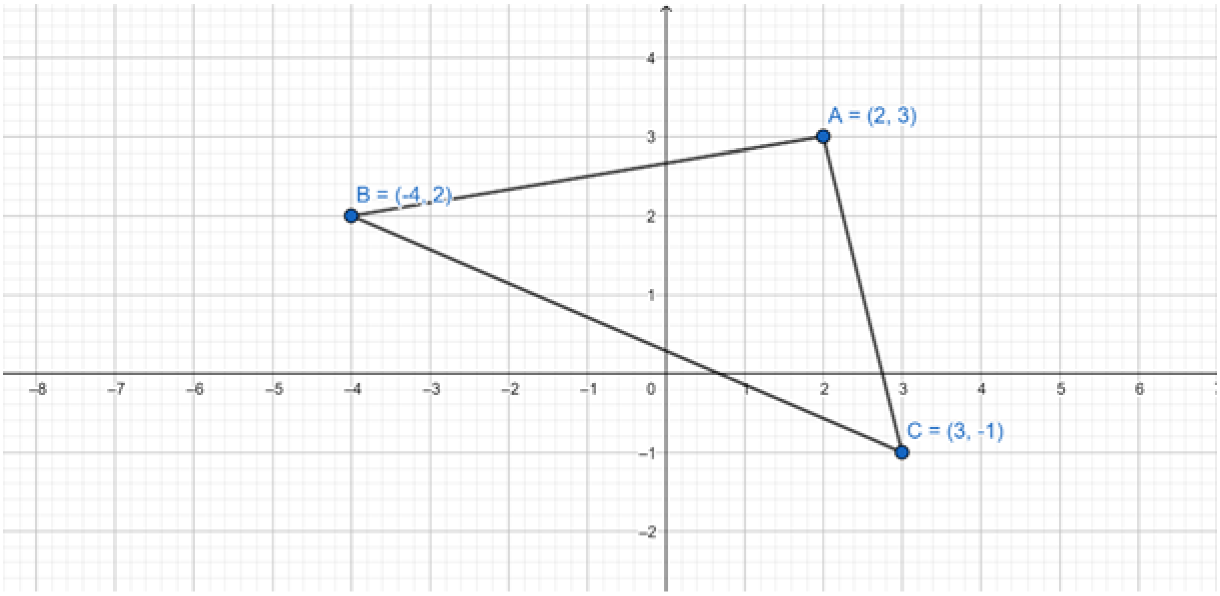
Answer

It is easy to draw a triangle when it's all vertices are given. We have to just locate all the given points on the graph and join them with a line as shown in the graph below.

Step 1. Locate all the vertices on the graph.



Step 2. Joins all the vertices with a line and it will form a triangle.

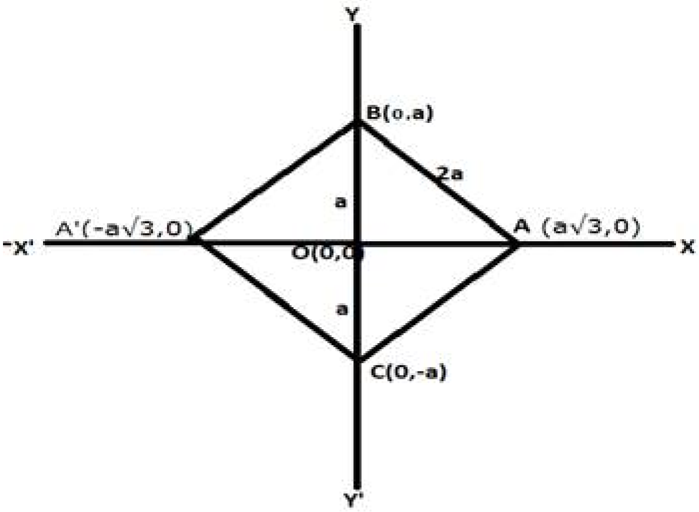


6. Question

The base of an equilateral triangle with side 2a lies along the y-axis such that the mid-point of the base is at the origin. Find

the vertices of the triangle.

Answer



Given that the base of the equilateral triangle is on the y-axis and mid-point of the base is at the origin, so its figure will be like this as shown.

So here, O(0,0) is the midpoint of the base.

An equilateral triangle has all sides equal so if O is the mid-point of the base BC, so B and C are the two vertices of the triangle. Now we have two vertices of the triangle, which is the base of equilateral triangle lying on the y-axis. Now if base in on y-axis then x-axis are as bisector of the base and so our third vertices will be on the x-axis either left or right.

So now in right ΔBOA

Pythagoras Theorem: In a right-angled triangle the square of the biggest side(hypotenuse) equals the sum of the squares of the other two sides(Perpendicular and base).

$BO^2 + OA^2 = AB^2$ { By Pythagoras theorem}

$a^2 + OA^2 = (2a)^2$

$OA^2 = 4a^2 - a^2$

$OA^2 = 3a^2$

$OA = \pm a\sqrt{3}$

So vertices of triangle are $A(\pm a\sqrt{3},0)$ $B(0,a)$and $C(0,-a)$.

7. Question

Let ABCD be a rectangle such that AB = 10 units and BC = 8 units. Taking AB and AD as x and y-axes respectively, find the coordinates of A, B, C and D.

Answer

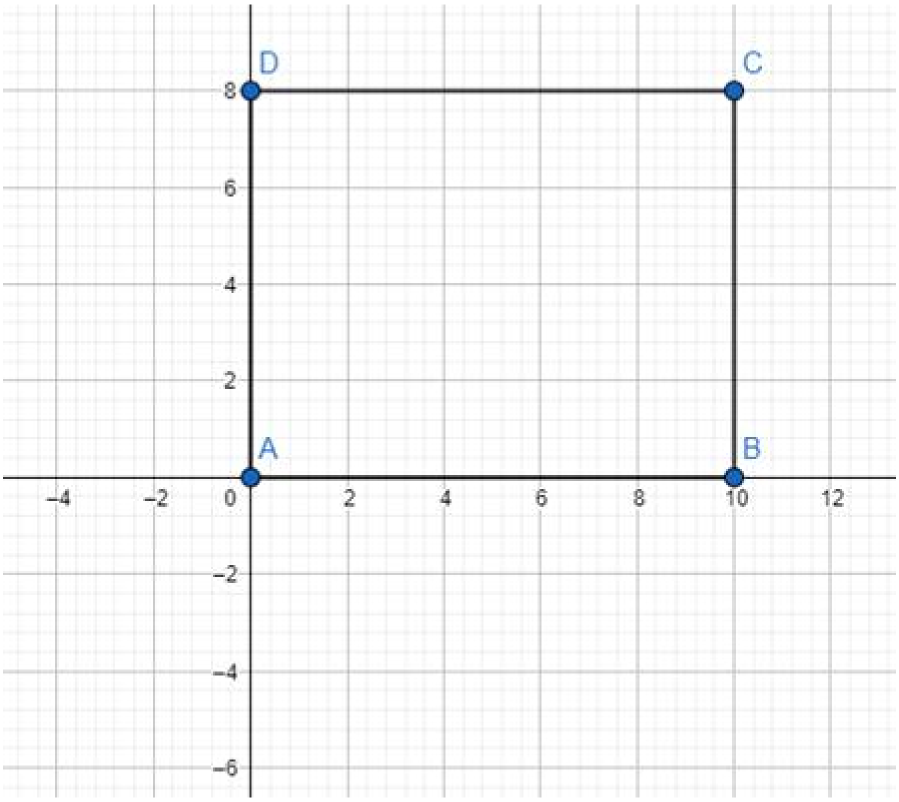
Since AB and AD both have as an endpoint, we can find the coordinate of A by finding the intersection of the two sides.

So the coordinates of A will be where the x-axis and y-axis intersect.

As we know AB lies on the x-axis so the coordinates of B can be found by using the coordinates of A and changing the x-coordinate by the measure of AB.

As we know the opposite side of a rectangle, AD and BC are congruent. Now if we have a measure of BC, we can simply find the y-coordinate of D.

Since AB and AD are the x and y-axes, A is at(0,0), B is at (10,0), C is at (10,8), and D is at (0,8).



8. Question

ABCD is a square having a length of a side 20 units. Taking the centre of the square as the origin and x and y-axes parallel to AB and AD respectively, find the coordinates of A, B, C and D.

Answer

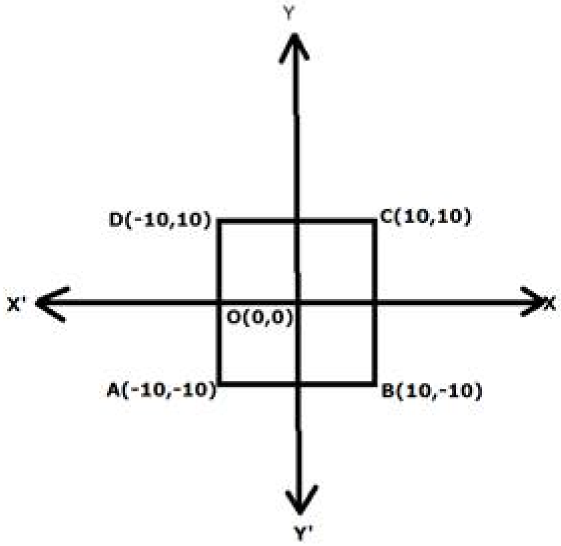
Square ABCD. Center = O(0,0) Origin.

AB = BC = 20 units.

Y-coordinates of AB = $\frac{0-20}{2}$ = -10

Y-coordinates of AD = $\frac{0+20}{2}$ = 10

∴the coordinates are :-



A(-10,-10)

B(10,-10)

C(10,10)

D(-10,10)

Exercise 10.2

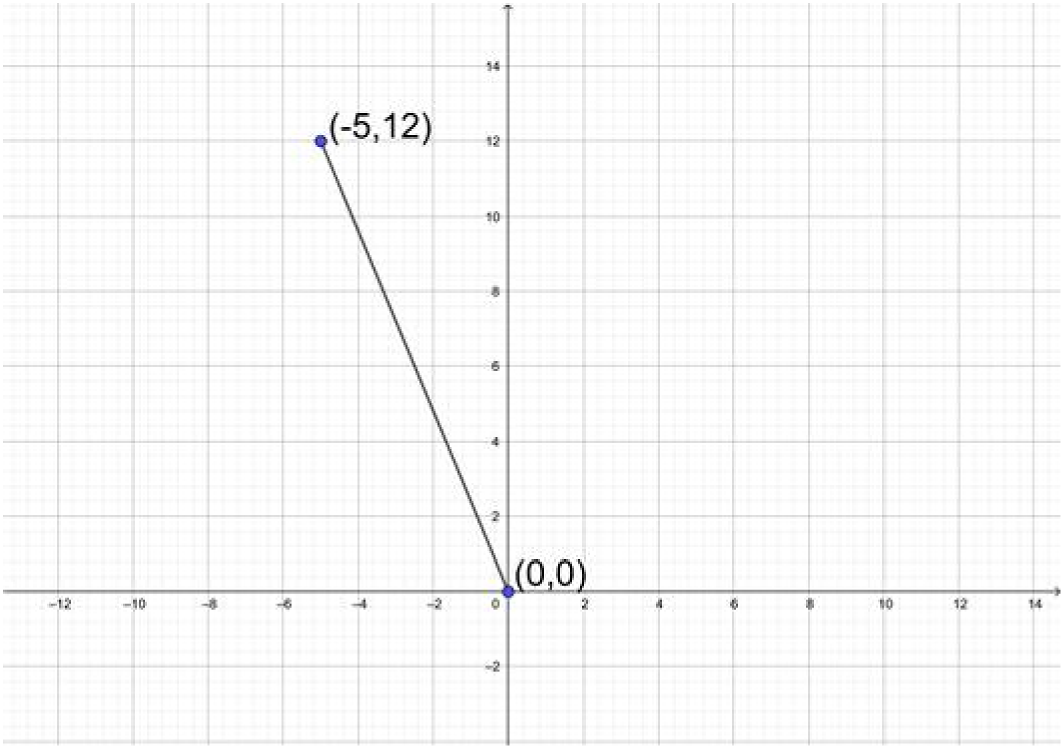
1 A. Question

Find the distance between the following pair of points:

(0, 0), (- 5, 12)

Answer

Given points are (0, 0) and (- 5, 12)



We need to find the distance between these two points.

We know that distance(S) between the points (x_1, y_1) and (x_2, y_2) is

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$\Rightarrow S = \sqrt{(0 - (-5))^2 + (0 - 12)^2}$$

$$\Rightarrow S = \sqrt{(5)^2 + (-12)^2}$$

$$\Rightarrow S = \sqrt{25 + 144}$$

$\Rightarrow S = \sqrt{169}$

$\Rightarrow S = 13$

∴ The distance between the points (0, 0) and (- 5, 12) is 13 units.

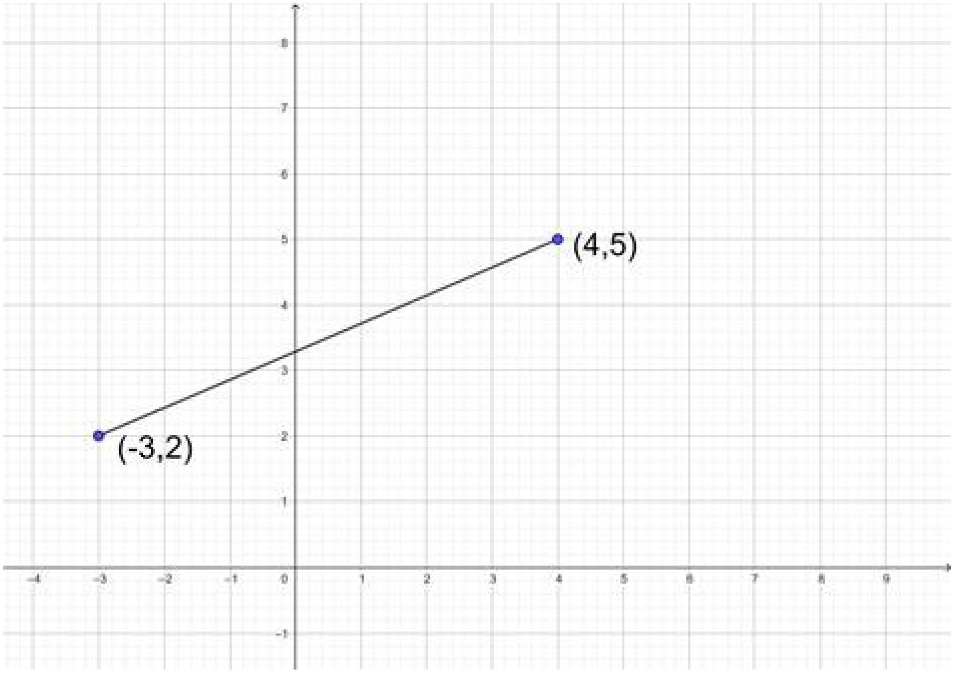
1 B. Question

Find the distance between the following pair of points:

(4, 5), (- 3, 2)

Answer

Given points are (4, 5) and (- 3, 2)



We need to find the distance between these two points.

We know that distance(S) between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$$\Rightarrow S = \sqrt{\left(4 - (-3)\right)^2 + (5 - 2)^2}$$

$$\Rightarrow S = \sqrt{(7)^2 + (3)^2}$$

$$\Rightarrow S = \sqrt{49 + 9}$$

$\Rightarrow S = \sqrt{58}$

∴ The distance between the points (4, 5) and (- 3, 2) is $\sqrt{58}$ units.

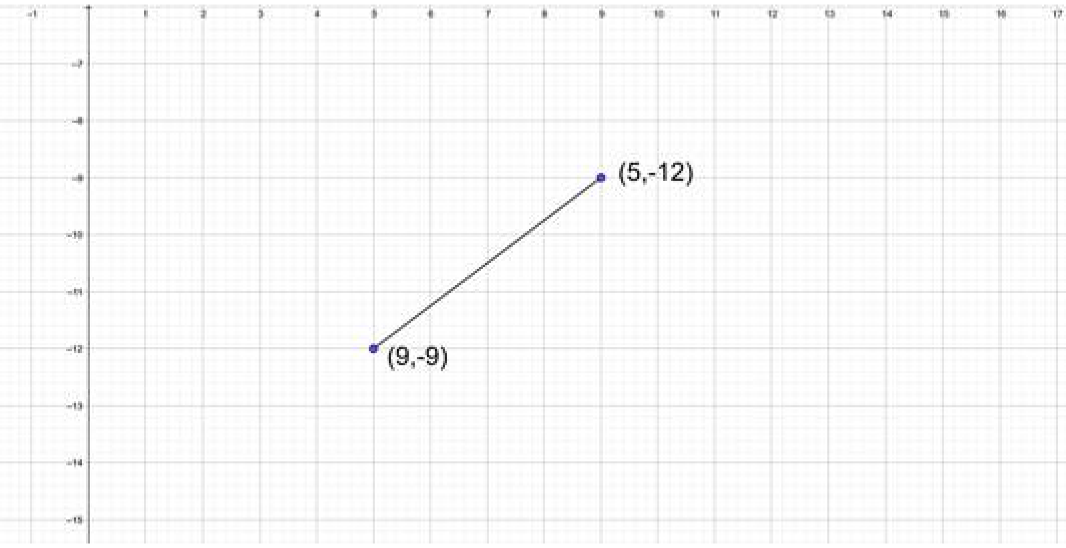
1 C. Question

Find the distance between the following pair of points:

(5, - 12), (9, - 9)

Answer

Given points are (5, - 12) and (9, - 9)



We need to find the distance between these two points.

We know that distance(S) between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$$\Rightarrow S = \sqrt{(5 - 9)^2 + (-12 - (-9))^2}$$

$$\Rightarrow S = \sqrt{(-4)^2 + (-3)^2}$$

$$\Rightarrow S = \sqrt{16 + 9}$$

$\Rightarrow S = \sqrt{25}$

$\Rightarrow S = 5$

∴ The distance between the points (5, - 12) and (9, - 9) is 5 units.

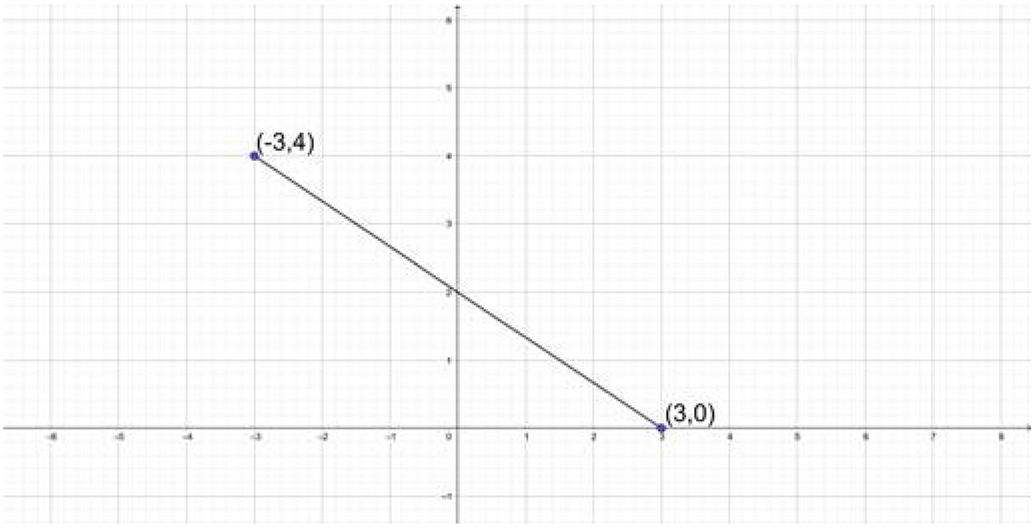
1 D. Question

Find the distance between the following pair of points:

(- 3, 4), (3, 0)

Answer

Given points are (- 3, 4) and (3, 0)



We need to find the distance between these two points.

We know that distance(S) between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$\Rightarrow S = \sqrt{(-3-3)^2 + (4-0)^2}$

$\Rightarrow S = \sqrt{(-6)^2 + (4)^2}$

$\Rightarrow S = \sqrt{36 + 16}$

$\Rightarrow S = \sqrt{52}$

$\Rightarrow S = \sqrt{4 \times 13}$

$\Rightarrow S = 2\sqrt{13}$

∴ The distance between the points (- 3, 4) and (3, 0) is $2\sqrt{13}$ units.

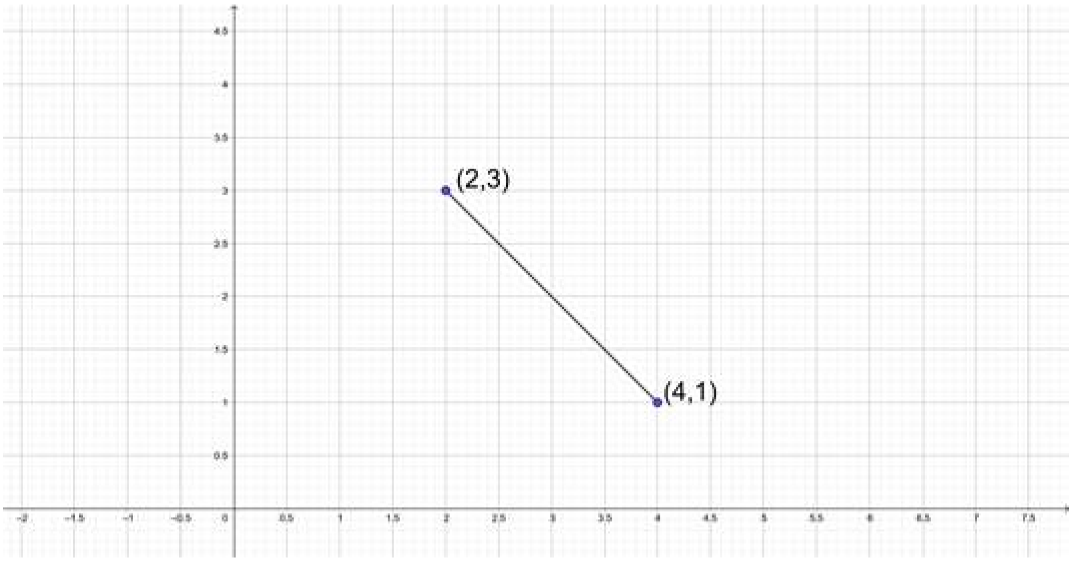
1 E. Question

Find the distance between the following pair of points:

(2, 3), (4, 1)

Answer

Given points are (2, 3) and (4, 1)



We need to find the distance between these two points.

We know that distance(S) between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$\Rightarrow S = \sqrt{(2-4)^2 + (3-1)^2}$

$\Rightarrow S = \sqrt{(-2)^2 + (2)^2}$

$\Rightarrow S = \sqrt{4 + 4}$

$\Rightarrow S = \sqrt{8}$

$\Rightarrow S = \sqrt{4 \times 2}$

$\Rightarrow S = 2\sqrt{2}$

∴ The distance between the points (2, 3) and (4, 1) is $2\sqrt{2}$ units.

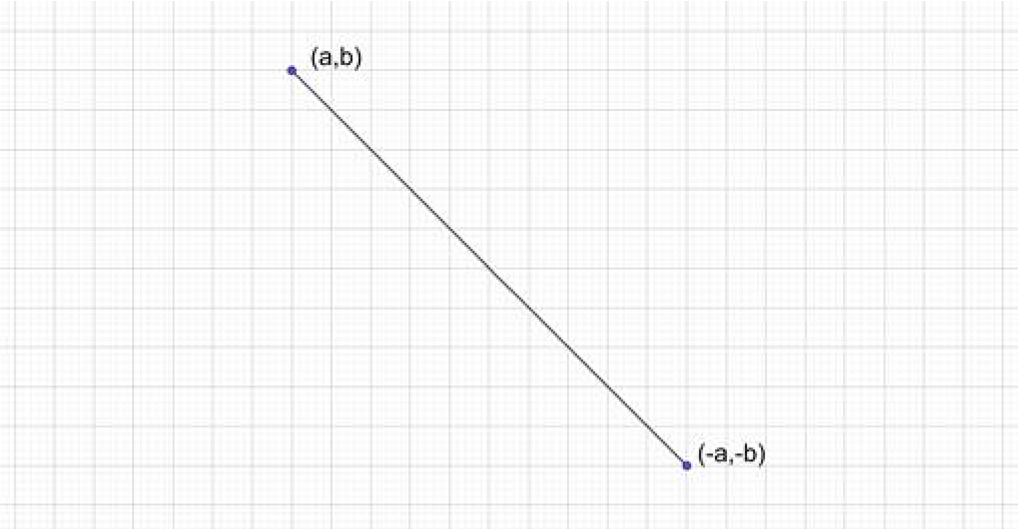
1 F. Question

Find the distance between the following pair of points:

(a, b), (- a, - b)

Answer

Given points are (a, b) and (- a, - b)



We need to find the distance between these two points.

We know that distance(S) between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$$\Rightarrow S = \sqrt{(a - (-a))^2 + (b - (-b))^2}$$

$$\Rightarrow S = \sqrt{(2a)^2 + (2b)^2}$$

$$\Rightarrow S = \sqrt{4a^2 + 4b^2}$$

$$\Rightarrow S = \sqrt{4 \times (a^2 + b^2)}$$

$$\Rightarrow S = 2\sqrt{a^2 + b^2}$$

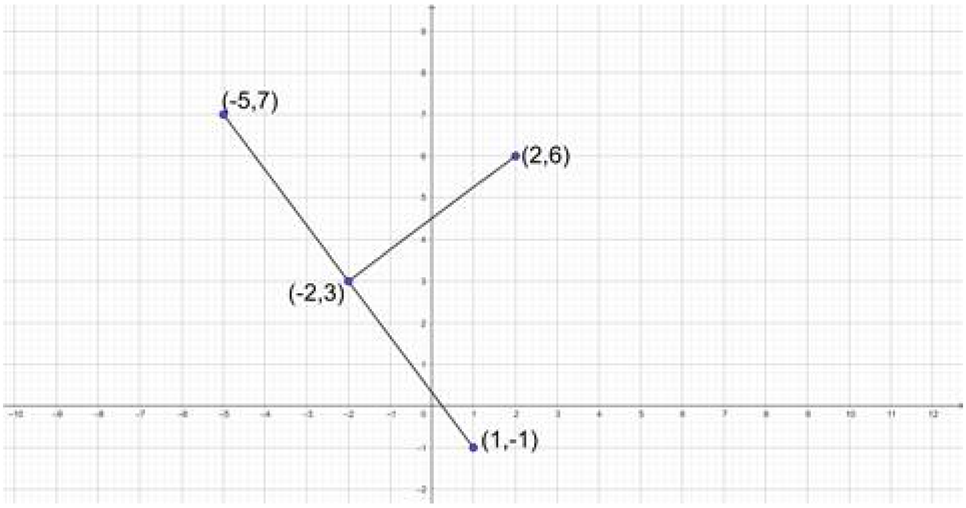
∴ The distance between the points (a, b) and (- a, - b) is $2\sqrt{a^2 + b^2}$ units.

2. Question

Examine whether the points (1, - 1), (- 5, 7) and (2, 6) are equidistant from the point (- 2, 3)?

Answer

Given that we need to show that the points (1, - 1), (- 5, 7) and (2, 6) are equidistant from the point (- 2, 3).



We know that distance between two points (x_1, y_1) and (x_2, y_2) is

$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Let S_1 be the distance between the points (1, - 1) and (- 2, 3)

$\Rightarrow S_1 = \sqrt{(1-(-2))^2 + (-1-3)^2}$

$\Rightarrow S_1 = \sqrt{(3)^2 + (-4)^2}$

$\Rightarrow S_1 = \sqrt{9 + 16}$

$\Rightarrow S_1 = \sqrt{25}$

$\Rightarrow S_1 = 5$ ..... (1)

Let S_2 be the distance between the points (- 5, 7) and (- 2, 3)

$\Rightarrow S_2 = \sqrt{(-5-(-2))^2 + (7-3)^2}$

$\Rightarrow S_2 = \sqrt{(-3)^2 + (4)^2}$

$\Rightarrow S_2 = \sqrt{9 + 16}$

$\Rightarrow S_2 = \sqrt{25}$

$\Rightarrow S_2 = 5$ ..... (2)

Let S_3 be the distance between the points (2, 6) and (- 2, 3)

$\Rightarrow S_3 = \sqrt{(2-(-2))^2 + (6-3)^2}$

$\Rightarrow S_3 = \sqrt{(4)^2 + (3)^2}$

$\Rightarrow S_3 = \sqrt{16 + 9}$

$\Rightarrow S_3 = \sqrt{25}$

$\Rightarrow S_3 = 5$ ..... (3)

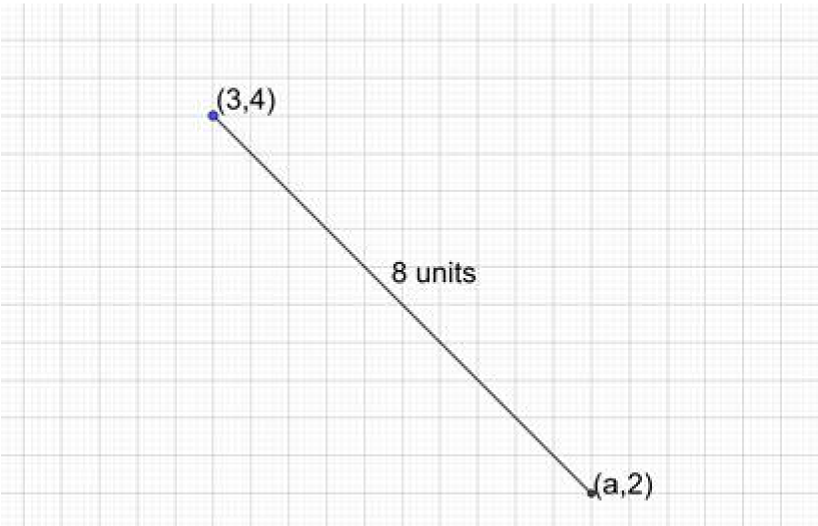
From (1), (2), and (3) we got $S_1 = S_2 = S_3$ which tells us that (1, - 1), (5, 7) and (2, 5) are equidistant from (- 2, 3).

3 A. Question

Find a if the distance between (a, 2) and (3, 4) is 8.

Answer

Given that the distance between the points (a, 2) and (3, 4) is 8.



We need to find the value of a.

We know that distance(S) between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$\Rightarrow 8 = \sqrt{(a - 3)^2 + (2 - 4)^2}$

$\Rightarrow 8^2 = (a - 3)^2 + (- 2)^2$

$\Rightarrow 64 = (a - 3)^2 + 4$

$\Rightarrow (a - 3)^2 = 60$

$\Rightarrow a - 3 = \pm\sqrt{60}$

$\Rightarrow a = 3 \pm \sqrt{60}$

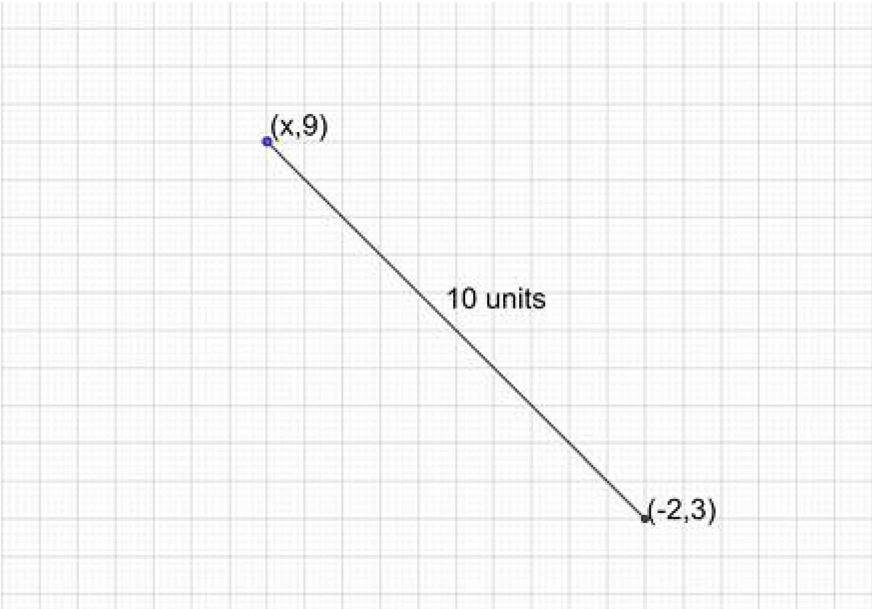
∴ The values of a are 3±√60.

3 B. Question

A line is of length 10 units and one of its ends is (- 2, 3). If the ordinate of the other end is 9, prove that the absicca of the other end is 6 or - 10.

Answer

Given that the line has length of 10 units and one of its ends is (- 2, 3).



It is also given that the ordinate of the other end is 9. Let us assume the other end is (x, 9).

We know that distance(S) between the points (x_1, y_1) and (x_2, y_2) is

$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$\Rightarrow 10 = \sqrt{(-2 - x)^2 + (3 - 9)^2}$

$\Rightarrow 10 = (x + 2)^2 + (-6)^2$

$\Rightarrow 100 = (x + 2)^2 + 36$

$\Rightarrow (x + 2)^2 = 64$

$\Rightarrow x + 2 = \pm 8$

$\Rightarrow x = -2 + 8$ (or) $x = -2 - 8$

$\Rightarrow x = 6$ or 10

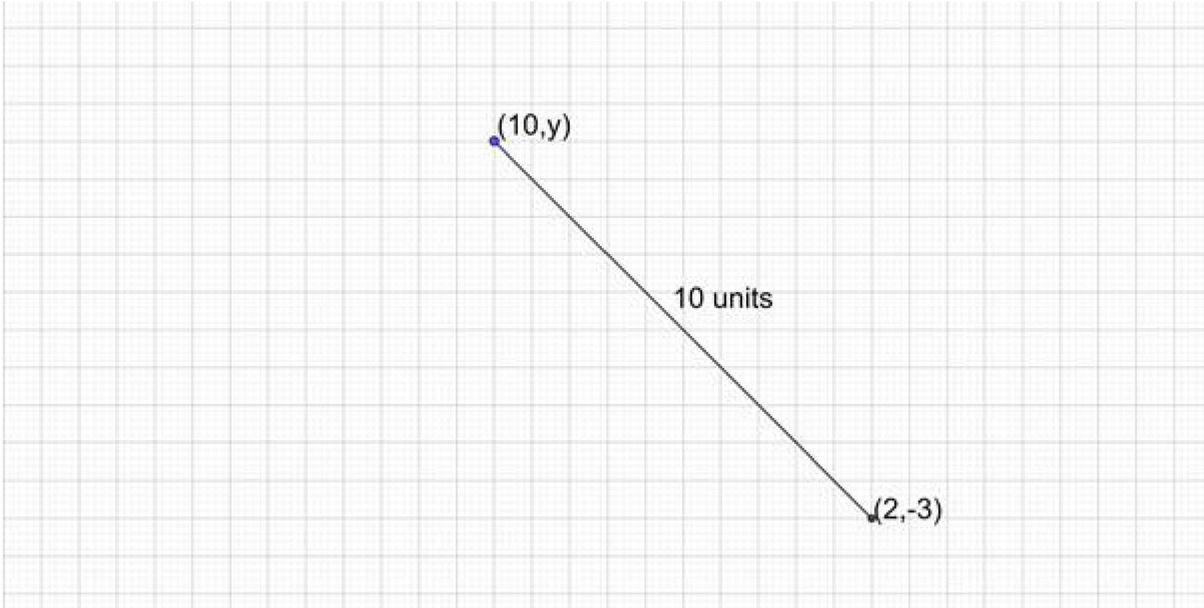
∴ Thus proved.

3 C. Question

Find the value of y for which the distance between the points P(2, - 3) and Q(10, y) is 10 units.

Answer

Given that the distance between the points P(2, - 3) and Q(10, y) is 10.



We need to find the value of y.

We know that distance(S) between the points (x_1, y_1) and (x_2, y_2) is

$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$\Rightarrow 10 = \sqrt{(2 - 10)^2 + (-3 - y)^2}$

$\Rightarrow 10^2 = (-8)^2 + (3 + y)^2$

$\Rightarrow 100 = 64 + (3 + y)^2$

$\Rightarrow (3 + y)^2 = 36$

$\Rightarrow 3 + y = \pm 6$

$\Rightarrow y = 3 + 6$ (or) $y = 3 - 6$

$\Rightarrow y = 9$ (or) $y = -3$

∴ The values of y are 9, - 3.

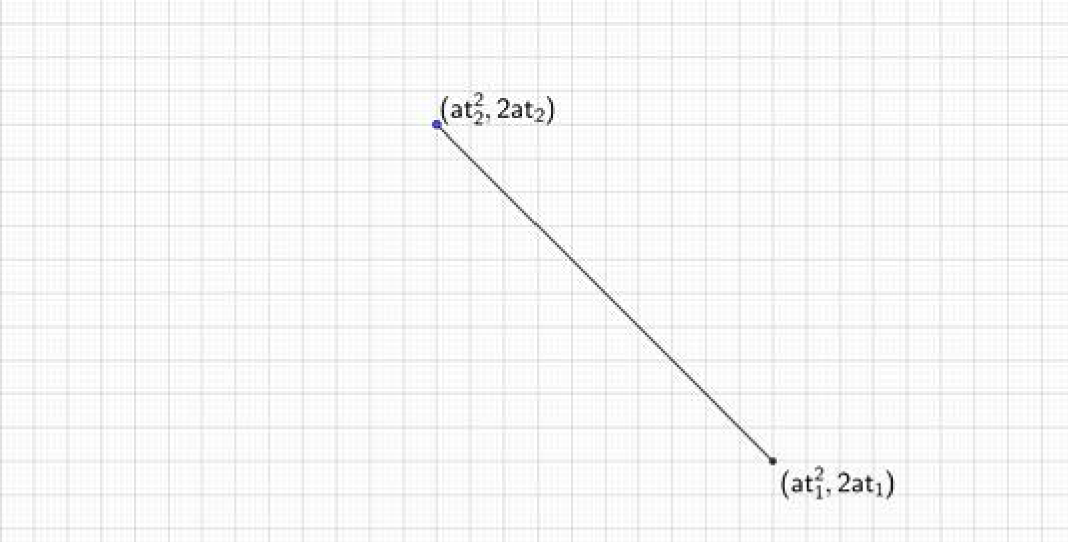
4 A. Question

Find the distance between the points:

$(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$

Answer

Given points are $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$.



We need to find the distance between these two points.

We know that distance(S) between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$$\Rightarrow S = \sqrt{(at_1^2 - at_2^2)^2 + (2at_1 - 2at_2)^2}$$

$$\Rightarrow S = \sqrt{(a(t_1 - t_2)(t_1 + t_2))^2 + (2a(t_1 - t_2))^2}$$

$$\Rightarrow S = (a(t_1 + t_2))\sqrt{(t_1 + t_2)^2 + 2^2}$$

$$\Rightarrow S = (a(t_1 + t_2))\sqrt{(t_1 + t_2)^2 + 4}$$

∴ The distance between the points $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ is $(a(t_1 + t_2))\sqrt{(t_1 + t_2)^2 + 4}$.

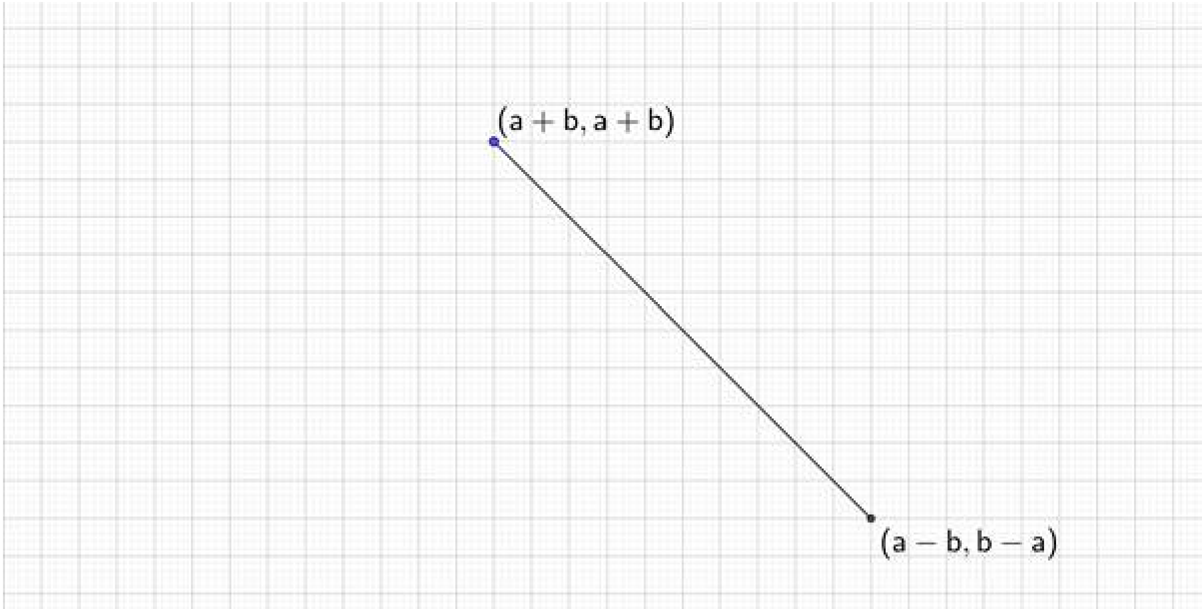
4 B. Question

Find the distance between the points:

(a - b, b - a) and (a + b, a + b)

Answer

Given points are (a - b, b - a) and (a + b, a + b).



We need to find the distance between these two points.

We know that distance(S) between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$\Rightarrow S = \sqrt{((a - b) - (a + b))^2 + ((b - a) - (a + b))^2}$

$\Rightarrow S = \sqrt{(-2b)^2 + (-2a)^2}$

$\Rightarrow S = \sqrt{4(a^2 + b^2)}$

$\Rightarrow S = 2\sqrt{a^2 + b^2}$

∴ The distance between the points (a - b, b - a) and (a + b, a + b) is $2\sqrt{a^2 + b^2}$.

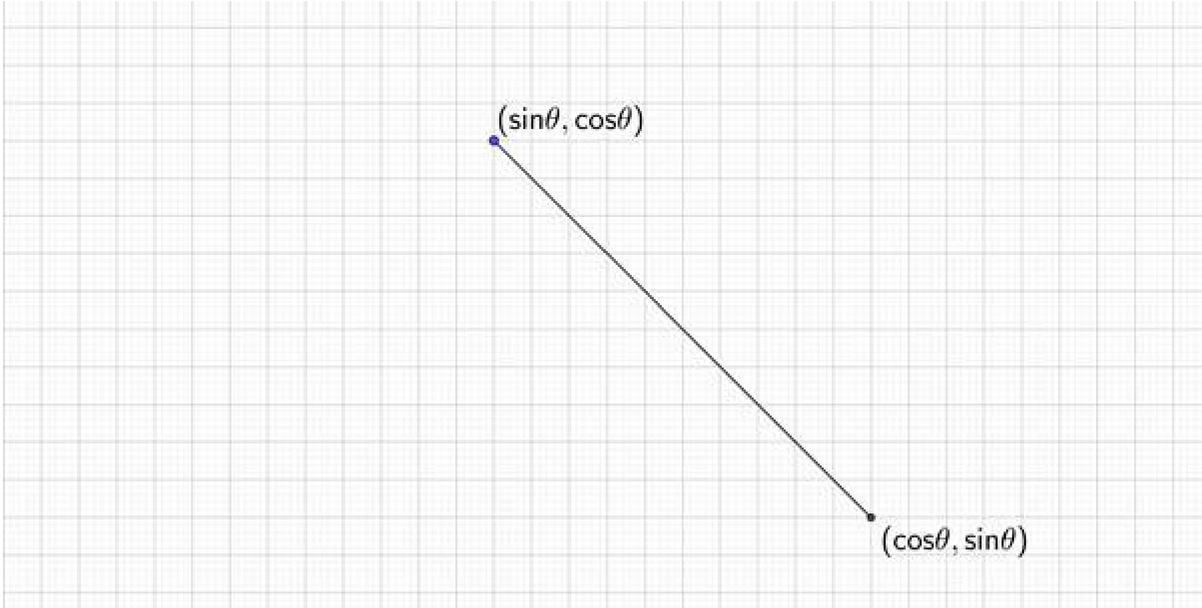
4 C. Question

Find the distance between the points:

(cosθ, sinθ) and (sinθ, cosθ)

Answer

Given points are (cosθ, sinθ) and (sinθ, cosθ).



We need to find the distance between these two points.

We know that distance(S) between the points (x_1, y_1) and (x_2, y_2) is
$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$\Rightarrow S = \sqrt{(\cos\theta - \sin\theta)^2 + (\sin\theta - \cos\theta)^2}$

$\Rightarrow S = \sqrt{\cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta + \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta}$

$\Rightarrow S = \sqrt{2(\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta)}$

$\Rightarrow S = \sqrt{2(\sin\theta - \cos\theta)^2}$

$\Rightarrow S = \sqrt{2}(\sin\theta - \cos\theta)$

∴ The distance between the points (cosθ, sinθ) and (sinθ, cosθ) is
$\sqrt{2}(\sin\theta - \cos\theta)$.

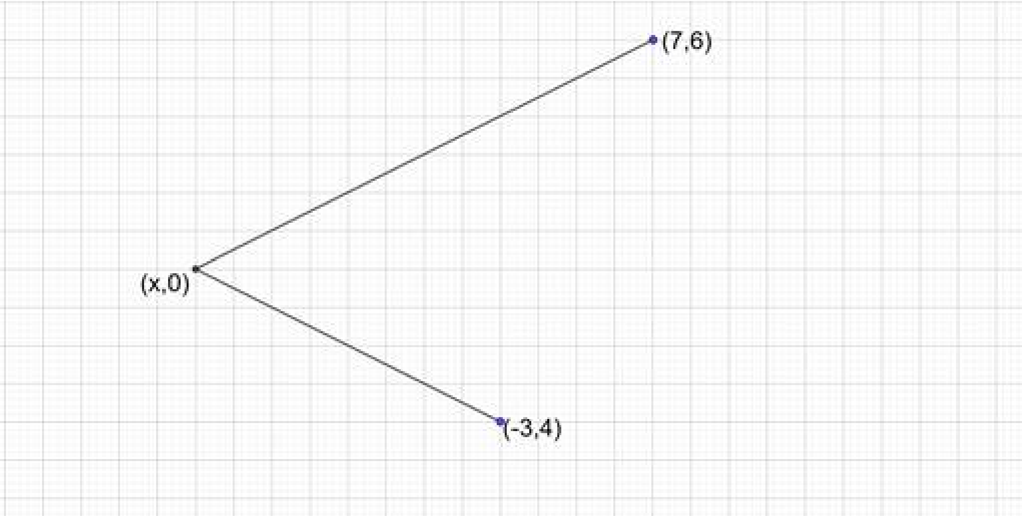
5 A. Question

Find the point on x - axis which is equidistant from the following pair of points:

(7, 6) and (- 3, 4)

Answer

Given points are A(7, 6) and B(- 3, 4).



We need to find a point on x - axis which is equidistant from these points.

Let us assume the point on x - axis be S(x, o).

We know that distance between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

From the problem,

$\Rightarrow$ SA = SB

$\Rightarrow SA^2 = SB^2$

$\Rightarrow (x - 7)^2 + (0 - 6)^2 = (x - (- 3))^2 + (0 - 4)^2$

$\Rightarrow (x - 7)^2 + (- 6)^2 = (x + 3)^2 + (- 4)^2$

$\Rightarrow x^2 - 14x + 49 + 36 = x^2 + 6x + 9 + 16$

$\Rightarrow 20x = 60$

$\Rightarrow x = \frac{60}{20}$

$\Rightarrow x = 3$

∴ The point on x - axis is (3, 0).

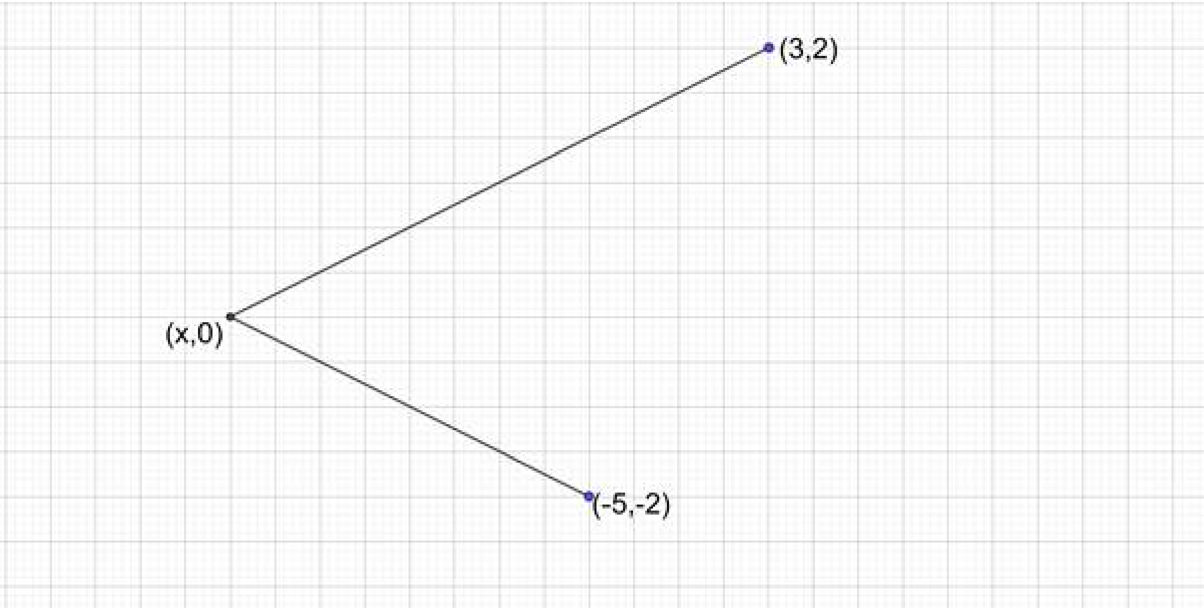
5 B. Question

Find the point on x - axis which is equidistant from the following pair of points:

(3, 2) and (- 5, - 2)

Answer

Given points are A(3, 2) and B(- 5, - 2).



We need to find a point on x - axis which is equidistant from these points.

Let us assume the point on x - axis be S(x, o).

We know that distance between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

From the problem,

⇒ SA = SB

⇒ $SA^2 = SB^2$

⇒ $(x - 3)^2 + (0 - 2)^2 = (x + 5)^2 + (0 - (- 2))^2$

⇒ $(x - 3)^2 + (- 2)^2 = (x + 5)^2 + (2)^2$

⇒ $x^2 - 6x + 9 + 4 = x^2 + 10x + 25 + 4$

⇒ 16x = - 16

⇒ $x = \frac{-16}{16}$

⇒ x = - 1

∴ The point on x - axis is (- 1, 0).

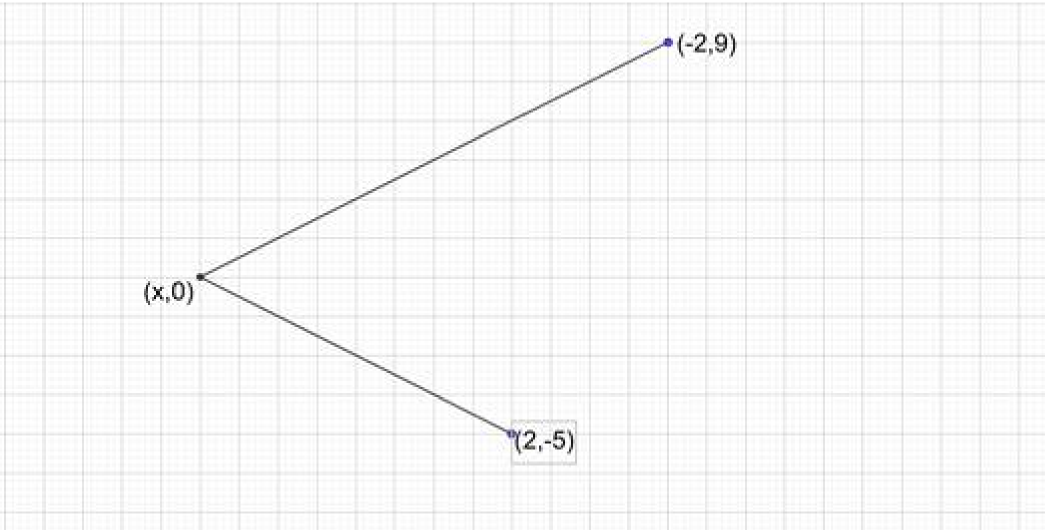
5 C. Question

Find the point on x - axis which is equidistant from the following pair of points:

(2, - 5) and (- 2, 9)

Answer

Given points are A(2, - 5) and B(- 2, 9).



We need to find a point on x - axis which is equidistant from these points.

Let us assume the point on x - axis be S(x, o).

We know that distance between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

From the problem,

⇒ SA = SB

⇒ $SA^2 = SB^2$

⇒ $(x - 2)^2 + (0 - (- 5))^2 = (x - (- 2))^2 + (0 - 9)^2$

⇒ $(x - 2)^2 + (5)^2 = (x + 2)^2 + (- 9)^2$

⇒ $x^2 - 4x + 4 + 25 = x^2 + 4x + 4 + 81$

⇒ 8x = - 56

⇒ $x = \frac{-56}{8}$

⇒ x = - 7

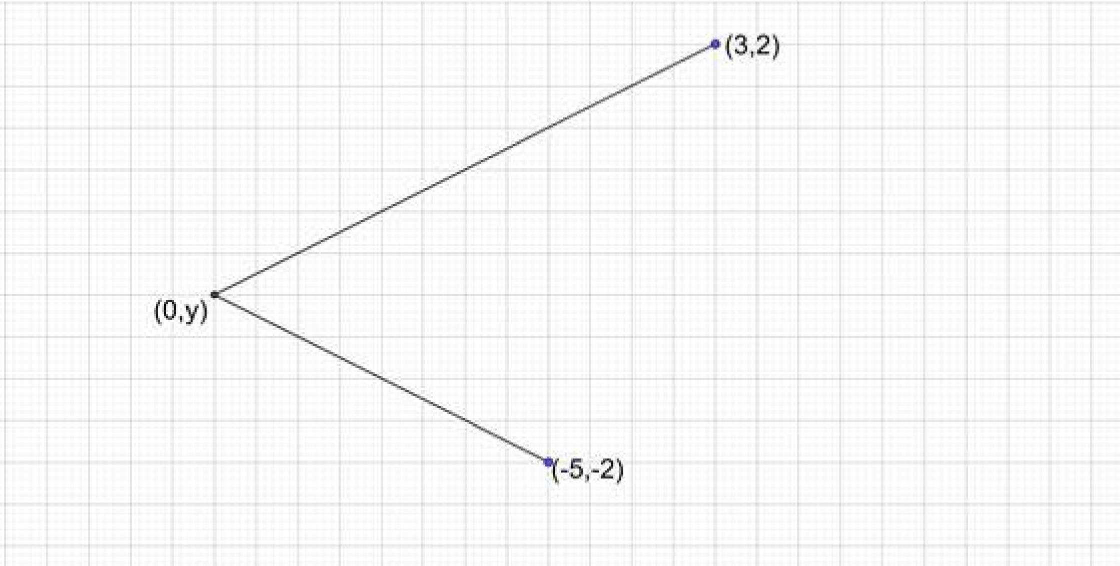
∴ The point on x - axis is (- 7, 0).

6 A. Question

Find the point on y - axis which is equidistant from point (- 5, - 2) and (3, 2).

Answer

Given points are A(- 5, - 2) and B(3, 2).



We need to find a point on y - axis which is equidistant from these points.

Let us assume the point on y - axis be S(0, y).

We know that distance between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

From the problem,

⇒ SA = SB

⇒ $SA^2 = SB^2$

⇒ $(0 - (-5))^2 + (y - (-2))^2 = (0 - 3)^2 + (y - 2)^2$

⇒ $(5)^2 + (y + 2)^2 = (-3)^2 + (y - 2)^2$

⇒ $25 + y^2 + 4y + 4 = 9 + y^2 - 4y + 4$

⇒ 8y = - 16

⇒ $y = \frac{-16}{8}$

⇒ y = - 2

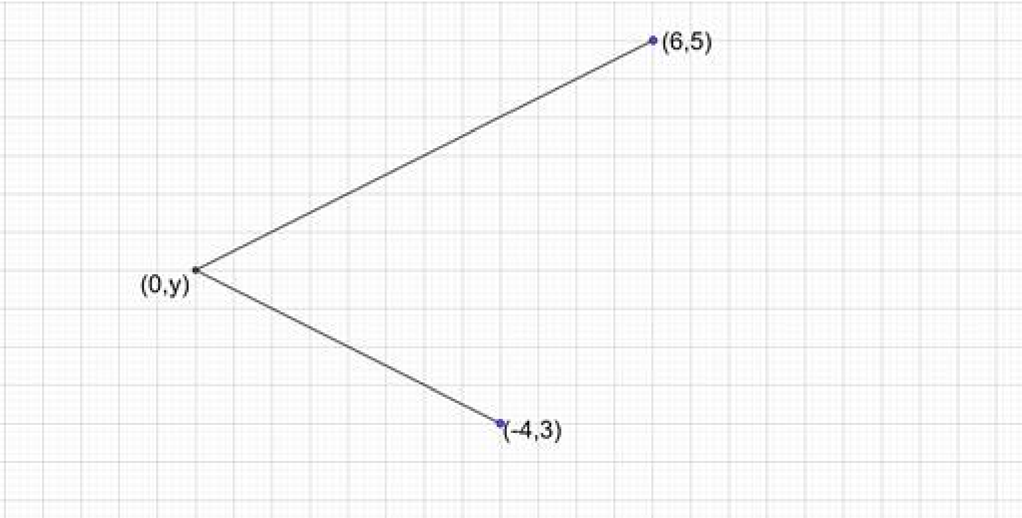
∴ The point on y - axis is (0, - 2).

6 B. Question

Find the point on y - axis which is equidistant from the points A(6, 5) and B(-4, 3).

Answer

Given points are A(6, 5) and B(- 4, 3).



We need to find a point on y - axis which is equidistant from these points.

Let us assume the point on y - axis be S(0, y).

We know that distance between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

From the problem,

⇒ SA = SB

⇒ $SA^2 = SB^2$

⇒ $(0 - 6)^2 + (y - 5)^2 = (0 - (- 4))^2 + (y - 3)^2$

⇒ $(- 6)^2 + (y - 5)^2 = (4)^2 + (y - 3)^2$

⇒ $36 + y^2 - 10y + 25 = 16 + y^2 - 6y + 9$

⇒ 4y = 36

⇒ $y = \frac{36}{4}$

⇒ y = 9

∴ The point on y - axis is (0, 9).

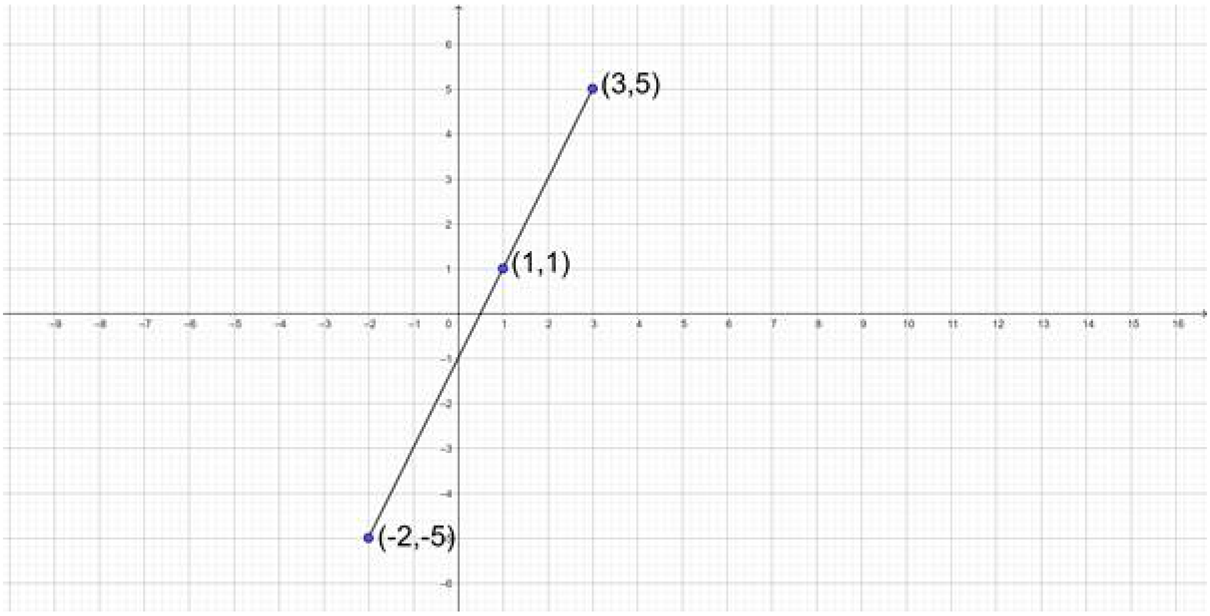
7 A. Question

Using distance formula, examine whether the following sets of points are collinear?

(3, 5), (1, 1), (- 2, - 5)

Answer

Given points are A(3, 5), B(1, 1) and C(- 2, - 5).



We need to check whether these points are collinear.

We know that for three points A, B and C to be collinear, the criteria to be satisfied is AC = AB + BC.

Let us find the distances first,

We know that distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$$\Rightarrow AC = \sqrt{(3 - (-2))^2 + (5 - (-5))^2}$$

$$\Rightarrow AC = \sqrt{(5)^2 + (10)^2}$$

$$\Rightarrow AC = \sqrt{25 + 100}$$

$$\Rightarrow AC = \sqrt{125}$$

$$\Rightarrow AC = \sqrt{5 \times 25}$$

$\Rightarrow$ AC = $5\sqrt{5}$ ..... (1)

$$\Rightarrow AB = \sqrt{(3 - 1)^2 + (5 - 1)^2}$$

$$\Rightarrow AB = \sqrt{(2)^2 + (4)^2}$$

$\Rightarrow AB = \sqrt{4 + 16}$

$\Rightarrow AB = \sqrt{20}$

$\Rightarrow AB = \sqrt{4 \times 5}$

$\Rightarrow AB = 2\sqrt{5}$ ..... (2)

$\Rightarrow BC = \sqrt{(1-(-2))^2 + (1-(-5))^2}$

$\Rightarrow BC = \sqrt{(3)^2 + (6)^2}$

$\Rightarrow BC = \sqrt{9 + 36}$

$\Rightarrow BC = \sqrt{45}$

$\Rightarrow BC = \sqrt{9 \times 5}$

$\Rightarrow BC = 3\sqrt{5}$ ..... (3)

From (1), (2), (3) we can see that AB + BC = AC.

∴ The three points are collinear.

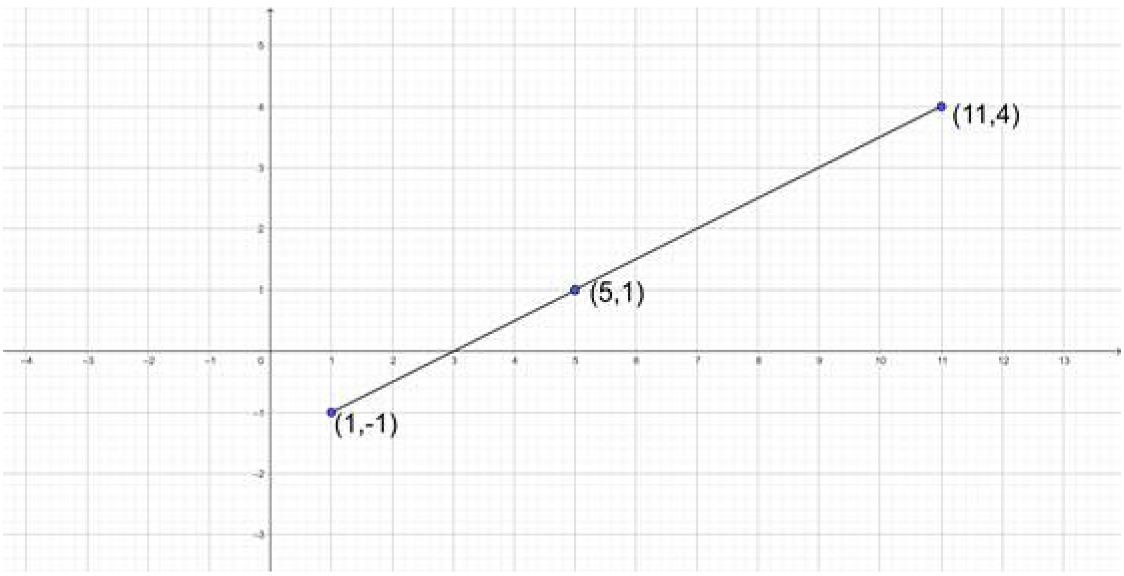
7 B. Question

Using distance formula, examine whether the following sets of points are collinear?

(5, 1), (1, - 1), (11, 4)

Answer

Given points are A(5, 1), B(1, - 1) and C(11, 4).



We need to check whether these points are collinear.

We know that for three points A, B and C to be collinear, the criteria to be satisfied is a linear relationship between AB, BC and AC.

Let us find the distances first,

We know that distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$\Rightarrow AC = \sqrt{(5 - 11)^2 + (1 - 4)^2}$

$\Rightarrow AC = \sqrt{(-6)^2 + (-3)^2}$

$\Rightarrow AC = \sqrt{36 + 9}$

$\Rightarrow AC = \sqrt{45}$

$\Rightarrow AC = \sqrt{9 \times 5}$

$\Rightarrow AC = 3\sqrt{5}$ ..... (1)

$\Rightarrow AB = \sqrt{(5 - 1)^2 + (1 - (-1))^2}$

$\Rightarrow AB = \sqrt{(4)^2 + (2)^2}$

$\Rightarrow AB = \sqrt{16 + 4}$

$\Rightarrow AB = \sqrt{20}$

$\Rightarrow AB = \sqrt{4 \times 5}$

$\Rightarrow AB = 2\sqrt{5}$ ..... (2)

$\Rightarrow BC = \sqrt{(1 - 11)^2 + (-1 - 4)^2}$

$\Rightarrow BC = \sqrt{(-10)^2 + (-5)^2}$

$\Rightarrow BC = \sqrt{100 + 25}$

$\Rightarrow BC = \sqrt{125}$

$\Rightarrow BC = \sqrt{25 \times 5}$

$\Rightarrow BC = 5\sqrt{5}$ ..... (3)

From (1), (2), (3) we can see that AB + AC = BC.

∴ The three points are collinear.

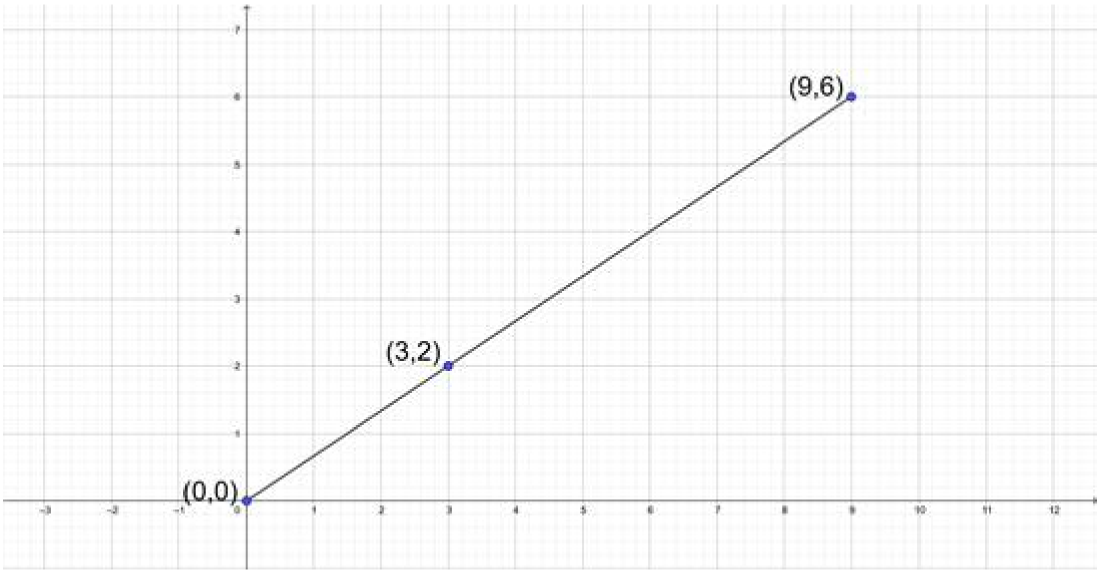
7 C. Question

Using distance formula, examine whether the following sets of points are collinear?

(0, 0), (9, 6), (3, 2)

Answer

Given points are A(0, 0), B(9, 6) and C(3, 2).



We need to check whether these points are collinear.

We know that for three points A, B and C to be collinear, the criteria to be satisfied is a linear relationship between AB, BC and AC.

Let us find the distances first,

We know that distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$\Rightarrow AC = \sqrt{(0-3)^2 + (0-2)^2}$

$\Rightarrow AC = \sqrt{(-3)^2 + (-2)^2}$

$\Rightarrow AC = \sqrt{9 + 4}$

$\Rightarrow AC = \sqrt{13}$ ..... (1)

$\Rightarrow AB = \sqrt{(0-9)^2 + (0-6)^2}$

$\Rightarrow AB = \sqrt{(-9)^2 + (-6)^2}$

$\Rightarrow AB = \sqrt{81 + 36}$

$\Rightarrow AB = \sqrt{117}$

$\Rightarrow AB = \sqrt{9 \times 13}$

$\Rightarrow AB = 3\sqrt{13}$ ..... (2)

$\Rightarrow BC = \sqrt{(9-3)^2 + (6-2)^2}$

$\Rightarrow BC = \sqrt{(6)^2 + (4)^2}$

$\Rightarrow BC = \sqrt{36 + 16}$

$\Rightarrow BC = \sqrt{52}$

$\Rightarrow BC = \sqrt{4 \times 13}$

$\Rightarrow BC = 2\sqrt{13}$ ..... (3)

From (1), (2), (3) we can see that AB = BC + AC.

∴ The three points are collinear.

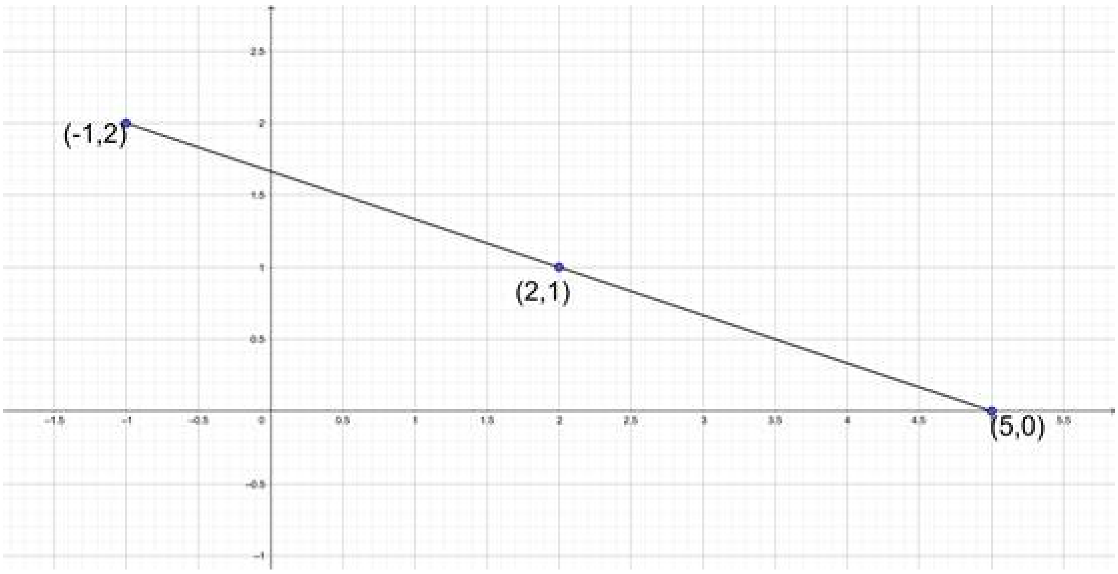
7 D. Question

Using distance formula, examine whether the following sets of points are collinear?

(- 1, 2), (5, 0), (2, 1)

Answer

Given points are A(- 1, 2), B(5, 0) and C(2, 1).



We need to check whether these points are collinear.

We know that for three points A, B and C to be collinear, the criteria to be satisfied is a linear relationship between AB, BC and AC.

Let us find the distances first,

We know that distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$\Rightarrow AC = \sqrt{(-1-2)^2 + (2-1)^2}$

$\Rightarrow AC = \sqrt{(-3)^2 + (1)^2}$

$\Rightarrow AC = \sqrt{9 + 1}$

$\Rightarrow AC = \sqrt{10}$ ..... (1)

$\Rightarrow AB = \sqrt{(-1-5)^2 + (2-0)^2}$

$\Rightarrow AB = \sqrt{(-6)^2 + (2)^2}$

$\Rightarrow AB = \sqrt{36 + 4}$

$\Rightarrow AB = \sqrt{40}$

$\Rightarrow AB = \sqrt{4 \times 10}$

$\Rightarrow AB = 2\sqrt{10}$ ..... (2)

$\Rightarrow BC = \sqrt{(5-2)^2 + (0-1)^2}$

$\Rightarrow BC = \sqrt{(3)^2 + (-1)^2}$

$\Rightarrow BC = \sqrt{9 + 1}$

$\Rightarrow BC = \sqrt{10}$ ..... (3)

From (1), (2), (3) we can see that AC + BC = AB.

∴ The three points are collinear.

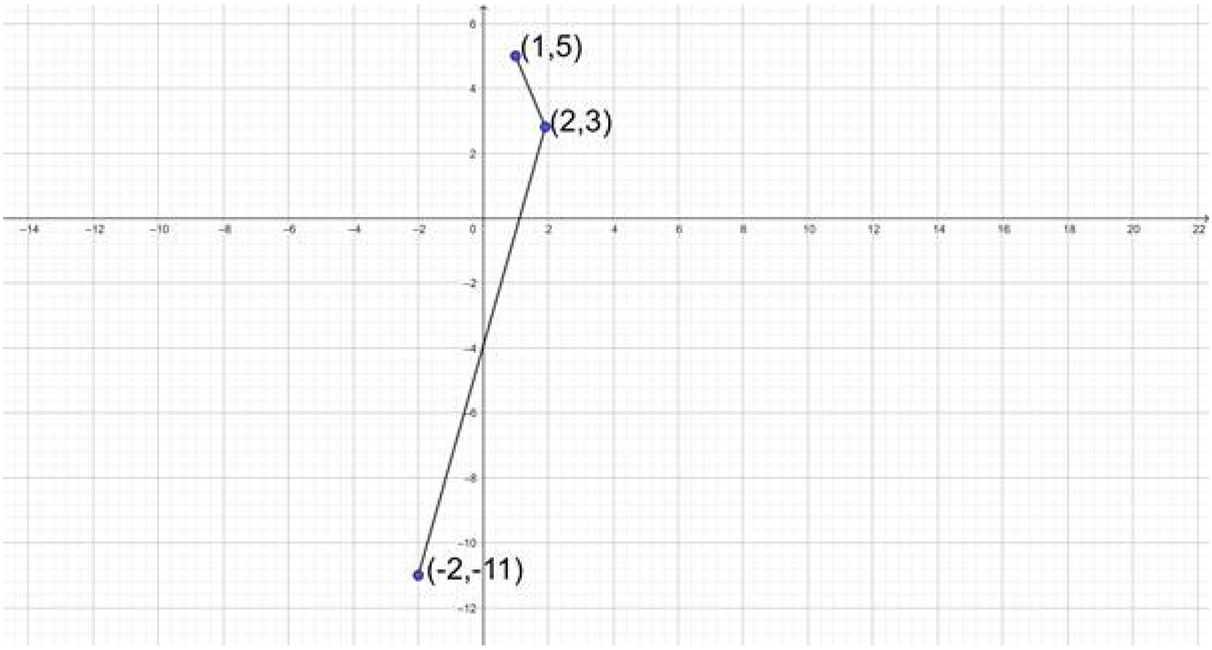
7 E. Question

Using distance formula, examine whether the following sets of points are collinear?

(1, 5), (2, 3), (- 2, - 11)

Answer

Given points are A(1, 5), B(2, 3) and C(- 2, - 11).



We need to check whether these points are collinear.

We know that for three points A, B and C to be collinear, the criteria to be satisfied is AC = AB + BC.

Let us find the distances first,

We know that distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$$\Rightarrow AC = \sqrt{(1-(-2))^2 + (5-(-11))^2}$$

$$\Rightarrow AC = \sqrt{(3)^2 + (16)^2}$$

$$\Rightarrow AC = \sqrt{9 + 256}$$

$\Rightarrow AC = \sqrt{265}$ ..... (1)

$$\Rightarrow AB = \sqrt{(1-2)^2 + (5-3)^2}$$

$$\Rightarrow AB = \sqrt{(-1)^2 + (2)^2}$$

$$\Rightarrow AB = \sqrt{1 + 4}$$

$\Rightarrow AB = \sqrt{5}$ ..... (2)

$\Rightarrow$ BC $= \sqrt{(2-(-2))^2 + (3-(-11))^2}$

$\Rightarrow$ BC $= \sqrt{(4)^2 + (14)^2}$

$\Rightarrow$ BC $= \sqrt{16 + 196}$

$\Rightarrow$ BC $= \sqrt{212}$

$\Rightarrow$ BC $= \sqrt{4 \times 53}$

$\Rightarrow$ BC = $2\sqrt{53}$ ..... (3)

From (1), (2), (3) we can see that we cannot get any linear relationship.

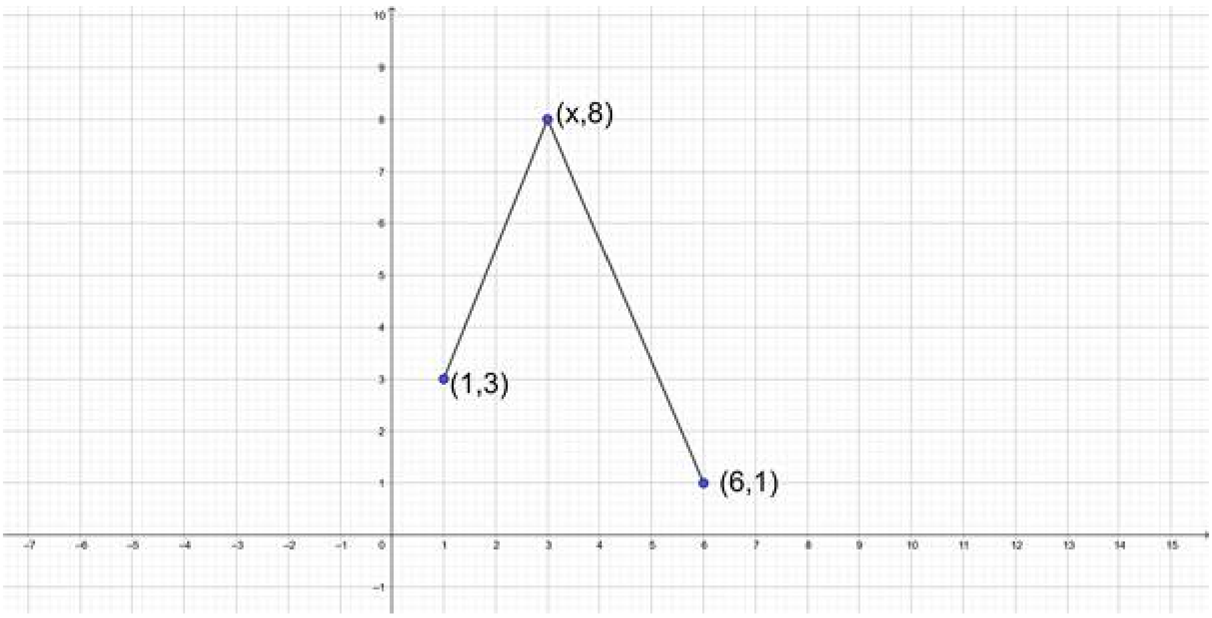
∴ The three points are not collinear.

8. Question

If A = (6, 1), B = (1, 3) and C = (x, 8), find the value of x such that AB = BC.

Answer

Given points are A(6, 1), B(1, 3) and C(x, 8). We need to find the value of x such that AB = BC.



We know that the distance between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$\Rightarrow$ AB = BC

$\Rightarrow$ $AB^2 = BC^2$

$\Rightarrow$ $(6 - 1)^2 + (1 - 3)^2 = (1 - x)^2 + (3 - 8)^2$

$\Rightarrow (5)^2 + (-2)^2 = (1 - x)^2 + (-5)^2$

$\Rightarrow 25 + 4 = 1 - 2x + x^2 + 25$

$\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow x^2 - 3x + x - 3 = 0$

$\Rightarrow x(x - 3) + 1(x - 3) = 0$

$\Rightarrow (x + 1)(x - 3) = 0$

$\Rightarrow x + 1 = 0$ (or) $x - 3 = 0$

$\Rightarrow x = -1$ (or) $x = 3$

∴ The values of x are - 1 or 3.

9. Question

Prove that the distance between the points (a + rcosθ, b + rsinθ) and (a, b) is independent of θ.

Answer

Given points are A(a + rcosθ, b + rsinθ) and B(a, b).

We know that the distance between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$\Rightarrow AB = \sqrt{(a + r\cos\theta - a)^2 + (b + r\sin\theta - b)^2}$

$\Rightarrow AB = \sqrt{(r\cos\theta)^2 + (r\sin\theta)^2}$

$$\Rightarrow AB = \sqrt{r^2 \cos^2\theta + r^2 \sin^2\theta}$$

$$\Rightarrow AB = \sqrt{r^2(\cos^2\theta + \sin^2\theta)}$$

$$\Rightarrow AB = \sqrt{r^2(1)}$$

$\Rightarrow AB = r$

We can see that AB is independent of θ.

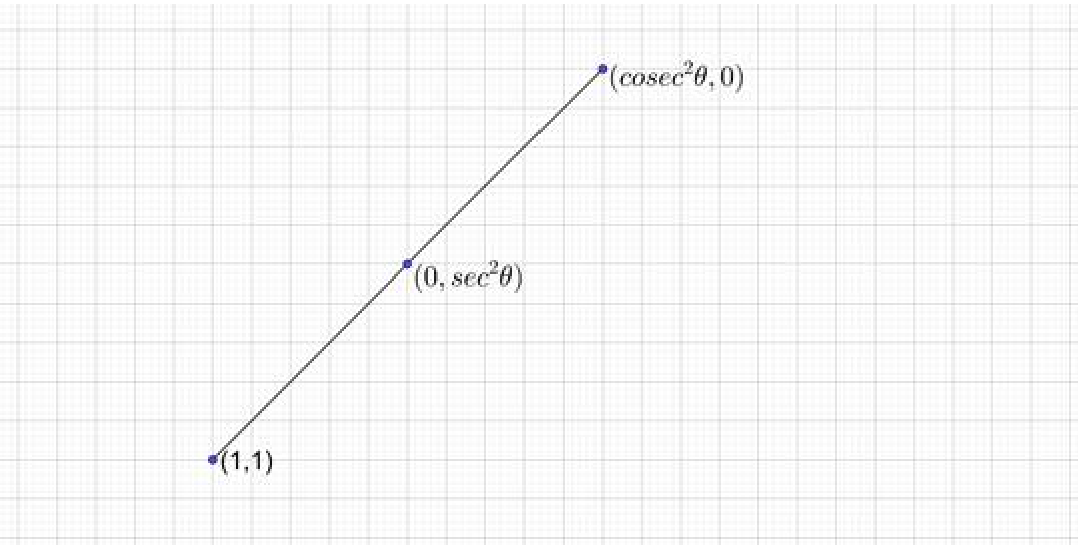
∴ Thus proved.

10 A. Question

use distance formula to show that the points ($cosec^2\theta$, 0), (0, $sec^2\theta$) and (1, 1) are collinear.

Answer

Given points are A($cosec^2\theta$, 0), B(0, $sec^2\theta$) and C(1, 1).



We need to check whether these points are collinear.

We know that for three points A, B and C to be collinear, the criteria to be satisfied is AB = AC + BC.

Let us find the distances first,

We know that distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$$\Rightarrow AC = \sqrt{(cosec^2\theta - 1)^2 + (0 - 1)^2}$$

$$\Rightarrow AC = \sqrt{(\cot^2\theta)^2 + 1^2}$$

$$\Rightarrow AC = \sqrt{\cot^4\theta + 1}$$

$$\Rightarrow AC = \sqrt{\frac{1}{\tan^4\theta} + 1}$$

$$\Rightarrow AC = \sqrt{\frac{1 + \tan^4\theta}{\tan^4\theta}}$$

$$\Rightarrow AC = \frac{\sqrt{1 + \tan^4\theta}}{\tan^2\theta} \text{ ..... } (1)$$

$$\Rightarrow AB = \sqrt{(\operatorname{cosec}^2\theta - 0)^2 + (0 - \sec^2\theta)^2}$$

$$\Rightarrow AB = \sqrt{(1 + \cot^2\theta)^2 + (1 + \tan^2\theta)^2}$$

$$\Rightarrow AB = \sqrt{\left(1 + \frac{1}{\tan^2\theta}\right)^2 + (1 + \tan^2\theta)^2}$$

$$\Rightarrow AB = \sqrt{(1 + \tan^2\theta)^2 \times \left(\frac{1}{\tan^4\theta} + 1\right)}$$

$$\Rightarrow AB = (1 + \tan^2\theta)\sqrt{\left(\frac{1 + \tan^4\theta}{\tan^4\theta}\right)}$$

$$\Rightarrow AB = \left(\frac{\sec^2\theta}{\tan^2\theta}\right)\sqrt{(1 + \tan^4\theta)} \text{ .... - } (2)$$

$$\Rightarrow BC = \sqrt{(0 - 1)^2 + (\sec^2\theta - 1)^2}$$

$$\Rightarrow BC = \sqrt{1 + (\tan^2\theta)^2}$$

$$\Rightarrow BC = \sqrt{1 + \tan^4\theta} \text{ ..... } (3)$$

Now,

$$\Rightarrow AC + BC = \left(\frac{1}{\tan^2\theta}\right)\sqrt{(1 + \tan^4\theta)} + \sqrt{1 + \tan^4\theta}$$

$$\Rightarrow AC + BC = \sqrt{1 + \tan^4\theta}\left(\frac{1}{\tan^2\theta} + 1\right)$$

$$\Rightarrow AC + BC = \sqrt{1 + \tan^4\theta}\left(\frac{1 + \tan^2\theta}{\tan^2\theta}\right)$$

$$\Rightarrow AC + BC = \sqrt{1 + \tan^4\theta}\left(\frac{\sec^2\theta}{\tan^2\theta}\right)$$

⇒ AC + BC = AB

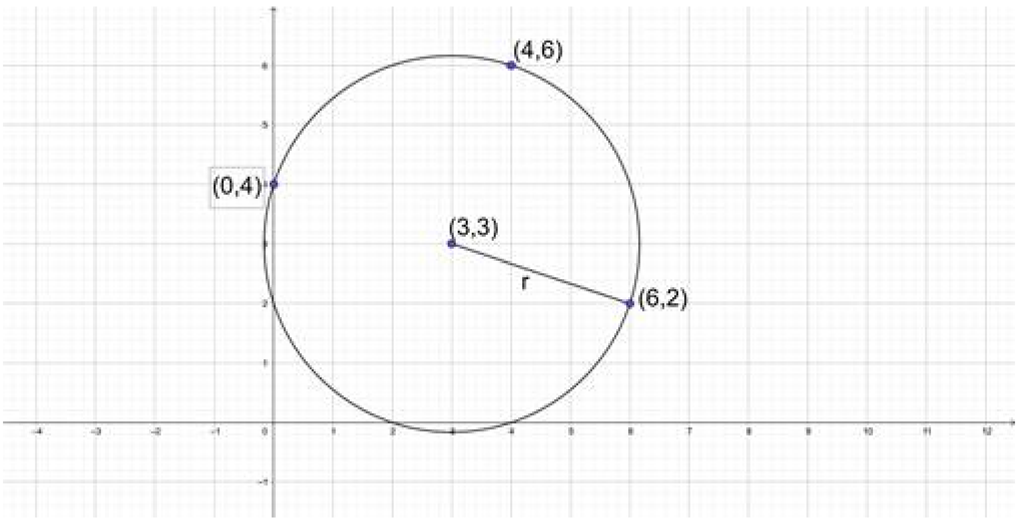
∴ The three points are collinear.

10 B. Question

Using distance formula show that (3, 3) is the centre of the circle passing through the points (6, 2), (0, 4) and (4, 6). Find the radius of the circle.

Answer

Given that circle passes through the points A(6, 2), B(0, 4), C(4, 6).



Let us assume O(x, y) be the centre of the circle.

We know that distance from the centre to any point on h circle is equal.

So, OA = OB = OC

We know that distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Now,

⇒ OA = OB

⇒ $OA^2 = OB^2$

⇒ $(x - 6)^2 + (y - 2)^2 = (x - 0)^2 + (y - 4)^2$

⇒ $x^2 - 12x + 36 + y^2 - 4y + 4 = x^2 + y^2 - 8y + 16$

⇒ $12x - 4y = 24$

⇒ $3x - y = 6$ ..... (1)

Now,

⇒ OB = OC

⇒ $OB^2 = OC^2$

⇒ $(x - 0)^2 + (y - 4)^2 = (x - 4)^2 + (y - 6)^2$

⇒ $x^2 + y^2 - 8y + 16 = x^2 - 8x + 16 + y^2 - 12y + 36$

⇒ $8x + 4y = 36$

⇒ $2x + y = 9$ .... - (2)

On solving (1) and (2), we get

⇒ x = 3 and y = 3

∴ (3, 3) is the centre of the circle.

We know radius is the distance between the centre and any point on the circle.

Let 'r' be the radius of the circle.

⇒ $r = OA = \sqrt{(3-6)^2 + (3-2)^2}$

⇒ $r = \sqrt{(-3)^2 + (1)^2}$

⇒ $r = \sqrt{9 + 1}$

⇒ $r = \sqrt{10}$

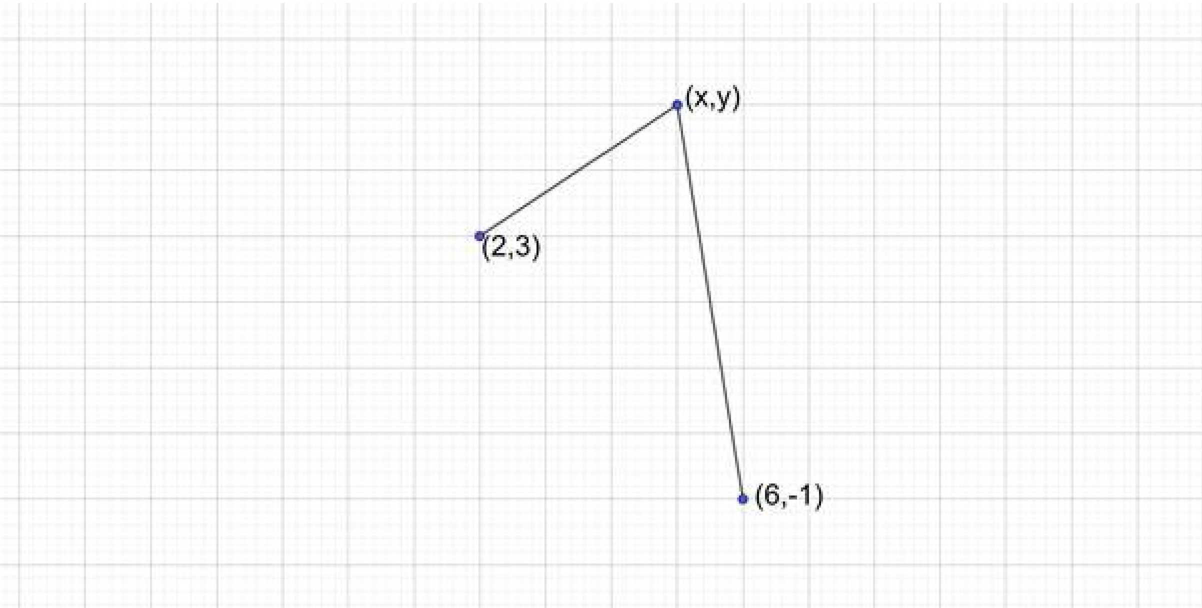
∴ The radius of the circle is $\sqrt{10}$.

11 A. Question

If the point (x, y) on the tangent is equidistant from the points (2, 3) and (6, - 1), find the relation between x and y.

Answer

Given points are A(2, 3) and B(6, - 1). It is told that S(x, y) is equidistant from A and B.



So, we get SA = SB,

We know that distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Now,

⇒ SA = SB

⇒ $SA^2 = SB^2$

⇒ $(x - 2)^2 + (y - 3)^2 = (x - 6)^2 + (y - (- 1))^2$

⇒ $(x - 2)^2 + (y - 3)^2 = (x - 6)^2 + (y + 1)^2$

⇒ $x^2 - 4x + 4 + y^2 - 6y + 9 = x^2 - 12x + 36 + y^2 + 2y + 1$

⇒ 8x - 8y = 24

⇒ x - y = 3

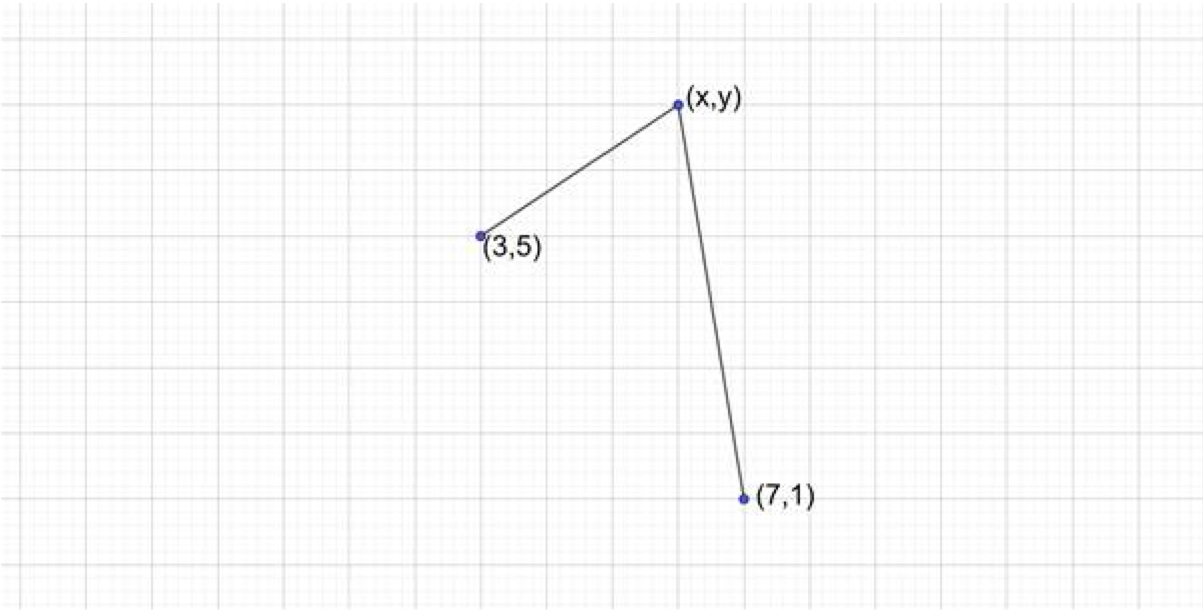
∴ The relation between x and y is x - y = 3.

11 B. Question

Find a relation between x and y such that the point (x, y) is equidistant from points (7, 1) and (3, 5).

Answer

Given points are A(7, 1) and B(3, 5). It is told that S(x, y) is equidistant from A and B.



So, we get SA = SB,

We know that distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Now,

$\Rightarrow$ SA = SB

$\Rightarrow SA^2 = SB^2$

$\Rightarrow (x - 7)^2 + (y - 1)^2 = (x - 3)^2 + (y - 5)^2$

$\Rightarrow x^2 - 14x + 49 + y^2 - 2y + 1 = x^2 - 6x + 9 + y^2 - 10y + 25$

$\Rightarrow 8x - 8y = 16$

$\Rightarrow x - y = 2$

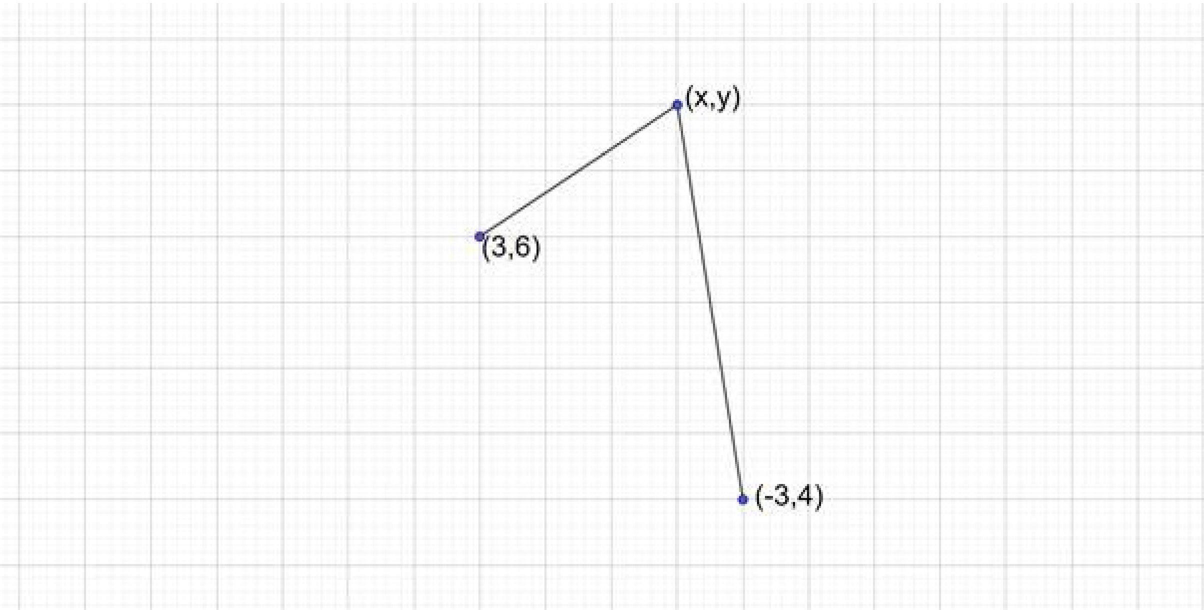
$\therefore$ The relation between x and y is $x - y = 2$.

12 A. Question

If the distances of P(x, y) from points A(3, 6) and B(- 3, 4) are equal, prove that $3x + y = 5$

Answer

Given points are A(3, 6) and B(- 3, 4). It is told that S(x, y) is equidistant from A and B.



So, we get SA = SB,

We know that distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Now,

⇒ SA = SB

⇒ $SA^2 = SB^2$

⇒ $(x - 3)^2 + (y - 6)^2 = (x - (- 3))^2 + (y - 4)^2$

⇒ $(x - 3)^2 + (y - 6)^2 = (x + 3)^2 + (y - 4)^2$

⇒ $x^2 - 6x + 9 + y^2 - 12y + 36 = x^2 + 6x + 9 + y^2 - 8y + 16$

⇒ 12x + 4y = 20

⇒ 3x + y = 5

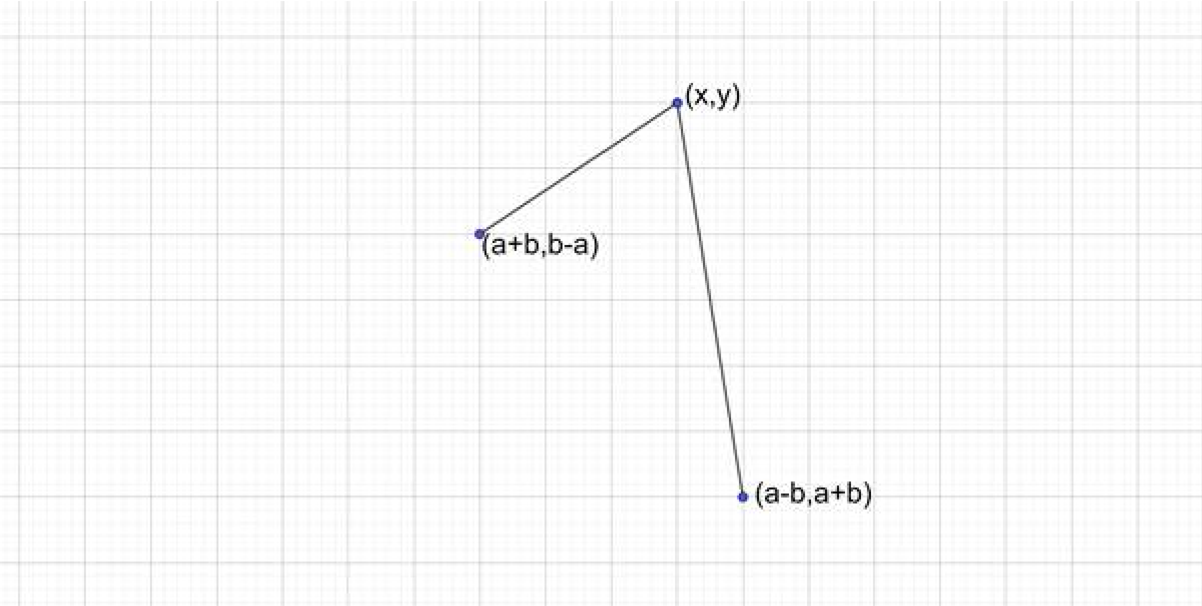
∴ Thus proved.

12 B. Question

If the point (x, y) be equidistant from the points (a + b, b - a) and (a - b, a + b), prove that $\frac{a-b}{a+b}=\frac{x-y}{x+y}$

Answer

Given points are A(a + b, b - a) and B(a - b, a + b). It is told that S(x, y) is equidistant from A and B.



So, we get SA = SB,

We know that distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Now,

⇒ SA = SB

⇒ $SA^2 = SB^2$

⇒ $(x - (a + b))^2 + (y - (b - a))^2 = (x - (a - b))^2 + (y - (a + b))^2$

⇒ $x^2 - 2(a + b)x + (a + b)^2 + y^2 - 2(b - a)y + (b - a)^2 = x^2 - 2(a - b)x + (a - b)^2 + y^2 - 2(a + b)y + (a + b)^2$

⇒ x(- 2a - 2b + 2a - 2b) = y(2b - 2a - 2a - 2b)

⇒ x(- 4b) = y(- 4a)

⇒ x(b) = y(a)

⇒ $\frac{x}{y} = \frac{a}{b}$

Applying componendo and dividendo,

⇒ $\frac{x-y}{x+y} = \frac{a-b}{a+b}$

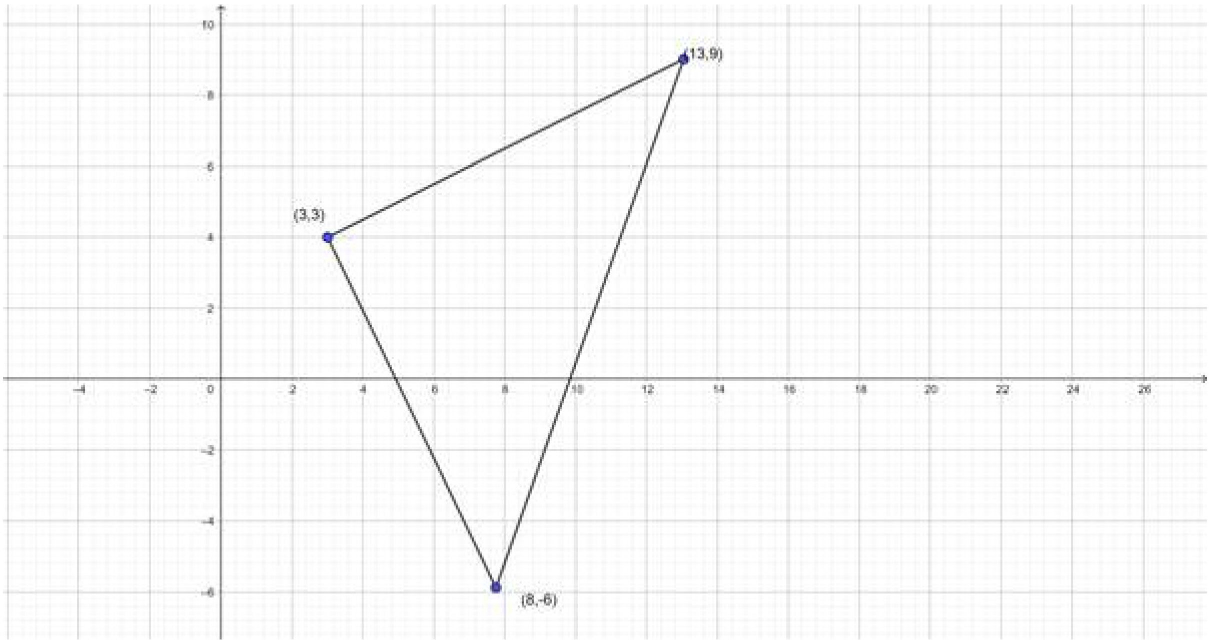
∴ Thus proved.

13. Question

Prove that the points (3, 4), (8, - 6) and (13, 9) are the vertices of a right angled triangle.

Answer

Given points are A(3, 4), B(8, - 6) and C(13, 9).



Let us find the distance between sides AB, BC and CA.

We know that distance between the two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

$$\Rightarrow AB = \sqrt{(3-8)^2 + \left(4-(-6)\right)^2}$$

$$\Rightarrow AB = \sqrt{(3-8)^2 + (4+6)^2}$$

$$\Rightarrow AB = \sqrt{(-5)^2 + (10)^2}$$

$$\Rightarrow AB = \sqrt{25 + 100}$$

$\Rightarrow AB = \sqrt{125}$

$$\Rightarrow BC = \sqrt{(8-13)^2 + (-6-9)^2}$$

$$\Rightarrow BC = \sqrt{(-5)^2 + (-15)^2}$$

$$\Rightarrow BC = \sqrt{25 + 225}$$

$\Rightarrow BC = \sqrt{250}$

$$\Rightarrow CA = \sqrt{(13-3)^2 + (9-4)^2}$$

$$\Rightarrow CA = \sqrt{(10)^2 + (5)^2}$$

$\Rightarrow CA = \sqrt{100 + 25}$

$\Rightarrow CA = \sqrt{125}$

Now,

$\Rightarrow AB^2 + CA^2 = (\sqrt{125})^2 + (\sqrt{125})^2$

$\Rightarrow AB^2 + CA^2 = 125 + 125$

$\Rightarrow AB^2 + CA^2 = 250$

$\Rightarrow AB^2 + CA^2 = (\sqrt{250})^2$

$\Rightarrow AB^2 + CA^2 = BC^2$

∴ The given points form a right angled isosceles triangle.

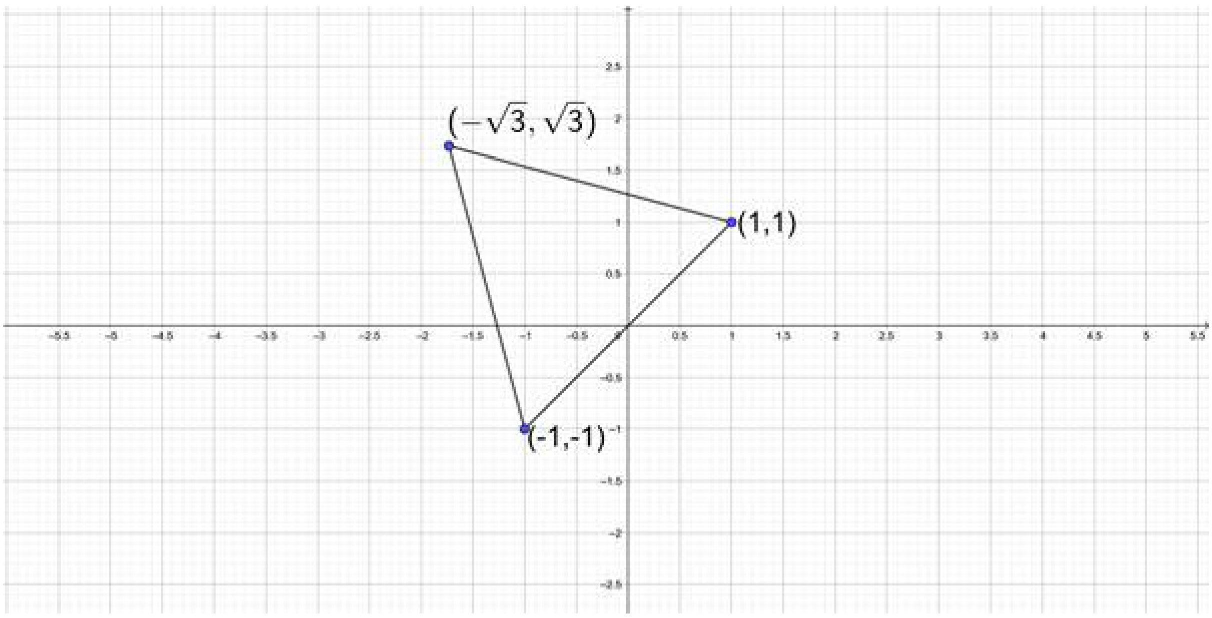
14 A. Question

Determine the type (isosceles, right angled, right angled isosceles, equilateral, scalene) of the following triangles whose vertices are:

(1, 1), (- $\sqrt{3}$, $\sqrt{3}$), (- 1, - 1)

Answer

Given points are A(1, 1), B(- $\sqrt{3}$, $\sqrt{3}$) and C(- 1, - 1).



Let us find the distance between sides AB, BC and CA.

We know that the distance between the two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$\Rightarrow AB = \sqrt{\left(1-(-\sqrt{3})\right)^2 + \left(1-\sqrt{3}\right)^2}$

$\Rightarrow AB = \sqrt{\left(1 + \sqrt{3}\right)^2 + \left(1-\sqrt{3}\right)^2}$

$\Rightarrow AB = \sqrt{1 + 2\sqrt{3} + 3 + 1 - 2\sqrt{3} + 3}$

$\Rightarrow AB = \sqrt{8}$

$\Rightarrow BC = \sqrt{\left(-\sqrt{3}-(-1)\right)^2 + \left(\sqrt{3}-(-1)\right)^2}$

$\Rightarrow BC = \sqrt{\left(1-\sqrt{3}\right)^2 + \left(1 + \sqrt{3}\right)^2}$

$\Rightarrow BC = \sqrt{1 - 2\sqrt{3} + 3 + 1 + 2\sqrt{3} + 3}$

$\Rightarrow BC = \sqrt{8}$

$\Rightarrow CA = \sqrt{(-1-1)^2 + (-1-1)^2}$

$\Rightarrow CA = \sqrt{(-2)^2 + (-2)^2}$

$\Rightarrow CA = \sqrt{4 + 4}$

$\Rightarrow CA = \sqrt{8}$

We got AB = BC = CA

∴ The given points form an equilateral triangle.

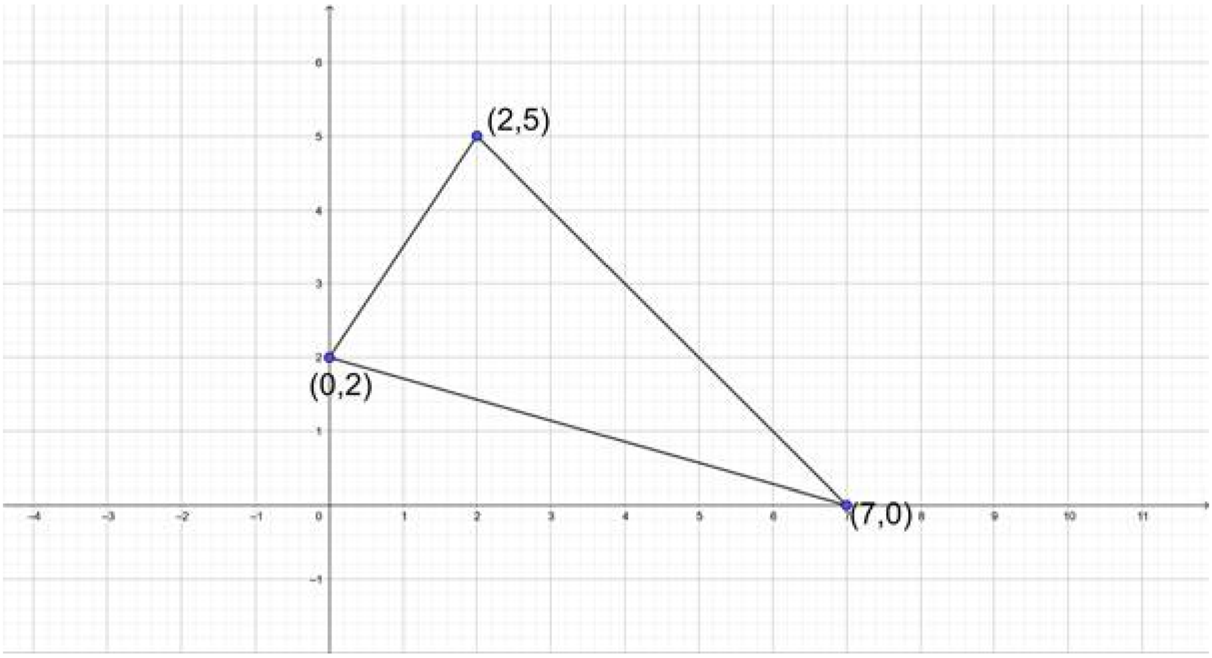
14 B. Question

Determine the type (isosceles, right angled, right angled isosceles, equilateral, scalene) of the following triangles whose vertices are:

(0, 2), (7, 0), (2, 5)

Answer

Given points are A(0, 2), B(7, 0) and C(2, 5).



Let us find the distance between sides AB, BC and CA.

We know that the distance between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

$\Rightarrow AB = \sqrt{(0-7)^2 + (2-0)^2}$

$\Rightarrow AB = \sqrt{(-7)^2 + (2)^2}$

$\Rightarrow AB = \sqrt{49 + 4}$

$\Rightarrow AB = \sqrt{53}$

$\Rightarrow BC = \sqrt{(7-2)^2 + (0-5)^2}$

$\Rightarrow BC = \sqrt{(5)^2 + (-5)^2}$

$\Rightarrow BC = \sqrt{25 + 25}$

$\Rightarrow BC = \sqrt{50}$

$\Rightarrow CA = \sqrt{(2-0)^2 + (5-2)^2}$

$\Rightarrow CA = \sqrt{(2)^2 + (3)^2}$

$\Rightarrow CA = \sqrt{4 + 9}$

$\Rightarrow CA = \sqrt{13}$

We got $AB \neq BC \neq CA$

∴ The given points form a scalene triangle.

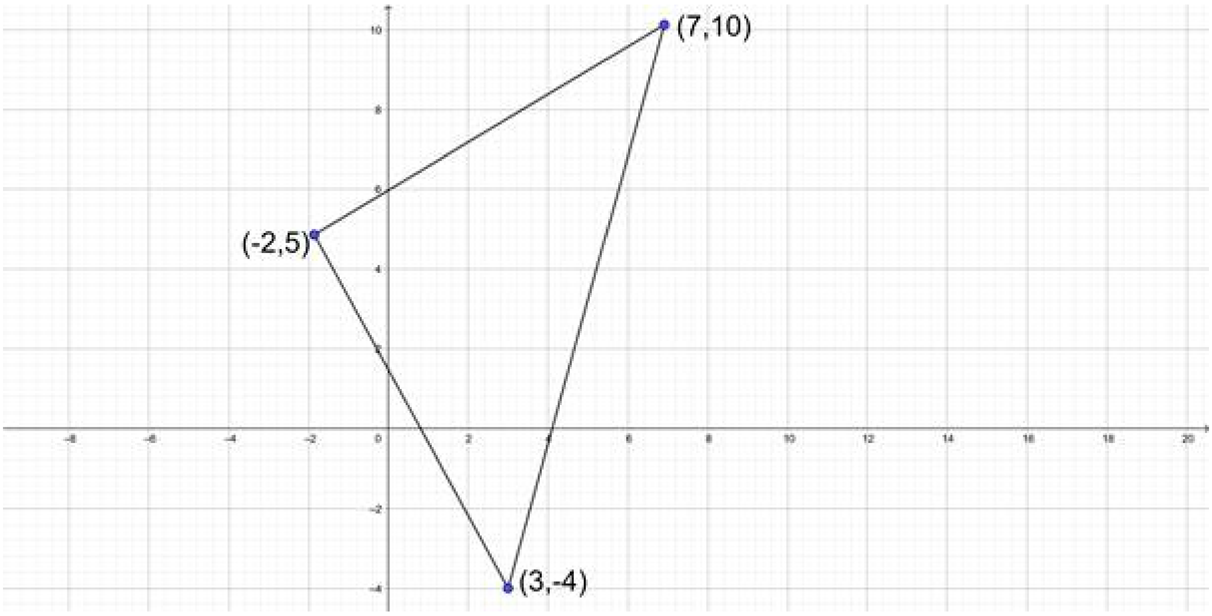
14 C. Question

Determine the type (isosceles, right angled, right angled isosceles, equilateral, scalene) of the following triangles whose vertices are:

(- 2, 5), (7, 10), (3, - 4)

Answer

Given points are A(- 2, 5), B(7, 10) and C(3, - 4).



Let us find the distance between sides AB, BC and CA.

We know that the distance between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

$\Rightarrow AB = \sqrt{(-2-7)^2 + (5-10)^2}$

$\Rightarrow AB = \sqrt{(-9)^2 + (-5)^2}$

$\Rightarrow AB = \sqrt{81 + 25}$

$\Rightarrow AB = \sqrt{106}$

$\Rightarrow BC = \sqrt{(7-3)^2 + (10-(-4))^2}$

$\Rightarrow BC = \sqrt{(7-3)^2 + (10+4)^2}$

$\Rightarrow BC = \sqrt{4^2 + 14^2}$

$\Rightarrow BC = \sqrt{16 + 196}$

$$\Rightarrow BC = \sqrt{212}$$

$$\Rightarrow CA = \sqrt{\left(3-(-2)\right)^2 + (-4-5)^2}$$

$$\Rightarrow CA = \sqrt{(3+2)^2 + (-4-5)^2}$$

$$\Rightarrow CA = \sqrt{(5)^2 + (-9)^2}$$

$$\Rightarrow CA = \sqrt{25 + 81}$$

$$\Rightarrow CA = \sqrt{106}$$

We got AB = CA

Now,

$$\Rightarrow AB^2 + CA^2 = \left(\sqrt{106}\right)^2 + \left(\sqrt{106}\right)^2$$

$\Rightarrow AB^2 + CA^2 = 106 + 106$

$\Rightarrow AB^2 + CA^2 = 212$

$$\Rightarrow AB^2 + CA^2 = \left(\sqrt{212}\right)^2$$

$\Rightarrow AB^2 + CA^2 = BC^2$

∴ The given points form a right angles isosceles triangle.

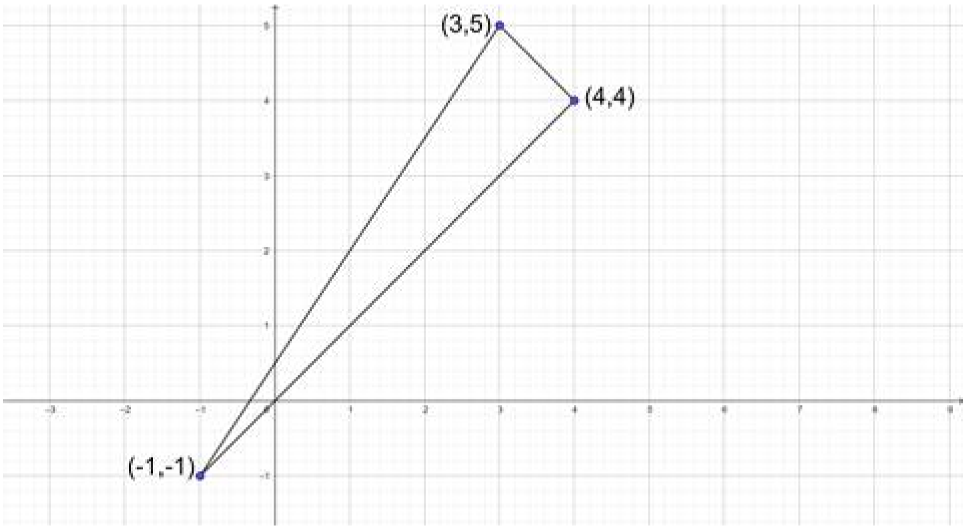
14 D. Question

Determine the type (isosceles, right angled, right angled isosceles, equilateral, scalene) of the following triangles whose vertices are:

(4, 4), (3, 5), (- 1, - 1)

Answer

Given points are A(4, 4), B(3, 5) and C(- 1, - 1).



Let us find the distance between sides AB, BC and CA.

We know that the distance between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

$\Rightarrow AB = \sqrt{(4-3)^2 + (4-5)^2}$

$\Rightarrow AB = \sqrt{(1)^2 + (-1)^2}$

$\Rightarrow AB = \sqrt{1 + 1}$

$\Rightarrow AB = \sqrt{2}$

$\Rightarrow BC = \sqrt{(3-(-1))^2 + (5-(-1))^2}$

$\Rightarrow BC = \sqrt{(3 + 1)^2 + (5 + 1)^2}$

$\Rightarrow BC = \sqrt{(4)^2 + (6)^2}$

$\Rightarrow BC = \sqrt{16 + 36}$

$\Rightarrow BC = \sqrt{52}$

$\Rightarrow CA = \sqrt{(-1-4)^2 + (-1-4)^2}$

$\Rightarrow CA = \sqrt{(-5)^2 + (-5)^2}$

$\Rightarrow CA = \sqrt{25 + 25}$

$\Rightarrow CA = \sqrt{50}$

Now,

$\Rightarrow AB^2 + CA^2 = (\sqrt{2})^2 + (\sqrt{50})^2$

$\Rightarrow AB^2 + CA^2 = 2 + 50$

$\Rightarrow AB^2 + CA^2 = 52$

$\Rightarrow AB^2 + CA^2 = (\sqrt{52})^2$

$\Rightarrow AB^2 + CA^2 = BC^2$

∴ The given points form a right - angled triangle.

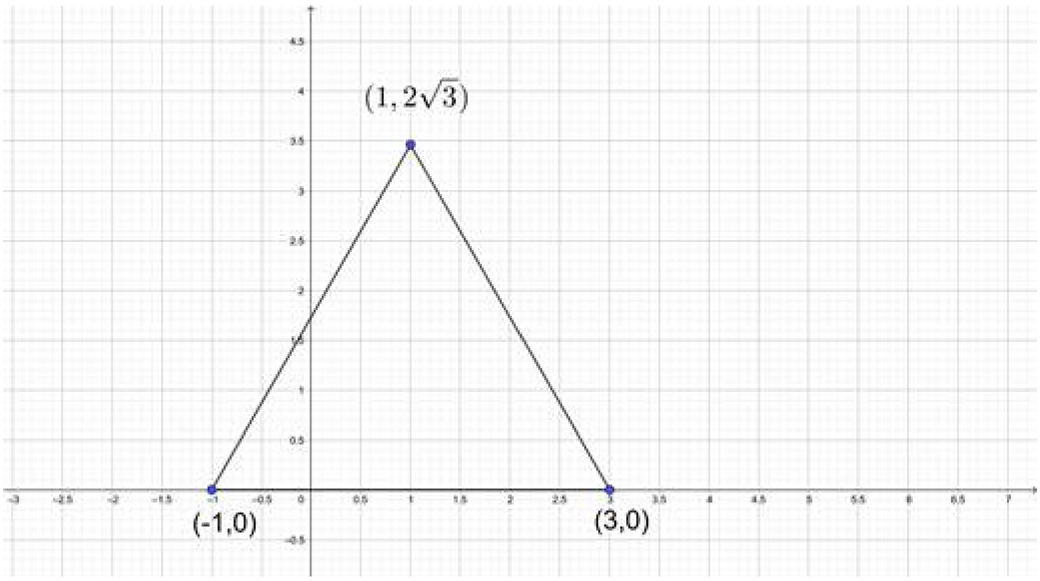
14 E. Question

Determine the type (isosceles, right angled, right angled isosceles, equilateral, scalene) of the following triangles whose vertices are:

(1, 2$\sqrt{3}$), (3, 0), (- 1, 0)

Answer

Given points are A(1, 2$\sqrt{3}$), B(3, 0) and C(- 1, 0).



Let us find the distance between sides AB, BC and CA.

We know that the distance between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

$\Rightarrow AB = \sqrt{(1 - 3)^2 + (2\sqrt{3} - 0)^2}$

$\Rightarrow AB = \sqrt{(-2)^2 + (2\sqrt{3})^2}$

$\Rightarrow AB = \sqrt{4 + 12}$

$\Rightarrow AB = \sqrt{16}$

$\Rightarrow$ AB = 4

$\Rightarrow BC = \sqrt{(3 - (-1))^2 + (0 - 0)^2}$

$\Rightarrow BC = \sqrt{(3 + 1)^2 + (0 - 0)^2}$

$\Rightarrow BC = \sqrt{4^2}$

$\Rightarrow BC = 4$

$\Rightarrow CA = \sqrt{(-1 - 1)^2 + \left(0 - 2\sqrt{3}\right)^2}$

$\Rightarrow CA = \sqrt{(-2)^2 + \left(-2\sqrt{3}\right)^2}$

$\Rightarrow CA = \sqrt{4 + 12}$

$\Rightarrow CA = \sqrt{16}$

$\Rightarrow$ CA = 4

We got AB = BC = CA

∴ The given points form an equilateral triangle.

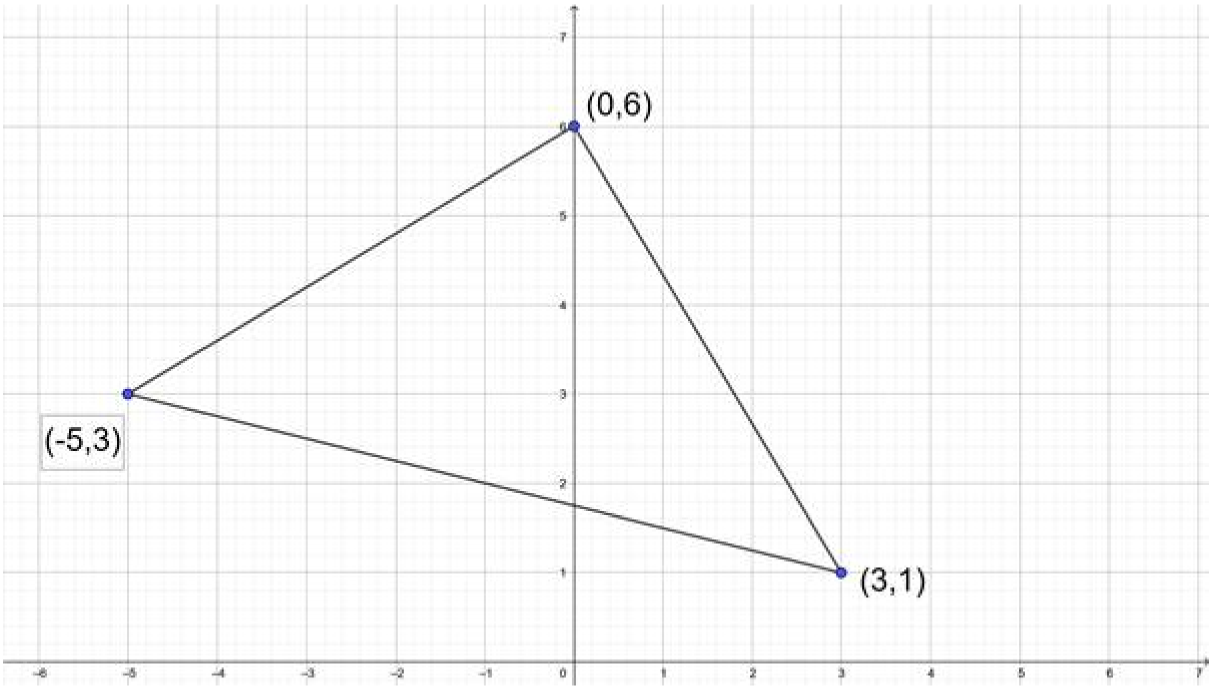
14 F. Question

Determine the type (isosceles, right angled, right angled isosceles, equilateral, scalene) of the following triangles whose vertices are:

(0, 6), (- 5, 3), (3, 1)

Answer

Given points are A(0, 6), B(- 5, 3) and C(3, 1).



Let us find the distance between sides AB, BC and CA.

We know that the distance between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

$\Rightarrow AB = \sqrt{(0 - (-5))^2 + (6 - 3)^2}$

$\Rightarrow AB = \sqrt{(0 + 5)^2 + (6 - 3)^2}$

$\Rightarrow AB = \sqrt{5^2 + 3^2}$

$\Rightarrow AB = \sqrt{25 + 9}$

$\Rightarrow AB = \sqrt{34}$

$\Rightarrow BC = \sqrt{(-5 - 3)^2 + (3 - 1)^2}$

$\Rightarrow BC = \sqrt{(-8)^2 + (2)^2}$

$\Rightarrow BC = \sqrt{64 + 4}$

$\Rightarrow BC = \sqrt{68}$

$\Rightarrow CA = \sqrt{(3 - 0)^2 + (1 - 6)^2}$

$\Rightarrow CA = \sqrt{(3)^2 + (-5)^2}$

$\Rightarrow CA = \sqrt{9 + 25}$

$\Rightarrow CA = \sqrt{34}$

We got AB = CA

Now,

$\Rightarrow AB^2 + CA^2 = (\sqrt{34})^2 + (\sqrt{34})^2$

$\Rightarrow AB^2 + CA^2 = 34 + 34$

$\Rightarrow AB^2 + CA^2 = 68$

$\Rightarrow AB^2 + CA^2 = (\sqrt{68})^2$

$\Rightarrow AB^2 + CA^2 = BC^2$

∴ The given points form a right - angled isosceles triangle.

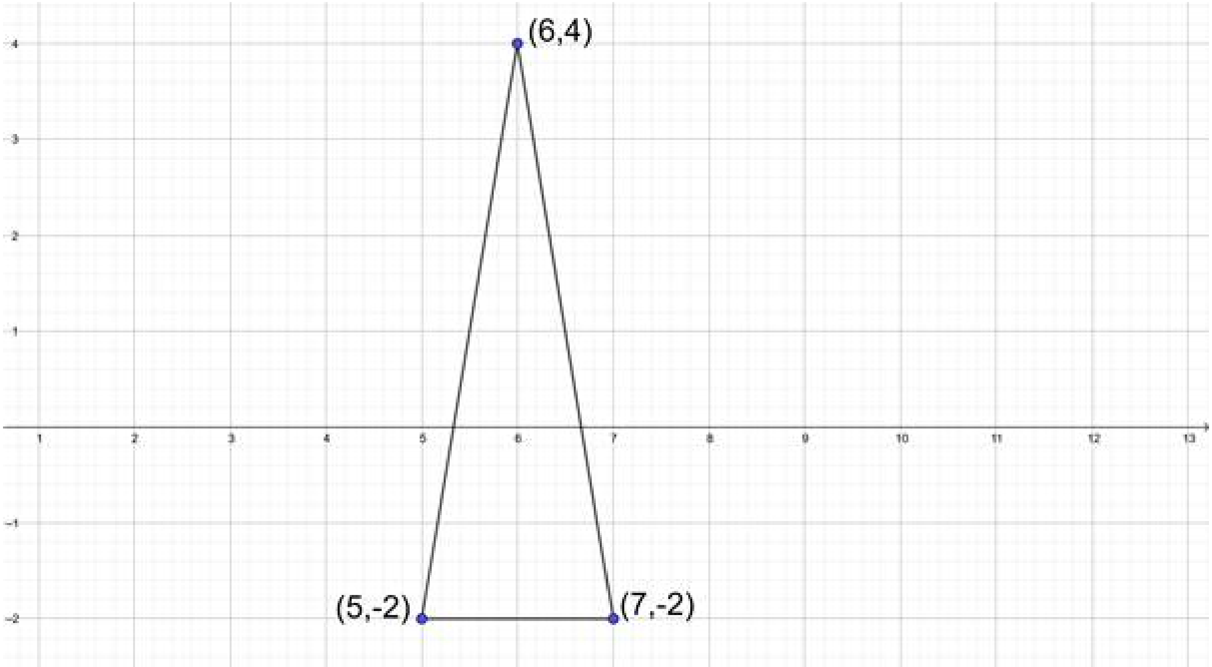
14 G. Question

Determine the type (isosceles, right angled, right angled isosceles, equilateral, scalene) of the following triangles whose vertices are:

(5, - 2), (6, 4), (7, - 2)

Answer

Given points are A(5, - 2), B(6, 4) and C(7, - 2).



Let us find the distance between sides AB, BC and CA.

We know that the distance between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

$\Rightarrow AB = \sqrt{(5-6)^2 + (-2-4)^2}$

$\Rightarrow AB = \sqrt{(-1)^2 + (-6)^2}$

$\Rightarrow AB = \sqrt{1 + 36}$

$\Rightarrow AB = \sqrt{37}$

$\Rightarrow BC = \sqrt{(6-7)^2 + (4-(-2))^2}$

$\Rightarrow BC = \sqrt{(6-7)^2 + (4+2)^2}$

$\Rightarrow BC = \sqrt{(-1)^2 + (6)^2}$

$\Rightarrow BC = \sqrt{1 + 36}$

$\Rightarrow BC = \sqrt{37}$

$\Rightarrow CA = \sqrt{(7-5)^2 + (-2-(-2))^2}$

$\Rightarrow CA = \sqrt{(7-5)^2 + (-2+2)^2}$

$\Rightarrow CA = \sqrt{(2)^2 + (0)^2}$

$\Rightarrow CA = \sqrt{4}$

$\Rightarrow CA = 2$

We got AB = BC≠CA

∴ The given points form an isosceles triangle.

15. Question

If $A(at^2, 2at)$, $B\left(\frac{a}{t^2}, \frac{2a}{t}\right)$ and C(a, 0) be any three points, show that $\frac{1}{AC} + \frac{1}{BC}$ is independent of t.

Answer

Given points are $A(at^2, 2at)$, $B\left(\frac{a}{t^2}, \frac{2a}{t}\right)$ and C(a, 0).

Let us find the distance between sides AB, BC and CA.

We know that distance between the two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

$$\Rightarrow AC = \sqrt{(at^2 - a)^2 + (2at - 0)^2}$$

$$\Rightarrow AC = \sqrt{a^2t^4 + a^2 - 2a^2t^2 + 4a^2t^2}$$

$$\Rightarrow AC = \sqrt{a^2t^4 + 2a^2t^2 + a^2}$$

$$\Rightarrow AC = \sqrt{(at^2 + a)^2}$$

$\Rightarrow AC = at^2 + a$

$$\Rightarrow BC = \sqrt{\left(\frac{a}{t^2} - a\right)^2 + \left(\frac{2a}{t} - 0\right)^2}$$

$$\Rightarrow BC = \sqrt{\frac{a^2}{t^4} + a^2 - \frac{2a^2}{t^2} + \frac{4a^2}{t^2}}$$

$$\Rightarrow BC = \sqrt{\frac{a^2}{t^4} + \frac{2a^2}{t^2} + a^2}$$

$$\Rightarrow BC = \sqrt{\left(\frac{a}{t^2} + a\right)^2}$$

$$\Rightarrow BC = \frac{a}{t^2} + a$$

$$\Rightarrow BC = \frac{a + at^2}{t^2}$$

Now,

$$\Rightarrow \frac{1}{AC} + \frac{1}{BC} = \frac{1}{at^2 + a} + \frac{1}{\frac{a + at^2}{t^2}}$$

$$\Rightarrow \frac{1}{AC} + \frac{1}{BC} = \frac{1}{at^2 + a} + \frac{t^2}{a + at^2}$$

$$\Rightarrow \frac{1}{AC} + \frac{1}{BC} = \frac{1 + t^2}{a(1 + t^2)}$$

$\Rightarrow \frac{1}{AC} + \frac{1}{BC} = \frac{1}{a}$

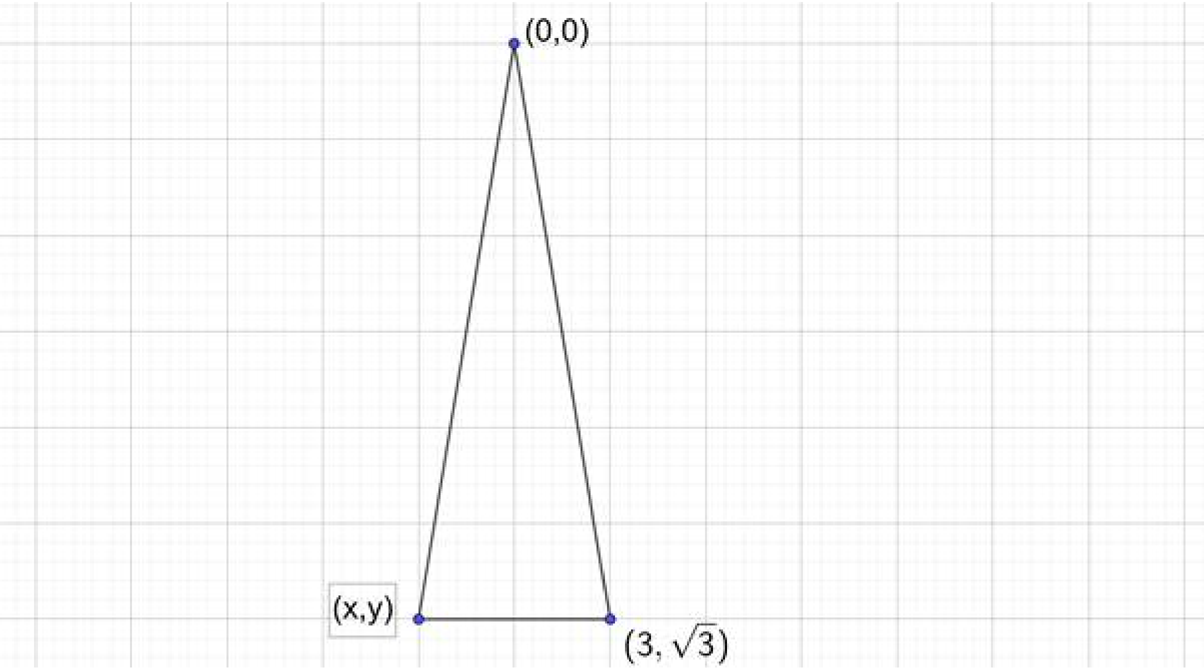
$\therefore \frac{1}{AC} + \frac{1}{BC}$ is independent of t.

16. Question

If two vertices of an equilateral triangle be (0, 0) and (3, $\sqrt{3}$), find the co - ordinates of the third vertex.

Answer

Given that A(0, 0) and B(3, $\sqrt{3}$) are two vertices of an equilateral triangle.



Let us assume C(x, y) be the third vertex of the triangle.

We have AB = BC = CA

We know that the distance between the two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Now,

$\Rightarrow$ BC = CA

$\Rightarrow BC^2 = CA^2$

$\Rightarrow (3 - x)^2 + (\sqrt{3} - y)^2 = (x - 0)^2 + (y - 0)^2$

$\Rightarrow x^2 - 6x + 9 + 3 + y^2 - 2\sqrt{3}y = x^2 + y^2$

$\Rightarrow 6x = 12 - 2\sqrt{3}y$

$\Rightarrow x = \frac{12-2\sqrt{3}y}{6}$ ..... - (1)

$\Rightarrow AB = BC$

$\Rightarrow AB^2 = BC^2$

$\Rightarrow (0 - 3)^2 + (0 - \sqrt{3})^2 = (3 - x)^2 + (\sqrt{3} - y)^2$

$\Rightarrow 9 + 3 = 9 - 6x + x^2 + 3 - 2\sqrt{3}y + y^2$

From (1)

$$\Rightarrow y^2 - 2\sqrt{3}y + \left(\frac{12 - 2\sqrt{3}y}{6}\right)^2 - 6\left(\frac{12 - 2\sqrt{3}y}{6}\right) = 0$$

$$\Rightarrow y^2 - 2\sqrt{3}y + \left(\frac{144 + 12y^2 - 48\sqrt{3}y}{36}\right) - 12 + 2\sqrt{3}y = 0$$

$\Rightarrow 36y^2 - 432 + 144 + 12y^2 - 48\sqrt{3}y = 0$

$\Rightarrow 48y^2 - 48\sqrt{3}y - 288 = 0$

$\Rightarrow y^2 - \sqrt{3}y - 6 = 0$

$\Rightarrow y^2 - 2\sqrt{3}y + \sqrt{3}y - 6 = 0$

$\Rightarrow y(y - 2\sqrt{3}) + \sqrt{3}(y - 2\sqrt{3}) = 0$

$\Rightarrow (y + \sqrt{3})(y - 2\sqrt{3}) = 0$

$\Rightarrow y + \sqrt{3} = 0$ (or) $y - 2\sqrt{3} = 0$

$\Rightarrow y = -\sqrt{3}$ (or) $y = 2\sqrt{3}$

From (1), for $y = \sqrt{3}$

$$\Rightarrow x = \frac{12 - 2\sqrt{3}(-\sqrt{3})}{6}$$

$$\Rightarrow x = \frac{12 + 6}{6}$$

$$\Rightarrow x = \frac{18}{6}$$

$\Rightarrow x = 3$

From (1), for $y = 2\sqrt{3}$

$$\Rightarrow\ x = \frac{12 - 2\sqrt{3}(2\sqrt{3})}{6}$$

$$\Rightarrow\ x = \frac{12 - 12}{6}$$

⇒ x = 0

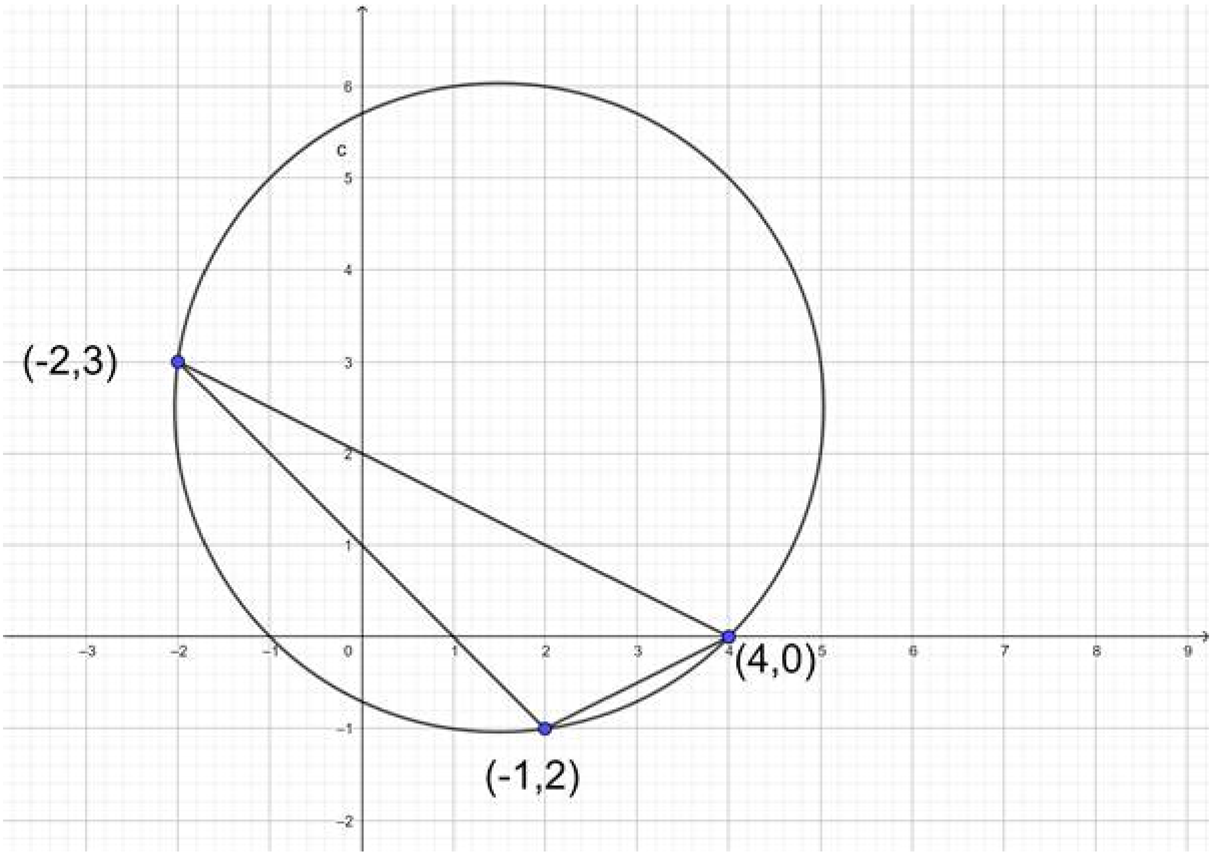
∴ The third vertex of equilateral triangle is (0, 2√3) and (3, √3).

17 A. Question

Find the circum - centre and circum - radius of the triangle whose vertices are (- 2, 3), (2, - 1) and (4, 0).

Answer

Given that we need to find the circum - centre and circum - radius of the triangle whose vertices are A(- 2, 3), B(2, - 1), C(4, 0).



Let us assume O(x, y) be the Circum - centre of the circle.

We know that distance from circum - centre to any vertex is equal.

So, OA = OB = OC

We know that distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Now,

$\Rightarrow$ OA = OB

$\Rightarrow OA^2 = OB^2$

$\Rightarrow (x - (-2))^2 + (y - 3)^2 = (x - 2)^2 + (y - (-1))^2$

$\Rightarrow (x + 2)^2 + (y - 3)^2 = (x - 2)^2 + (y + 1)^2$

$\Rightarrow x^2 + 4x + 4 + y^2 - 6y + 9 = x^2 - 4x + 4 + y^2 + 2y + 1$

$\Rightarrow 8x - 8y = -8$

$\Rightarrow x - y = -1$ ..... (1)

Now,

$\Rightarrow$ OB = OC

$\Rightarrow OB^2 = OC^2$

$\Rightarrow (x - 2)^2 + (y - (-1))^2 = (x - 4)^2 + (y - 0)^2$

$\Rightarrow (x - 2)^2 + (y + 1)^2 = (x - 4)^2 + (y)^2$

$\Rightarrow x^2 - 4x + 4 + y^2 + 2y + 1 = x^2 - 8x + 16 + y^2$

$\Rightarrow 4x + 2y = 11$ .... - (2)

On solving (1) and (2), we get

$\Rightarrow x = \frac{3}{2}$ and $y = \frac{5}{2}$

$\therefore \left(\frac{3}{2}, \frac{5}{2}\right)$ is the centre of the circle.

We know radius is the distance between the centre and any point on the circle.

Let ‘r’ be the circum - radius of the circle.

$$\Rightarrow r = OA = \sqrt{\left(\frac{3}{2} - (-2)\right)^2 + \left(\frac{5}{2} - 3\right)^2}$$

$$\Rightarrow r = \sqrt{\left(\frac{7}{2}\right)^2 + \left(-\frac{1}{2}\right)^2}$$

$$\Rightarrow r = \sqrt{\frac{49}{4} + \frac{1}{4}}$$

$$\Rightarrow r = \sqrt{\frac{50}{4}}$$

$$\Rightarrow r = \sqrt{\frac{25 \times 2}{4}}$$

$$\Rightarrow r = \frac{5\sqrt{2}}{2}$$

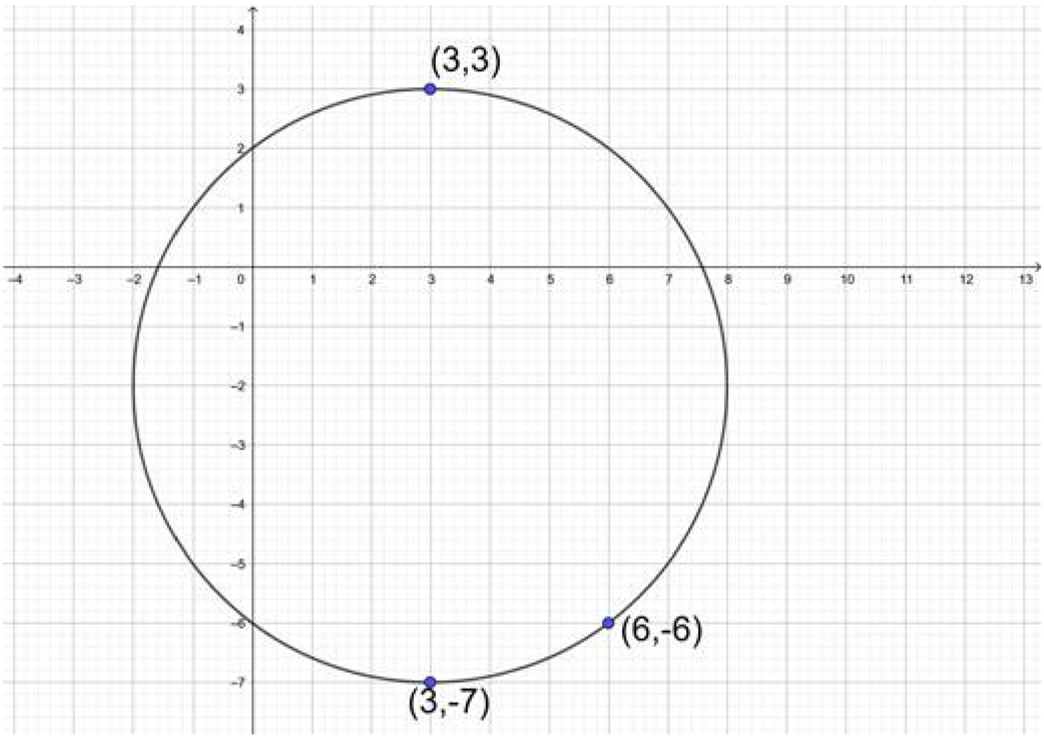
∴ The radius of the circle is $\frac{5\sqrt{2}}{2}$.

17 B. Question

Find the centre of a circle passing through the points (6, - 6), (3, - 7) and (3, 3).

Answer

Given that circle passes through the points A(6, - 6), B(3, - 7), C(3, 3).



Let us assume O(x, y) be the centre of the circle.

We know that distance from the centre to any point on h circle is equal.

So, OA = OB = OC

We know that distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Now,

$\Rightarrow$ OA = OB

$\Rightarrow OA^2 = OB^2$

$\Rightarrow (x - 6)^2 + (y - (- 6))^2 = (x - 3)^2 + (y - (- 7))^2$

$\Rightarrow (x - 6)^2 + (y + 6)^2 = (x - 3)^2 + (y + 7)^2$

$\Rightarrow x^2 - 12x + 36 + y^2 + 12y + 36 = x^2 - 6x + 9 + y^2 + 14y + 49$

$\Rightarrow 6x + 2y = 14$

$\Rightarrow 3x + y = 7$ ..... (1)

Now,

$\Rightarrow$ OB = OC

$\Rightarrow OB^2 = OC^2$

$\Rightarrow (x - 3)^2 + (y - (- 7))^2 = (x - 3)^2 + (y - 3)^2$

$\Rightarrow (x - 3)^2 + (y + 7)^2 = (x - 3)^2 + (y - 3)^2$

$\Rightarrow x^2 - 6x + 9 + y^2 + 14y + 49 = x^2 - 6x + 9 + y^2 - 6y + 9$

$\Rightarrow 20y = - 40$

$\Rightarrow y = - 2$ .... - (2)

Substituting (2) in (1), we get

$\Rightarrow x = 3$

∴ (3, - 2) is the centre of the circle.

We know radius is the distance between the centre and any point on the circle.

Let ‘r’ be the radius of the circle.

$\Rightarrow r = OA = \sqrt{(3 - 6)^2 + (-2 - (-6))^2}$

$\Rightarrow r = \sqrt{(-3)^2 + (4)^2}$

$\Rightarrow r = \sqrt{(9 + 16)}$

⇒ r = √25

⇒ r = 5

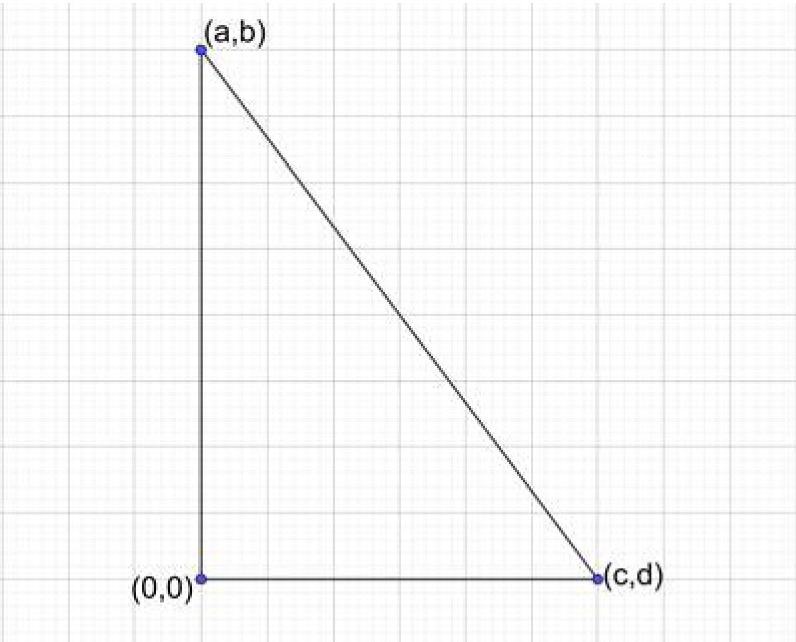
∴ The radius of the circle is 5.

18. Question

If the line segment joining the points A(a, b) and B(c, d) subtends a right angle at the origin, show that ac + bd = 0.

Answer

Given that the line segment joining the points A(a, b) and B(c, d) subtends a right angle at the origin O(0, 0)



So, AOB is a right angled triangle with right angle at O.

We got $OA^2 + OB^2 = AB^2$ [By Pythagoras Theorem]

We know that the distance between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

⇒ $OA^2 + OB^2 = AB^2$

⇒ $(0 - a)^2 + (0 - b)^2 + (0 - c)^2 + (0 - d)^2 = (a - c)^2 + (b - d)^2$

⇒ $a^2 + b^2 + c^2 + d^2 = a^2 + c^2 - 2ac + b^2 + d^2 - 2bd$

⇒ $2ac + 2bd = 0$

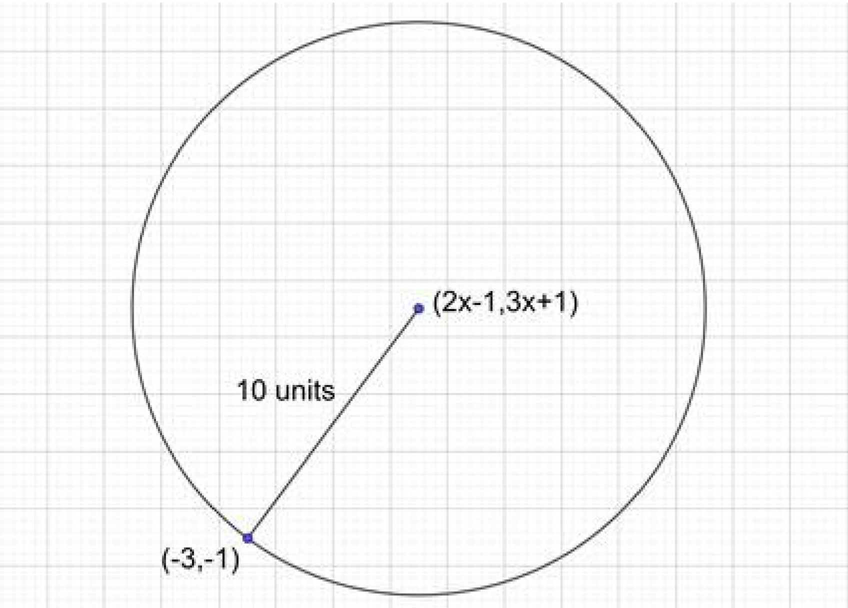
⇒ $ac + bd = 0$

19. Question

The centre of the circle is (2x - 1, 3x + 1) and radius is 10 units. Find the value of x if the circle passes through the point (- 3, - 1).

Answer

Given that the circle has centre O(2x - 1, 3x + 1) and passes through the point A(- 3, - 1) and has a radius(r) of 10 units.



We know that the radius of the circle is the distance between the centre and any point on the circle.

So, we have r = OA

$\Rightarrow$ OA = 10

$\Rightarrow OA^2 = 100$

$\Rightarrow (2x - 1 - (- 3))^2 + (3x + 1 - (- 1))^2 = 100$

$\Rightarrow (2x + 2)^2 + (3x + 2)^2 = 100$

$\Rightarrow 4x^2 + 8x + 4 + 9x^2 + 12x + 4 = 100$

$\Rightarrow 13x^2 + 20x - 92 = 0$

$\Rightarrow 13x^2 - 26x + 46x - 92 = 0$

$\Rightarrow 13x(x - 2) + 46(x - 2) = 0$

$\Rightarrow (13x + 46)(x - 2) = 0$

$\Rightarrow 13x + 46 = 0$ (or) $x - 2 = 0$

$\Rightarrow 13x = - 46$ (or) $x = 2$

$\Rightarrow x = \frac{-46}{13}$ (or) $x = 2$

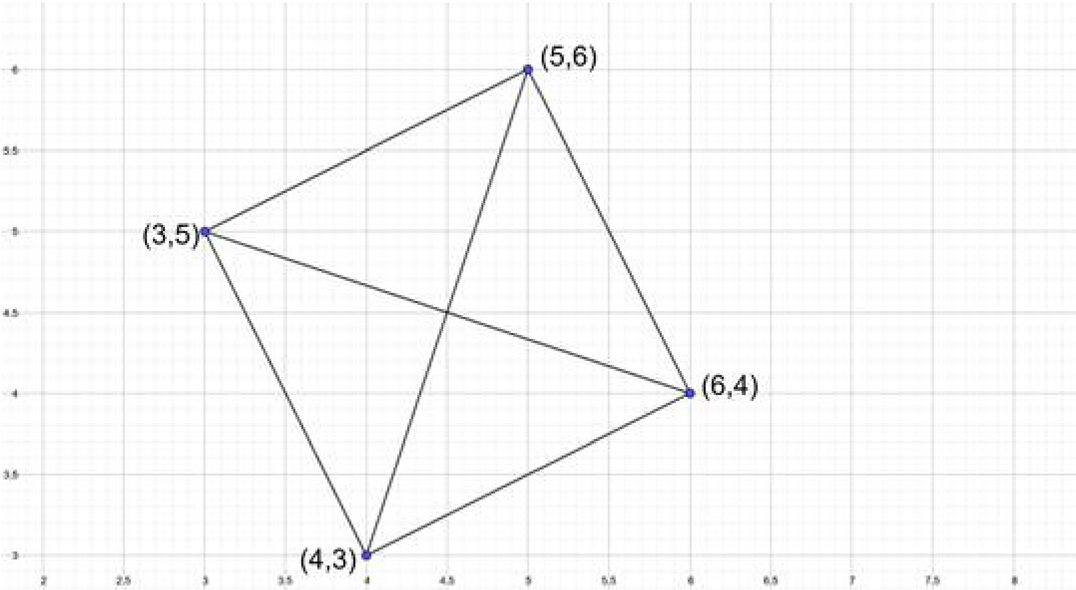
∴ The values of the x are $\frac{-46}{13}$ or 2.

20 A. Question

Prove that the points (4, 3), (6, 4), (5, 6) and (3, 5) are the vertices of a square.

Answer

Given points are A(4, 3), B(6, 4), C(5, 6) and D(3, 5).



We need to prove that these are the vertices of a square.

We know that in the lengths of all sides are equal and the lengths of the diagonals are equal.

Let us find the lengths of the sides.

We know that the distance between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Now,

$\Rightarrow AB = \sqrt{(4-6)^2 + (3-4)^2}$

$\Rightarrow AB = \sqrt{(-2)^2 + (-1)^2}$

$\Rightarrow AB = \sqrt{4 + 1}$

$\Rightarrow AB = \sqrt{5}$

$\Rightarrow BC = \sqrt{(6-5)^2 + (4-6)^2}$

$\Rightarrow BC = \sqrt{(1)^2 + (-2)^2}$

$\Rightarrow BC = \sqrt{1 + 4}$

⇒ BC = √5

$\Rightarrow CD = \sqrt{(5-3)^2 + (6-5)^2}$

$\Rightarrow CD = \sqrt{(2)^2 + (1)^2}$

$\Rightarrow CD = \sqrt{4 + 1}$

⇒ CD = √5

$\Rightarrow DA = \sqrt{(3-4)^2 + (5-3)^2}$

$\Rightarrow DA = \sqrt{(-1)^2 + (2)^2}$

$\Rightarrow DA = \sqrt{1 + 4}$

⇒ DA = √5

We got AB = BC = CD = DA, this may be square (or) rhombus.

Now we find the lengths of the diagonals.

$\Rightarrow AC = \sqrt{(4-5)^2 + (3-6)^2}$

$\Rightarrow AC = \sqrt{(-1)^2 + (-3)^2}$

$\Rightarrow AC = \sqrt{1 + 9}$

⇒ AC = √10

$\Rightarrow BD = \sqrt{(6-3)^2 + (4-5)^2}$

$\Rightarrow BD = \sqrt{(3)^2 + (-1)^2}$

$\Rightarrow BD = \sqrt{9 + 1}$

⇒ BD = √10

We got AC = BD.

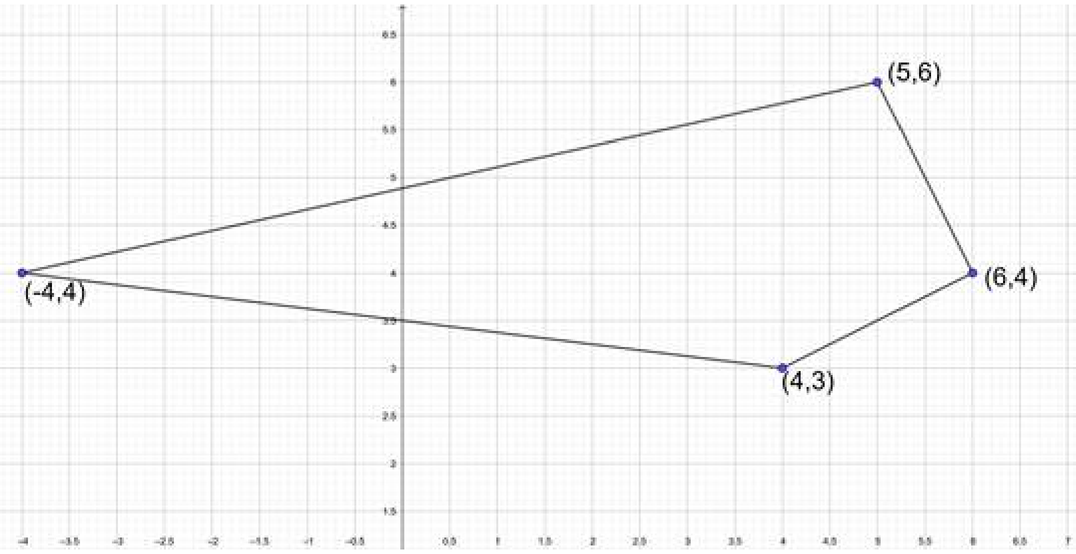
∴ The points form a square.

20 B. Question

Prove that the points (4, 3), (6, 4), (5, 6) and (- 4, 4) are the vertices of a square.

Answer

Given points are A(4, 3), B(6, 4), C(5, 6) and D(- 4, 4).



We need to prove that these are the vertices of a square.

We know that in the lengths of all sides are equal and the lengths of the diagonals are equal.

Let us find the lengths of the sides.

We know that the distance between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Now,

$\Rightarrow AB = \sqrt{(4-6)^2 + (3-4)^2}$

$\Rightarrow AB = \sqrt{(-2)^2 + (-1)^2}$

$\Rightarrow AB = \sqrt{4 + 1}$

$\Rightarrow AB = \sqrt{5}$

$\Rightarrow BC = \sqrt{(6-5)^2 + (4-6)^2}$

$\Rightarrow BC = \sqrt{(1)^2 + (-2)^2}$

$\Rightarrow BC = \sqrt{1 + 4}$

$\Rightarrow BC = \sqrt{5}$

$\Rightarrow CD = \sqrt{(5-(-4))^2 + (6-4)^2}$

$\Rightarrow CD = \sqrt{(9)^2 + (2)^2}$

$\Rightarrow CD = \sqrt{81 + 4}$

$\Rightarrow CD = \sqrt{85}$

$\Rightarrow DA = \sqrt{(-4-4)^2 + (4-3)^2}$

$\Rightarrow DA = \sqrt{(-8)^2 + (1)^2}$

$\Rightarrow DA = \sqrt{64 + 1}$

$\Rightarrow DA = \sqrt{65}$

We got AB = BC≠CD≠DA,

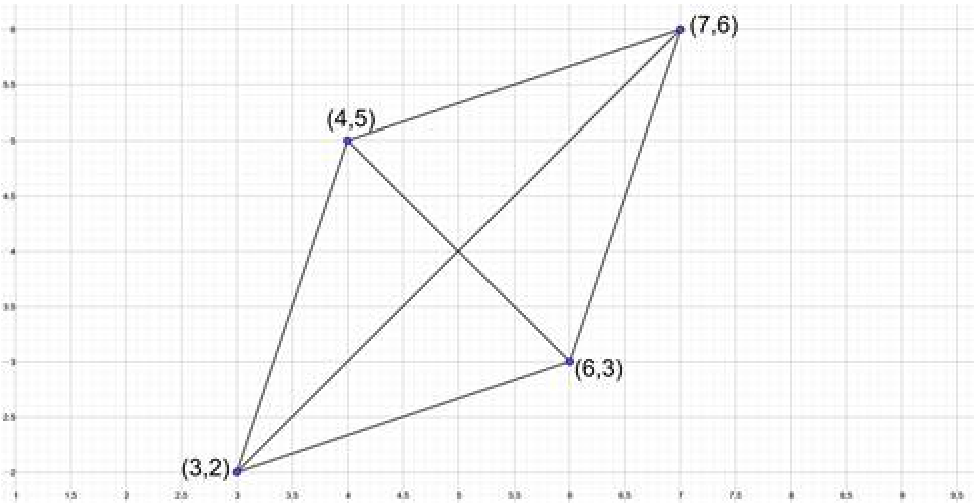
∴ The points doesn't form a square.

21. Question

Prove that the points (3, 2), (6, 3), (7, 6), (4, 5) are the vertices of a parallelogram. Is it a rectangle?

Answer

Given points are A(3, 2), B(6, 3), C(7, 6) and D(4, 5).



We need to prove that these are the vertices of a parallelogram.

We know that in the lengths of opposite sides are equal in a parallelogram.

Let us find the lengths of the sides.

We know that the distance between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Now,

$\Rightarrow AB = \sqrt{(3-6)^2 + (2-3)^2}$

$\Rightarrow AB = \sqrt{(-3)^2 + (-1)^2}$

$\Rightarrow AB = \sqrt{9 + 1}$

$\Rightarrow AB = \sqrt{10}$

$\Rightarrow BC = \sqrt{(6-7)^2 + (3-6)^2}$

$\Rightarrow BC = \sqrt{(-1)^2 + (-3)^2}$

$\Rightarrow BC = \sqrt{1 + 9}$

$\Rightarrow BC = \sqrt{10}$

$\Rightarrow CD = \sqrt{(7-4)^2 + (6-5)^2}$

$\Rightarrow CD = \sqrt{(3)^2 + (1)^2}$

$\Rightarrow CD = \sqrt{9 + 1}$

$\Rightarrow CD = \sqrt{10}$

$\Rightarrow DA = \sqrt{(4-3)^2 + (5-2)^2}$

$\Rightarrow DA = \sqrt{(1)^2 + (3)^2}$

$\Rightarrow DA = \sqrt{1 + 9}$

$\Rightarrow DA = \sqrt{10}$

We got AB = CD and BC = DA, these are the vertices of a parallelogram.

Now we find the lengths of the diagonals.

$\Rightarrow AC = \sqrt{(3-7)^2 + (2-6)^2}$

$\Rightarrow AC = \sqrt{(-4)^2 + (-4)^2}$

$\Rightarrow AC = \sqrt{16 + 16}$

$\Rightarrow$ AC = $\sqrt{32}$

$\Rightarrow \; BD = \sqrt{(6-4)^2 + (3-5)^2}$

$\Rightarrow \; BD = \sqrt{(2)^2 + (-2)^2}$

$\Rightarrow \; BD = \sqrt{4 + 4}$

$\Rightarrow$ BD = $\sqrt{8}$

We got AC≠BD.

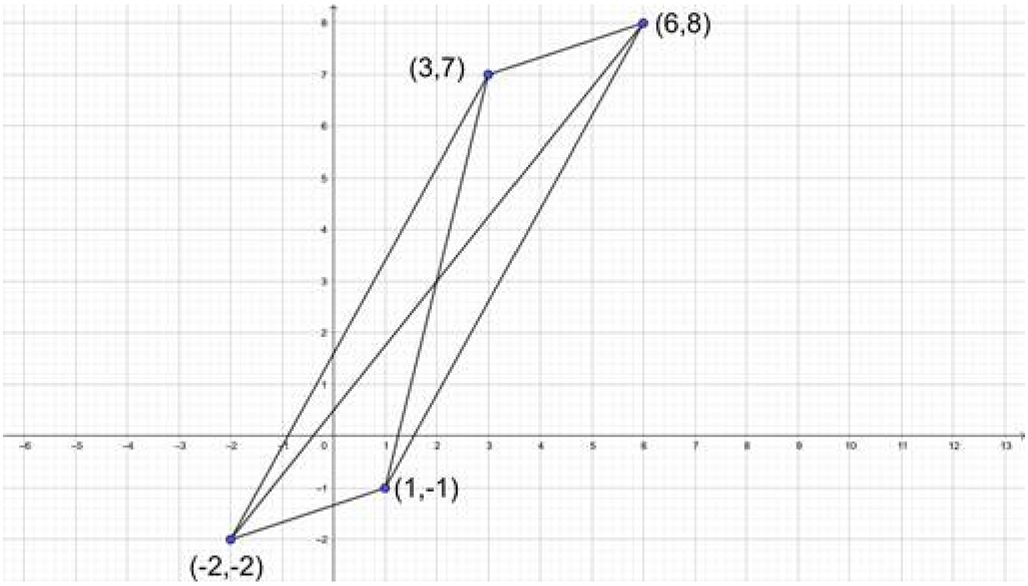
∴ The points doesn't form a rectangle.

22. Question

Prove that the points (6, 8), (3, 7), (- 2, - 2), (1, - 1) are the vertices of a parallelogram.

Answer

Given points are A(6, 8), B(3, 7), C(- 2, - 2) and D(1, - 1).



We need to prove that these are the vertices of a parallelogram.

We know that in the lengths of opposite sides are equal in a parallelogram and the lengths of diagonals are not equal.

Let us find the lengths of the sides.

We know that the distance between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Now,

$$\Rightarrow AB = \sqrt{(6-3)^2 + (8-7)^2}$$

$$\Rightarrow AB = \sqrt{(3)^2 + (1)^2}$$

$$\Rightarrow AB = \sqrt{9 + 1}$$

$$\Rightarrow AB = \sqrt{10}$$

$$\Rightarrow BC = \sqrt{\left(3-(-2)\right)^2 + (7-(-2))^2}$$

$$\Rightarrow BC = \sqrt{(5)^2 + (9)^2}$$

$$\Rightarrow BC = \sqrt{25 + 81}$$

$$\Rightarrow BC = \sqrt{106}$$

$$\Rightarrow CD = \sqrt{(-2-1)^2 + (-2-(-1))^2}$$

$$\Rightarrow CD = \sqrt{(-3)^2 + (-1)^2}$$

$$\Rightarrow CD = \sqrt{9 + 1}$$

$$\Rightarrow CD = \sqrt{10}$$

$$\Rightarrow DA = \sqrt{(1-6)^2 + (-1-8)^2}$$

$$\Rightarrow DA = \sqrt{(-5)^2 + (-9)^2}$$

$$\Rightarrow DA = \sqrt{25 + 81}$$

$$\Rightarrow DA = \sqrt{106}$$

We got AB = CD and BC = DA, these are the vertices of a parallelogram or rectangle.

Now we find the lengths of the diagonals.

$$\Rightarrow AC = \sqrt{\left(6-(-2)\right)^2 + (8-(-2))^2}$$

$$\Rightarrow AC = \sqrt{(8)^2 + (10)^2}$$

$$\Rightarrow AC = \sqrt{64 + 100}$$

$$\Rightarrow AC = \sqrt{164}$$

$\Rightarrow BD = \sqrt{(3-1)^2 + (7-(-1))^2}$

$\Rightarrow BD = \sqrt{(2)^2 + (8)^2}$

$\Rightarrow BD = \sqrt{4 + 64}$

$\Rightarrow BD = \sqrt{68}$

We got AC≠BD.

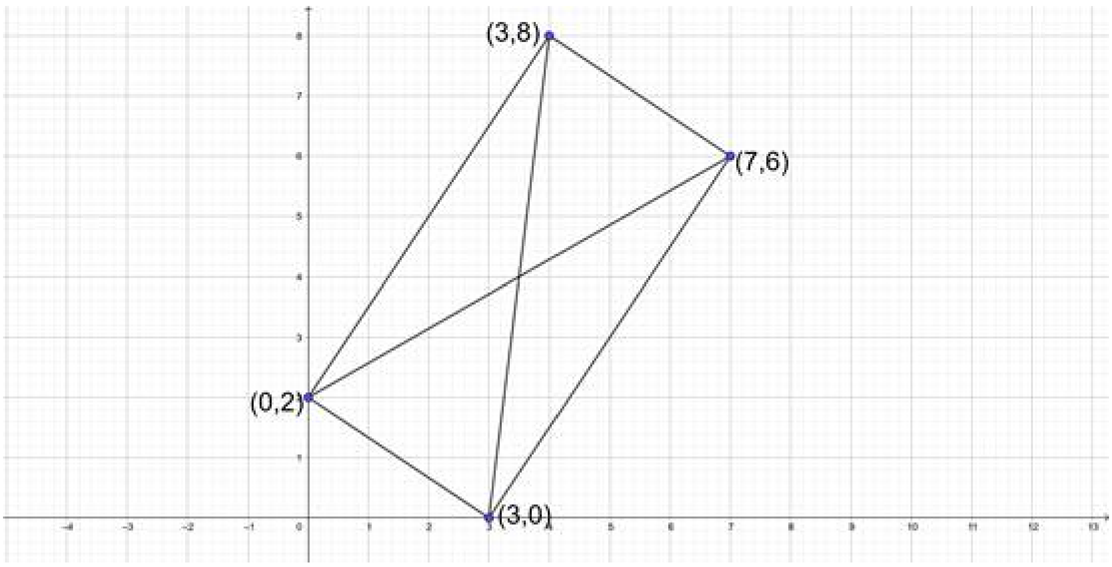
∴ The points form a parallelogram.

23. Question

Prove that the points (4, 8), (0, 2), (3, 0) and (7, 6) are the vertices of a rectangle.

Answer

Given points are A(4, 8), B(0, 2), C(3, 0) and D(7, 6).



We need to prove that these are the vertices of a rectangle.

We know that in the lengths of opposite sides and lengths of diagonals are equal in a rectangle.

Let us find the lengths of the sides.

We know that the distance between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Now,

$\Rightarrow AB = \sqrt{(4-0)^2 + (8-2)^2}$

$$\Rightarrow AB = \sqrt{(4)^2 + (6)^2}$$

$$\Rightarrow AB = \sqrt{16 + 36}$$

$\Rightarrow$ AB = $\sqrt{52}$

$$\Rightarrow BC = \sqrt{(0-3)^2 + (2-0)^2}$$

$$\Rightarrow BC = \sqrt{(-3)^2 + (2)^2}$$

$$\Rightarrow BC = \sqrt{9 + 4}$$

$\Rightarrow$ BC = $\sqrt{13}$

$$\Rightarrow CD = \sqrt{(3-7)^2 + (0-6)^2}$$

$$\Rightarrow CD = \sqrt{(-4)^2 + (-6)^2}$$

$$\Rightarrow CD = \sqrt{16 + 36}$$

$\Rightarrow$ CD = $\sqrt{52}$

$$\Rightarrow DA = \sqrt{(7-4)^2 + (6-8)^2}$$

$$\Rightarrow DA = \sqrt{(3)^2 + (-2)^2}$$

$$\Rightarrow DA = \sqrt{9 + 4}$$

$\Rightarrow$ DA = $\sqrt{13}$

We got AB = CD and BC = DA, these are the vertices of a parallelogram or a rectangle.

Now we find the lengths of the diagonals.

$$\Rightarrow AC = \sqrt{(4-3)^2 + (8-0)^2}$$

$$\Rightarrow AC = \sqrt{(1)^2 + (8)^2}$$

$$\Rightarrow AC = \sqrt{1 + 64}$$

$$\Rightarrow AC = \sqrt{65}$$

$$\Rightarrow BD = \sqrt{(0-7)^2 + (2-6)^2}$$

$$\Rightarrow BD = \sqrt{(-7)^2 + (-4)^2}$$

$\Rightarrow$ BD $= \sqrt{49 + 16}$

$\Rightarrow$ BD $= \sqrt{65}$

We got AC = BD.

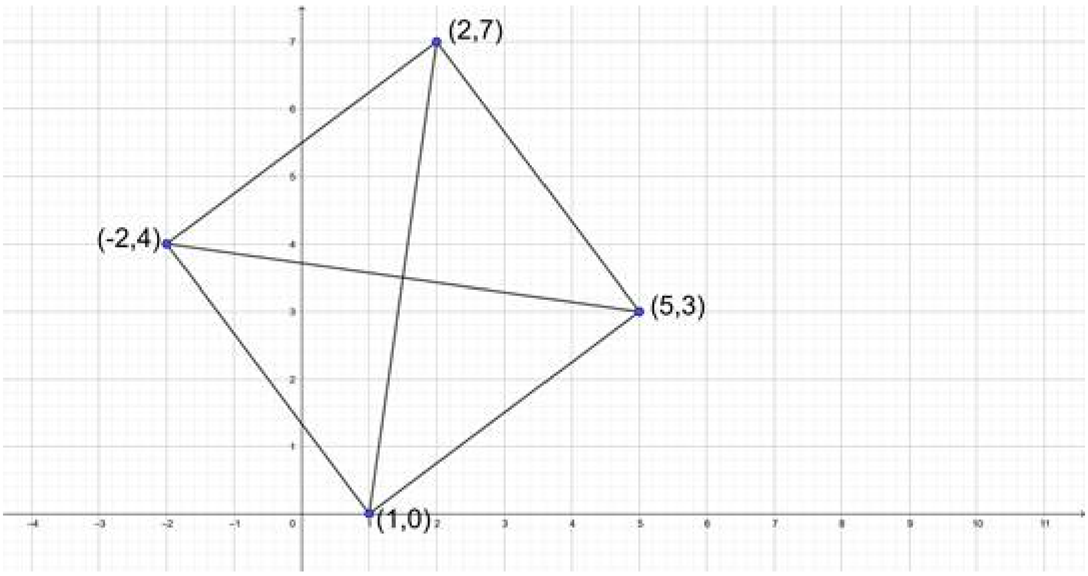
∴ The points form a rectangle.

24. Question

Show that the points A(1, 0), B(5, 3), C(2, 7) and D(- 2, 4) are the vertices of a rhombus.

Answer

Given points are A(1, 0), B(5, 3), C(2, 7) and D(- 2, 4).



We need to prove that these are the vertices of a rhombus.

We know that in the lengths of sides are equal in a rhombus and the length of diagonals are not equal.

Let us find the lengths of the sides.

We know that the distance between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Now,

$\Rightarrow$ AB $= \sqrt{(1-5)^2 + (0-3)^2}$

$\Rightarrow$ AB $= \sqrt{(-4)^2 + (-3)^2}$

$\Rightarrow$ AB $= \sqrt{16 + 9}$

$\Rightarrow AB = \sqrt{25}$

$\Rightarrow$ AB = 5

$\Rightarrow BC = \sqrt{(5-2)^2 + (3-7)^2}$

$\Rightarrow BC = \sqrt{(3)^2 + (4)^2}$

$\Rightarrow BC = \sqrt{9 + 16}$

$\Rightarrow BC = \sqrt{25}$

$\Rightarrow$ BC = 5

$\Rightarrow CD = \sqrt{(2-(-2))^2 + (7-4)^2}$

$\Rightarrow CD = \sqrt{(4)^2 + (3)^2}$

$\Rightarrow CD = \sqrt{16 + 9}$

$\Rightarrow CD = \sqrt{25}$

$\Rightarrow$ CD = 5

$\Rightarrow DA = \sqrt{(-2-1)^2 + (4-0)^2}$

$\Rightarrow DA = \sqrt{(-3)^2 + (4)^2}$

$\Rightarrow DA = \sqrt{9 + 16}$

$\Rightarrow DA = \sqrt{25}$

$\Rightarrow$ DA = 5

We got AB = BC = CD = DA, these are the vertices of a square or a rhombus.

Now we find the lengths of the diagonals.

$\Rightarrow AC = \sqrt{(1-2)^2 + (0-7)^2}$

$\Rightarrow AC = \sqrt{(-1)^2 + (-7)^2}$

$\Rightarrow AC = \sqrt{1 + 49}$

$\Rightarrow AC = \sqrt{50}$

$$\Rightarrow BD = \sqrt{(5-(-2))^2 + (3-4)^2}$$

$$\Rightarrow BD = \sqrt{(7)^2 + (-1)^2}$$

$$\Rightarrow BD = \sqrt{49 + 1}$$

$$\Rightarrow BD = \sqrt{50}$$

We got AC = BD.

∴ The points form a square not rhombus.

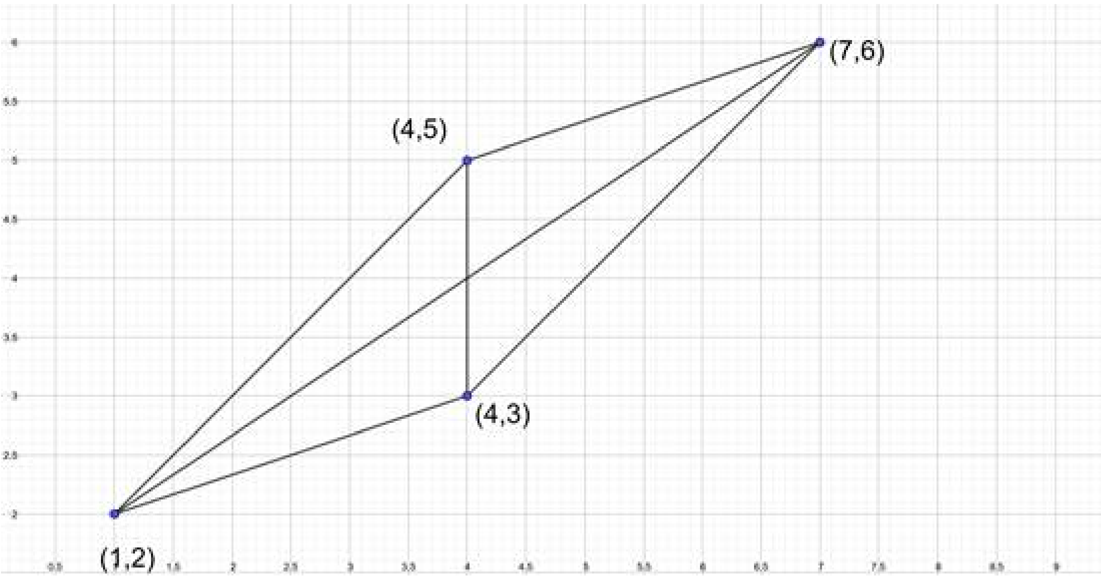
25 A. Question

Name the type or quadrilateral formed, if any, by the following points and give reasons for your answer:

(4, 5), (7, 6), (4, 3), (1, 2)

Answer

Given points are A(4, 5), B(7, 6), C(4, 3) and D(1, 2).



Let us find the lengths of the sides.

We know that the distance between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Now,

$$\Rightarrow AB = \sqrt{(4-7)^2 + (5-6)^2}$$

$$\Rightarrow AB = \sqrt{(-3)^2 + (-1)^2}$$

$\Rightarrow AB = \sqrt{9 + 1}$

$\Rightarrow AB = \sqrt{10}$

$\Rightarrow BC = \sqrt{(7-4)^2 + (6-3)^2}$

$\Rightarrow BC = \sqrt{(3)^2 + (3)^2}$

$\Rightarrow BC = \sqrt{9 + 9}$

$\Rightarrow BC = \sqrt{18}$

$\Rightarrow CD = \sqrt{(4-1)^2 + (3-2)^2}$

$\Rightarrow CD = \sqrt{(3)^2 + (1)^2}$

$\Rightarrow CD = \sqrt{9 + 1}$

$\Rightarrow CD = \sqrt{10}$

$\Rightarrow DA = \sqrt{(1-4)^2 + (2-5)^2}$

$\Rightarrow DA = \sqrt{(-3)^2 + (-3)^2}$

$\Rightarrow DA = \sqrt{9 + 9}$

$\Rightarrow DA = \sqrt{18}$

We got AB = CD and BC = DA, this may be a parallelogram or a rectangle.

Now we find the lengths of the diagonals.

$\Rightarrow AC = \sqrt{(4-4)^2 + (5-3)^2}$

$\Rightarrow AC = \sqrt{(0)^2 + (2)^2}$

$\Rightarrow AC = \sqrt{4}$

$\Rightarrow AC = 2$

$\Rightarrow BD = \sqrt{(7-1)^2 + (6-2)^2}$

$\Rightarrow BD = \sqrt{(6)^2 + (4)^2}$

$\Rightarrow BD = \sqrt{36 + 16}$

$\Rightarrow BD = \sqrt{52}$

We got AC≠BD.

∴ The points form a parallelogram.

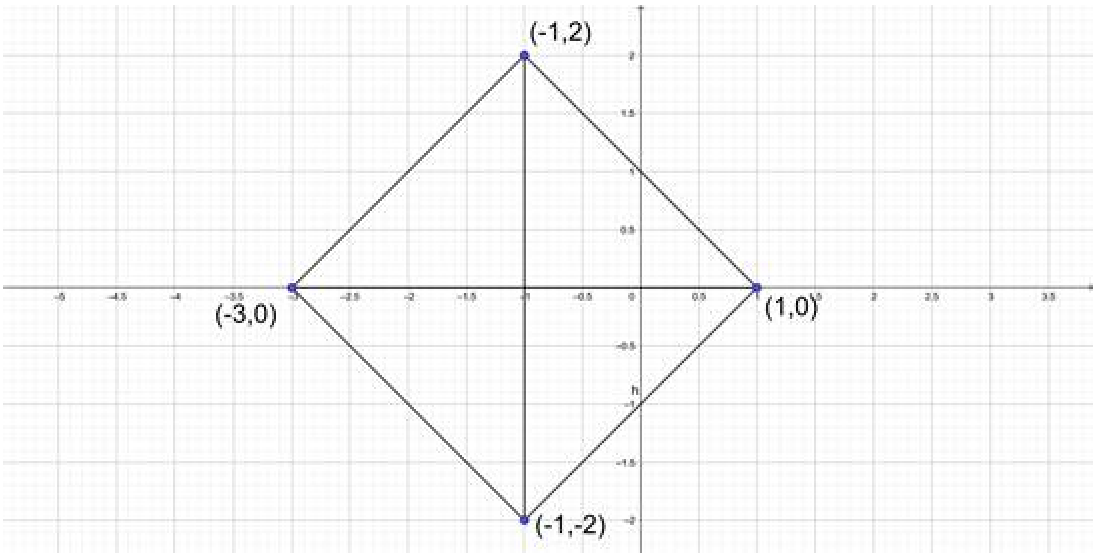
25 B. Question

Name the type or quadrilateral formed, if any, by the following points and give reasons for your answer:

(- 1, - 2), (1, 0), (- 1, 2), (- 3, 0)

Answer

Given points are A(- 1, - 2), B(1, 0), C(- 1, 2) and D(- 3, 0).



Let us find the lengths of the sides.

We know that the distance between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Now,

$\Rightarrow AB = \sqrt{(-1-1)^2 + (-2-0)^2}$

$\Rightarrow AB = \sqrt{(-2)^2 + (-2)^2}$

$\Rightarrow AB = \sqrt{4 + 4}$

$\Rightarrow AB = \sqrt{8}$

$\Rightarrow BC = \sqrt{(1-(-1))^2 + (0-2)^2}$

$\Rightarrow BC = \sqrt{(2)^2 + (-2)^2}$

$\Rightarrow BC = \sqrt{4 + 4}$

$\Rightarrow BC = \sqrt{8}$

$\Rightarrow CD = \sqrt{(-1-(-3))^2 + (2-0)^2}$

$\Rightarrow CD = \sqrt{(2)^2 + (2)^2}$

$\Rightarrow CD = \sqrt{4 + 4}$

$\Rightarrow CD = \sqrt{8}$

$\Rightarrow DA = \sqrt{(-3-(-1))^2 + (0-(-2))^2}$

$\Rightarrow DA = \sqrt{(-2)^2 + (2)^2}$

$\Rightarrow DA = \sqrt{4 + 4}$

$\Rightarrow DA = \sqrt{8}$

We got AB = BC = CD = DA, this may be square (or) rhombus.

Now we find the lengths of the diagonals.

$\Rightarrow AC = \sqrt{(-1-(-1))^2 + (-2-2)^2}$

$\Rightarrow AC = \sqrt{(0)^2 + (-4)^2}$

$\Rightarrow AC = \sqrt{16}$

$\Rightarrow AC = 4$

$\Rightarrow BD = \sqrt{(1-(-3))^2 + (0-0)^2}$

$\Rightarrow BD = \sqrt{(4)^2 + (0)^2}$

$\Rightarrow BD = \sqrt{16}$

$\Rightarrow BD = 4$

We got AC = BD.

∴ The points form a square.

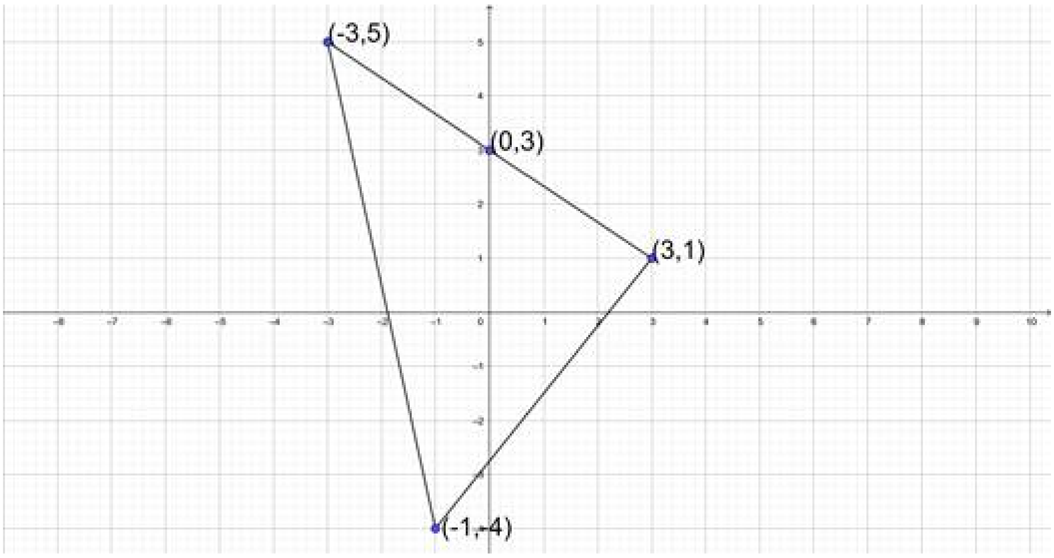
25 C. Question

Name the type or quadrilateral formed, if any, by the following points and give reasons for your answer:

(- 3, 5), (3, 1), (0, 3), (- 1, - 4)

Answer

Given points are A(- 3, 5), B(3, 1), C(0, 3) and D(- 1, - 4).



Let us find the lengths of the sides.

We know that the distance between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Now,

$\Rightarrow AB = \sqrt{(-3-3)^2 + (5-1)^2}$

$\Rightarrow AB = \sqrt{(-6)^2 + (4)^2}$

$\Rightarrow AB = \sqrt{36 + 16}$

$\Rightarrow AB = \sqrt{52}$

$\Rightarrow AB = \sqrt{4 \times 13}$

$\Rightarrow AB = 2\sqrt{13}$

$\Rightarrow BC = \sqrt{(3-0)^2 + (1-3)^2}$

$\Rightarrow BC = \sqrt{(3)^2 + (-2)^2}$

$\Rightarrow BC = \sqrt{9 + 4}$

$\Rightarrow BC = \sqrt{13}$

$\Rightarrow CD = \sqrt{(0-(-1))^2 + (3-(-4))^2}$

$\Rightarrow CD = \sqrt{(1)^2 + (7)^2}$

$\Rightarrow CD = \sqrt{1 + 49}$

$\Rightarrow CD = \sqrt{50}$

$\Rightarrow DA = \sqrt{(-1-(-3))^2 + (-4-5)^2}$

$\Rightarrow DA = \sqrt{(2)^2 + (-9)^2}$

$\Rightarrow DA = \sqrt{4 + 81}$

$\Rightarrow DA = \sqrt{85}$

We got AB≠BC≠CD≠DA, this may be a quadrilateral which is not of standard shape.

Now we find the lengths of the diagonals.

$\Rightarrow AC = \sqrt{(-3-0)^2 + (5-3)^2}$

$\Rightarrow AC = \sqrt{(-3)^2 + (2)^2}$

$\Rightarrow AC = \sqrt{9 + 4}$

$\Rightarrow AC = \sqrt{13}$

$\Rightarrow BD = \sqrt{(3-(-1))^2 + (1-(-4))^2}$

$\Rightarrow BD = \sqrt{(4)^2 + (5)^2}$

$\Rightarrow BD = \sqrt{16 + 25}$

$\Rightarrow BD = \sqrt{41}$

$\Rightarrow AC + BC = \sqrt{13} + \sqrt{13}$

$\Rightarrow AC + BC = 2\sqrt{13}$

$\Rightarrow AC + BC = AB$

We got points ABC are collinear.

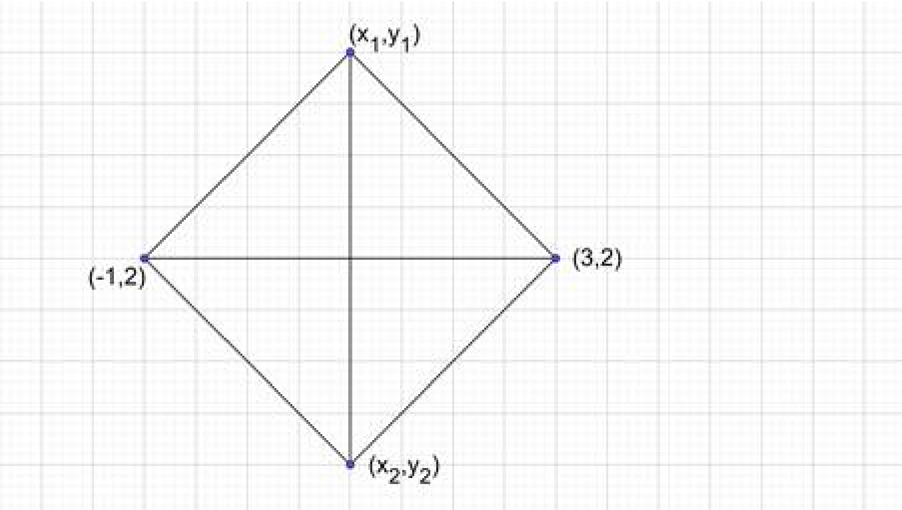
∴ The points doesn't form a quadrilateral.

26. Question

Two opposite vertices of a square are (- 1, 2) and (3, 2). Find the coordinates of other two vertices.

Answer

Given that A(- 1, 2) and C(3, 2) are the opposite vertices of a square.



Let us assume the other two vertices be $B(x_1, y_1)$ and $D(x_2, y_2)$ and the midpoint be M

We know that midpoint of AC = Midpoint of BD = M

$$\Rightarrow M = \left(\frac{-1 + 3}{2}, \frac{2 + 2}{2}\right)$$

$$\Rightarrow M = \left(\frac{2}{2}, \frac{4}{2}\right)$$

$\Rightarrow$ M = (1, 2)

$$\Rightarrow \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = (1, 2)$$

$\Rightarrow x_1 + x_2 = 2$ ..... (1)

$\Rightarrow y_1 + y_2 = 4$ ..... (2)

We know that lengths of the sides of the square are equal.

AB = BC = CD = DA.

We know that distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$\Rightarrow$ AB = BC

$\Rightarrow AB^2 = BC^2$

$\Rightarrow (x_1 - (-1))^2 + (y_1 - 2)^2 = (x_1 - 3)^2 + (y_1 - 2)^2$

$\Rightarrow x_1^2 + 1 + 2x_1 + y_1^2 + 4 - 4y_1 = x_1^2 - 6x_1 + 9 + y_1^2 + 4 - 4y_1$

$\Rightarrow 8x_1 = 8$

$$\Rightarrow x_1 = \frac{8}{8}$$

$\Rightarrow x_1 = 1$ ..... (3)

From (1)

$\Rightarrow x_2 = 2 - 1 = 1$ ..... (4)

We know that points ABC form right angled isosceles triangle.

We have $AB^2 + BC^2 = AC^2$

$\Rightarrow 2AB^2 = (-1 - 3)^2 + (2 - 2)^2$

$\Rightarrow 2((1 - (-1))^2 + (y_1 - 2)^2) = (-4)^2 + (0)^2$

$\Rightarrow 2(2^2 + (y_1 - 2)^2) = 8$

$\Rightarrow 4 + (y_1 - 2)^2 = 8$

$\Rightarrow (y_1 - 2)^2 = 4$

$\Rightarrow y_1 - 2 = \pm 2$

$\Rightarrow y_1 = 2 - 2$ (or) $y_1 = 2 + 2$

$\Rightarrow y_1 = 0$ (or) $y_1 = 4$

From (2)

$\Rightarrow y_2 = 4 - 0$

$\Rightarrow y_2 = 4$

$\Rightarrow y_2 = 4 - 4$

$\Rightarrow y_2 = 0$

It is clear that the other two points are (1, 0) and (1, 4).

∴ The other two points are (1, 0) and (1, 4).

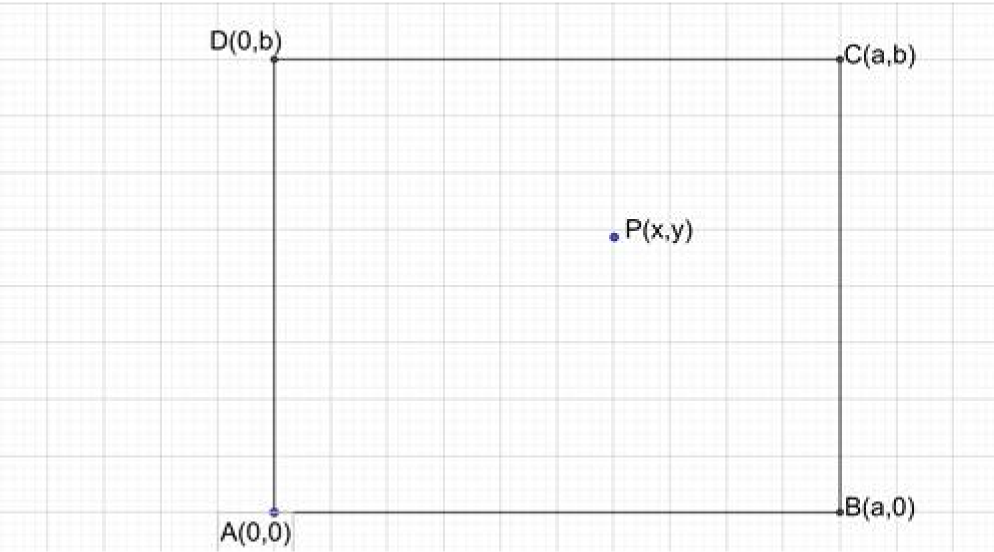
27. Question

If ABCD be a rectangle and P be any point in a plane of the rectangle, then prove that $PA^2 + PC^2 = PB^2 + PD^2$.

Answer

[Hint: Take A as the origin and AB and AD as x and y - axis respectively. Let AB = a, AD = b**]**

Let us assume A as the origin (0, 0) and AB and AD as x and y axis with length a and b units.



Then we get points B to be (a, 0), D to be (0, b) and C to be (a, b).

Let us assume P(x, y) be any point in a plane of the rectangle.

We need to prove $PA^2 + PC^2 = PB^2 + PD^2$.

We know that distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Let us assume L.H.S,

$\Rightarrow PA^2 + PC^2 = ((x - 0)^2 + (y - 0)^2) + ((x - a)^2 + (y - b)^2)$

$\Rightarrow PA^2 + PC^2 = x^2 + y^2 + x^2 - 2ax + a^2 + y^2 - 2by + b^2$

$\Rightarrow PA^2 + PC^2 = (x^2 - 2ax + a^2 + y^2) + (x^2 + y^2 - 2by + b^2)$

$\Rightarrow PA^2 + PC^2 = ((x - a)^2 + (y - 0)^2) + ((x - 0)^2 + (y - b)^2)$

$\Rightarrow PA^2 + PC^2 = PB^2 + PD^2$

$\Rightarrow$ L.H.S = R.H.S

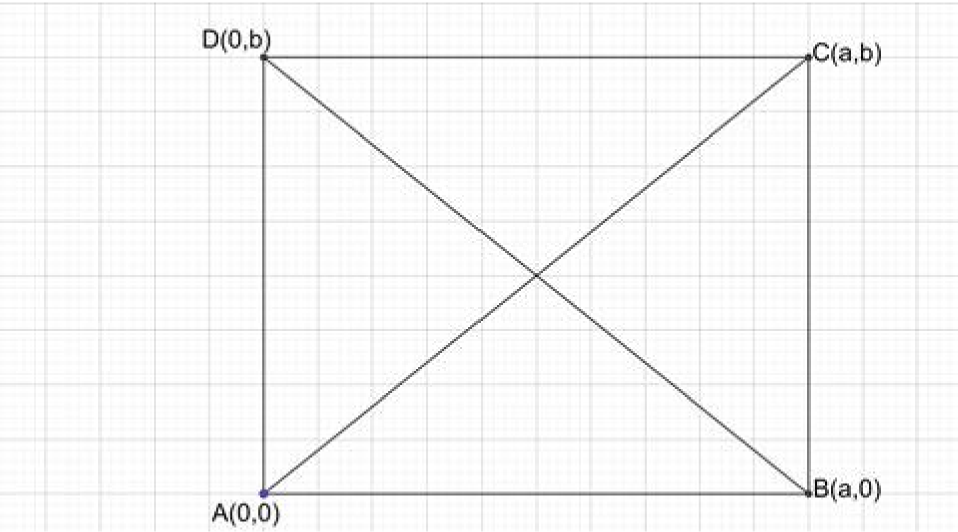
$\therefore$ Thus proved.

28. Question

Prove, using co - ordinates that diagonals of a rectangle are equal.

Answer

Let us assume ABCD be a rectangle with A as the origin and AB and AD as x and y - axes having lengths a and b units.



We get the vertices of the rectangle as follows.

⇒ A = (0, 0)

⇒ B = (a, 0)

⇒ C = (a, b)

⇒ D = (0, b)

We need to prove the lengths of the diagonals are equal.

i.e., AC = BD

We know that distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Let us find the individual lengths of diagonals,

$\Rightarrow AC = \sqrt{(0 - a)^2 + (0 - b)^2}$

$\Rightarrow AC = \sqrt{a^2 + b^2}$ ..... (1)

$\Rightarrow BD = \sqrt{(a - 0)^2 + (0 - b)^2}$

$\Rightarrow BD = \sqrt{a^2 + b^2}$ ..... (2)

From (1) and (2), we can clearly say that AC = BD.

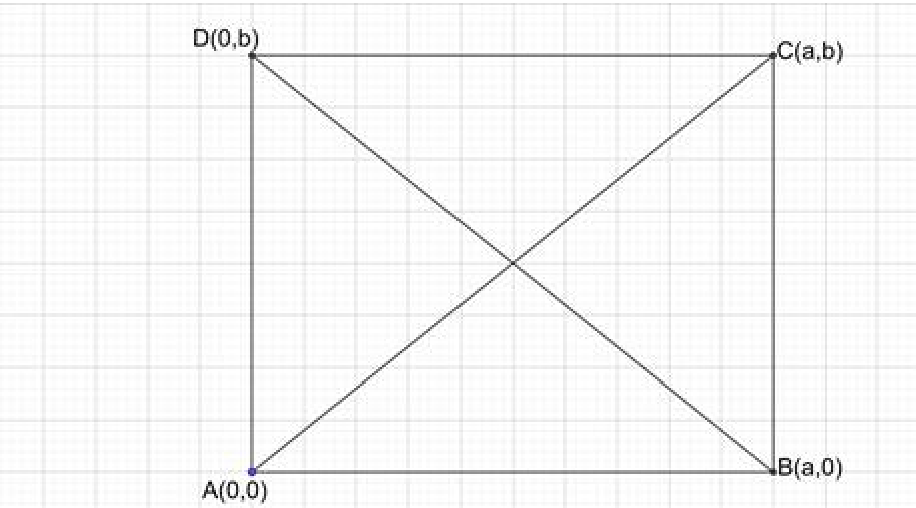
∴ The diagonals of a rectangle are equal.

29. Question

Prove, using coordinates that the sum of squares of the diagonals of a rectangle is equal to the sum of squares of its sides.

Answer

Let us assume ABCD be a rectangle with A as the origin and AB and AD as x and y - axes having lengths a and b units.



We get the vertices of the rectangle as follows.

⇒ A = (0, 0)

⇒ B = (a, 0)

⇒ C = (a, b)

⇒ D = (0, b)

We need to prove that the sum of squares of the diagonals of a rectangle is equal to the sum of squares of its sides.

i.e., $AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2$

We know that distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Assume L.H.S

⇒ $AC^2 + BD^2 = ((0 - a)^2 + (0 - b)^2) + ((a - 0)^2 + (0 - b)^2)$

⇒ $AC^2 + BD^2 = a^2 + b^2 + a^2 + b^2$

$\Rightarrow AC^2 + BD^2 = 2(a^2 + b^2)$ ..... - (1)

Assume R.H.S

$\Rightarrow AB^2 + BC^2 + CD^2 + DA^2 = ((0 - a)^2 + (0 - 0)^2) + ((a - a)^2 + (0 - b)^2) + ((a - 0)^2 + (b - b)^2) + ((0 - 0)^2 + (b - 0)^2)$

$\Rightarrow AB^2 + BC^2 + CD^2 + DA^2 = a^2 + 0 + 0 + b^2 + a^2 + 0 + 0 + b^2$

$\Rightarrow AB^2 + BC^2 + CD^2 + DA^2 = 2(a^2 + b^2)$ .... (2)

From (1) and (2), we can clearly say that,

$\Rightarrow AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2$

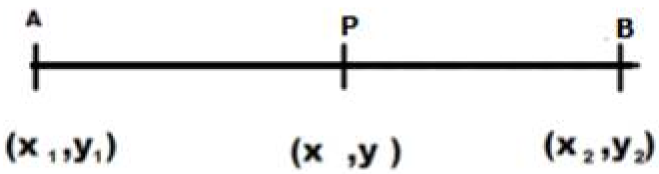
∴ Thus proved.

Exercise 10.3

1 A. Question

Find the coordinates of the point which divides the line segment joining (2,4) and (6,8) in the ratio 1:3 internally and externally.

Answer



Let P(x,y) be the point which divides the line segment internally.

Using the **section formula** for the internal division, i.e.

$$(x,y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right) \text{...(i)}$$

Here, $m_1 = 1$, $m_2 = 3$

$(x_1, y_1) = (2, 4)$ and $(x_2, y_2) = (6, 8)$

Putting the above values in the above formula, we get

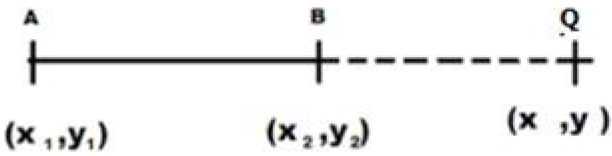
$$\Rightarrow x = \frac{1(6) + 3(2)}{1 + 3}, y = \frac{1(8) + 3(4)}{1 + 3}$$

$$\Rightarrow x = \frac{6 + 6}{4}, y = \frac{8 + 12}{4}$$

$\Rightarrow x = \frac{12}{4}, y = \frac{20}{4}$

⇒ x = 3, y = 5

Hence, (3,5) is the point which divides the line segment internally.



Now, Let Q(x,y) be the point which divides the line segment externally.

Using the **section formula** for the external division, i.e.

$$(x, y) = \left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}\right) \text{ ...(i)}$$

Here, $m_1 = 1$, $m_2 = 3$

$(x_1, y_1) = (2, 4)$ and $(x_2, y_2) = (6, 8)$

Putting the above values in the above formula, we get

$$\Rightarrow x = \frac{1(6) - 3(2)}{1 - 3}, y = \frac{1(8) - 3(4)}{1 - 3}$$

$$\Rightarrow x = \frac{6 - 6}{-2}, y = \frac{8 - 12}{-2}$$

$$\Rightarrow x = \frac{0}{-2}, y = \frac{-4}{-2}$$

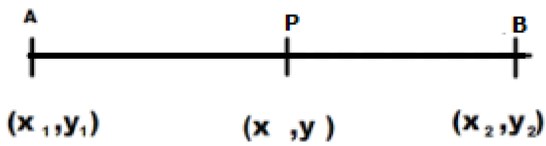
⇒ x = 0, y = 2

Hence, (0,2) is the point which divides the line segment externally.

1 B. Question

Find the coordinates of the point which divides the join of (-1,7) and (4,-3) internally in the ratio 2:3.

Answer



Let P(x,y) be the point which divides the line segment internally.

Using the **section formula** for the internal division, i.e.

$$(x, y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right) \text{...(i)}$$

Here, $m_1 = 2$, $m_2 = 3$

$(x_1, y_1) = (-1, 7)$ and $(x_2, y_2) = (4, -3)$

Putting the above values in the above formula, we get

$$\Rightarrow x = \frac{2(4)+3(-1)}{2+3}, y = \frac{2(-3)+3(7)}{2+3}$$

$$\Rightarrow x = \frac{8-3}{5}, y = \frac{-6+21}{5}$$

$$\Rightarrow x = \frac{5}{5}, y = \frac{15}{5}$$

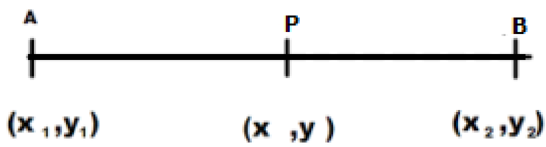
$\Rightarrow x = 1, y = 3$

Hence, (1,3) is the point which divides the line segment internally.

1 C. Question

Find the coordinates of the point which divides the line segment joining the points (4,-3) and (8,5) in the ratio 3:1 internally.

Answer



Let P(x,y) be the point which divides the line segment internally.

Using the **section formula** for the internal division, i.e.

$$(x, y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right) \text{...(i)}$$

Here, $m_1 = 3$, $m_2 = 1$

$(x_1, y_1) = (4, -3)$ and $(x_2, y_2) = (8, 5)$

Putting the above values in the above formula, we get

$$\Rightarrow \ x = \frac{3(8)+1(4)}{3+1}, y = \frac{3(5)+1(-3)}{3+1}$$

$$\Rightarrow \ x = \frac{24+4}{4}, y = \frac{15-3}{4}$$

$$\Rightarrow \ x = \frac{28}{4}, y = \frac{12}{4}$$

⇒ x = 7, y = 3

Hence, (7,3) is the point which divides the line segment internally.

2 A. Question

Find the coordinates of the points which trisect the line segment joining the points (2,3) and (6,5).

Answer

Let P and Q be the points of trisection of AB, i.e. AP = PQ = QB

∴ P divides AB internally in the ratio 1: 2.

∴ the coordinates of P, by applying the section formula, are

$$(x,y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right) \text{ ...(i)}$$

Here, $m_1 = 1$, $m_2 = 2$

$(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (6, 5)$

Putting the above values in the above formula, we get

$$\Rightarrow \ x = \frac{1(6)+2(2)}{1+2}, y = \frac{1(5)+2(3)}{1+2}$$

$$\Rightarrow \ x = \frac{6+4}{3}, y = \frac{5+6}{3}$$

$$\Rightarrow \ x = \frac{10}{3}, y = \frac{11}{3}$$

Now, Q also divides AB internally in the ratio 2: 1. So, the coordinates of Q are

$(x, y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$ ...(i)

Here, $m_1 = 2, m_2 = 1$

$(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (6, 5)$

Putting the above values in the above formula, we get

$$\Rightarrow\ x = \frac{2(6)+1(2)}{2+1}, y = \frac{2(5)+1(3)}{2+1}$$

$$\Rightarrow\ x = \frac{12+2}{3}, y = \frac{10+3}{3}$$

$$\Rightarrow\ x = \frac{14}{3}, y = \frac{13}{3}$$

Therefore, the coordinates of the points of trisection of the line segment joining A and B are $\left(\frac{10}{3}, \frac{11}{3}\right)$ and $\left(\frac{14}{3}, \frac{13}{3}\right)$

2 B. Question

Find the coordinates of the point of trisection of the line segment joining (1,-2) and (-3,4).

Answer

Let P and Q be the points of trisection of AB, i.e. AP = PQ = QB

∴ P divides AB internally in the ratio 1: 2.

∴ the coordinates of P, by applying the section formula, are

$(x, y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$ ...(i)

Here, $m_1 = 1, m_2 = 2$

$(x_1, y_1) = (1, -2)$ and $(x_2, y_2) = (-3, 4)$

Putting the above values in the above formula, we get

$$\Rightarrow\ x = \frac{1(-3)+2(1)}{1+2}, y = \frac{1(4)+2(-2)}{1+2}$$

$$\Rightarrow x = \frac{-3+2}{3}, y = \frac{4-4}{3}$$

$$\Rightarrow x = \frac{-1}{3}, y = 0$$

Now, Q also divides AB internally in the ratio 2: 1. So, the coordinates of Q are

$(x, y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$ ...(i)

Here, $m_1 = 2$, $m_2 = 1$

$(x_1, y_1) = (1, -2)$ and $(x_2, y_2) = (-3, 4)$

Putting the above values in the above formula, we get

$$\Rightarrow x = \frac{2(-3)+1(1)}{2+1}, y = \frac{2(4)+1(-2)}{2+1}$$

$$\Rightarrow x = \frac{-6+1}{3}, y = \frac{8-2}{3}$$

$$\Rightarrow x = \frac{-5}{3}, y = \frac{6}{3} = 2$$

Therefore, the coordinates of the points of trisection of the line segment joining A and B are $\left(\frac{-1}{3}, 0\right)$ and $\left(\frac{-5}{3}, 2\right)$

3 A. Question

The coordinates of A and B are (1,2) and (2,3) respectively, If P lies on AB, find the coordinates of P such that $\frac{AP}{PB} = \frac{4}{3}$

Answer

Given:

$$\frac{AP}{PB} = \frac{4}{3}$$

$\Rightarrow m_1 = 4$ and $m_2 = 3$

and $(x_1, y_1) = (1, 2)$; $(x_2, y_2) = (2, 3)$

Using the **section formula** for the internal division, i.e.

$$(x, y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right) \ldots\text{(i)}$$

$$\Rightarrow\ x = \frac{4(2)+3(1)}{4+3}, y = \frac{4(3)+3(2)}{4+3}$$

$$\Rightarrow\ x = \frac{8+3}{7}, y = \frac{12+6}{7}$$

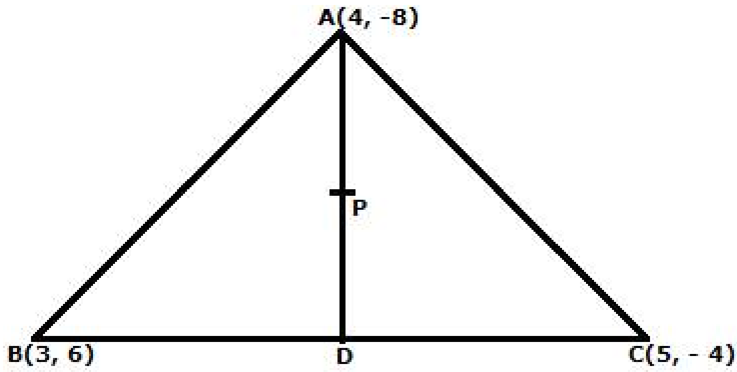
$$\Rightarrow\ x = \frac{11}{7}, y = \frac{18}{7}$$

Hence, the coordinates of P are $P(x, y)\ =\ P\left(\frac{11}{7}, \frac{18}{7}\right)$

3 B. Question

If A (4,-8), B (3,6) and C(5,-4) are the vertices of a ΔABC, D is the mid-point of BC and P is a point on AD joined such that $\frac{AP}{PD} = 2$, find the coordinates of P.

Answer



Given: D is the midpoint of BC. So, BD = DC

Then the coordinates of D are

$$x = \left(\frac{3+5}{2}\right), y = \left(\frac{6+(-4)}{2}\right)$$

$$\Rightarrow\ x = \frac{8}{2}, y = \frac{2}{2}$$

$\Rightarrow$ x = 4 and y = 1

So, coordinates of D are (4, 1)

Now, we have to find the coordinates of P.

Given:

$$\frac{AP}{PD} = \frac{2}{1}$$

⇒ $m_1 = 2$ and $m_2 = 1$

and $(x_1, y_1) = (4, -8)$; $(x_2, y_2) = (4, 1)$

Using the **section formula** for the internal division, i.e.

$$(x,y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right) \text{ ...(i)}$$

$$\Rightarrow\ x = \frac{2(4)+1(4)}{2+1}, y = \frac{2(1)+1(-8)}{2+1}$$

$$\Rightarrow\ x = \frac{8+4}{3}, y = \frac{2-8}{3}$$

$$\Rightarrow\ x = \frac{12}{3}, y = \frac{-6}{3}$$

⇒ $x = 4, y = -2$

Hence, the coordinates of P are P(x,y) = P(4, -2)

3 C. Question

If p divides the join of A (-2,-2) and B (2,-4) such that $\frac{AP}{AB} = \frac{3}{7}$, find the coordinates of P.

Answer

Given:

$$\frac{AP}{AB} = \frac{3}{7}$$

$$\Rightarrow\ AP = \frac{3}{7}AB$$

$$\Rightarrow\ AP = \frac{3}{7}(AP + PB)$$

⇒ 7AP = 3AP + 3PB

⇒ 7AP – 3AP = 3PB

⇒ 4AP = 3PB

$$\Rightarrow \frac{AP}{PB} = \frac{3}{4}$$

Hence, the point P divides AB in the ratio of 3:4

⇒ $m_1 = 3$ and $m_2 = 4$

and $(x_1, y_1) = (-2, -2)$; $(x_2, y_2) = (2, -4)$

Using the **section formula** for the internal division, i.e.

$$(x, y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right) \text{ ...(i)}$$

$$\Rightarrow x = \frac{3(2)+4(-2)}{3+4}, y = \frac{3(-4)+4(-2)}{3+4}$$

$$\Rightarrow x = \frac{6-8}{7}, y = \frac{-12-8}{7}$$

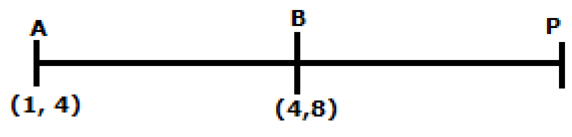
$$\Rightarrow x = \frac{-2}{7}, y = \frac{-20}{7}$$

Hence, the coordinates of P are $P(x, y) = P\left(\frac{-2}{7}, \frac{-20}{7}\right)$

3 D. Question

A (1,4) and B (4,8) are two points. P is a point on AB such that AP = AB + BP. If AP = 10 find the coordinates of P.

Answer



Given: AP = AB + BP and AP = 10

Firstly, we find the distance between A and B

$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(4 - 1)^2 + (8 - 4)^2}$

$= \sqrt{(3)^2 + (4)^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

= 5

So, AB = 5

It is given that AP = AB + BP

⇒ 10 = 5 + BP

⇒ 10 – 5 = BP

⇒ BP = 5

⇒ A, B and P are collinear

and since AB = BP

⇒ B is the midpoint of AP

Let the coordinates of P = (x,y)

$$\Rightarrow \left(\frac{x+1}{2}, \frac{y+4}{2}\right) = (4,8)$$

$$\Rightarrow \frac{x+1}{2} = 4 \text{ and } \frac{y+4}{2} = 8$$

⇒ x + 1 = 8 and y + 4 = 16

⇒ x = 7 and y = 12

Hence, the coordinates of P are (7, 12)

4. Question

The line segment joining A (2,3) and B(-3,5) is extended through each end by a length equal to its original length. Find the coordinates of the new ends.

Answer

Let P and Q be the required new ends

Coordinates of P

Let AP = k

∴ AB = AP = k

and PB = AP + AB = k + k = 2k

$$\therefore \frac{AP}{PB} = \frac{k}{2k} = \frac{1}{2}$$

∴ P divides AB externally in the ratio 1:2

Using the **section formula** for the external division, i.e.

$$(x, y) = \left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}\right) \text{ ...(i)}$$

Here, m_1 = 1, m_2 = 2

(x_1, y_1) = (2, 3) and (x_2, y_2) = (-3, 5)

Putting the above values in the above formula, we get

$$\Rightarrow \ x = \frac{1(-3) - 2(2)}{1 - 2}, y = \frac{1(5) - 2(3)}{1 - 2}$$

$$\Rightarrow \ x = \frac{-3 - 4}{-1}, y = \frac{5 - 6}{-1}$$

$$\Rightarrow \ x = \frac{-7}{-1}, y = \frac{-1}{-1}$$

⇒ x = 7, y = 1

∴Coordinates of P are (7, 1)

Coordinates of Q.

Q divides AB externally in the ratio 2:1

Again, Using the **section formula** for the external division, i.e.

$$(x, y) = \left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}\right) \text{ ...(i)}$$

Here, m_1 = 2, m_2 = 1

(x_1, y_1) = (2, 3) and (x_2, y_2) = (-3, 5)

Putting the above values in the above formula, we get

$$\Rightarrow \ x = \frac{2(-3) - 1(2)}{2 - 1}, y = \frac{2(5) - 1(3)}{2 - 1}$$

$$\Rightarrow\ x = \frac{-6-2}{1}, y = \frac{10-3}{1}$$

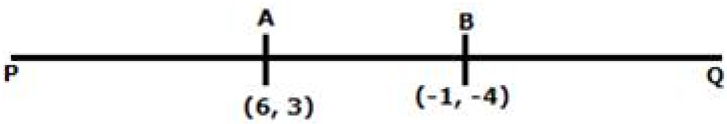
$$\Rightarrow\ x = \frac{-8}{1}, y = \frac{7}{1}$$

∴Coordinates of Q are (-8, 7)

5. Question

The line segment joining A(6,3) to B(-1,-4) is doubled in length by having half its length added to each end. Find the coordinates of the new ends.

Answer



Let P and Q be the required new ends

Coordinates of P

Let AP = k

∴ AB = 2AP = 2k

and PB = AP + AB = k + 2k = 3k

$$\therefore \frac{AP}{PB} = \frac{k}{3k} = \frac{1}{3}$$

∴ P divides AB externally in the ratio 1:3

Using the **section formula** for the external division, i.e.

$$(x, y) = \left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}\right) \ldots(i)$$

Here, $m_1 = 1$, $m_2 = 3$

$(x_1, y_1) = (6, 3)$ and $(x_2, y_2) = (-1, -4)$

Putting the above values in the above formula, we get

$$\Rightarrow\ x = \frac{1(-1) - 3(6)}{1-3}, y = \frac{1(-4) - 3(3)}{1-3}$$

$$\Rightarrow\ x = \frac{-1-18}{-2}, y = \frac{-4-9}{-2}$$

$\Rightarrow \ x = \frac{-19}{-2}, y = \frac{-13}{-2}$

$\Rightarrow \ x = \frac{19}{2}, y = \frac{13}{2}$

∴Coordinates of P are $\left(\frac{19}{2}, \frac{13}{2}\right)$

Coordinates of Q.

Q divides AB externally in the ratio 3:1

Again, Using the **section formula** for the external division, i.e.

$(x, y) = \left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}\right)$ ...(i)

Here, $m_1 = 3$, $m_2 = 1$

$(x_1, y_1) = (6, 3)$ and $(x_2, y_2) = (-1, -4)$

Putting the above values in the above formula, we get

$\Rightarrow \ x = \frac{3(-1) - 1(6)}{3 - 1}, y = \frac{3(-4) - 1(3)}{3 - 1}$

$\Rightarrow \ x = \frac{-3 - 6}{2}, y = \frac{-12 - 3}{2}$

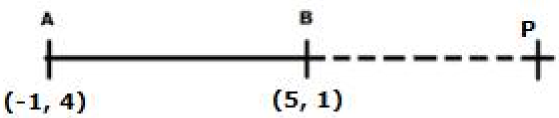
$\Rightarrow \ x = \frac{-9}{2}, y = \frac{-15}{2}$

∴Coordinates of Q are $\left(\frac{-9}{2}, \frac{-15}{2}\right)$

6. Question

The coordinates of two points A and B are (-1,4) and (5,1) respectively. Find the coordinates of the point P which lies on extended line AB such that it is three times as far from B as from A.

Answer



Now, Let P(x,y) be the point which lies on extended line AB

Using the **section formula** for the external division, i.e.

$(x, y) = \left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}\right)$ ...(i)

Here, $m_1 = 1$, $m_2 = 3$

$(x_1, y_1) = (-1, 4)$ and $(x_2, y_2) = (5, 1)$

Putting the above values in the above formula, we get

$$\Rightarrow x = \frac{1(5) - 3(-1)}{1 - 3}, y = \frac{1(1) - 3(4)}{1 - 3}$$

$$\Rightarrow x = \frac{5 + 3}{-2}, y = \frac{1 - 12}{-2}$$

$$\Rightarrow x = \frac{8}{-2}, y = \frac{-11}{-2}$$

$$\Rightarrow x = -4, y = \frac{11}{2}$$

7. Question

Find the distances of that point from the origin which divides the line segment joining the points (5,-4) and (3,-2) in the ration 4:3.

Answer

Let the coordinates of the point be (x,y)

Let A = (5, -4) and B = (3, -2)

Here, the point divides the line segment in the ratio 4:3

So, $m_1 = 4$ and $m_2 = 3$

Using section formula,

$(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$ ...(i)

$$\Rightarrow x = \frac{4(3) + 3(5)}{4 + 3}, y = \frac{4(-2) + 3(-4)}{4 + 3}$$

$$\Rightarrow x = \frac{12 + 15}{7}, y = \frac{-8 - 12}{7}$$

$$\Rightarrow x = \frac{27}{7}, y = \frac{-20}{7}$$

Hence, the coordinates of P are $P(x, y) = P\left(\frac{27}{7}, \frac{-20}{7}\right)$

Now, the distance from the origin (0,0) is

$$D = \sqrt{\left(\frac{27}{7} - 0\right)^2 + \left(\frac{-20}{7} - 0\right)^2}$$

$$= \sqrt{\frac{729}{49} + \frac{400}{49}}$$

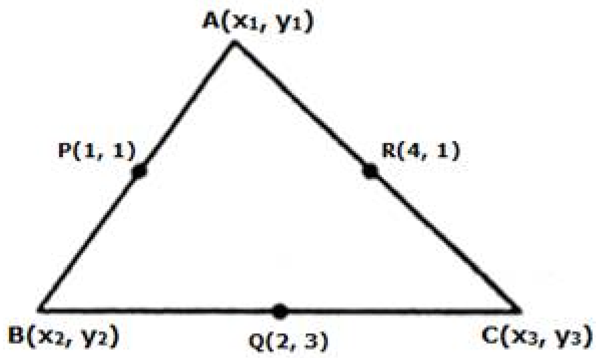
$$= \sqrt{\frac{1129}{49}}$$

$$= \frac{\sqrt{1129}}{7}$$

8 A. Question

The coordinates of the middle points of the sides of a triangle are (1,1), (2,3) and (4,1), find the coordinates of its vertices.

Answer



Consider a ΔABC with $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$. If P(1, 1), Q(2, 3) and R(4, 1) are the midpoints of AB, BC, and CA. Then,

$1 = \frac{x_1 + x_2}{2} \Rightarrow x_1 + x_2 = 2$ ...(i)

$1 = \frac{y_1 + y_2}{2} \Rightarrow y_1 + y_2 = 2$ ...(ii)

$2 = \frac{x_2 + x_3}{2} \Rightarrow x_2 + x_3 = 4$ ...(iii)

$3 = \frac{y_2 + y_3}{2} \Rightarrow y_2 + y_3 = 6$ ...(iv)

$4 = \frac{x_1 + x_3}{2} \Rightarrow x_1 + x_3 = 8$ ...(v)

$1 = \frac{y_1 + y_3}{2} \Rightarrow y_1 + y_3 = 2$ ...(vi)

Adding (i), (iii) and (v), we get

$x_1 + x_2 + x_2 + x_3 + x_1 + x_3 = 2 + 4 + 8$

$\Rightarrow 2(x_1 + x_2 + x_3) = 14$

$\Rightarrow x_1 + x_2 + x_3 = 7$ ...(vii)

From (i) and (vii), we get

$x_3 = 7 – 2 = 5$

From (iii) and (vii), we get

$x_1 = 7 – 4 = 3$

From (v) and (vii), we get

$x_2 = 7 – 8 = -1$

Now adding (ii), (iv) and (vi), we get

$y_1 + y_2 + y_2 + y_3 + y_1 + y_3 = 2 + 6 + 2$

$\Rightarrow 2(y_1 + y_2 + y_3) = 10$

$\Rightarrow y_1 + y_2 + y_3 = 5$ ...(viii)

From (ii) and (viii), we get

$y_3 = 5 – 2 = 3$

From (iv) and (vii), we get

$y_1 = 5 – 6 = -1$

From (vi) and (vii), we get

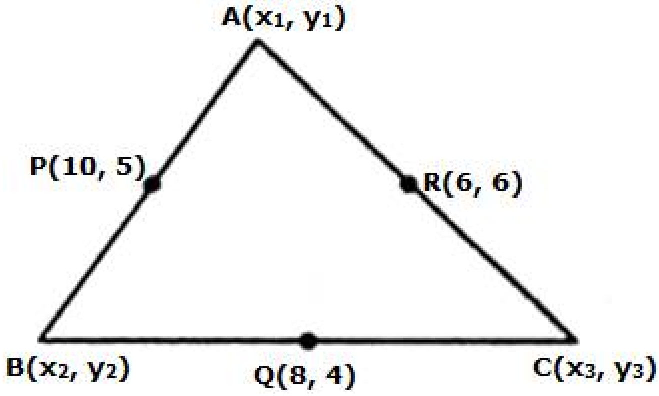
$y_2 = 5 – 2 = 3$

Hence, the vertices of ΔABC are A(3, -1), B(-1, 3) and C(5, 3)

8 B. Question

If the points (10,5),(8,4) and (6,6) are the mid-points of the sides of a triangle, find its vertices.

Answer



Consider a ΔABC with $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$. If P(10, 5), Q(8, 4) and R(6, 6) are the midpoints of AB, BC, and CA. Then,

$10 = \frac{x_1+x_2}{2} \Rightarrow x_1 + x_2 = 20$ ...(i)

$5 = \frac{y_1+y_2}{2} \Rightarrow y_1 + y_2 = 10$ ...(ii)

$8 = \frac{x_2+x_3}{2} \Rightarrow x_2 + x_3 = 16$ ...(iii)

$4 = \frac{y_2+y_3}{2} \Rightarrow y_2 + y_3 = 8$ ...(iv)

$6 = \frac{x_1+x_3}{2} \Rightarrow x_1 + x_3 = 12$ ...(v)

$6 = \frac{y_1+y_3}{2} \Rightarrow y_1 + y_3 = 12$ ...(vi)

Adding (i), (iii) and (v), we get

$x_1 + x_2 + x_2 + x_3 + x_1 + x_3 = 20 + 16 + 12$

$\Rightarrow 2(x_1 + x_2 + x_3) = 48$

$\Rightarrow x_1 + x_2 + x_3 = 24$ ...(vii)

From (i) and (vii), we get

$x_3 = 24 – 20 = 4$

From (iii) and (vii), we get

$x_1 = 24 – 16 = 8$

From (v) and (vii), we get

$x_2 = 24 – 12 = 12$

Now adding (ii), (iv) and (vi), we get

$y_1 + y_2 + y_2 + y_3 + y_1 + y_3 = 10 + 8 + 12$

$\Rightarrow 2(y_1 + y_2 + y_3) = 30$

$\Rightarrow y_1 + y_2 + y_3 = 15$ ...(viii)

From (ii) and (viii), we get

$y_3 = 15 - 10 = 5$

From (iv) and (vii), we get

$y_1 = 15 - 8 = 7$

From (vi) and (vii), we get

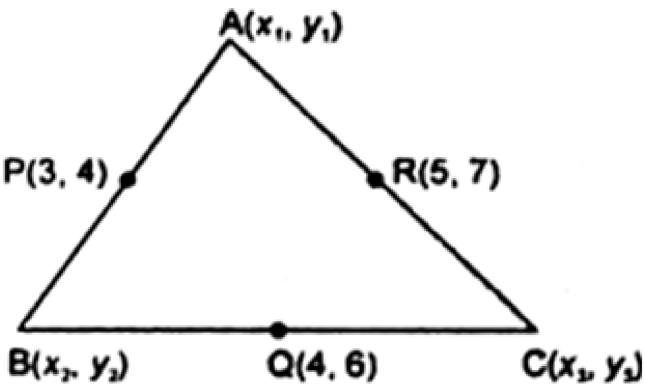
$y_2 = 15 - 12 = 3$

Hence, the vertices of ΔABC are A(8, 7), B(12, 3) and C(4, 5)

8 C. Question

The mid-points of the sides of a triangle are (3,4),(4,6) and (5,7). Find the coordinates of the vertices of the triangle.

Answer



Consider a ΔABC with $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$. If P(3, 4), Q(4, 6) and R(5, 7) are the midpoints of AB, BC, and CA. Then,

$3 = \frac{x_1 + x_2}{2} \Rightarrow x_1 + x_2 = 6$ ...(i)

$4 = \frac{y_1 + y_2}{2} \Rightarrow y_1 + y_2 = 8$ ...(ii)

$4 = \frac{x_2 + x_3}{2} \Rightarrow x_2 + x_3 = 8$ ...(iii)

$5 = \frac{y_2 + y_3}{2} \Rightarrow y_2 + y_3 = 10$ ...(iv)

$6 = \frac{x_1 + x_3}{2} \Rightarrow x_1 + x_3 = 12$ ...(v)

$7 = \frac{y_1 + y_3}{2} \Rightarrow y_1 + y_3 = 14$ ...(vi)

Adding (i), (iii) and (v), we get

$x_1 + x_2 + x_2 + x_3 + x_1 + x_3 = 6 + 8 + 10$

$\Rightarrow 2(x_1 + x_2 + x_3) = 24$

$\Rightarrow x_1 + x_2 + x_3 = 12$ ...(vii)

From (i) and (vii), we get

$x_3 = 12 - 6 = 6$

From (iii) and (vii), we get

$x_1 = 12 - 8 = 4$

From (v) and (vii), we get

$x_2 = 12 - 10 = 2$

Now adding (ii), (iv) and (vi), we get

$y_1 + y_2 + y_2 + y_3 + y_1 + y_3 = 8 + 12 + 14$

$\Rightarrow 2(y_1 + y_2 + y_3) = 34$

$\Rightarrow y_1 + y_2 + y_3 = 17$ ...(viii)

From (ii) and (viii), we get

$y_3 = 17 - 8 = 9$

From (iv) and (vii), we get

$y_1 = 17 - 12 = 5$

From (vi) and (vii), we get

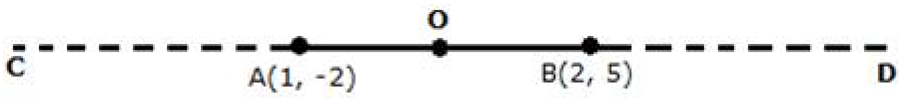
$y_2 = 17 - 14 = 3$

Hence, the vertices of ΔABC are A(4, 5), B(2, 3) and C(6, 9)

9. Question

A(1,-2) and B(2,5) are two points. The lines OA, OB are produced to C and D respectively such that OC = 2OA and OD = 2OB. Find CD.

Answer



Given:

A(1, -2) and B(2, 5) are two points.

OC = 2OA …(i)

and OD = 2OB …(ii)

Adding (i) and (ii), we get

OC + OD = 2OA + 2OB

⇒ CD = 2[OA + OB]

⇒ CD = 2[AB] …(iii)

Now, we find the distance between A and B

$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(2 - 1)^2 + \{5 - (-2)\}^2}$

$= \sqrt{(1)^2 + (5 + 2)^2}$

$= \sqrt{1 + 49}$

$= \sqrt{50}$

$= 5\sqrt{2}$

Putting the value in eq. (iii), we get

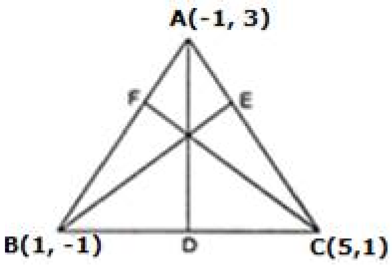
$CD = 2 \times 5\sqrt{2}$

$= 10\sqrt{2}$

10. Question

Find the length of the medians of the triangle whose vertices are (-1,3),(1,-1) and (5,1).

Answer



Let the given points of a triangle be A(-1, 3), B(1, -1) and C(5,1)

Let D, E and F are the midpoints of the sides BC, CA and AB respectively.

The coordinates of D are:

$$D = \left[\frac{5+1}{2}, \frac{1+(-1)}{2}\right]$$

$$D = \left[\frac{6}{2}, \frac{0}{2}\right]$$

D = (3, 0)

The coordinates of E are:

$$E = \left[\frac{5+(-1)}{2}, \frac{1+3}{2}\right]$$

$$E = \left[\frac{4}{2}, \frac{4}{2}\right]$$

E = (2, 2)

The coordinates of F are:

$$F = \left[\frac{1+(-1)}{2}, \frac{-1+3}{2}\right]$$

$$F = \left[\frac{0}{2}, \frac{2}{2}\right]$$

F = (0, 1)

Now, we have to find the lengths of the medians.

$d(A,D) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{\{3 - (-1)^2\} + \{0 - 3\}^2}$

$= \sqrt{(3 + 1)^2 + (-3)^2}$

$= \sqrt{16 + 9}$

$= \sqrt{25}$

= 5 units

$d(B,E) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(2 - 1)^2 + \{2 - (-1)\}^2}$

$= \sqrt{(1)^2 + (2 + 1)^2}$

$= \sqrt{1 + 9}$

$= \sqrt{10}$ units

$d(C,F) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(5 - 0)^2 + \{1 - 1\}^2}$

$= \sqrt{(5)^2 + (0)^2}$

$= \sqrt{25}$

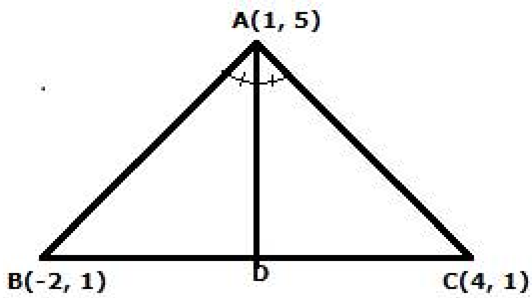
= 5 units

Hence, the length of the medians AD, BE and CF are 5, $\sqrt{10}$, 5 units respectively.

11. Question

If A(1,5), B (-2,1) and C(4,1) be the vertices of ΔABC and the internal bisector of ∠A meets BC and D, find AD.

Answer



Given: A(1, 5), B(-2, 1) and C(4,1) are the vertices of ΔABC

Using **angle bisector theorem**, which states that:

The ratio of the length of the line segment BD to the length of segment DC is equal to the ratio of the length of side AB to the length of side AC: {\displaystyle {\frac {|BD|}{|DC|}}={\frac {|AB|}{|AC|}},}

$$\frac{|BD|}{|DC|} = \frac{|AB|}{|AC|}$$

$$\Rightarrow \frac{BD}{DC} = \frac{\sqrt{(-2-1)^2 + (1-5)^2}}{\sqrt{(4-1)^2 + (1-5)^2}}$$

$$\Rightarrow \frac{BD}{DC} = \frac{\sqrt{9+16}}{\sqrt{9+16}}$$

$$\Rightarrow \frac{BD}{DC} = \frac{1}{1}$$

⇒ BD = DC

⇒ D is the midpoint of BC

So, the coordinates of D are:

$$D = \left[\frac{-2+4}{2}, \frac{1+1}{2}\right]$$

$$D = \left[\frac{2}{2}, \frac{2}{2}\right]$$

D = (1, 1)

Now, AD = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

= $\sqrt{(1 - 1)^2 + \{5 - 1\}^2}$

= $\sqrt{(0)^2 + (4)^2}$

= $\sqrt{16}$

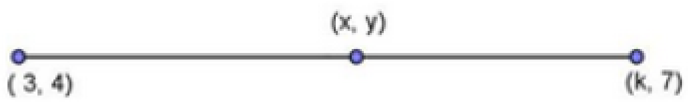
= 4 units

Hence, AD = 4 units

12. Question

If the middle point of the line segment joining (3,4) and (k,7) is (x,y) and 2x+2y+1=0, find the value of k.

Answer



Let P be the midpoint of the line segment joining (3, 4) and (k, 7)

So, the coordinates of P are:

$$x = \frac{3+k}{2}, y = \frac{4+7}{2}$$

$$x = \frac{3+k}{2}, y = \frac{11}{2}$$

Again,

2x + 2y + 1 = 0

$$\Rightarrow\ 2\left(\frac{3+k}{2}\right) + 2\left(\frac{11}{2}\right) + 1 = 0$$

⇒ 3 + k + 11 + 1 = 0

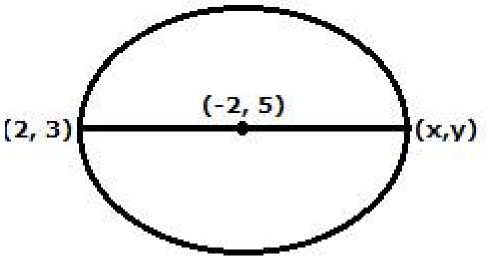
⇒ k + 15 = 0

⇒ k = -15

13 A. Question

one end of a diameter of a circle is at (2,3) and the center is (-2,5), find the coordinates of the other end of the diameter.

Answer



Let the coordinates of the other end be (x,y).

Since (-2, 5) is the midpoint of the line joining (2,3) and (x,y)

$$\therefore (-2, 5) = \left(\frac{2+x}{2}, \frac{3+y}{2}\right)$$

$$\Rightarrow \frac{2+x}{2} = -2 \text{ and } \frac{3+y}{2} = 5$$

⇒ x + 2 = -4 and y + 3 = 10

⇒ x = -4 – 2 and y = 10 – 3

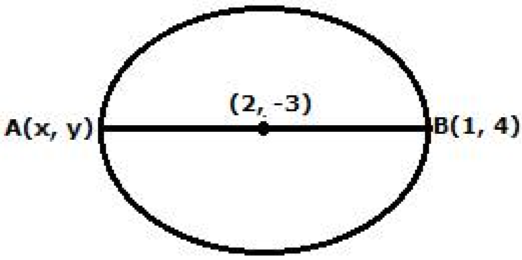
⇒ x = -6 and y = 7

Hence, the coordinates of the other end are (-6, 7)

13 B. Question

Find the coordinates of a point A, where AB is the diameter of a circle whose center is (2,-3), and B is (1,4)

Answer



Let the coordinates of the A be (x,y).

Since 2, -3) is the midpoint of the line joining (1, 4) and (x,y)

$$\therefore (2,-3) = \left(\frac{1+x}{2}, \frac{4+y}{2}\right)$$

$$\Rightarrow \frac{1+x}{2} = 2 \text{ and } \frac{4+y}{2} = -3$$

⇒ x + 1 = 4 and y + 4 = -6

⇒ x = 4 – 1 and y = -6 – 4

⇒ x = 3 and y = -10

Hence, the coordinates of A are (3, -10)

14. Question

If the point C (-1,2) divides internally the line segment joining A (2,5) and B in the ratio 3:4. Find the coordinates of B.

Answer

Let the coordinates of B are (x, y)

It is given that the line segment divide in the ratio 3:4

So, m_1 = 3 and m_2 = 4

and (x', y') =(-1, 2); (x_1, y_1) = (2,5); (x_2, y_2) = (x, y)

Using section formula for the internal division, we get

$$(x',y') = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right) \text{ ...(i)}$$

$$\Rightarrow\ (-1,2) = \frac{3(x)+4(2)}{3+4}, y = \frac{3(y)+4(5)}{3+4}$$

$$\Rightarrow\ -1 = \frac{3x+8}{7}, 2 = \frac{3y+20}{7}$$

⇒ 3x + 8 = -7 and 3y + 20 = 14

⇒ 3x = -7 – 8 and 3y = 14 – 20

⇒ 3x = -15 and 3y = -6

⇒ x = -5 and y = -2

Hence, the coordinates of B are (-5, -2)

15 A. Question

Find the ratio in which (-8,3) divides the line segment joining the points (2,-2) and (-4,1).

Answer

Let C(-8, 3) divides the line segment AB in the ratio m:n

$$\therefore x = \frac{mx_2 + nx_1}{m+n} \text{ and } y = \frac{my_2 + ny_1}{m+n}$$

Here, (x, y) = (-8, 3); (x_1, y_1) = (2, -2) and (x_2, y_2) = (-4,1)

$$\text{So, } -8 = \frac{m(-4)+n(2)}{m+n} \text{ and } 3 = \frac{m(1)+n(-2)}{m+n}$$

⇒ -8m -8n = -4m + 2n and 3m + 3n = m - 2n

⇒ -8m + 4m - 8n - 2n = 0 and 3m – m + 3n + 2n = 0

⇒ -4m – 10n = 0 and 2m + 5n = 0

⇒ -2m – 5n = 0 and 2m + 5n = 0

⇒ 2m + 5n = 0 and 2m + 5n = 0

⇒ 2m = -5n

$$\Rightarrow\ \frac{m}{n} = \frac{-5}{2}$$

Hence, the ratio is 5:2 and this negative sign shows that the division is external.

15 B. Question

In what ratio does the point (-4,6) divide the line segment joining the point A(-6,10) and B (3,-8)?

Answer

Let C(-4, 6) divides the line segment AB in the ratio m:n

$$\therefore x = \frac{mx_2 + nx_1}{m + n} \text{ and } y = \frac{my_2 + ny_1}{m + n}$$

Here, (x, y) = (-4, 6); (x_1, y_1) = (-6, 10) and (x_2, y_2) = (3, -8)

$$\text{So, } -4 = \frac{m(3)+n(-6)}{m+n} \text{ and } 6 = \frac{m(-8)+n(10)}{m+n}$$

⇒ -4m -4n = 3m - 6n and 6m + 6n = -8m + 10n

⇒ -4m – 3m – 4n + 6n = 0 and 6m + 8m + 6n – 10n = 0

⇒ -7m + 2n = 0 and 14m - 4n = 0

⇒ -7m = -2n and 14m = 4n

⇒ 7m = 2n and 7m = 2n

$$\Rightarrow \frac{m}{n} = \frac{2}{7}$$

Hence, the ratio is 2:7 and the division is internal.

15 C. Question

Find the ratio in which the line segment joining (-3,10) and (6,-8) is divided by (-1,6)

Answer

Let C(-1, 6) divides the line segment AB in the ratio m:n

$$\therefore x = \frac{mx_2 + nx_1}{m + n} \text{ and } y = \frac{my_2 + ny_1}{m + n}$$

Here, (x, y) = (-1, 6); (x_1, y_1) = (-3, 10) and (x_2, y_2) = (6, -8)

$$\text{So, } -1 = \frac{m(6)+n(-3)}{m+n} \text{ and } 6 = \frac{m(-8)+n(10)}{m+n}$$

⇒ -m – n = 6m - 3n and 6m + 6n = -8m + 10n

⇒ -m – 6m – n + 3n = 0 and 6m + 8m + 6n – 10n = 0

⇒ -7m + 2n = 0 and 14m - 4n = 0

⇒ 2n = 7m and 4n = 14m

⇒ 7m = 2n

$\Rightarrow \frac{m}{n} = \frac{2}{7}$

Hence, the ratio is 2:7 and the division is internal.

15 D. Question

Find the ratio in which the line segment joining (-3,-4) and (3,5) is divided by (x,2). Also, find x.

Answer

Let C(x, 2) divides the line segment AB in the ratio m:n

$\therefore x = \frac{mx_2 + nx_1}{m + n}$ and $y = \frac{my_2 + ny_1}{m + n}$

Here, (x, y) = (x, 2); (x_1, y_1) = (-3, -4) and (x_2, y_2) = (3, 5)

So, $2 = \frac{m(5)+n(-4)}{m+n}$

⇒ 2m + 2n = 5m – 4n

⇒ 2m – 5m = -4n – 2n

⇒ -3m = -6n

⇒ m = 2n

$\Rightarrow \frac{m}{n} = \frac{2}{1}$

Now, the ratio is 2:1

Now, $x = \frac{m(3)+n(-3)}{m+n}$

$\Rightarrow x = \frac{2(3) + 1(-3)}{2 + 1}$

⇒ 3x = 6 – 3

⇒ 3x = 3

⇒ x = 1

Hence, the ratio is 2:1 and the division is internal and the value of x = 1

16 A. Question

In what ratio does the x-axis divide the line segment joining the points (2,-3) and (5,6).

Answer

Let the line segment A(2, -3) and B(5, 6) is divided at point P(x,0) by x-axis in ratio m:n

$$\therefore x = \frac{mx_2 + nx_1}{m + n} \text{ and } y = \frac{my_2 + ny_1}{m + n}$$

Here, (x, y) = (x, 0); (x_1, y_1) = (2, -3) and (x_2, y_2) = (5, 6)

So, $0 = \frac{m(6)+n(-3)}{m+n}$

⇒ 0 = 6m – 3n

⇒ -6m = -3n

$$\Rightarrow \frac{m}{n} = \frac{1}{2}$$

Hence, the ratio is 1:2 and the division is internal.

16 B. Question

Find the ratio in which the line segment joining A(1,-5) and B(-4,5) is divided by the x-axis. Also, find the coordinates of the point of division.

Answer

Let the line segment A(1, -5) and B(-4, 5) is divided at point P(x,0) by x-axis in ratio m:n

$$\therefore x = \frac{mx_2 + nx_1}{m + n} \text{ and } y = \frac{my_2 + ny_1}{m + n}$$

Here, (x, y) = (x, 0); (x_1, y_1) = (1, -5) and (x_2, y_2) = (-4, 5)

So, $0 = \frac{m(5)+n(-5)}{m+n}$

⇒ 0 = 5m – 5n

⇒ 5m = 5n

$$\Rightarrow \frac{m}{n} = \frac{1}{1}$$

Hence, the ratio is 1:1 and the division is internal.

Now,

$$x = \frac{mx_2 + nx_1}{m + n}$$

$$\Rightarrow x = \frac{1(-4) + 1(1)}{1 + 1}$$

$$\Rightarrow x = \frac{-3}{2}$$

Hence, the coordinates of the point of division is $\left(\frac{-3}{2}, 0\right)$

16 C. Question

Find the ratio in which the y-axis divides the line segment joining points (5,-6) and (-1,-4). Also, find the point of intersection.

Answer

Let the line segment A(5, -6) and B(-1, -4) is divided at point P(0, y) by y-axis in ratio m:n

$$\therefore x = \frac{mx_2 + nx_1}{m + n} \text{ and } y = \frac{my_2 + ny_1}{m + n}$$

Here, (x, y) = (0, y); (x_1, y_1) = (5, -6) and (x_2, y_2) = (-1, -4)

So, $0 = \frac{m(-1)+n(5)}{m+n}$

⇒ 0 = -m + 5n

⇒ m = 5n

$$\Rightarrow \frac{m}{n} = \frac{5}{1}$$

Hence, the ratio is 5:1 and the division is internal.

Now,

$$y = \frac{mx_2 + nx_1}{m + n}$$

$$\Rightarrow\ y = \frac{5(-4) + 1(-6)}{5 + 1}$$

$$\Rightarrow\ y = \frac{-20 - 6}{6} = \frac{-26}{6} = \frac{-13}{3}$$

Hence, the coordinates of the point of division is $\left(0, \frac{-13}{3}\right)$

17. Question

Find the centroid of the triangle whose vertices are (2,4), (6,4), (2,0).

Answer

Here, $x_1 = 2, x_2 = 6, x_3 = 2$

and $y_1 = 4, y_2 = 4, y_3 = 0$

Let the coordinates of the centroid be(x,y)

So,

$$\text{Centroid of triangle } (x, y) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

$$= \left(\frac{2 + 6 + 2}{3}, \frac{4 + 4 + 0}{3}\right)$$

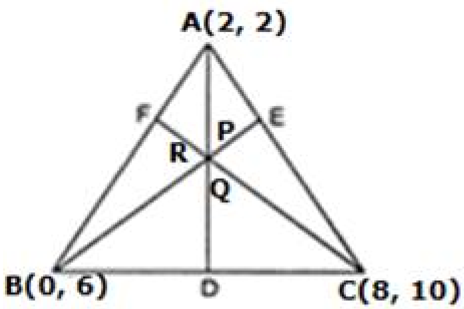
$$= \left(\frac{10}{3}, \frac{8}{3}\right)$$

Hence, the centroid of a triangle is $\left(\frac{10}{3}, \frac{8}{3}\right)$

18. Question

The vertices of a triangle are at (2,2), (0,6) and (8,10). Find the coordinates of the trisection point of each median which is nearer the opposite side.

Answer



Let (2, 2), (0, 6) and (8, 10) be the vertices A, B and C of the triangle respectively. Let AD, BE, CF be the medians

The coordinates of D are:

$$D = \left[\frac{0+8}{2}, \frac{6+10}{2}\right]$$

$$D = \left[\frac{8}{2}, \frac{16}{2}\right]$$

D = (4, 8)

The coordinates of E are:

$$E = \left[\frac{8+2}{2}, \frac{10+2}{2}\right]$$

$$E = \left[\frac{10}{2}, \frac{12}{2}\right]$$

E = (5, 6)

The coordinates of F are:

$$F = \left[\frac{2+0}{2}, \frac{2+6}{2}\right]$$

$$F = \left[\frac{2}{2}, \frac{8}{2}\right]$$

F = (1, 4)

Let P be the trisection point of the median AD which is nearer to the opposite side BC

∴ P divides DA in the ratio 1:2 internally

$$\therefore P = \left(\frac{1(2)+2(4)}{1+2}, \frac{1(2)+2(8)}{1+2}\right)$$

$$= \left(\frac{2+8}{3}, \frac{2+16}{3}\right)$$

$$= \left(\frac{10}{3}, 6\right)$$

Let Q be the trisection point of the median BE which is nearer to the opposite side CA

∴ Q divides EB in the ratio 1:2 internally

$$\therefore Q = \left(\frac{1(0)+2(5)}{1+2}, \frac{1(6)+2(6)}{1+2}\right)$$

$$= \left(\frac{0+10}{3}, \frac{6+12}{3}\right)$$

$$= \left(\frac{10}{3}, 6\right)$$

Let R be the trisection point of the median CF which is nearer to the opposite side AB

∴ R divides FC in the ratio 1:2 internally

$$\therefore R = \left(\frac{1(8)+2(1)}{1+2}, \frac{1(10)+2(4)}{1+2}\right)$$

$$= \left(\frac{8+2}{3}, \frac{10+8}{3}\right)$$

$$= \left(\frac{10}{3}, 6\right)$$

Therefore, Coordinates of required trisection points are $\left(\frac{10}{3}, 6\right), \left(\frac{10}{3}, 6\right)$ and $\left(\frac{10}{3}, 6\right)$

19. Question

Two vertices of a triangle are (1,4) and (5,2). If its centroid is (0,-3), find the third vertex.

Answer

Let the third vertex of a triangle be(x,y)

Here, $x_1 = 1$, $x_2 = 5$, $x_3 = x$

and $y_1 = 4$, $y_2 = 2$, $y_3 = y$

and the coordinates of the centroid is (0, -3)

We know that

$$\text{Centroid of triangle } (x, y) = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

$$(0, -3) = \left(\frac{1+5+x}{3}, \frac{4+2+y}{3}\right)$$

$$(0,-3)=\left(\frac{6+x}{3},\frac{6+y}{3}\right)$$

$$\Rightarrow\ 0=\frac{6+x}{3}\text{ and }-3=\frac{6+y}{3}$$

⇒ 6 + x = 0 and 6 + y = -9

⇒ x = -6 and y = -15

Hence, the third vertex of a triangle is (-6, -15)

20. Question

The coordinates of the centroid of a triangle are (√3,2), and two of its vertices are (2√3,-1) and (2√3,5). Find the third vertex of the triangle.

Answer

Let the third vertex of a triangle be (x, y)

Here, $x_1 = 2\sqrt{3}$, $x_2 = 2\sqrt{3}$, $x_3 = x$

and $y_1 = -1$, $y_2 = 5$, $y_3 = y$

and the coordinates of the centroid is (√3, 2)

We know that

$$\text{Centroid of triangle }(x,y)=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$$

$$(\sqrt{3},2)=\left(\frac{2\sqrt{3}+2\sqrt{3}+x}{3},\frac{-1+5+y}{3}\right)$$

$$(\sqrt{3},2)=\left(\frac{4\sqrt{3}+x}{3},\frac{4+y}{3}\right)$$

$$\Rightarrow\ \sqrt{3}=\frac{4\sqrt{3}+x}{3}\text{ and }2=\frac{4+y}{3}$$

⇒ 4√3 + x = 3√3 and 4 + y = 6

⇒ x = -√3 and y = 2

Hence, the third vertex of a triangle is (-√3, 2)

21. Question

Find the centroid of the triangle ABC whose vertices are A (9,2), B(1,10) and C(-7,-6). Find the coordinates of the middle points of its sides and hence find the centroid of the triangle formed by joining these middle points. Do the two triangles have the same centroid?

Answer

The vertices of a triangle are A (9,2), B(1,10) and C(-7,-6)

Here, $x_1 = 9, x_2 = 1, x_3 = -7$

and $y_1 = 2, y_2 = 10, y_3 = -6$

Let the coordinates of the centroid be(x,y)

So,

$$\text{Centroid of triangle } (x, y) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

$$= \left(\frac{9 + 1 + (-7)}{3}, \frac{2 + 10 + (-6)}{3}\right)$$

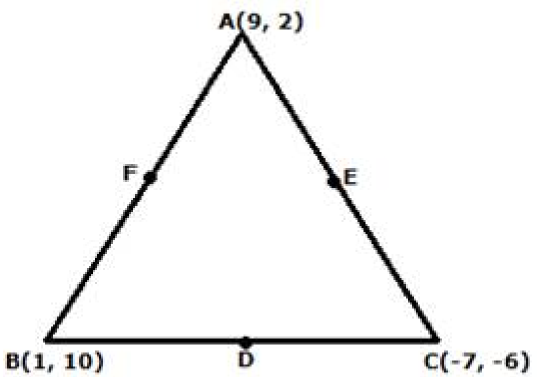
$$= \left(\frac{10 - 7}{3}, \frac{12 - 6}{3}\right)$$

$$= \left(\frac{3}{3}, \frac{6}{3}\right)$$

= (1,2)

Hence, the centroid of a triangle is (1, 2)

Now,



Let D, E and F are the midpoints of the sides BC, CA and AB respectively.

The coordinates of D are:

$$D = \left[\frac{-7 + 1}{2}, \frac{-6 + 10}{2}\right]$$

$$D = \left[\frac{-6}{2}, \frac{4}{2}\right]$$

D = (-3, 2)

The coordinates of E are:

$$E = \left[\frac{-7+9}{2}, \frac{-6+2}{2}\right]$$

$$E = \left[\frac{2}{2}, \frac{-4}{2}\right]$$

E = (1, -2)

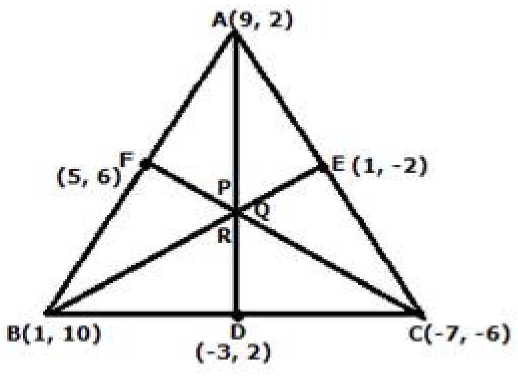
The coordinates of F are:

$$F = \left[\frac{1+9}{2}, \frac{10+2}{2}\right]$$

$$F = \left[\frac{10}{2}, \frac{12}{2}\right]$$

F = (5, 6)

Now, we find the centroid of a triangle formed by joining these middle points D, E, and F as shown in figure



Let P be the trisection point of the median AD which is nearer to the opposite side BC

∴ P divides DA in the ratio 1:2 internally

$$\therefore P = \left(\frac{1(9)+2(-3)}{1+2}, \frac{1(2)+2(2)}{1+2}\right)$$

$$= \left(\frac{9-6}{3}, \frac{2+4}{3}\right)$$

$$= \left(\frac{3}{3}, \frac{6}{3}\right)$$

= (1, 2)

Let Q be the trisection point of the median BE which is nearer to the opposite side CA

∴ Q divides EB in the ratio 1:2 internally

$$\therefore \text{Q} = \left(\frac{1(1) + 2(1)}{1 + 2}, \frac{1(10) + 2(-2)}{1 + 2}\right)$$

$$= \left(\frac{1 + 2}{3}, \frac{10 - 4}{3}\right)$$

$$= \left(\frac{3}{3}, \frac{6}{3}\right)$$

= (1, 2)

Let R be the trisection point of the median CF which is nearer to the opposite side AB

∴ R divides FC in the ratio 1:2 internally

$$\therefore \text{R} = \left(\frac{1(-7) + 2(5)}{1 + 2}, \frac{1(-6) + 2(6)}{1 + 2}\right)$$

$$= \left(\frac{-7 + 10}{3}, \frac{-6 + 12}{3}\right)$$

$$= \left(\frac{3}{3}, \frac{6}{3}\right)$$

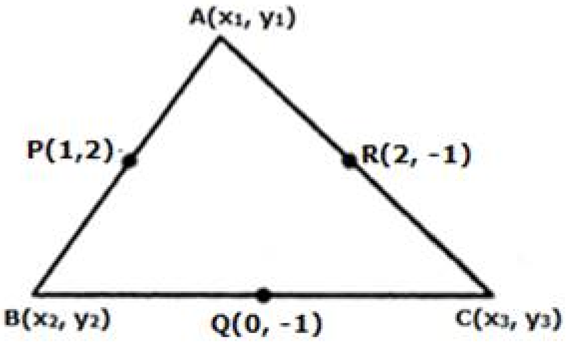
= (1, 2)

Yes, the triangle has the same centroid, i.e. (1,2)

22. Question

If (1,2), (0,-1) and (2,-1) are the middle points of the sides of the triangle, find the coordinates of its centroid.

Answer



Consider a ΔABC with $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$. If P(1, 2), Q(0, -1) and R(2, -1) are the midpoints of AB, BC and CA. Then,

$1 = \frac{x_1+x_2}{2} \Rightarrow x_1 + x_2 = 2$ ...(i)

$2 = \frac{y_1+y_2}{2} \Rightarrow y_1 + y_2 = 4$ ...(ii)

$0 = \frac{x_2+x_3}{2} \Rightarrow x_2 + x_3 = 0$ ...(iii)

$-1 = \frac{y_2+y_3}{2} \Rightarrow y_2 + y_3 = -2$ ...(iv)

$2 = \frac{x_1+x_3}{2} \Rightarrow x_1 + x_3 = 4$ ...(v)

$-1 = \frac{y_1+y_3}{2} \Rightarrow y_1 + y_3 = -2$ ...(vi)

Adding (i), (iii) and (v), we get

$x_1 + x_2 + x_2 + x_3 + x_1 + x_3 = 2 + 0 + 4$

$\Rightarrow 2(x_1 + x_2 + x_3) = 6$

$\Rightarrow x_1 + x_2 + x_3 = 3$ ...(vii)

From (i) and (vii), we get

$x_3 = 3 - 2 = 1$

From (iii) and (vii), we get

$x_1 = 3 - 0 = 3$

From (v) and (vii), we get

$x_2 = 3 - 4 = -1$

Now adding (ii), (iv) and (vi), we get

$y_1 + y_2 + y_2 + y_3 + y_1 + y_3 = 4 + (-2) + (-2)$

$\Rightarrow 2(y_1 + y_2 + y_3) = 0$

$\Rightarrow y_1 + y_2 + y_3 = 0$ ...(viii)

From (ii) and (viii), we get

$y_3 = 0 - 4 = -4$

From (iv) and (vii), we get

$y_1 = 0 - (-2) = 2$

From (vi) and (vii), we get

$y_2 = 0 - (-2) = 2$

Hence, the vertices of ΔABC are A(3, 2), B(-1, 2) and C(1, -4)

Now, we have to find the centroid of a triangle

The vertices of a triangle are A(3, 2), B(-1, 2) and C(1, -4)

Here, $x_1 = 3, x_2 = -1, x_3 = 1$

and $y_1 = 2, y_2 = 2, y_3 = -4$

Let the coordinates of the centroid be(x,y)

So,

$$\text{Centroid of triangle } (x, y) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

$$= \left(\frac{3 + (-1) + 1}{3}, \frac{2 + 2 + (-4)}{3}\right)$$

$$= \left(\frac{3}{3}, \frac{0}{3}\right)$$

= (1,0)

Hence, the centroid of a triangle is (1, 0)

23. Question

Show that A(-3,2), B(-5,-5), C(2,-3) and D(4,4) are the vertices of a rhombus.

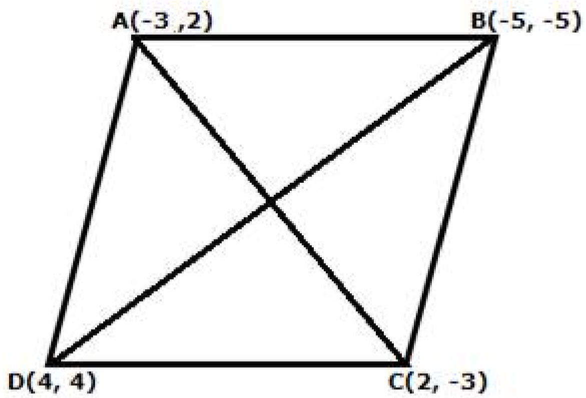
Answer

Note that to show that a quadrilateral is a rhombus, it is sufficient to show that

(a) ABCD is a parallelogram, i.e., AC and BD have the same midpoint.

(b) a pair of adjacent edges are equal

(c) the diagonal AC and BD are not equal.



Let A(-3, 2), B(-5,-5), C(2,-3) and D(4,4) are the vertices of a rhombus.

Coordinates of the midpoint of AC are

$$\left(\frac{-3+2}{2}, \frac{2-3}{2}\right) = \left(\frac{-1}{2}, \frac{-1}{2}\right)$$

Coordinates of the midpoint of BD are

$$\left(\frac{-5+4}{2}, \frac{-5+4}{2}\right) = \left(\frac{-1}{2}, \frac{-1}{2}\right)$$

Thus, AC and BD have the same midpoint.

Hence, ABCD is a parallelogram

Now, using Distance Formula

d(A,B)= AB = $\sqrt{(-5 + 3)^2 + (-5 - 2)^2}$

⇒ AB = $\sqrt{(-2)^2 + (-7)^2}$

⇒ AB = $\sqrt{4 + 49}$

⇒ AB = $\sqrt{53}$ units

d(B,C)= BC = $\sqrt{(-5 - 2)^2 + (-5 + 3)^2}$

⇒ BC = $\sqrt{(-7)^2 + (-2)^2}$

⇒ BC = $\sqrt{49 + 4}$

⇒ BC = $\sqrt{53}$ units

d(C,D) = CD = $\sqrt{(4 - 2)^2 + (4 + 3)^2}$

$\Rightarrow$ CD = $\sqrt{(2)^2 + (7)^2}$

$\Rightarrow$ CD = $\sqrt{4 + 49}$

$\Rightarrow$ CD = $\sqrt{53}$ units

d(A,D) = AD = $\sqrt{(4 + 3)^2 + (4 - 2)^2}$

$\Rightarrow$ AD = $\sqrt{(7)^2 + (2)^2}$

$\Rightarrow$ AD = $\sqrt{49 + 4}$

$\Rightarrow$ AD = $\sqrt{53}$ units

Therefore, AB = BC = CD = AD = $\sqrt{53}$ units

Now, check for the diagonals

AC = $\sqrt{(2 + 3)^2 + (-3 - 2)^2}$

= $\sqrt{(5)^2 + (-5)^2}$

= $\sqrt{25 + 25}$

= $\sqrt{50}$

and

BD = $\sqrt{(4 + 5)^2 + (4 + 5)^2}$

$\Rightarrow$ BD = $\sqrt{(9)^2 + (9)^2}$

$\Rightarrow$ BD = $\sqrt{81 + 81}$

$\Rightarrow$ BD = $\sqrt{162}$

$\Rightarrow$ Diagonal AC $\neq$ Diagonal BD

Hence, ABCD is a rhombus.

24. Question

Show that the point (3,2),(0,5),(-3,2) and (0,-1) are the vertices of a square.

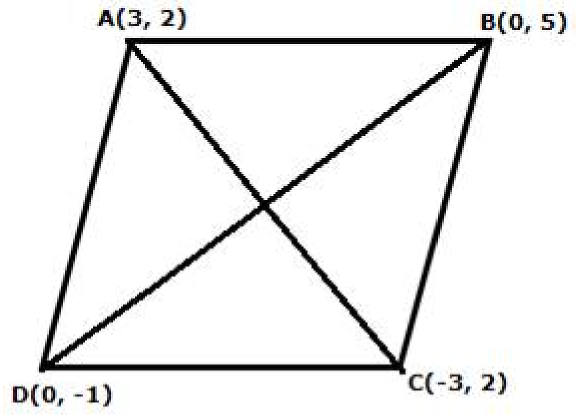
Answer

Note that to show that a quadrilateral is a square, it is sufficient to show that

(a) ABCD is a parallelogram, i.e., AC and BD bisect each other

(b) a pair of adjacent edges are equal

(c) the diagonal AC and BD are equal.



Let the vertices of a quadrilateral are A(3, 2), B(0,5), C(-3, 2) and D(0, -1).

Coordinates of the midpoint of AC are

$$\left(\frac{3+(-3)}{2}, \frac{2+2}{2}\right) = \left(\frac{0}{2}, \frac{4}{2}\right) = (0, 2)$$

Coordinates of the midpoint of BD are

$$\left(\frac{0+0}{2}, \frac{5+(-1)}{2}\right) = \left(\frac{0}{2}, \frac{4}{2}\right) = (0,2)$$

Thus, AC and BD have the same midpoint.

Hence, ABCD is a parallelogram

Now, Using Distance Formula, we get

AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

= $\sqrt{[(0 - 3)^2 + (5 - 2)^2]}$

= $\sqrt{(-3)^2 + (3)^2}$

= $\sqrt{(9 + 9)}$

= $\sqrt{18}$ units

BC = $\sqrt{[(-3 - 0)^2 + (2 - 5)^2]}$

= $\sqrt{(-3)^2 + (-3)^2}$

= $\sqrt{(9 + 9)}$

= $\sqrt{18}$ units

Therefore, AB = BC = $\sqrt{18}$ units

Now, check for the diagonals

AC = $\sqrt{(-3 - 3)^2 + (2 - 2)^2}$

= $\sqrt{(-6)^2 + (0)^2}$

= $\sqrt{36}$

= 6 units

and

BD = $\sqrt{(0 - 0)^2 + (-1 - 5)^2}$

⇒ BD = $\sqrt{(0)^2 + (-6)^2}$

⇒ BD = $\sqrt{36}$

⇒ BD = 6 units

∴ AC = BD

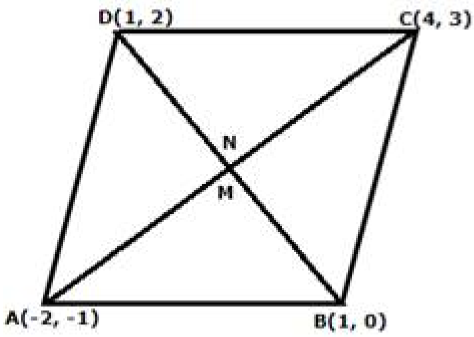
Hence, ABCD is a square.

25. Question

Prove that the points (-2,-1), (1,0),(4,3) and (1,2) are the vertices of a parallelogram.

Answer

Note that to show that a quadrilateral is a parallelogram, it is sufficient to show that the diagonals of the quadrilateral bisect each other.



Let A(-2, -1), B(1, 0), C(4, 3) and D(1, 2) are the vertices of a parallelogram.

Let M be the midpoint of AC, then the coordinates of M are given by

$$\left(\frac{-2+4}{2}, \frac{-1+3}{2}\right) = \left(\frac{2}{2}, \frac{2}{2}\right) = (1,1)$$

Let N be the midpoint of BD, then the coordinates of N are given by

$$\left(\frac{1+1}{2}, \frac{2+0}{2}\right) = \left(\frac{2}{2}, \frac{2}{2}\right) = (1,1)$$

Thus, AC and BD have the same midpoint.

In other words, AC and BD bisect each other.

Hence, ABCD is a parallelogram.

26. Question

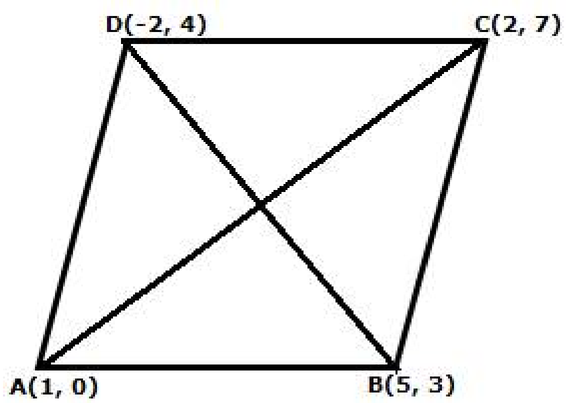
Show that the points A(1,0), B(5,3), C(2,7) and D(-2,4) are the vertices of a rhombus.

Answer

Note that to show that a quadrilateral is a rhombus, it is sufficient to show that

(a) ABCD is a parallelogram, i.e., AC and BD have the same midpoint.

(b) a pair of adjacent edges are equal



Let A(1, 0), B(5, 3), C(2, 7) and D(-2, 4) are the vertices of a rhombus.

Coordinates of the midpoint of AC are

$$\left(\frac{1+2}{2}, \frac{0+7}{2}\right) = \left(\frac{3}{2}, \frac{7}{2}\right)$$

Coordinates of the midpoint of BD are

$$\left(\frac{5-2}{2}, \frac{3+4}{2}\right) = \left(\frac{3}{2}, \frac{7}{2}\right)$$

Thus, AC and BD have the same midpoint.

Hence, ABCD is a parallelogram

Now, using Distance Formula

$d(A,B) = AB = \sqrt{(5 - 1)^2 + (3 - 0)^2}$

$\Rightarrow AB = \sqrt{(4)^2 + (3)^2}$

$\Rightarrow AB = \sqrt{16 + 9}$

$\Rightarrow AB = \sqrt{25} = 5$ units

$d(B,C) = BC = \sqrt{(2 - 5)^2 + (7 - 3)^2}$

$\Rightarrow BC = \sqrt{(-3)^2 + (4)^2}$

$\Rightarrow BC = \sqrt{9 + 16}$

$\Rightarrow BC = \sqrt{25} = 5$ units

Therefore, adjacent sides are equal.

Hence, ABCD is a rhombus.

27. Question

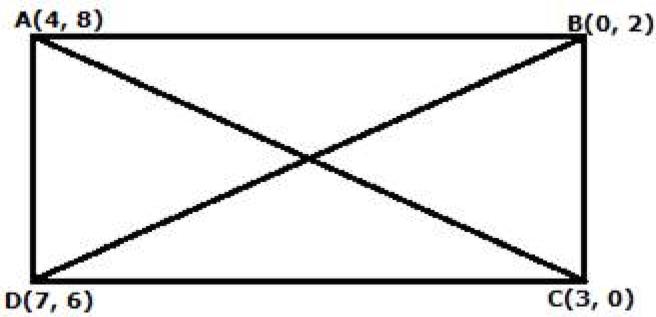
Prove that the point (4,8), (0,2), (3,0) and (7,6) are the vertices of a rectangle.

Answer

Note that to show that a quadrilateral is a rectangle, it is sufficient to show that

(a) ABCD is a parallelogram, i.e., AC and BD bisect each other and,

(b) the diagonal AC and BD are equal



Let A(4, 8), B(0, 2), C(3, 0) and D(7, 6) are the vertices of a rectangle.

Coordinates of the midpoint of AC are

$$\left(\frac{4+3}{2}, \frac{8+0}{2}\right) = \left(\frac{7}{2}, 4\right)$$

Coordinates of the midpoint of BD are

$$\left(\frac{0+7}{2}, \frac{2+6}{2}\right) = \left(\frac{7}{2}, 4\right)$$

Thus, AC and BD have the same midpoint.

Hence, ABCD is a parallelogram

Now, check for the diagonals by using the distance formula

AC = $\sqrt{(3 - 4)^2 + (0 - 8)^2}$

= $\sqrt{(-1)^2 + (-8)^2}$

= $\sqrt{1 + 64}$

= $\sqrt{65}$ units

and

BD = $\sqrt{(7 - 0)^2 + (6 - 2)^2}$

⇒ BD = $\sqrt{(7)^2 + (4)^2}$

⇒ BD = $\sqrt{49 + 16}$

⇒ BD = $\sqrt{65}$ units

∴ AC = BD

Hence, ABCD is a rectangle.

28. Question

Prove that the points (4,3), (6,4), (5,6) and (3,5) are the vertices of a square.

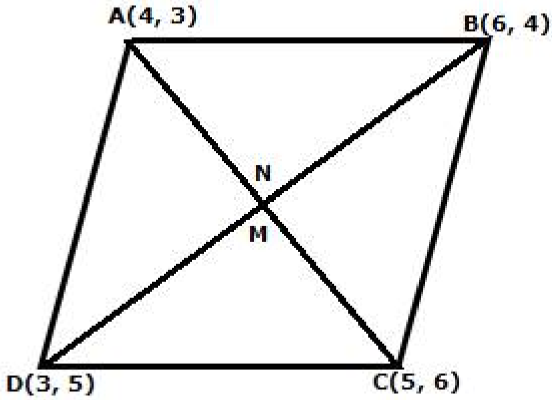
Answer

Note that to show that a quadrilateral is a square, it is sufficient to show that

(a) ABCD is a parallelogram, i.e., AC and BD bisect each other

(b) a pair of adjacent edges are equal

(c) the diagonal AC and BD are equal.



Let the vertices of a quadrilateral are A(4, 3), B(6, 4), C(5, 6) and D(3, 5).

Coordinates of the midpoint of AC are

$$\left(\frac{4+5}{2}, \frac{3+6}{2}\right) = \left(\frac{9}{2}, \frac{9}{2}\right)$$

Coordinates of the midpoint of BD are

$$\left(\frac{6+3}{2}, \frac{4+5}{2}\right) = \left(\frac{9}{2}, \frac{9}{2}\right)$$

Thus, AC and BD have the same midpoint.

Hence, ABCD is a parallelogram

Now, Using Distance Formula, we get

AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

= $\sqrt{[(6 - 4)^2 + (4 - 3)^2]}$

= $\sqrt{(2)^2 + (1)^2}$

= $\sqrt{(4 + 1)}$

= $\sqrt{5}$ units

BC = $\sqrt{[(5 - 6)^2 + (6 - 4)^2]}$

= $\sqrt{(-1)^2 + (2)^2}$

= $\sqrt{(1 + 4)}$

= $\sqrt{5}$ units

Therefore, AB = BC = $\sqrt{5}$ units

Now, check for the diagonals

$AC = \sqrt{(5 - 4)^2 + (6 - 3)^2}$

$= \sqrt{(1)^2 + (3)^2}$

$= \sqrt{1 + 9}$

$= \sqrt{10}$ units

and

$BD = \sqrt{(3 - 6)^2 + (5 - 4)^2}$

$\Rightarrow BD = \sqrt{(-3)^2 + (1)^2}$

$\Rightarrow BD = \sqrt{9 + 1}$

$\Rightarrow BD = \sqrt{10}$ units

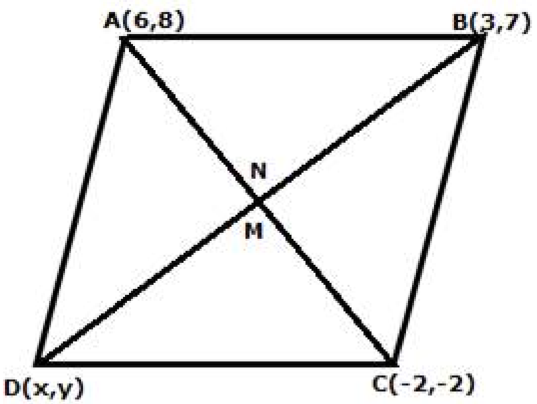
$\therefore$ AC = BD

Hence, ABCD is a square.

29. Question

If (6,8), (3,7) and (-2,-2) be the coordinates of the three consecutive vertices of a parallelogram, find coordinates of the fourth vertex.

Answer



Let the coordinates of the fourth vertex D be (x, y).

We know that diagonals of a parallelogram bisect each other.

$\therefore$ Midpoint of AC = Midpoint of BD ...(i)

Coordinates of the midpoint of AC are

$$\left(\frac{6-2}{2}, \frac{8-2}{2}\right) = \left(\frac{4}{2}, \frac{6}{2}\right) = (2,3)$$

Coordinates of the midpoint of BD are

$$\left(\frac{3+x}{2}, \frac{7+y}{2}\right)$$

So, according to eq. (i), we have

$$\Rightarrow\ (2,3) = \left(\frac{3+x}{2}, \frac{7+y}{2}\right)$$

$$\Rightarrow\ 2 = \frac{3+x}{2} \text{ and } 3 = \frac{7+y}{2}$$

⇒ 3 + x = 4 and 7 + y = 6

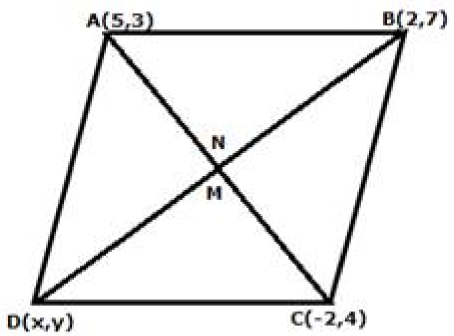
⇒ x = 1 and y = -1

Thus, the coordinates of the vertex D are (1, -1)

30. Question

Three consecutive vertices of a rhombus are (5,3), (2,7) and (-2,4). Find the fourth vertex.

Answer



Let the coordinates of the fourth vertex D be (x, y).

We know that diagonals of a rhombus bisect each other.

∴ Midpoint of AC = Midpoint of BD ...(i)

Coordinates of the midpoint of AC are

$$\left(\frac{5-2}{2}, \frac{3+4}{2}\right) = \left(\frac{3}{2}, \frac{7}{2}\right)$$

Coordinates of the midpoint of BD are

$$\left(\frac{2+x}{2}, \frac{7+y}{2}\right)$$

So, according to eq. (i), we have

$\Rightarrow \left(\frac{3}{2}, \frac{7}{2}\right) = \left(\frac{2+x}{2}, \frac{7+y}{2}\right)$

$\Rightarrow \frac{3}{2} = \frac{2+x}{2}$ and $\frac{7}{2} = \frac{7+y}{2}$

⇒ 2 + x = 3 and 7 + y = 7

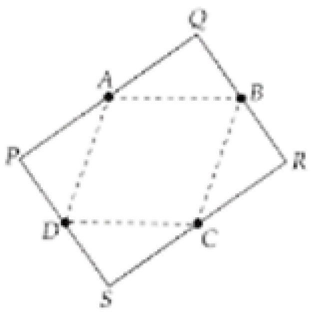
⇒ x = 1 and y = 0

Thus, the coordinates of the vertex D are (1, 0)

31. Question

A quadrilateral has the vertices at the point (-4,2), (2,6), (8,5) and (9,-7). Show that the mid-point of the sides of this quadrilateral are the vertices of a parallelogram.

Answer



Let the vertices of quadrilateral be P(-4,2), Q(2,6), R(8,5) and S(9,-7)

Let A, B, C and D are the midpoints of PQ, QR, RS and SP respectively.

Now, since A is the midpoint of P(-4, 2) and Q(2, 6)

∴ Coordinates of A are

$$\left(\frac{-4+2}{2}, \frac{2+6}{2}\right) = \left(\frac{-2}{2}, \frac{8}{2}\right) = (-1,4)$$

Coordinates of B are

$$\left(\frac{2+8}{2}, \frac{6+5}{2}\right) = \left(\frac{10}{2}, \frac{11}{2}\right) = \left(5, \frac{11}{2}\right)$$

Coordinates of C are

$$\left(\frac{8+9}{2}, \frac{5-7}{2}\right) = \left(\frac{17}{2}, \frac{-2}{2}\right) = \left(\frac{17}{2}, -1\right)$$

and

Coordinates of D are

$$\left(\frac{9-4}{2}, \frac{-7+2}{2}\right) = \left(\frac{5}{2}, \frac{-5}{2}\right)$$

Now,

we find the distance between A and B

$$d(A, B) = \sqrt{(-1-5)^2 + \left(4 - \frac{11}{2}\right)^2}$$

$$= \sqrt{36 + \frac{9}{4}} = \sqrt{\frac{144+9}{4}} = \sqrt{\frac{153}{4}}$$

$$d(C, D) = \sqrt{\left(\frac{17}{2} - \frac{5}{2}\right)^2 + \left(-1 + \frac{5}{2}\right)^2}$$

$$= \sqrt{36 + \frac{9}{4}} = \sqrt{\frac{144+9}{4}} = \sqrt{\frac{153}{4}}$$

$$d(A, D) = \sqrt{\left(-1 - \frac{5}{2}\right)^2 + \left(4 + \frac{5}{2}\right)^2}$$

$$= \sqrt{\frac{49}{4} + \frac{169}{4}} = \sqrt{\frac{218}{4}}$$

$$d(B, C) = \sqrt{\left(5 - \frac{17}{2}\right)^2 + \left(\frac{11}{2} + 1\right)^2}$$

$$= \sqrt{\frac{49}{4} + \frac{169}{4}} = \sqrt{\frac{218}{4}}$$

Now, since length of opposite sides of the quadrilateral formed by the midpoints of the given quadrilateral are equal .i.e.

AB = CD and AD = BC

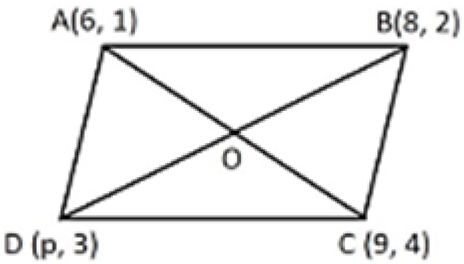
∴ it is a parallelogram

Hence Proved

32. Question

If the points A(6,1), B(8,2), C(9,4) and D(p,3) are the vertices of a parallelogram taken in order, find the value of p.

Answer



Let the points be A(6,1), B(8,2), C(9,4) and D(p,3)

We know that diagonals of parallelogram bisect each other.

∴ Midpoint of AC = Midpoint of BD ...(i)

Coordinates of the midpoint of AC are

$$\left(\frac{6+9}{2}, \frac{1+4}{2}\right) = \left(\frac{15}{2}, \frac{5}{2}\right)$$

Coordinates of the midpoint of BD are

$$\left(\frac{8+p}{2}, \frac{2+3}{2}\right) = \left(\frac{8+p}{2}, \frac{5}{2}\right)$$

So, according to eq. (i), we have

$$\Rightarrow \left(\frac{15}{2}, \frac{5}{2}\right) = \left(\frac{8+p}{2}, \frac{5}{2}\right)$$

$$\Rightarrow \frac{15}{2} = \frac{8+p}{2}$$

$\Rightarrow 8 + p = 15$

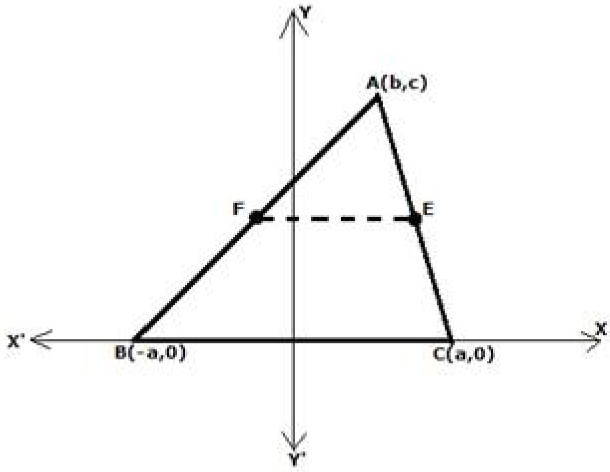
$\Rightarrow p = 15 - 8 = 7$

Hence, the value of p is 7

33. Question

Prove that the line segment joining the middle points of two sides of a triangle is half the third side.

Answer



We take O as the origin and OX and OY as the x and y axis respectively.

Let BC = 2a, then B = (-a, 0) and C = (a, 0)

Let A = (b, c), if E and F are the midpoints of sides AC and AB respectively.

Coordinates of midpoint of AC are

$$\left(\frac{b+a}{2}, \frac{c+0}{2}\right) = \left(\frac{a+b}{2}, \frac{c}{2}\right)$$

Coordinates of the midpoint of AB are

$$\left(\frac{b-a}{2}, \frac{c-0}{2}\right) = \left(\frac{b-a}{2}, \frac{c}{2}\right)$$

Now, distance between F and E is

$d(F,E) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{\left(\frac{a+b}{2} - \frac{b-a}{2}\right)^2 + \left(\frac{c}{2} - \frac{c}{2}\right)^2}$$

$$= \sqrt{\left(\frac{a+b-b+a}{2}\right)^2}$$

$$= \sqrt{\left(\frac{2a}{2}\right)^2}$$

= a ...(i)

and Length of BC = 2a ...(ii)

From (i) and (ii), we can say that

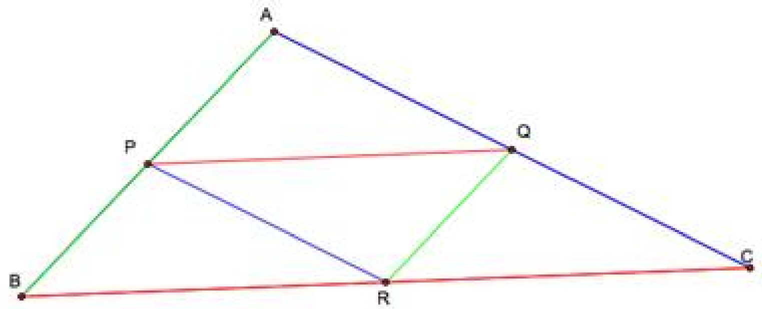
$$FE = \frac{1}{2}BC$$

Hence Proved

34. Question

If P,Q,R divide the side BC,CA and AB of ΔABC in the same ratio, prove that the centroid of the triangle ABC and PQR coincide.

Answer



Let P, Q, R be the midpoints of sides BC, CA and AB respectively

Construct a ΔPQR by joining these three midpoints of the sides.

This is called the medial triangle

Since, PQ, QR and PR are midsegments of BC, AB and AC respectively

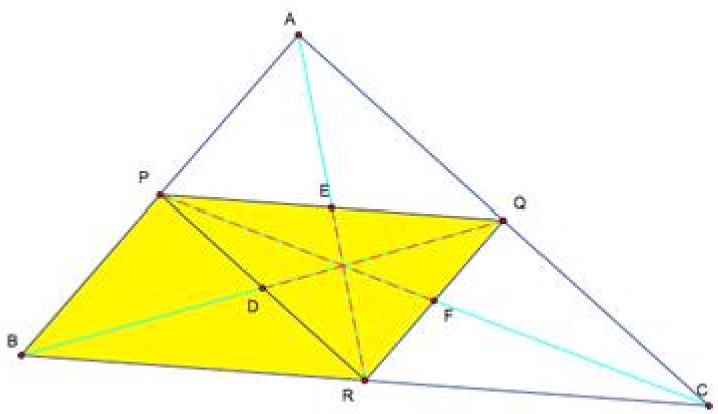
So,

$$PQ = \frac{1}{2}BC; QR = \frac{1}{2}AB \text{ and } PR = \frac{1}{2}AC$$

Since the corresponding sides are proportional

∴ ΔPQR ≅ ΔABC

Now, we have to prove that the centroid of the triangle ABC and PQR coincide.



For that we must show that the medians of ΔABC pass through the midpoints of three sides of the medial triangle ΔPQR.

Since PQ is a midsegment of ΔABC,

⇒ PQ || BC, so PQ || BR.

And since QR is a midsegment of AB,

⇒ AB || QR, so QR || PB.

By definition, a quadrilateral PQRB is a parallelogram.

The medians BQ and CP are in fact the diagonals of the parallelogram PQRB.

And we know that the diagonals of a parallelogram bisect each other, so PD = DR.

In other words, D is the midpoint of PR.

In the similar manner, we can show that F and E are midpoints of RQ and PQ respectively.

Hence, the centroid of the triangle ABC and PQR coincide.

Exercise 10.4

1 A. Question

Find the area of the triangle whose vertices are

(3, -4), (7, 5), (-1, 10)

Answer

Given: (3, -4), (7, 5), (-1, 10)

Let us Assume $A(x_1, y_1) = (3, -4)$

Let us Assume $B(x_2, y_2) = (7, 5)$

Let us Assume $C(x_3, y_3) = (-1, 10)$

Area of triangle $= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Now,

Area of given triangle $= \frac{1}{2}[\{3(5 - (10))\} + (7)\{(10 + 4)\} - 1(-4 - 5)\}]$

$$= \frac{1}{2}\{-15 + 98 + 9\}$$

$= \frac{92}{2}$

= 46 square units

1 B. Question

Find the area of the triangle whose vertices are

(-1.5, 3), (6, -2), (-3, 4)

Answer

Given (-1.5, 3), (6, -2), (-3, 4)

Let us Assume $A(x_1, y_1)$ = (-1.5, 3)

Let us Assume $B(x_2, y_2)$ = (6, -2)

Let us Assume $C(x_3, y_3)$ = (-3, 4)

Area of triangle $= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Now,

Area of given triangle $= \frac{1}{2}[\{-1.5(-2-(4))\} + (6)\{(4-3)\} \ -3(3+2)\}]$

$= \frac{1}{2}\{+9 + 6 - 15\}$

$= \frac{0}{2}$

= 0 square units

1 C. Question

Find the area of the triangle whose vertices are

(-5, -1), (3, -5), (5, 2)

Answer

Given (-5, -1), (3, -5), (5, 2)

Let us Assume $A(x_1, y_1)$ = (-5, -1)

Let us Assume $B(x_2, y_2)$ = (3, -5)

Let us Assume $C(x_3, y_3)$ = (5, 2)

Area of triangle $= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Now,

Area of given triangle $= \frac{1}{2}[\{-5(-5-2)\} + 3\{(2+1)\} + 5(-1+5)\}]$

$= \frac{1}{2}\{35 + 9 + 20\}$

$= \frac{64}{2}$

= 32 square units

1 D. Question

Find the area of the triangle whose vertices are

(5, 2), (4, 7), (7, -4)

Answer

Given (5, 2), (4, 7), (7, -4)

Let us Assume $A(x_1, y_1) = (5, 2)$

Let us Assume $B(x_2, y_2) = (4, 7)$

Let us Assume $C(x_3, y_3) = (7, -4)$

Area of triangle $= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Now,

Area of given triangle $= \frac{1}{2}[\{5(7-(-4))\} + 4\{(-4-2) + 7(2-7)\}]$

$= \frac{1}{2}\{55 - 24 - 35\}$

$= \frac{4}{2}$

= 2 square units

1 E. Question

Find the area of the triangle whose vertices are

(2, 3), (-1, 0), (2, -4)

Answer

Given (2, 3), (-1, 0), (2, -4)

Let us Assume $A(x_1, y_1)$ = (2, 3)

Let us Assume $B(x_2, y_2)$ = (-1, 0)

Let us Assume $C(x_3, y_3)$ = (2, -4)

Area of triangle $= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Now,

Area of given triangle $= \frac{1}{2}[\{2(0 - (-4))\} + (-1)\{(-4 - 3) + 2(3 - 0)\}]$

$= \frac{1}{2}\{8 + 7 + 6\}$

$= \frac{21}{2}$ Square units

1 F. Question

Find the area of the triangle whose vertices are

(1, -1), (-4, 6), (-3, -5)

Answer

Given (1, -1), (-4, 6), (-3, -5)

Let us Assume $A(x_1, y_1)$ = (1, -1)

Let us Assume $B(x_2, y_2)$ = (-4, 6)

Let us Assume $C(x_3, y_3)$ = (-3, -5)

Area of triangle $= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Now,

Area of given triangle
$= \frac{1}{2}[\{1(6 - (-5))\} + (-4)\{(-5 + 1) - 3(-1 - 6)\}]$

$= \frac{1}{2}\{30 + 16 + 21\}$

$= \frac{67}{2}$ Square units

1 G. Question

Find the area of the triangle whose vertices are

$(at_1^2, 2at_1), (at_2^2, 2at_2), (at_3^2, 2at_3)$

Answer

Given $(at_1^2, 2at_1), (at_2^2, 2at_2), (at_3^2, 2at_3)$

Let us Assume $A(x_1, y_1) = (at_1^2, 2at_1)$

Let us Assume $B(x_2, y_2) = (at_2^2, 2at_2)$

Let us Assume $C(x_3, y_3) = (at_3^2, 2at_3)$

Area of triangle $= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Now,

Area of given triangle
$= \frac{1}{2}[\{at_1^2(2at_2 - 2at_3)\} + at_2^2\{(2at_3 - 2at_1) - at_3^2(2at_1 - 2at_2)\}]$

$= \frac{1}{2}\{30 + 16 + 21\}$

$= \frac{67}{2}$ Square units

1 H. Question

Find the area of the triangle whose vertices are

(-5, 7), (-4, -5), (4, 5)

Answer

Given (-5, 7), (-4, -5), (4, 5)

Let us Assume $A(x_1, y_1) = (-5, 7)$

Let us Assume $B(x_2, y_2) = (-4, -5)$

Let us Assume $C(x_3, y_3) = (4, 5)$

Area of triangle $= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Now,

Area of given triangle $= \frac{1}{2}[\{-5(-5-(5))\} + (-4)\{(5-7) + 4(7+5)\}]$

$= \frac{1}{2}\{50 + 8 + 48\}$

$= \frac{106}{2}$

= 53 square units

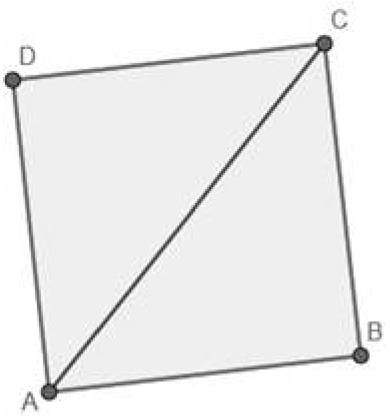
2 A. Question

Find the area of the quadrilateral whose vertices are

(1, 1), (7, -3), (12, 2) and (7, 21)

Answer

Given (1, 1), (7, -3), (12, 2) and (7, 21)



Let us Assume $A(x_1, y_1)$ = (1, 1)

Let us Assume $B(x_2, y_2)$ = (7, -3)

Let us Assume $C(x_3, y_3)$ = (12, 2)

Let us Assume $D(x_4, y_4)$ = (7, 21)

Let us join Ac to from two triangles ΔABC and ΔACD

Now

Area of triangle $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

Then,

Area of given triangle ABC $= \frac{1}{2}[1(-3-2) + 7(2-1) + 12(1+3)]$

$= \frac{1}{2}[-5 + 7 + 48]$

$= \frac{50}{2}$

= 25 square units

Area of given triangle ABC $= \frac{1}{2}[1(2-21) + 12(21-1) + 7(1-2)]$

$= \frac{1}{2}[-19 + 240 - 7]$

$= \frac{214}{2}$

= 107 square units

Area of quadrilateral ABCD = Area of ABC + Area of ACD

= 25 + 107

= 132 sq units.

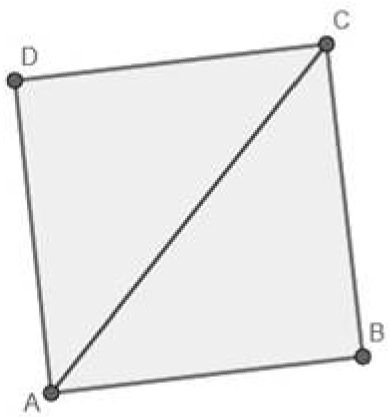
2 B. Question

Find the area of the quadrilateral whose vertices are

(-4, 5), (0, 7), (5, -5), and (-4, -2)

Answer

Given (-4, 5), (0, 7), (5, -5), and (-4, -2)



To Find: Find the area of quadrilateral.

Let us Assume A(x_1, y_1) = (-4, 5)

Let us Assume B(x_2, y_2) = (0, 7)

Let us Assume $C(x_3, y_3) = (5, -5)$

Let us Assume $D(x_4, y_4) = (4, -2)$

Let us join Ac to from two triangles ΔABC and ΔACD

Now

Area of triangle $= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Then,

Area of given triangle ABC $= \frac{1}{2}|-4(7+5) + 0(-5-5) + 5(5-7)|$

$= \frac{1}{2}[-48 + 0 - 10]$

$= \frac{58}{2}$

Area of given triangle ABC $= \frac{1}{2}|-4(-5+2) + 5(-2-5) - 4(5+5)|$

$= \frac{1}{2}|-4(-3) + 5(-7) - 4(10)|$

$= \frac{1}{2}|12 - 35 - 40|$

$= \frac{1}{2}|-63|$

$= \frac{63}{2}$ square units

Area of quadrilateral ABCD = Area of ABC + Area of ACD

$= \left|\frac{58}{2} + \frac{63}{2}\right|$

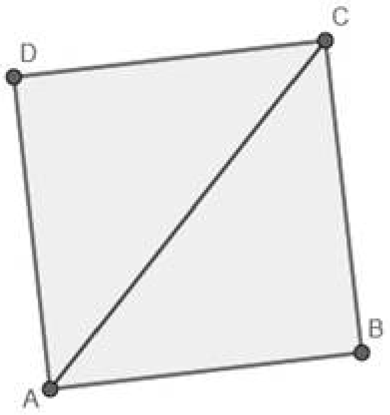
Hence, Area of Quadrilateral ABCD $= \frac{121}{2}$ sq units.

2 C. Question

Find the area of the quadrilateral whose vertices are

Given (-5, 7), (-4, -5), (-1, -6) and (4, 5)

Answer



To Find: Find the area of quadrilateral.

Let us Assume A(x_1, y_1) = (-5, 7)

Let us Assume B(x_2, y_2) = (-4, -5)

Let us Assume C(x_3, y_3) = (-1, -6)

Let us Assume D(x_4, y_4) = (4, 5)

Let us join Ac to from two triangles ΔABC and ΔACD

Now

Area of triangle $= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Then,

Area of given triangle ABC $= \frac{1}{2}| - 5(-5 + 6) - 4(-6 - 7) - 1(7 + 5)|$

$$= \frac{1}{2}| - 5(1) - 4(-13) - 1(12)|$$

$$= \frac{1}{2}| - 5 + 52 - 12|$$

$$= \frac{35}{2}$$

Area of given triangle ABC $= \frac{1}{2}| - 5(-6 - 5) - 1(5 - 7) + 4(7 + 6)|$

$$= \frac{1}{2}| - 5(-11) - 1(-2) + 4(13)|$$

$$= \frac{1}{2}|55 + 2 + 52|$$

$= \frac{112}{2}$

= 56 square units

Area of quadrilateral ABCD = Area of ABC + Area of ACD

$= \left|\frac{35}{2} + 56\right|$

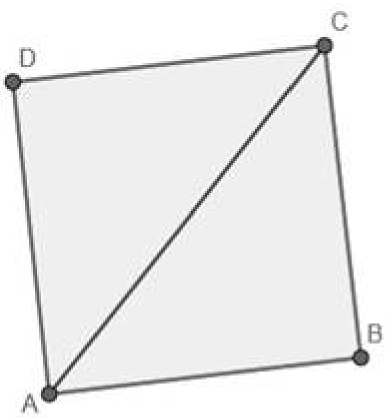
Hence, Area $= \frac{147}{2}$ sq units.

2 D. Question

Find the area of the quadrilateral whose vertices are

Given (0, 0), (6, 0), (4, 3), and (0, 3)

Answer



To Find: Find the area of quadrilateral.

Let us Assume $A(x_1, y_1) = (0, 0)$

Let us Assume $B(x_2, y_2) = (6, 0)$

Let us Assume $C(x_3, y_3) = (4, 3)$

Let us Assume $D(x_4, y_4) = (0, 3)$

Let us join Ac to from two triangles ΔABC and ΔACD

Now

Area of triangle $= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Then,

Area of given triangle ABC $= \frac{1}{2}|0(0-3)+6(3-0)+4(0-0)|$

$= \frac{1}{2}|0+18+0|$

$= \frac{1}{2}|18|$

= 9 Square units

Area of given triangle ABC $= \frac{1}{2}|0(3-3)+4(3-0)+0(0-3)|$

$= \frac{1}{2}|0+12+0|$

$= \frac{1}{2}|12|$

= 6 square units

Area of quadrilateral ABCD = Area of ABC + Area of ACD

= 9 + 6

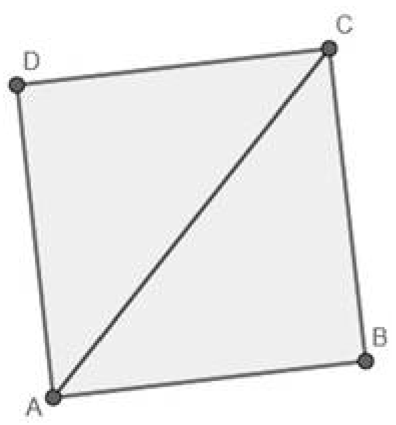
Hence, Area = 15 sq units.

2 E. Question

Find the area of the quadrilateral whose vertices are

Given (1, 0), (5, 3), (2, 7) and (-2, 4)

Answer



To Find: Find the area of the quadrilateral.

Let us Assume $A(x_1, y_1) = (1, 0)$

Let us Assume $B(x_2, y_2) = (5, 3)$

Let us Assume $C(x_3, y_3) = (2, 7)$

Let us Assume $D(x_4, y_4) = (-2, 4)$

Let us join Ac to from two triangles ΔABC and ΔACD

Now

Area of triangle $= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Then,

Area of given triangle ABC $= \frac{1}{2}|1(3-7) + 5(7-0) + 2(0-3)|$

$= \frac{1}{2}|-4 + 35 - 6|$

$= \frac{1}{2}|25|$

$= \frac{25}{2}$ Square units

Area of given triangle ABC $= \frac{1}{2}|1(7-4) + 2(4-0) - 2(0-7)$

$= \frac{1}{2}|3 + 8 + 14|$

$= \frac{1}{2}|25|$

$= \frac{25}{2}$ square units

Area of quadrilateral ABCD = Area of ABC + Area of ACD

$= \frac{25}{2} + \frac{25}{2}$

Hence, Area = 25 sq units.

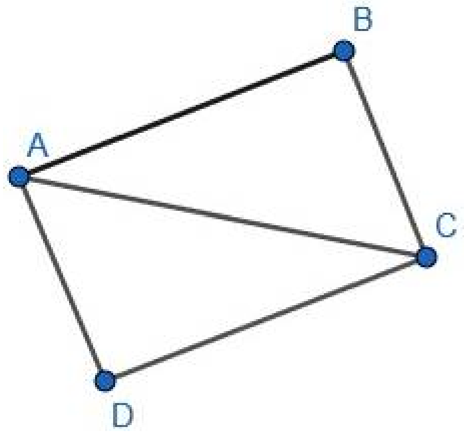
3. Question

Find the area of the quadrilateral whose vertices taken in order are (- 4, -2), (-3, -5), (3, -2) and (2, 3).

Answer

Given: The vertices of the quadrilateral be A(-4, -2), B(-3, -5), C(3, -2) and D(2, 3).

Let join AC to form two triangles,



Now, We know that

Area of triangle $= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Then,

Area of triangle ABC

$= \frac{1}{2}|(-4)\{(-5+2)\} + (-3)\{((-2)+2)\} + 3\{((-2)+2)\}]|$

$= \frac{1}{2}[12 + 0 + 9]$

$= \frac{21}{2}$ Square Units

Now, Area of triangle ACD

$= \frac{1}{2}|[(-4)\{(-2+3)\} + (3)\{((3)+2)\} + 2\{((-2)+2)\}]|$

$= \frac{1}{2}[20 + 15 + 0]$

$= \frac{35}{2}$ Square Units

Area of quadrilateral ABCD = Area of triangle ABC+ Area of triangle ACD

$= \frac{21}{2} + \frac{35}{2}$

$= \frac{56}{2}$

Hence, Area of quadrilateral ABCD = 28 square Units

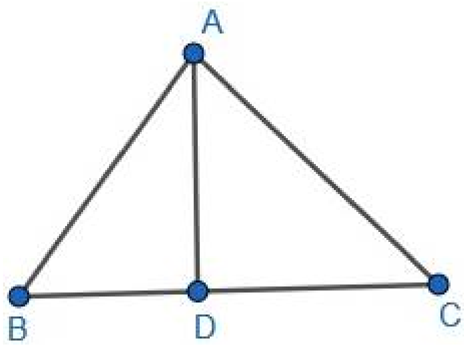
4. Question

A median of a triangle divides it into two triangles of equal area. Verify this result for $\triangle$ ABC whose vertices are A(1, 2), B(2, 5), C(3, 1).

Answer

Given a triangle whose vertices A(1, 2), B(2, 5), C(3, 1)

Let AD is the median on side BC



D will be the mid-point of segment BC. Therefore,

Coordinate of D $= \left[\frac{x_1+x_2}{2}, \frac{y_1 y_2}{2}\right]$

$= \left[\frac{2+3}{2}, \frac{5+1}{2}\right]$

$= \left[\frac{5}{2}, \frac{6}{2}\right]$

$= \left[\frac{5}{2}, 3\right]$

Area of triangle $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

Then,

Area of triangle ABD $= \frac{1}{2}\left|\left[1(5-3) + 2(3-2) + \frac{5}{2}(2-5)\right]\right|$

$= \frac{1}{2}\left[2 + 2 - \frac{15}{2}\right]$

$= \frac{1}{2}\left|-\frac{7}{2}\right|$

$= \frac{7}{4}$ sq units

Area of triangle ACD $= \frac{1}{2}\left|\left[1(1-3) + 3(3-2) + \frac{5}{2}(2-1)\right]\right|$

$= \frac{1}{2}\left|\left[-2+3+\frac{5}{2}\right]\right|$

$= \frac{1}{2}\left|\frac{7}{2}\right|$

$= \frac{7}{4}$ sq units

Hence, ΔABD = ΔACD

5. Question

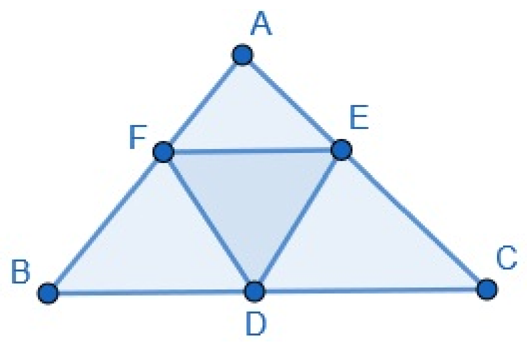
If A, B, C are the points (-1, 5), (3, 1), (5, 7) respectively and D, E, F are the middle points of BC, CA and AB respectively, prove that ΔABC = 4ΔDEF.

Answer

Given: ABC is a triangle with points (-1, 5), (3, 1), (5, 7)

To Find Δ ABC=4 Δ DEF

We know that



Area of triangle $= \frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Then,

Area of triangle ABC $= \frac{1}{2}|(-1)\{(1-7)\}+3(7-5)+5(5-1)|$

$= \frac{1}{2}[6+6+20]$

$= \frac{32}{2}$

= 16

Now we have to find point D, E, and F.

Hence D is the midpoint of side BC then, Coordinates of D $= \left[\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right]$

$= \left[\frac{3+5}{2}, \frac{1+7}{2}\right]$

= (4, 4)

Hence E is the midpoint of side AC then,

Coordinates of E $= \left[\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right]$

$= \left[\frac{-1+5}{2}, \frac{5+7}{2}\right]$

= (2, 6)

Hence F is the midpoint of side AB then,

Coordinates of F $= \left[\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right]$

$= \left[\frac{-1+3}{2}, \frac{5+1}{2}\right]$

= (1, 3)

Area of triangle $= \frac{1}{2}\{x_1(y_2-y_3)+x_2(y_3-y_1)+1(y_1-y_2)\}$

Now Area of triangle DEF $= \frac{1}{2}\ [4(6-3)+2(3-4)+1(4-6)]$

$= \frac{1}{2}[12-2-2]$

$= \frac{1}{2}[8]$

= 4

Therefore Area of Δ ABC= 4 Area of Δ DEF.

Hence Proved.

6 A. Question

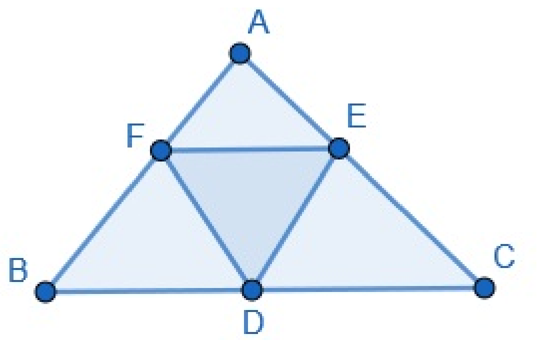
Three vertices of a triangle are A(1, 2), B(-3, 6) and C(5, 4). If D, E, and C, respectively, show that the area of triangle ABC is four times the area of triangle DEF.

Answer

Given: ABC is a triangle with points (1, 2), (-3, 6), (5, 4)

To prove: The area of triangle ABC is four times the area of triangle DEF

We know that



Area of triangle $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

Then,

Area of triangle ABC $= \frac{1}{2}|(1)\{(6-4)\} - 3(4-2) + 5(2-6)|$

$= \frac{1}{2}|2 - 6 - 20|$

$= \frac{-24}{2}$

= 12

Now we have to find point D, E, F

Hence D is the midpoint of side BC then,

Coordinates of D $= \left[\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right]$

$= \left[\frac{-3+5}{2}, \frac{6+4}{2}\right]$

= (1, 5)

Hence E is the midpoint of side AC then,

Coordinates of E $= \left[\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right]$

$= \left[\frac{1+5}{2}, \frac{2+4}{2}\right]$

= (3, 3)

Hence F is the midpoint of side AB then,

Coordinates of F $= \left[\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right]$

$= \left[\frac{1-3}{2}, \frac{2+6}{2}\right]$

= (-1, 4)

Area of triangle $= \frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Now Area of triangle DEF $= \frac{1}{2}|1(3-4)+3(4-5)-1(5-3)|$

$= \frac{1}{2}[-1-3-2]$

$= \frac{1}{2}|-6|$

= 3

Therefore Area of Δ ABC = 4 Area of Δ DEF.

Hence, Proved.

6 B. Question

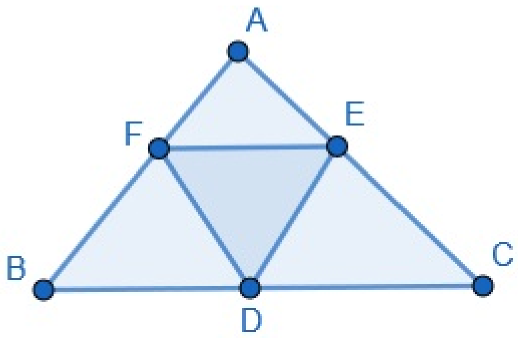
Find the area of the triangle formed by joining the mid-points of the sides of the triangles whose vertices are (0, -1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

Answer

Let ABC is a triangle with points (0, -1), (2, 1), (0, 3)

To Find: Ratio of area of triangle ABC to triangle DEF

We know that



Area of triangle $= \frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Then,

Area of triangle ABC $= \frac{1}{2}|0(1-3) + 2(3+1) + 0(-1-1)|$

$= \frac{1}{2}[8]$

= 4

Now we have to find point D, E, and F.

Hence D is the midpoint of side BC then,

Coordinates of D $= \left[\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right]$

$= \left[\frac{2+0}{2}, \frac{1+3}{2}\right]$

= (1, 2)

Hence E is the midpoint of side AC then,

Coordinates of E $= \left[\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right]$

$= \left[\frac{0+0}{2}, \frac{-1+3}{2}\right]$

= (0, 1)

Hence F is the midpoint of side AB then,

Coordinates of F $= \left[\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right]$

$= \left[\frac{0+2}{2}, \frac{-1+1}{2}\right]$

= (1, 0)

Area of triangle $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

Now Area of triangle DEF $= \frac{1}{2}|1(1-0) + 0(0-2) + 1(2-1)|$

$= \frac{1}{2}[1+1]$

$= \frac{1}{2}[2]$

= 1

Therefore Area of $\triangle$ ABC= 4 Area of $\triangle$ DEF.

Then, The ratio of ΔDEF and ΔABC = 1:4

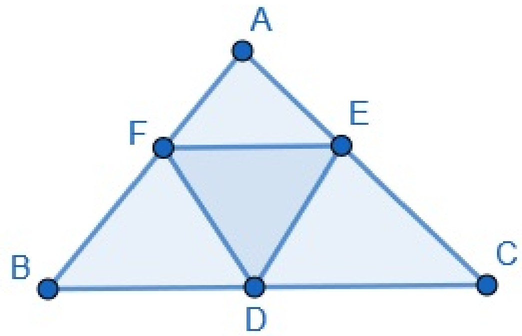
7. Question

Find the area of a triangle ABC if the coordinates of the middle points of the sides of the triangle are (-1, - 2), (6, 1) and (3, 5).

Answer

Given: Coordinates of middle points are D(-1, -2), E(6, 1) and F(3, 5).

To find: Area of triangle ABC



Area of triangle $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

Now Area of triangle DEF $= \frac{1}{2}|-1(1-5) + 6(5+2) + 3(-2-1)|$

$= \frac{1}{2}[4 + 42 - 9]$

$= \frac{1}{2}[37]$

$= \frac{37}{2}$ square units

Hence the area of ABC is $\frac{37}{2}$ square units

8. Question

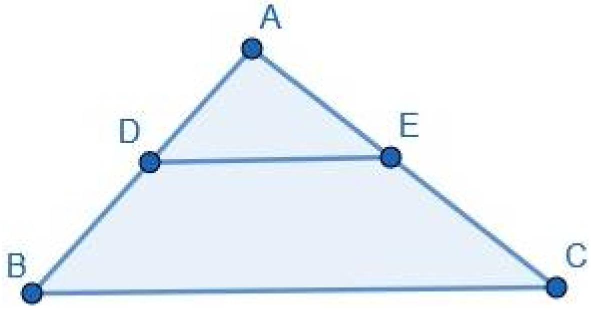
The vertices of ΔABC are A(3, 0), B(0, 6) and (6, 9). A straight line DE divides AB and AC in the ratio 1:2 at D and E respectively, prove that $\frac{\Delta ABC}{\Delta ADE} = 9$

Answer

Given, ABC is a triangle with vertices A(3, 0), B(0, 6) and C (6, 9)

To find: $\frac{\Delta ABC}{\Delta ADC} = 9$

We know that



Area of triangle $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

Now Area of triangle DEF $= \frac{1}{2}|3(6 - 9) + 0(9 - 0) + 6(0 - 6)|$

$= \frac{1}{2}[-9 - 36]$

$= \frac{1}{2}[-45]$

$= \frac{45}{2}$ square units

Now, According to the question,

DE internally divides AB in the ratio 1:2 hence

Coordinates of D $= \left[\frac{m_1x_2 + m_2x_1}{m1+m2}, \frac{m_1y_2 + m_2y_1}{m1+m2}\right]$

$= \left[\frac{1 \times 0 + 2 \times 3}{2}, \frac{1 \times 6 + 2 \times 0}{1 + 2}\right]$

$= \left[\frac{6}{3}, \frac{6}{3}\right]$

= (2, 2)

E internally divides AC in the ratio 1:2 hence

Coordinates of D $= \left[\frac{m_1x_2 + m_2x_1}{m1+m2}, \frac{m_1y_2 + m_2y_1}{m1+m2}\right]$

$= \left[\frac{1 \times 6 + 2 \times 3}{2 + 1}, \frac{1 \times 9 + 2 \times 0}{1 + 2}\right]$

$= \left[\frac{12}{3}, \frac{9}{3}\right]$

= (4, 3)

Now Area of triangle ADE $= \frac{1}{2}|3(2-3)+2(3-0)+4(0-2)|$

$= \frac{1}{2}[-3+6-8]$

$= \frac{1}{2}[-5]$

$= \frac{5}{2}$ square units

Therefore, Area of ΔABC $= \frac{45}{2}$ sq. units

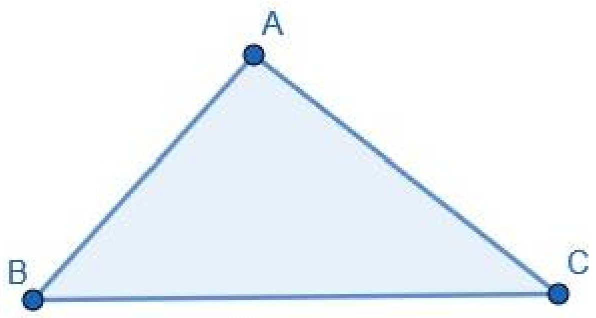
Hence, Area of ABC = 9. Area of ADE

9. Question

If (t, t - 2), (t + 3, t) and (t + 2, t + 2) are the vertices of a triangle, show that its area is independent of t.

Answer

Given a triangle with vertices (t, t-2), (t+3, t) and (t+2, t+2)



Area of triangle $= \frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

$= \frac{1}{2}|t(t-(t+2))+(t+3)\big((t+2)-(t-2)\big)+(t+2)((t-2)-t)|$

$= \frac{1}{2}[t(2)+(t+2)\{0\}+(t+2)\{-2\}]$

$= \frac{1}{2}[2t-2t-4]$

$= \frac{4}{2}$

= 2 sq units

Hence, t is not dependant variable in the triangle.

10. Question

If A(x, y), B(1, 2) and C(2, 1) are the vertices of a triangle of area 6 square unit, show that x+y=15 or-9

Answer

Given: A triangle with vertices A(x, y), B(1, 2) and C (2, 1)

To find: x + y = 15 or -9

The area is 6 square units.

Area of triangle $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$$\frac{1}{2}|x(2-1) + 1(1-y) + 2(y-2)| = 6$$

[x + 1 – y + 2y - 4] = 12

[x + y - 3] = 12

x + y =15

Hence, Proved

11. Question

Prove that the points (a, b+c), (b, c+a) and (c, a+b) are collinear.

Answer

Given: A(a, b + c), B(b, c + a) and C(c, a + b)

To prove : Given points are collinear

We know the points are collinear if area(ΔABC)=0

Area of triangle $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 0$

Then,

Area
$= \frac{1}{2}[a\{(c+a) - (a+b)\} + b\{(a+b) - (b+c)\} + c\{(b+c) - (c+a)\}]$

$$= \frac{1}{2}[a\{c + a - a - b\} + b\{a + b - b - c\} + c\{b + c - c - a\}$$

$= \frac{1}{2}[a\{c - b\} + b\{a - c\} + c\{b - a\}]$

= 0

Hence, Points are collinear.

12. Question

If the points (x_1y_1), (x_2, y_2) and (x_3, y_3) be collinear, show that

$$\frac{y_2 - y_3}{x_2x_3} + \frac{y_3 - y_1}{x_3y_1} + \frac{y_1 - y_2}{x_1x_2} = 0$$

Answer

Given : $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$

We know the points are collinear if area(ΔABC)=0

Area of triangle $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 0$

Then, Area $= \frac{1}{2}\{x_1y_2 - x_1y_3 + x_2y_3 - x_2y_1 + x_3y_1 - x_3y_2\} = 0$

Now, Divide by $x_1x_2x_3$

$$\Rightarrow \frac{1}{2}\left\{\frac{(x_1y_2 - x_1y_3)}{x_1x_2x_3} + \frac{(x_2y_3 - x_2y_1)}{x_1x_2x_3} + \frac{(x_3y_1 - x_3y_2)}{x_1x_2x_3}\right\} = 0$$

Taking common x_1,x_2 and x_3 respectively

$$\Rightarrow \frac{1}{2}\left\{\frac{x_1(y_2 - y_3)}{x_1x_2x_3} + \frac{x_2(y_3 - y_1)}{x_1x_2x_3} + \frac{x_3(y_1 - y_2)}{x_1x_2x_3}\right\} = 0$$

$$\Rightarrow \left\{\frac{(y_2 - y_3)}{x_2x_3} + \frac{(y_3 - y_1)}{x_1x_3} + \frac{(y_1 - y_2)}{x_1x_2}\right\}$$

Hence, Proved.

13. Question

If the points (a, b), (a_1, b_1) and $(a-a_1, b-b_1)$ are collinear, show that $\frac{a}{a_1} = \frac{b}{b_1}$

Answer

Given (a, b), (a_1, b_1) and $(a-a_1, b-b_1)$

We know the points are collinear if area(ΔABC)=0

Area of triangle $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 0$

Then, Area

$= \frac{1}{2}\{a(b_1 - (b - b_1) + a_1\{(b - b_1) - b\} + (a - a_1)(b - b_1\} = 0$

$\frac{1}{2}\{ab_1 - (ab - ab_1) + a_1b - a_1b_1 - a_1b\} + (ab - ab_1 - a_1b + a_1b_1\} = 0$

$\{ab + a_1 b_1\} = ab - ab_1 - a_1 b + a_1 b_1$

$- ab_1 - a_1b = 0$

$-ab_1 = a_1b$

Therofe, We can write as

$\frac{a}{a_1} = \frac{b}{b_1}$

Hence, Proved.

14. Question

Show that the point (a, 0), (0, b) and (1, 1) are collinear if $\frac{1}{a} + \frac{1}{b} = 1$

Answer

Given: (a, 0), (0, b) and (1, 1)

We know the points are collinear if area(ΔABC)=0

Area of triangle $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 0$

Then, Area $= \frac{1}{2}\{a(b - 1) + 0\{1 - 0\} + 1(0 - b)\} = 0$

$\frac{1}{2}[ab - a + 0 + 0 - b] = 0$

$ab - a - b = 0$

$ab = a + b$

Since, $\frac{1}{a} + \frac{1}{b} = 1$

$$\frac{a+b}{ab} = 1$$

Then, a + b = ab

$$\frac{1}{a} + \frac{1}{b} = 1$$

Hence, Proved.

15 A. Question

Find the values of x if the points (2x, 2x), (3, 2x+1) and (1, 0) are collinear.

Answer

Given (2x, 2x), (3, 2x+1) and (1, 0)

We know the points are collinear if the area(ΔABC)=0

Area of triangle $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 0$

Then, Area $= \frac{1}{2}|2x(2x + 1 - 0) + 3(0 - 2x) + 1(2x - 2x - 1)| = 0$

$$\frac{1}{2}[4x^2 + 2x - 6x - 1] = 0$$

$4x^2 - 4x - 1 = 0$

$4x^2 - 2x - 2x - 1 = 0$

2x(2x - 1) - 1(2x - 1) = 0

(2x-1)(2x-1)=0

Hence, $x = \frac{1}{2}$

15 B. Question

Find the value of K if the points A(2, 3), B(4, k) and C(6, -3) are collinear.

Answer

Given A(2, 3), B(4, k) and C(6, -3) are collinear

To find: Find the value of K

So, The given points are collinear, if are (ΔABC)=0

= Area of triangle $= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\} = 0$

Then, $\frac{1}{2}[\{2(k+3)\}+\{4(-3-3)\}+\{6(3-k)\}]=0$

= 2k+6 - 24 +18 - 6k = 0

= -4k = 0

Hence, K = 0

15 C. Question

Find the value of K for which the points (7, -2), (5, 1), (3, k) are collinear.

Answer

Given A(7, -2), B(5, 1) and C(3, k) are collinear

To find: Find the value of K

So, The given points are collinear, if are (ΔABC)=0

= Area of triangle = $\frac{1}{2}\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\}=0$

Then, $\frac{1}{2}[\{7(1-k)\}+\{5(k+2)\}+\{3(-2-1)\}]=0$

= 7 – 7k +5k+10-9 = 0

= -2k + 8 = 0

Hence, K = 4

15 D. Question

Find the value of K for which the points (8, 1), (k, -4), (2, -5) are collinear?

Answer

Given A(8, 1), B(k, -4) and C(2, -5) are collinear

To Find: Find the value of k

So, The given points are collinear, if are (ΔABC)=0

= Area of triangle = $\frac{1}{2}\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\}=0$

Then, $\frac{1}{2}[\{8(-4+5)\}+\{k(-5-1)\}+\{2(-1+4)\}]=0$

= 8(1)+k(-6)+2(3) = 0

= 8 - 6k + 6 = 0

= -6k = -14

Hence, K = $\frac{7}{3}$

15 E. Question

Find the value of P are the points (2, 1), (p, -1) and (-1, 3)collinear?

Answer

Given A(2, 1), B(p, -1) and C(-1, 3) are collinear

To find: Find the value of p

So, The given points are collinear, if are (ΔABC)=0

= Area of triangle = $\frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\} = 0$

Then, $\frac{1}{2}[\{2(-1-3)\} + \{p(3-1)\} + \{-1(1+1)\}] = 0$

= 2(-4)+p(2)-2=0

= -8 +2p -2 = 0

= 2p = 10

Hence, p = 5

16. Question

Show that the straight line joining the points A(0, -1) and B(15, 2) divides the line joining the points C(-1, 2)and D(4, -5) internally in the ratio 2:3.

Answer

Given, A(0, -1) B (15, 2) divides the line on points C(-1, 2) and D(4, -5)

To Prove. Straight line divides in the ratio 2:3 internally

The equation of line = $(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

Now, Equation of line BC = $(y + 1) = \frac{2+1}{15+0}(x - 0)$

$\Rightarrow (y + 1) = \frac{3x}{15}$

$\Rightarrow (y + 1) = \frac{x}{5}$

⇒ 5y + 5 = x

Therefore, x – 5y = 5 ---(1)

Now, Equation of line BC = $(y-2) = \frac{-5-2}{4+1}(x+1)$

$\Rightarrow (y-2) = \frac{-7}{5}(x+1)$

⇒ 5(y - 2) = -7(x + 1)

⇒ 5y - 10 = -7x - 7

Therefore, 7x +5y = 3 ---(2)

On solving equation (1) and (2)

X = 1 y = $-\frac{4}{5}$

Now, Point of the intersection of AB and CD is O (1, $-\frac{4}{5}$)

Let us Assume that AB divides CD at O in the ratio m:n, then

x coordinate of O = $\frac{m.x_2+n.x_1}{m+n}$

1 = $\frac{4m-n}{m+n}$

= 4m – n = m+n

= 4m – m = n+n

= 3m = 2n

= $\frac{m}{n} = \frac{2}{3}$ Hence Proved

17. Question

Find the area of the triangle whose vertices are

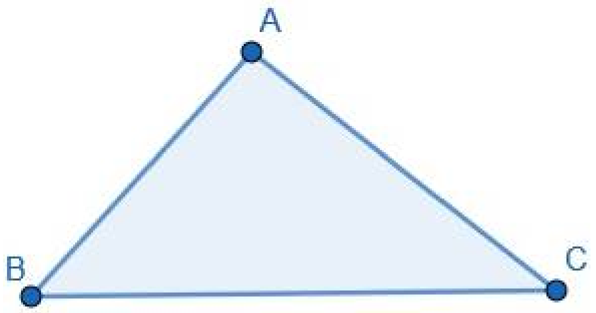
((a+1) (a+2), (a+2)), ((a+2) (a+3), (a+3)) and ((a, 3) (a+4), (a+4))

Answer

Given, A triangle whose vertices are A ((a+1)(a+2), (a+2))

B ((a+2)(a+3), (a+3)) and C = ((a+b)(a+4), (a+4)).

To find: Find the area of a triangle.



Since, Area of triangle = $\frac{1}{2}\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\}=0$

Then, =
$\frac{1}{2}[(a+1)(a+2)\{(a+3)-(a+4)\}+(a+2)(a+3)\{(a+4)-(a+2)\}+(a+3)(a+4)\{(a+2)-(a+3)\}]$

$=\frac{1}{2}[(a^2+3a+2)(-1)+(a^2+5a+6)(2)+(a^2+7a+12)(-1)]$

$=\frac{1}{2}[-a^2-3a-2+2a^2+10a+12-a^2+7a+12]$

Common terms will be canceled out

$=\frac{1}{2}$ [2]

Hence, = 1 sq unit

18. Question

The point A divides the join of P(-5, 1) and Q(3, 5) in the ratio k:1. Find the two values of k for which the area of ΔABC, where B is (1, 5) and C is (7, - 2) is equal to 2 units in magnitude.

Answer

Given: A divides the join of P(-5, 1) and Q(3, 5) in the ratio k:1

To Find: Two values of K

A divides join of PQ in the ratio k:1 hence

Coordinates of A $=\left[\frac{m_1x_2+m_2x_1}{m1+m2},\frac{m_1y_2+m_2y_1}{m1+m2}\right]$

$=\left[\frac{3k-5}{k+1},\frac{5k+1}{k+1}\right]$

Now, We have A $\left[\frac{3k-5}{k+1},\frac{5k+1}{k+1}\right]$, B (1, 5), and C(7, -2)

Now, The area of ABC is equal to the magnitude 2 (Given)

Area of ABC $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$$\Rightarrow \frac{1}{2}\left|\frac{3k-5}{k+1}(5+2) + 1\left(-2 - \left(\frac{5k+1}{k+1}\right)\right) + 7\left(\left(\frac{5k+1}{k+1}\right) - 5\right)\right| = 2$$

$$\Rightarrow \frac{1}{2}\left|\frac{3k-5}{k+1}(7) + 1\left(\frac{-2k-1-5k-1}{k+1}\right) + 7\left(\left(\frac{5k+1-5k-5}{k+1}\right)\right)\right| = 2$$

$$\Rightarrow \left|\frac{21k-35}{k+1} + \left(\frac{-7k-2}{k+1}\right) + 7\left(\left(\frac{-4}{k+1}\right)\right)\right| = 4$$

$$\Rightarrow \left|\frac{21k-35}{k+1} + \left(\frac{-7k-2}{k+1}\right) + \left(\left(\frac{-28}{k+1}\right)\right)\right| = 4$$

$$\Rightarrow \left|\frac{21k-35+(-7k-2)(-28)}{k+1}\right| = 4$$

14k - 66 = 4k + 4

14k - 66 = -4k - 4

10k = 70

18k = 62

Hence, k = 7 and $k = \frac{31}{9}$

19. Question

The coordinates of A, B, C, D are (6, 3), (-3, 5), (4, -2) and (x, 3x) respectively. If $\frac{\Delta DBC}{\Delta ABC} = \frac{1}{2}$, find x.

Answer

Given A, B, C, and D are (6, 3), (-3, 5), (4, -2) and (x, 3x) respectively.

and ΔDBC = 2ΔABC

To find: Find x.

Since Area of triangle $= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Now, The area of ΔDBC $= \frac{1}{2}[x(5+2) - 3(-2-3x) + 4(3x-5)$

$$= \frac{1}{2}[5x + 2x + 6 + 3x + 12x - 20]$$

$$= \frac{1}{2}[22x - 14]$$

= 11x – 7 sq units

Now, The area of ΔDBC = $\frac{1}{2}[6(5 + 2) - 3(-2 - 3) + 4(3 - 5)$

$$= \frac{1}{2}[42 + 15 - 8]$$

$$= \frac{1}{2}[49]$$

$$= \frac{49}{2}$$

According to question, ΔDBC = 2ΔABC

$$\frac{49}{2} = 2(11x - 7)$$

⇒ 49 = 4(11x – 7)

⇒ 49 = 44x – 28

⇒ 44x = 77

$$\Rightarrow x = \frac{77}{44}$$

Hence, $x = \frac{7}{4}$

20. Question

If the area of the quadrilateral whose angular points taken in order are (1, 2), (-5, 6), (7, -4) and (h, -2) be zero, show that h=3.

Answer

Given: vertices of the quadrilateral be A(1, 2), B(-5, 6), C(7, -4) and D(h, -2).

Let join AC to form two triangles,

Now, We know that

Area of triangle = $\frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Then,

Area of triangle ABC = $\frac{1}{2}[1(6+4)-5(-4-2)+7(2-6)]$

= $\frac{1}{2}[10+30-28]$

= $\frac{12}{2}$

= 6 sq units

Now, Area of triangle ADC = $\frac{1}{2}[1(-2+4)+h(-4-2)+7(2+2)]$

= $\frac{1}{2}[2-6h+28]$

= $\frac{1}{2}[-6h+30]$

= 3h - 15

Area of quadrilateral ABCD = Area of triangle ABC+ Area of triangle ADC

= 3h – 15 + 6

= 3h = 9

= h = 3

Hence, h is 3

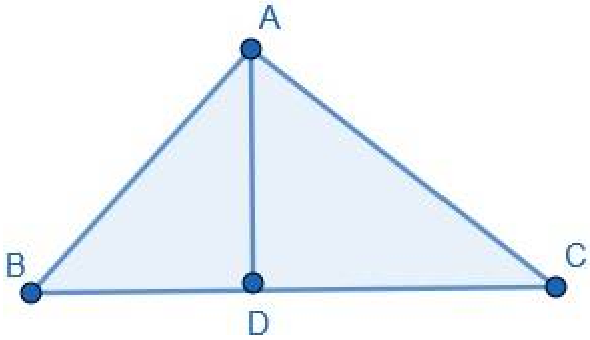
21. Question

Find the area of the triangle whose vertices A, B, C are (3, 4) (-4, 3), (8, 6) respectively and hence find the length of the perpendicular from A to BC.

Answer

Given: A triangle whose vertices A (3, 4) B(-4, 3), C(8, 6)

To find: Find the area of Triangle and length of AD



Area of triangle = $\frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Then,

Area of triangle ABC = $\frac{1}{2}|3(3-6) - 4(6-4) + 8(4-3)|$

= $\frac{1}{2}|-9 - 8 + 8|$

= $\frac{8}{2}$

= 4 square units

We need to find the length of AD on BC

Hence, We need to find the slope first,

The slope $= \frac{y_2 - y_1}{x_2 - x_1}$

Now the slope of BC = $\frac{8+2}{-1+3} = \frac{10}{2}$ = 5

If AD perpendicular BC then the slope of AD is $= -\frac{1}{5}$

Therefore, The equation of is $(y - y_1) = m(x - x_1)$

$(y + 1) = -\frac{1}{5}(x - 5)$

5y + 5 = - x + 5

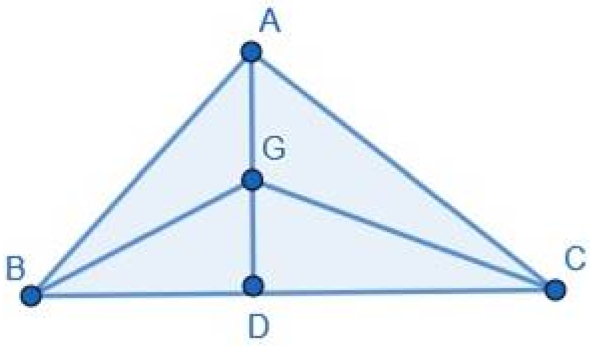
22. Question

The coordinates of the centroid of a triangle and those of two of its vertices are A(3, 1), B(1, -3) and the centroid of the triangle lies on the x-axis. Find the coordinates of the third vertex C.

Answer

Given: A(3, 1) and B(1, -3)

To find: Find the coordinate of the third vertex C.



Let Assume C = (a, b)

Centroid on C $= \left[\frac{3+1+a}{3}, \frac{1-3+b}{3}\right]$

$= \left[\frac{4+a}{3}, \frac{-2+b}{3}\right]$

Therefore, G(1, 0) as it lies on x-axis

⇒ 4 + a = 3

⇒ a = -1

⇒ - 2 + b = 0

⇒ b = 2

Hence, C is (-1, 2)

23. Question

The area of a triangle is 3 square units. Two of its vertices are A(3,1), B(1,-3) and the centroid of the triangle lies on x-axis. Find the coordinates of the third vertex C.

Answer

Given: Coordinates of Triangle are A(3, 1) and B(1, -3)

Centroid of triangle lies on x- axis.

Let the third coordinate be C(x, y)

Centroid of the triangle with vertices (x_1, y_1), (x_2, y_2), (x_3, y_3) is given by,

$C(X, Y) = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

The y coordinate of centroid will be 0, as it lies opn x – axis.

Therefore,

$y_1 + y_2 + y_3 = 0$

$1 - 3 + y_3 = 0$

$y_3 = 2$

So, C(x, y) becomes (x, 2)

We are given that the area of triangle = 3 square units.

We know that,

Area of triangle $= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Putting the values we get,

$3 = \frac{1}{2}[3(-3-2) + 1(2-3) + x(1+3)]$

$-15 - 1 + 4x = 6$

$4x = 22$

$x = \frac{22}{4}$

Hence, coordinates of the third vertex are $C\left(\frac{22}{4}, 2\right)$.

24. Question

The area of a parallelogram is 12 square units. Two of its vertices are the points A(-1, 3) and B (-2, 4). Find the other two vertices of the parallelogram, if the point of intersection of diagonals lies on x-axis on its positive side.

Answer

Given: The area of a parallelogram is 12. A(-1, 3) and B(-2, 4)

To find: Find the other two vertices of the parallelogram.

Let C is (x, y) and A(-1, 3)

Since, AC is bisected at P, y coordinate (when p = 0)

Then, $\frac{y+3}{2} = 0$

y = -3

So, Coordinate of C is (x, -3)

Now, Area of parallelogram ABCD = area of triangle ABC + Area of triangle BAD

Since, 2(Area of triangles) = area of parallelogram

We have, A(-1, 3) B(-2, 4) and C(x, -3)

Now, Area of triangle $= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Then,

Area of triangle ABC $= \frac{1}{2}[-1(4+3) - 2(-3-3) + x(3-4)]$

$6 = \frac{1}{2}[-7 + 12 - x]$

$6 = \frac{1}{2}[5 - x]$

12 = 5 - x

So, x = -7

Hence, Coordinate of C is (-7, -3)

In the same we will calculate for D

Let D is (x, y) and A(-2, 4)

Since, BD is bisected at Q, y coordinate (when Q = 0)

Then, $\frac{y+4}{2} = 0$

y = -4

So, Coordinate of C is (x, -4)

We have, A(-1, 3) B(-2, 4) and C(x, -4)

Now, Area of triangle $= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Then,

Area of triangle ABC $= \frac{1}{2}[-1(4+4) - 2(-4-3) + x(3-4)]$

$6 = \frac{1}{2}[-8 + 14 - x]$

$6 = \frac{1}{2}[6 - x]$

12 = 6 - x

So, x = -6

Hence, Coordinate of D is (-6, -4)

Hence, C (-7, -3) and D(-6, -4)

25. Question

Prove that the quadrilateral whose vertices are A(-2, 5), B(4, -1), C(9, 1) and D(3, 7) is a parallelogram and find its area. If E divides AC in the ratio 2:1, prove that D, E and the middle point F of BC are collinear.

Answer

Given: Let ABCD is a quadrilateral whose vertices A(-2, 5), B(4, -1), C(9, 1) and D(3, 7).

To prove: ABCD is a parallelogram .

We have to find |AD|, |AB|, |BC|, |DC|

The distance between two sides $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AD| = \sqrt{(3+2)^2 + (7-5)^2}$

$= \sqrt{29}$

$|AB| = \sqrt{(4+2)^2 + (1+5)^2}$

$= \sqrt{72}$

$|DC| = \sqrt{(9-3)^2 + (1-7)^2}$

$= \sqrt{72}$

$|BC| = \sqrt{(9-4)^2 + (1+1)^2}$

$= \sqrt{29}$

Therefore, AB = DC and AD = BC

Hence, ABCD is a parallelogram

Now, The Area of ABCD is $= |a \times b| = \begin{vmatrix} i & j & k \\ 6 & -6 & 0 \\ 5 & 2 & 0 \end{vmatrix}$

$= \begin{vmatrix} -6 & 0 \\ 2 & 0 \end{vmatrix} i - \begin{vmatrix} 6 & 0 \\ 5 & 0 \end{vmatrix} j + \begin{vmatrix} 6 & -6 \\ 5 & 2 \end{vmatrix} k$

= 0i – 0j+ 42 k

|a×b| = 42

Hence The area of parallelgram is 42

26. Question

Prove that points (-3, -1), (2, -1), (1, 1) and (-2, 1) taken in order are the vertices of a trapezium.

Answer

Given: Points of the quadrilatreral A(-3, -1), B(2, -1), C(1, 1), and D(-2, 1).

To Prove: ABCD is a trapezium

Proof:

For proving ABCD to be a trapezium, we need to prove that two of the sides are parallel.

Therefore, AB and CD are parallel.

For proving ABCD a trapezium,

Slope of AB = Slope of CD

Slope of AB $= \frac{-1+1}{2+5} = 0$

Slope of CD $= \frac{1-1}{-2-1} = 0$

Hence, the quadrilateral is a trapezium.